# Accelerating Universe: Theory versus Experiment

Moshe Carmeli \*

Department of Physics, Ben Gurion University, Beer Sheva 84105, Israel

The theory presented here, cosmological general relativity, uses a Riemannian four-dimensional presentation of gravitation in which the coordinates are those of Hubble, i.e. distances and velocity rather than the traditional space and time. We solve the field equations and show that there are three possibilities for the Universe to expand. The theory describes the Universe as having a three-phase evolution with a decelerating expansion, followed by a constant and an accelerating expansion, and it predicts that the Universe is now in the latter phase. It is shown, assuming  $\Omega_m = 0.245$ , that the time at which the Universe goes over from a decelerating to an accelerating expansion, i.e., the constant-expansion phase, occurs at 8.5 Gyr ago. Also, at that time the cosmic radiation temperature was 146K. Recent observations of distant supernovae imply, in defiance of expectations, that the Universe's growth is accelerating, contrary to what has always been assumed, that the expansion is slowing down due to gravity. Our theory confirms these recent experimental results by showing that the Universe now is definitely in a stage of accelerating expansion. The theory predicts also that now there is a positive pressure,  $p = 0.034q/cm^2$ , in the Universe. Although the theory has no cosmological constant, we extract from it its equivalence and show that  $\Lambda = 1.934 \times 10^{-35} s^{-2}$ . This value of  $\Lambda$  is in excellent agreement with the measurements obtained by the High-Z Supernova Team and the Supernova Cosmology Project. It is also shown that the three-dimensional space of the Universe is Euclidean, as the Boomerang experiment shows. Comparison with general relativity theory is finally made and it is shown that the classical experiments as well as the gravitational radiation prediction follow from the present theory, too.

PACS numbers: 04.50.+h, 11.25.Mj., 11.27.+d, 98.80Cq

\*Email: carmelim@bgumail.bgu.ac.il

#### 1 Preliminaries

As in classical general relativity we start our discussion in flat spacevelocity which will then be generalized to curved space.

The flat-spacevelocity cosmological metric is given by

$$ds^{2} = \tau^{2}dv^{2} - (dx^{2} + dy^{2} + dz^{2}).$$
(1)

Here  $\tau$  is Hubble's time, the inverse of Hubble's constant, as given by measurements in the limit of zero distances and thus zero gravity. As such,  $\tau$  is a constant, in fact a universal constant (its numerical value is given in Section 8,  $\tau = 12.486 \, \mathrm{Gyr}$ ). Its role in cosmology theory resembles that of c, the speed of light in vacuum, in ordinary special relativity. The velocity v is used here in the sense of cosmology, as in Hubble's law, and is usually not the time-derivative of the distance.

The Universe expansion is obtained from the metric (1) as a null condition, ds = 0. Using spherical coordinates r,  $\theta$ ,  $\phi$  for the metric (1), and the fact that the Universe is spherically symmetric ( $d\theta = d\phi = 0$ ), the null condition then yields  $dr/dv = \tau$ , or upon integration and using appropriate initial conditions, gives  $r = \tau v$  or  $v = H_0 r$ , i.e. the Hubble law in the zero-gravity limit.

Based on the metric (1) a cosmological special relativity (CSR) was presented in the text [1] (see Chapter 2). In this theory the receding velocities of galaxies and the distances between them in the Hubble expansion are united into a four-dimensional pseudo-Euclidean manifold, similarly to space and time in ordinary special relativity. The Hubble law is assumed and is written in an invariant way that enables one to derive a four-dimensional transformation which is similar to the Lorentz transformation. The parameter in the new transformation is the ratio between the cosmic time to  $\tau$  (in which the cosmic time is measured backward with respect to the present time). Accordingly, the new transformation relates physical quantities at different cosmic times in the limit of weak or negligible gravitation.

The transformation between the four variables x, y, z, v and x', y', z', v' (assuming y' = y and z' = z) is given by

$$x' = \frac{x - tv}{\sqrt{1 - t^2/\tau^2}}, \quad v' = \frac{v - tx/\tau^2}{\sqrt{1 - t^2/\tau^2}}, \quad y' = y, \ z' = z.$$
 (2)

Equations (2) are the cosmological transformation and very much resemble the well-known Lorentz transformation. In CSR it is the relative cosmic time which takes the role of the relative velocity in Einstein's special relativity. The transformation (2) leaves invariant the Hubble time  $\tau$ , just as the Lorentz transformation leaves invariant the speed of light in vacuum c.

# 2 Cosmology in spacevelocity

A cosmological general theory of relativity, suitable for the large-scale structure of the Universe, was subsequently developed [2-5]. In the framework of

cosmological general relativity (CGR) gravitation is described by a curved four-dimensional Riemannian spacevelocity. CGR incorporates the Hubble constant  $\tau$  at the outset. The Hubble law is assumed in CGR as a fundamental law. CGR, in essence, extends Hubble's law so as to incorporate gravitation in it; it is actually a distribution theory that relates distances and velocities between galaxies. The theory involves only measured quantities and it takes a picture of the Universe as it is at any moment. The following is a brief review of CGR as was originally given by the author in 1996 in Ref. 2.

The foundations of any gravitational theory are based on the principle of equivalence and the principle of general covariance [6]. These two principles lead immediately to the realization that gravitation should be described by a four-dimensional curved spacetime, in our theory spacevelocity, and that the field equations and the equations of motion should be written in a generally covariant form. Hence these principles were adopted in CGR also. Use is made in a four-dimensional Riemannian manifold with a metric  $g_{\mu\nu}$  and a line element  $ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}$ . The difference from Einstein's general relativity is that our coordinates are:  $x^0$  is a velocitylike coordinate (rather than a timelike coordinate), thus  $x^0 = \tau v$  where  $\tau$  is the Hubble time in the zero-gravity limit and v the velocity. The coordinate  $x^0 = \tau v$  is the comparable to  $x^0 = ct$  where c is the speed of light and t is the time in ordinary general relativity. The other three coordinates  $x^k$ , k = 1, 2, 3, are spacelike, just as in general relativity theory.

An immediate consequence of the above choice of coordinates is that the null condition ds=0 describes the expansion of the Universe in the curved spacevelocity (generalized Hubble's law with gravitation) as compared to the propagation of light in the curved spacetime in general relativity. This means one solves the field equations (to be given in the sequel) for the metric tensor, then from the null condition ds=0 one obtains immedialety the dependence of the relative distances between the galaxies on their relative velocities.

As usual in gravitational theories, one equates geometry to physics. The first is expressed by means of a combination of the Ricci tensor and the Ricci scalar, and follows to be naturally either the Ricci trace-free tensor or the Einstein tensor. The Ricci trace-free tensor does not fit gravitation in general, and the Einstein tensor is a natural candidate. The physical part is expressed by the energy-momentum tensor which now has a different physical meaning from that in Einstein's theory. More important, the coupling constant that relates geometry to physics is now also different.

Accordingly the field equations are

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu},\tag{3}$$

exactly as in Einstein's theory, with  $\kappa$  given by  $\kappa = 8\pi k/\tau^4$ , (in general relativity it is given by  $8\pi G/c^4$ ), where k is given by  $k = G\tau^2/c^2$ , with G being Newton's gravitational constant, and  $\tau$  the Hubble constant time. When the equations of motion will be written in terms of velocity instead of time, the constant k will replace G. Using the above equations one then has  $\kappa = 8\pi G/c^2\tau^2$ .

The energy-momentum tensor  $T^{\mu\nu}$  is constructed, along the lines of general relativity theory, with the speed of light being replaced by the Hubble constant time. If  $\rho$  is the average mass density of the Universe, then it will be assumed that  $T^{\mu\nu} = \rho u^{\mu}u^{\nu}$ , where  $u^{\mu} = dx^{\mu}/ds$  is the four-velocity. In general relativity theory one takes  $T_0^0 = \rho$ . In Newtonian gravity one has the Poisson equation  $\nabla^2 \phi = 4\pi G \rho$ . At points where  $\rho = 0$  one solves the vacuum Einstein field equations in general relativity and the Laplace equation  $\nabla^2 \phi = 0$  in Newtonian gravity. In both theories a null (zero) solution is allowed as a trivial case. In cosmology, however, there exists no situation at which  $\rho$  can be zero because the Universe is filled with matter. In order to be able to have zero on the right-hand side of Eq. (3) one takes  $T_0^0$  not as equal to  $\rho$ , but to  $\rho_{eff} = \rho - \rho_c$ , where  $\rho_c$  is the critical mass density, a constant in CGR given by  $\rho_c = 3/8\pi G \tau^2$ , whose value is  $\rho_c \approx 10^{-29} g/cm^3$ , a few hydrogen atoms per cubic meter. Accordingly one takes

$$T^{\mu\nu} = \rho_{eff} u^{\mu} u^{\nu}; \qquad \rho_{eff} = \rho - \rho_c \tag{4}$$

for the energy-momentum tensor.

In the next sections we apply CGR to obtain the accelerating expanding Universe and related subjects.

## 3 Gravitational field equations

In the four-dimensional spacevelocity the spherically symmetric metric is given by

$$ds^{2} = \tau^{2} dv^{2} - e^{\mu} dr^{2} - R^{2} \left( d\theta^{2} + \sin^{2} \theta d\phi^{2} \right), \tag{5}$$

where  $\mu$  and R are functions of v and r alone, and comoving coordinates  $x^{\mu}=(x^0,x^1,x^2,x^3)=(\tau v,r,\theta,\phi)$  have been used. With the above choice of coordinates, the zero-component of the geodesic equation becomes an identity, and since r,  $\theta$  and  $\phi$  are constants along the geodesics, one has  $dx^0=ds$  and therefore

$$u^{\alpha} = u_{\alpha} = (1, 0, 0, 0). \tag{6}$$

The metric (5) shows that the area of the sphere r = constant is given by  $4\pi R^2$  and that R should satisfy  $R' = \partial R/\partial r > 0$ . The possibility that R' = 0 at a point  $r_0$  is excluded since it would allow the lines r = constants at the neighboring points  $r_0$  and  $r_0 + dr$  to coincide at  $r_0$ , thus creating a caustic surface at which the comoving coordinates break down.

As has been shown in the previous sections the Universe expands by the null condition ds = 0, and if the expansion is spherically symmetric one has  $d\theta = d\phi = 0$ . The metric (5) then yields

$$\tau^2 dv^2 - e^{\mu} dr^2 = 0, (7)$$

thus

$$\frac{dr}{dv} = \tau e^{-\mu/2}. (8)$$

This is the differential equation that determines the Universe expansion. In the following we solve the gravitational field equations in order to find out the function  $\mu(r,v)$ .

The gravitational field equations (3), written in the form

$$R_{\mu\nu} = \kappa \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right), \tag{9}$$

where

$$T_{\mu\nu} = \rho_{eff} u_{\mu} u_{\nu} + p \left( u_{\mu} u_{\nu} - g_{\mu\nu} \right), \tag{10}$$

with  $\rho_{eff} = \rho - \rho_c$  and  $T = T_{\mu\nu}g^{\mu\nu}$ , are now solved. Using Eq. (6) one finds that the only nonvanishing components of  $T_{\mu\nu}$  are  $T_{00} = \tau^2 \rho_{eff}$ ,  $T_{11} = c^{-1}\tau p e^{\mu}$ ,  $T_{22} = c^{-1}\tau p R^2$  and  $T_{33} = c^{-1}\tau p R^2 \sin^2\theta$ , and that  $T = \tau^2 \rho_{eff} - 3c^{-1}\tau p$ .

The only nonvanishing components of the Ricci tensor yield (dots and primes denote differentiation with respect to v and r, respectively), using Eq. (9), the following field equations:

$$R_{00} = -\frac{1}{2}\ddot{\mu} - \frac{2}{R}\ddot{R} - \frac{1}{4}\dot{\mu}^2 = \frac{\kappa}{2} \left(\tau^2 \rho_{eff} + 3c^{-1}\tau p\right), \tag{11a}$$

$$R_{01} = \frac{1}{R}R'\dot{\mu} - \frac{2}{R}\dot{R}' = 0, \tag{11b}$$

$$R_{11} = e^{\mu} \left( \frac{1}{2} \ddot{\mu} + \frac{1}{4} \dot{\mu}^2 + \frac{1}{R} \dot{\mu} \dot{R} \right) + \frac{1}{R} \left( \mu' R' - 2R'' \right)$$
$$= \frac{\kappa}{2} e^{\mu} \left( \tau^2 \rho_{eff} - c^{-1} \tau p \right), \tag{11c}$$

$$R_{22} = R\ddot{R} + \frac{1}{2}R\dot{R}\dot{\mu} + \dot{R}^2 + 1 - e^{-\mu}\left(RR'' - \frac{1}{2}RR'\mu' + R'^2\right)$$
$$= \frac{\kappa}{2}R^2\left(\tau^2\rho_{eff} - c^{-1}\tau p\right), \tag{11d}$$

$$R_{33} = \sin^2 \theta R_{22} = \frac{\kappa}{2} R^2 \sin^2 \theta \left( \tau^2 \rho_{eff} - c^{-1} \tau p \right). \tag{11e}$$

The field equations obtained for the components 00, 01, 11, and 22 (the 33 component contributes no new information) are given by

$$-\ddot{\mu} - \frac{4}{R}\ddot{R} - \frac{1}{2}\dot{\mu}^2 = \kappa \left(\tau^2 \rho_{eff} + 3c^{-1}\tau p\right),\tag{12}$$

$$2\dot{R}' - R'\dot{\mu} = 0,\tag{13}$$

$$\ddot{\mu} + \frac{1}{2}\dot{\mu}^2 + \frac{2}{R}\dot{R}\dot{\mu} + e^{-\mu}\left(\frac{2}{R}R'\mu' - \frac{4}{R}R''\right) = \kappa\left(\tau^2\rho_{eff} - c^{-1}\tau p\right)$$
(14)

$$\frac{2}{R}\ddot{R} + 2\left(\frac{\dot{R}}{R}\right)^{2} + \frac{1}{R}\dot{R}\dot{\mu} + \frac{2}{R^{2}} + e^{-\mu}\left[\frac{1}{R}R'\mu' - 2\left(\frac{R'}{R}\right)^{2} - \frac{2}{R}R''\right]$$

$$= \kappa \left( \tau^2 \rho_{eff} - c^{-1} \tau p \right). \tag{15}$$

It is convenient to eliminate the term with the second velocity-derivative of  $\mu$  from the above equations. This can easily be done, and combinations of Eqs. (12)–(15) then give the following set of three independent field equations:

$$e^{\mu} \left( 2R\ddot{R} + \dot{R}^2 + 1 \right) - R'^2 = -\kappa \tau c^{-1} e^{\mu} R^2 p,$$
 (16)

$$2\dot{R}' - R'\dot{\mu} = 0,\tag{17}$$

$$e^{-\mu} \left[ \frac{1}{R} R' \mu' - \left( \frac{R'}{R} \right)^2 - \frac{2}{R} R'' \right] + \frac{1}{R} \dot{R} \dot{\mu} + \left( \frac{\dot{R}}{R} \right)^2 + \frac{1}{R^2}$$
$$= \kappa \tau^2 \rho_{eff}, \tag{18}$$

other equations being trivial combinations of (16)–(18).

## 4 Solution of the field equations

The solution of Eq. (17) satisfying the condition R' > 0 is given by

$$e^{\mu} = \frac{R^{2}}{1 + f(r)},\tag{19}$$

where f(r) is an arbitrary function of the coordinate r and satisfies the condition f(r) + 1 > 0. Substituting (19) in the other two field equations (16) and (18) then gives

$$2R\ddot{R} + \dot{R}^2 - f = -\kappa c^{-1} \tau R^2 p, \tag{20}$$

$$\frac{1}{RR'} \left( 2\dot{R}\dot{R}' - f' \right) + \frac{1}{R^2} \left( \dot{R}^2 - f \right) = \kappa \tau^2 \rho_{eff}, \tag{21}$$

respectively.

The simplest solution of the above two equations, which satisfies the condition R' = 1 > 0, is given by

$$R = r. (22)$$

Using Eq. (22) in Eqs. (20) and (21) gives

$$f(r) = \kappa c^{-1} \tau p r^2, \tag{23}$$

and

$$f' + \frac{f}{r} = -\kappa \tau^2 \rho_{eff} r, \tag{24}$$

respectively. The solution of Eq. (24) is the sum of the solutions of the homogeneous equation

$$f' + \frac{f}{r} = 0, (25)$$

and a particular solution of Eq. (24). These are given by

$$f_1 = -\frac{2Gm}{c^2r},\tag{26}$$

and

$$f_2 = -\frac{\kappa}{3}\tau^2 \rho_{eff} r^2. \tag{27}$$

The solution  $f_1$  represents a particle at the origin of coordinates and as such is not relevant to our problem. We take, accordingly,  $f_2$  as the general solution,

$$f(r) = -\frac{\kappa}{3}\tau^2 \rho_{eff} r^2 = -\frac{\kappa}{3}\tau^2 (\rho - \rho_c) r^2$$
$$= -\frac{\kappa}{3}\tau^2 \rho_c \left(\frac{\rho}{\rho_c} - 1\right) r^2. \tag{28}$$

Using the values of  $\kappa = 8\pi G/c^2\tau^2$  and  $\rho_c = 3/8\pi G\tau^2$ , we obtain

$$f(r) = \frac{1 - \Omega_m}{c^2 \tau^2} r^2,\tag{29}$$

where  $\Omega_m = \rho/\rho_c$ .

The two solutions given by Eqs. (23) and (29) for f(r) can now be equated, giving

$$p = \frac{1 - \Omega_m}{\kappa c \tau^3} = \frac{c}{\tau} \frac{1 - \Omega_m}{8\pi G} = 4.544 (1 - \Omega_m) \times 10^{-2} g/cm^2.$$
 (30)

Furthermore, from Eqs. (19) and (22) we find that

$$e^{-\mu} = 1 + f(r) = 1 + \tau c^{-1} \kappa p r^2 = 1 + \frac{1 - \Omega_m}{c^2 \tau^2} r^2.$$
 (31)

It will be recalled that the Universe expansion is determined by Eq. (8),  $dr/dv = \tau e^{-\mu/2}$ . The only thing that is left to be determined is the signs of  $(1 - \Omega_m)$  or the pressure p.

Thus we have

$$\frac{dr}{dv} = \tau \sqrt{1 + \kappa \tau c^{-1} p r^2} = \tau \sqrt{1 + \frac{1 - \Omega_m}{c^2 \tau^2}} r^2.$$
 (32)

For simplicity we confine ourselves to the linear approximation, thus Eq. (32) yields

$$\frac{dr}{dv} = \tau \left( 1 + \frac{\kappa}{2} \tau c^{-1} p r^2 \right) = \tau \left[ 1 + \frac{1 - \Omega_m}{2c^2 \tau^2} r^2 \right]. \tag{33}$$

#### 5 Classification of universes

The second term in the square bracket in the above equation represents the deviation due to gravity from the standard Hubble law. For without that term, Eq. (33) reduces to  $dr/dv = \tau$ , thus  $r = \tau v + const$ . The constant can be taken

zero if one assumes, as usual, that at r=0 the velocity should also vanish. Thus  $r=\tau v$ , or  $v=H_0r$  (since  $H_0\approx 1/\tau$ ). Accordingly, the equation of motion (33) describes the expansion of the Universe when  $\Omega_m=1$ , namely when  $\rho=\rho_c$ . The equation then coincides with the standard Hubble law.

The equation of motion (33) can easily be integrated exactly by the substitions

$$\sin \chi = \sqrt{\frac{(\Omega_m - 1)}{2}} \frac{r}{2c\tau}; \quad \Omega_m > 1, \tag{34a}$$

$$\sinh \chi = \sqrt{\frac{(1 - \Omega_m)}{2}} \frac{r}{2c\tau}; \qquad \Omega_m < 1. \tag{34b}$$

One then obtains, using Eqs. (33) and (34),

$$dv = cd\chi/\left(\Omega_m - 1\right)^{1/2}\cos\chi; \quad \Omega_m > 1, \tag{35a}$$

$$dv = cd\chi/\left(1 - \Omega_m\right)^{1/2} \cosh \chi; \qquad \Omega_m < 1. \tag{35b}$$

We give below the exact solutions for the expansion of the Universe for each of the cases,  $\Omega_m > 1$  and  $\Omega_m < 1$ . As will be seen, the case of  $\Omega_m = 1$  can be obtained at the limit  $\Omega_m \to 1$  from both cases.

The case  $\Omega_m > 1$ . From Eq. (35a) we have

$$\int dv = \frac{c}{\sqrt{(\Omega_m - 1)/2}} \int \frac{d\chi}{\cos \chi},\tag{36}$$

where  $\sin \chi = r/a$ , and  $a = c\tau \sqrt{(\Omega_m - 1)/2}$ . A simple calculation gives [7]

$$\int \frac{d\chi}{\cos \chi} = \ln \left| \frac{1 + \sin \chi}{\cos \chi} \right|. \tag{37}$$

A straightforward calculation then gives

$$v = \frac{a}{2\tau} \ln \left| \frac{1 + r/a}{1 - r/a} \right|. \tag{38}$$

As is seen, when  $r \to 0$  then  $v \to 0$  and using the L'Hospital lemma,  $v \to r/\tau$  as  $a \to 0$  (and thus  $\Omega_m \to 1$ ).

The case  $\Omega_m < 1$ . From Eq. (35b) we now have

$$\int dv = \frac{c}{\sqrt{(1 - \Omega_m)/2}} \int \frac{d\chi}{\cosh \chi},\tag{39}$$

where  $\sinh \chi = r/b$ , and  $b = c\tau \sqrt{(1 - \Omega_m)/2}$ . A straightforward calculation then gives [7]

$$\int \frac{d\chi}{\cosh \chi} = \arctan e^{\chi}. \tag{40}$$

We then obtain

$$\cosh \chi = \sqrt{1 + \frac{r^2}{b^2}},$$
(41)

$$e^{\chi} = \sinh \chi + \cosh \chi = \frac{r}{h} + \sqrt{1 + \frac{r^2}{h^2}}.$$
 (42)

Equations (39) and (40) now give

$$v = \frac{2c}{\sqrt{(1 - \Omega_m)/2}} \arctan e^{\chi} + K, \tag{43}$$

where K is an integration constant which is determined by the requirement that at r=0 then v should be zero. We obtain

$$K = -\pi c/2\sqrt{\left(1 - \Omega_m\right)/2},\tag{44}$$

and thus

$$v = \frac{2c}{\sqrt{(1-\Omega_m)/2}} \left(\arctan e^{\chi} - \frac{\pi}{4}\right). \tag{45}$$

A straightforward calculation then gives

$$v = \frac{b}{\tau} \left\{ 2 \arctan\left(\frac{r}{b} + \sqrt{1 + \frac{r^2}{b^2}}\right) - \frac{\pi}{2} \right\}. \tag{46}$$

As for the case  $\Omega_m > 1$  one finds that  $v \to 0$  when  $r \to 0$ , and again, using L'Hospital lemma,  $r = \tau v$  when  $b \to 0$  (and thus  $\Omega_m \to 1$ ).

## 6 Physical meaning

To see the physical meaning of these solutions, however, one does not need the exact solutions. Rather, it is enough to write down the solutions in the lowest approximation in  $\tau^{-1}$ . One obtains, by differentiating Eq. (33) with respect to v, for  $\Omega_m > 1$ ,

$$d^2r/dv^2 = -kr;$$
  $k = \frac{(\Omega_m - 1)}{2c^2},$  (47)

the solution of which is

$$r(v) = A\sin\alpha \frac{v}{c} + B\cos\alpha \frac{v}{c},\tag{48}$$

where  $\alpha^2 = (\Omega_m - 1)/2$  and A and B are constants. The latter can be determined by the initial condition r(0) = 0 = B and  $dr(0)/dv = \tau = A\alpha/c$ , thus

$$r\left(v\right) = \frac{c\tau}{\alpha}\sin\alpha\frac{v}{c}.\tag{49}$$

This is obviously a closed Universe, and presents a decelerating expansion.

For  $\Omega_m < 1$  we have

$$d^{2}r/dv^{2} = \frac{(1-\Omega_{m})r}{2c^{2}},$$
(50)

whose solution, using the same initial conditions, is

$$r\left(v\right) = \frac{c\tau}{\beta}\sinh\beta\frac{v}{c},\tag{51}$$

where  $\beta^2 = (1 - \Omega_m)/2$ . This is now an open accelerating Universe. For  $\Omega_m = 1$  we have, of course,  $r = \tau v$ .

## 7 The accelerating universe

We finally determine which of the three cases of expansion is the one at present epoch of time. To this end we have to write the solutions (49) and (51) in ordinary Hubble's law form  $v = H_0 r$ . Expanding Eqs. (49) and (51) into power series in v/c and keeping terms up to the second order, we obtain

$$r = \tau v \left( 1 - \alpha^2 v^2 / 6c^2 \right), \tag{52a}$$

$$r = \tau v \left( 1 + \beta^2 v^2 / 6c^2 \right), \tag{52b}$$

for  $\Omega_m > 1$  and  $\Omega_m < 1$ , respectively. Using now the expressions for  $\alpha$  and  $\beta$ , Eqs. (52) then reduce into the single equation

$$r = \tau v \left[ 1 + (1 - \Omega_m) v^2 / 6c^2 \right].$$
 (53)

Inverting now this equation by writing it as  $v = H_0 r$ , we obtain in the lowest approximation

$$H_0 = h \left[ 1 - (1 - \Omega_m) v^2 / 6c^2 \right],$$
 (54)

where  $h = \tau^{-1}$ . To the same approximation one also obtains

$$H_0 = h \left[ 1 - (1 - \Omega_m) z^2 / 6 \right] = h \left[ 1 - (1 - \Omega_m) r^2 / 6c^2 \tau^2 \right], \tag{55}$$

where z is the redshift parameter. As is seen, and it is confirmed by experiments,  $H_0$  depends on the distance it is being measured; it has physical meaning only at the zero-distance limit, namely when measured *locally*, in which case it becomes  $h = 1/\tau$ .

It follows that the measured value of  $H_0$  depends on the "short" and "long" distance scales [8]. The farther the distance  $H_0$  is being measured, the lower the value for  $H_0$  is obtained. By Eq. (55) this is possible only when  $\Omega_m < 1$ , namely when the Universe is accelerating. By Eq. (30) we also find that the pressure is positive.

The possibility that the Universe expansion is accelerating was first predicted using CGR by the author in 1996 [2] before the supernovae experiments results became known.

It will be noted that the constant expansion is just a transition stage between the decelerating and the accelerating expansions as the Universe evolves toward its present situation.

Figure 1 describes the Hubble diagram of the above solutions for the three types of expansion for values of  $\Omega_m$  from 100 to 0.245. The figure describes the three-phase evolution of the Universe. Curves (1)-(5) represent the stages of decelerating expansion according to Eq. (49). As the density of matter  $\rho$  decreases, the Universe goes over from the lower curves to the upper ones, but it does not have enough time to close up to a big crunch. The Universe subsequently goes over to curve (6) with  $\Omega_m = 1$ , at which time it has a constant expansion for a fraction of a second. This then followed by going to the upper curves (7) and (8) with  $\Omega_m < 1$ , where the Universe expands with acceleration

according to Eq. (51). Curve no. 8 fits the present situation of the Universe. For curves (1)-(4) in the diagram we use the cutoff when the curves were at their maximum. In Table 1 we present the cosmic times with respect to the big bang, the cosmic radiation temperature and the pressure for each of the curves in Fig. 1.

Figures 2 and 3 show the Hubble diagrams for the distance-redshift relationship predicted by the theory for the accelerating expanding Universe at the present time, and Figures 4 and 5 show the experimental results.

Our estimate for h, based on published data, is  $h \approx 80$  km/sec-Mpc. Assuming  $\tau^{-1} \approx 80$  km/sec-Mpc, Eq. (55) then gives

$$H_0 = h \left[ 1 - 1.3 \times 10^{-4} \left( 1 - \Omega_m \right) r^2 \right], \tag{56}$$

where r is in Mpc. A computer best-fit can then fix both h and  $\Omega_m$ .

To summarize, a theory of cosmology has been presented in which the dynamical variables are those of Hubble, i.e. distances and velocities. The theory descirbes the Universe as having a three-phase evolution with a decelerating expansion, followed by a constant and an accelerating expansion, and it predicts that the Universe is now in the latter phase. As the density of matter decreases, while the Universe is at the decelerating phase, it does not have enough time to close up to a big crunch. Rather, it goes to the constant-expansion phase, and then to the accelerating stage. As we have seen, the equation obtained for the Universe expansion, Eq. (51), is very simple.

## 8 Theory versus experiment

The Einstein gravitational field equations with the added cosmological term are [9]:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu},\tag{57}$$

where  $\Lambda$  is the cosmological constant, the value of which is supposed to be determined by experiment. In Eq. (57)  $R_{\mu\nu}$  and R are the Ricci tensor and scalar, respectively,  $\kappa = 8\pi G$ , where G is Newton's constant and the speed of light is taken as unity.

Recently the two groups (the Supernovae Cosmology Project and the High-Z Supernova Team) concluded that the expansion of the Universe is accelerating [10-16]. The two groups had discovered and measured moderately high redshift (0.3 < z < 0.9) supernovae, and found that they were fainter than what one would expect them to be if the cosmos expansion were slowing down or constant. Both teams obtained

$$\Omega_m \approx 0.3, \quad \Omega_\Lambda \approx 0.7,$$
(58)

and ruled out the traditional  $(\Omega_m, \Omega_{\Lambda})=(1, 0)$  Universe. Their value of the density parameter  $\Omega_{\Lambda}$  corresponds to a cosmological constant that is small but, nevertheless, nonzero and positive,

$$\Lambda \approx 10^{-52} \text{m}^{-2} \approx 10^{-35} \text{s}^{-2}.$$
 (59)

In previous sections a four-dimensional cosmological theory (CGR) was presented. Although the theory has no cosmological constant, it predicts that the Universe accelerates and hence it has the equivalence of a positive cosmological constant in Einstein's general relativity. In the framework of this theory (see Section 2) the zero-zero component of the field equations (3) is written as

$$R_0^0 - \frac{1}{2} \delta_0^0 R = \kappa \rho_{eff} = \kappa \left( \rho - \rho_c \right),$$
 (60)

where  $\rho_c = 3/\kappa \tau^2$  is the critical mass density and  $\tau$  is Hubble's time in the zero-gravity limit.

Comparing Eq. (60) with the zero-zero component of Eq. (57), one obtains the expression for the cosmological constant of general relativity,

$$\Lambda = \kappa \rho_c = 3/\tau^2. \tag{61}$$

To find out the numerical value of  $\tau$  we use the relationship between  $h = \tau^{-1}$  and  $H_0$  given by Eq. (55) (CR denote values according to Cosmological Relativity):

$$H_0 = h \left[ 1 - \left( 1 - \Omega_m^{CR} \right) z^2 / 6 \right],$$
 (62)

where z=v/c is the redshift and  $\Omega_m^{CR}=\rho_m/\rho_c$  with  $\rho_c=3h^2/8\pi G$ . (Notice that our  $\rho_c=1.194\times 10^{-29}g/cm^3$  is different from the standard  $\rho_c$  defined with  $H_0$ .) The redshift parameter z determines the distance at which  $H_0$  is measured. We choose z=1 and take for

$$\Omega_m^{CR} = 0.245,\tag{63}$$

its value at the present time (see Table 1) (corresponds to 0.32 in the standard theory), Eq. (62) then gives

$$H_0 = 0.874h.$$
 (64)

At the value z=1 the corresponding Hubble parameter  $H_0$  according to the latest results from HST can be taken [17] as  $H_0=70 \mathrm{km/s\text{-}Mpc}$ , thus  $h=(70/0.874)\mathrm{km/s\text{-}Mpc}$ , or

$$h = 80.092 \text{km/s-Mpc},$$
 (65)

and

$$\tau = 12.486Gyr = 3.938 \times 10^{17}s. \tag{66}$$

What is left is to find the value of  $\Omega_{\Lambda}^{CR}$ . We have  $\Omega_{\Lambda}^{CR}=\rho_c^{ST}/\rho_c$ , where  $\rho_c^{ST}=3H_0^2/8\pi G$  and  $\rho_c=3h^2/8\pi G$ . Thus  $\Omega_{\Lambda}^{CR}=(H_0/h)^2=0.874^2$ , or

$$\Omega_{\Lambda}^{CR} = 0.764. \tag{67}$$

As is seen from Eqs. (63) and (67) one has

$$\Omega_T = \Omega_m^{CR} + \Omega_{\Lambda}^{CR} = 0.245 + 0.764 = 1.009 \approx 1,$$
(68)

which means the Universe is Euclidean.

As a final result we calculate the cosmological constant according to Eq. (61). One obtains

$$\Lambda = 3/\tau^2 = 1.934 \times 10^{-35} s^{-2}. \tag{69}$$

Our results confirm those of the supernovae experiments and indicate on the existance of the dark energy as has recently received confirmation from the Boomerang cosmic microwave background experiment [18,19], which showed that the Universe is Euclidean.

#### 9 Some remarks

In this paper the cosmological general relativity, a relativistic theory in spacevelocity, has been presented and applied to the problem of the expansion of the Universe. The theory, which predicts a positive pressure for the Universe now, describes the Universe as having a three-phase evolution: decelerating, constant and accelerating expansion, but it is now in the latter stage. Furthermore, the cosmological constant that was extracted from the theory agrees with the experimental result. Finally, it has also been shown that the three-dimensional spatial space of the Universe is Euclidean, again in agreement with observations.

Recently [20,21], more confirmation to the Universe accelerating expansion came from the most distant supernova, SN 1997ff, that was recorded by the Hubble Space Telescope. As has been pointed out before, if we look back far enough, we should find a decelerating expansion (curves 1-5 in Figure 1). Beyond z=1 one should see an earlier time when the mass density was dominant. The measurements obtained from SN 1997ff's redshift and brightness provide a direct proof for the transition from past decelerating to present accelerating expansion (see Figures 6 and 7). The measurements also exclude the possibility that the acceleration of the Universe is not real but is due to other astrophysical effects such as dust.

Table 2 gives some of the cosmological parameters obtained here and in the standard theory.

## 10 Comparison with general relativity

In order to compare the present theory with general relativity, we now add the time coordinate. We then have a time-space-velocity Universe with two time-like and three space-like coordinates, with signature (+ - - - +). We will be concerned with the classical experiments of general relativity and the gravitational waves predicted by that theory. In the following we show that all these results are also obtained from the present theory. To this end we proceed as follows.

We first find the cosmological-equivalent of the Schwarzschild sphericallysymmetric solution in cosmology. It will be useful to change variables from the classical Schwarzschild metric to new variables as follows:

$$\sin^2 \chi = r_s/r, \quad dr = -2r_s \sin^{-3} \chi \cos \chi d\chi, \tag{70}$$

where  $r_s = 2GM/c^2$  is the Schwarzschild radius. We also change the time coordinate  $cdt = r_s d\eta$ , thus  $\eta$  is a time parameter. The classical Schwarzschild solution will thus have the following form:

$$ds^{2} = r_{s}^{2} \left[ \cos^{2} \chi d\eta^{2} - 4 \sin^{-6} \chi d\chi^{2} - \sin^{-4} \chi \left( d\theta^{2} + \sin^{2} \theta d\phi^{2} \right) \right]. \tag{71}$$

So far this is just the classical spherically symmetric solution of the Einstein field equations in four dimensions, though written in new variables. The non-zero Christoffel symbols are given by

$$\Gamma_{01}^{0} = -\sin\chi \cos^{-1}\chi, \qquad \Gamma_{00}^{1} = -\frac{1}{4}\sin^{7}\chi \cos\chi,$$

$$\Gamma_{11}^{1} = -3\sin^{-1}\chi \cos\chi, \qquad \Gamma_{22}^{1} = \frac{1}{2}\sin\chi \cos\chi,$$

$$\Gamma_{33}^{1} = \frac{1}{2}\sin\chi \cos\chi \sin^{2}\theta, \qquad \Gamma_{12}^{2} = -2\sin^{-1}\chi \cos\chi,$$

$$\Gamma_{33}^{2} = -\sin\theta \cos\theta, \qquad \Gamma_{13}^{3} = -2\sin^{-1}\chi \cos\chi, \qquad \Gamma_{23}^{3} = \sin^{-1}\theta \cos\theta.$$
(72)

It is very lengthy, but one can verify that all components of the Ricci tensor  $R_{\alpha\beta}$  are equal to zero identically.

We now extend this solution to cosmology. In order to conform with the standard notation, the zero component will be chosen as the time parameter, followed by the three space-like coordinates and then the fourth coordinate representing the velocity  $\tau dv$ . We will make one more change by choosing  $\tau dv = r_s du$ , thus u is the velocity parameter. The simplest way to have a cosmological solution of the Einstein field equation is using the so-called comoving coordinates in which:

$$ds^{2} = r_{s}^{2} \left[\cos^{2} \chi d\eta^{2} - 4\sin^{-6} \chi d\chi^{2} - \sin^{-4} \chi \left(d\theta^{2} + \sin^{2} \theta d\phi^{2}\right) + du^{2}\right].$$
 (73)

The coordinates are now  $x^0 = \eta$ ,  $x^1 = \chi$ ,  $x^2 = \theta$ ,  $x^3 = \phi$ , and  $x^4 = u$ , and  $r_s$  is now a function of the velocity u,  $r_s = r_s(u)$  to be determined by the Einstein field equations in five dimensions. Accordingly we have the following form for the metric:

$$g_{\mu\nu} = r_s^2 \begin{pmatrix} \cos^2 \chi & & & & 0\\ & -4\sin^{-6} \chi & & & \\ & & -\sin^{-4} \chi & & \\ & & & -\sin^{-4} \chi \sin^2 \theta & \\ 0 & & & 1 \end{pmatrix}, \quad (74a)$$

$$\sqrt{-g} = 2r_s^5 \sin^{-7} \chi \cos \chi \sin \theta. \tag{74b}$$

The non-zero Christoffel symbols are given by

$$\Gamma_{01}^{0} = -\sin\chi \cos^{-1}\chi, \qquad \Gamma_{04}^{0} = \dot{r_s}r_s^{-1},$$

$$\Gamma_{00}^{1} = -\frac{1}{4}\sin^{7}\chi \cos\chi, \qquad \Gamma_{11}^{1} = -3\sin^{-1}\chi \cos\chi,$$

$$\Gamma_{14}^{1} = \dot{r_s}r_s^{-1}, \qquad \Gamma_{22}^{1} = \frac{1}{2}\sin\chi \cos\chi, \qquad \Gamma_{33}^{1} = \frac{1}{2}\sin\chi \cos\chi \sin^{2}\theta,$$

$$\Gamma_{12}^{2} = -2\sin^{-1}\chi \cos\chi, \qquad \Gamma_{24}^{2} = \dot{r_s}r_s^{-1}, \qquad \Gamma_{33}^{2} = -\sin\theta \cos\theta, \qquad (75)$$

$$\Gamma_{13}^{3} = -2\sin^{-1}\chi \cos\chi, \qquad \Gamma_{23}^{3} = \sin^{-1}\theta \cos\theta, \qquad \Gamma_{34}^{3} = \dot{r_s}r_s^{-1},$$

$$\Gamma_{00}^{4} = -\dot{r_s}r_s^{-1}\cos^{2}\chi, \qquad \Gamma_{11}^{4} = 4\dot{r_s}r_s^{-1}\sin^{-6}\chi, \qquad \Gamma_{22}^{4} = \dot{r_s}r_s^{-1}\sin^{-4}\chi,$$

$$\Gamma_{33}^{4} = \dot{r_s}r_s^{-1}\sin^{-4}\chi\sin^{2}\theta, \qquad \Gamma_{44}^{4} = \dot{r_s}r_s^{-1},$$

where the dots denote derivatives with respect to the velocity parameter u.

The Ricci tensor components after a lengthy but straightforward calculation, are given by:

$$R_{00} = -\left(\ddot{r}_{s}r_{s}^{-1} + 2\dot{r}_{s}^{2}r_{s}^{-2}\right)\cos^{2}\chi,$$

$$R_{11} = 4\left(\ddot{r}_{s}r_{s}^{-1} + 2\dot{r}_{s}^{2}r_{s}^{-2}\right)\sin^{-6}\chi,$$

$$R_{22} = \left(\ddot{r}_{s}r_{s}^{-1} + 2\dot{r}_{s}^{2}r_{s}^{-2}\right)\sin^{-4}\chi,$$

$$R_{33} = \left(\ddot{r}_{s}r_{s}^{-1} + 2\dot{r}_{s}^{2}r_{s}^{-2}\right)\sin^{-4}\chi\sin^{2}\theta,$$

$$R_{44} = -4\left(\ddot{r}_{s}r_{s}^{-1} - \dot{r}_{s}^{2}r_{s}^{-2}\right).$$

$$(76)$$

All other components are identically zero.

We are interested in vacuum solution of the Einstein field equations for the spherically symmetric metric (generalized Schwarzschild to cosmology), the right-hand sides of the above equations should be taken zero. A simple calculation then shows that  $\dot{r}_s = 0$ ,  $\ddot{r}_s = 0$ . Accordingly the cosmological Schwarzschild metric is given by Eq. (74a) with a constant  $r_s = 2GM/c^2$ . The metric (74a) can then be written, using the coordinate transformations (70), as

$$g_{\mu\nu} = \begin{pmatrix} 1 - \frac{r_s}{r} & 0 \\ & -\left(1 - \frac{r_s}{r}\right)^{-1} & 0 \\ & & -r^2 & 0 \\ & & & -r^2 \sin^2 \theta \\ 0 & & & 1 \end{pmatrix}, \tag{77}$$

where the coordinates are now  $x^0 = ct$ ,  $x^1 = r$ ,  $x^2 = \theta$ ,  $x^3 = \phi$ , and  $x^4 = \tau v$ .

We are now in a position to compare the present theory with general relativity.

### 11 Gravitational redshift

We start with the simplest experiment of the gravitational redshift. Although this experiment is not considered as one of the proofs of general relativity (it can be derived from conservation laws and Newtonian theory).

Consider two clocks at rest at two points denoted by 1 and 2. The propagation of light is determined by ds at each point. Since at these points all spatial infinitesimal displacements and change in velocities vanish, one has  $ds^2 = g_{00}c^2dt^2$ . Hence at the two points we have

$$ds(1) = [g_{00}(1)]^{1/2} cdt, (78a)$$

$$ds(2) = [g_{00}(2)]^{1/2} cdt (78b)$$

for the proper time (see Fig. 8).

The ratio of the rates of similar clocks, located at different places in a gravitational field, is therefore given by

$$ds(2)/ds(1) = [g_{00}(2)/g_{00}(1)]^{1/2}.$$
(79)

The frequency  $\nu_0$  of an atom located at point 1, when measured by an observer located at point 2, is therefore given by

$$\nu = \nu_0 \left[ g_{00} \left( 1 \right) / g_{00} \left( 2 \right) \right]^{1/2}. \tag{80}$$

If the gravitational field is produced by a spherically symmetric mass distribution, then we may use the generalized Schwarzschild metric given above to calculate the above ratio at the two points. In this case  $g_{00}=1-2GM/c^2r$ , and therefore

$$\left[g_{00}\left(1\right)/g_{00}\left(2\right)\right]^{1/2} \approx 1 + \left(GM/c^{2}\right)\left(1/r_{2} - 1/r_{1}\right)$$

to first order in  $GM/c^2r$ . We thus obtain

$$\Delta \nu / \nu_0 = (\nu - \nu_0) / \nu_0 \approx - (GM/c^2) (1/r_1 - 1/r_2)$$

for the frequency shift per unit frequency. Taking now  $r_1$  to be the observed radius of the Sun and  $r_2$  the radius of the Earth's orbit around the Sun, then we find that

$$\Delta \nu / \nu_0 \approx -GM_{Sun}/c^2 r_{Sun},\tag{81}$$

where  $M_{Sun}$  and  $r_{Sun}$  are the mass and radius of the Sun. Accordingly we obtain  $\Delta \nu / \nu_0 \approx -2.12 \times 10^{-6}$  for the frequency shift per unit frequency of the light emitted from the Sun. The calculation made above amounts to neglecting completely the Earth's gravitational field. The above result is the standard gravitational redshift (also known as the gravitational time dilation).

# 12 Motion in a centrally symmetric gravitational field

We assume that small test particles move along geodesics in the gravitational field. We also assume that planets have small masses as compared with the mass of the Sun, to the extent that they can be considered as test particles moving in the gravitational field of the Sun. As a result of these assumptions, the geodesic equation in the cosmological Schwarzschild field will be taken to describe the equation of motion of a planet moving in the gravitational field of the Sun. In fact, we do not need the exact solution of the generalized Schwarzschild metric (77), but just its first approximation. We obtain in the first approximation the following expressions for the components of the metric tensor:

$$g_{00} = 1 - r_s/r, g_{0m} = 0, g_{04} = 0,$$
  
 $g_{mn} = -\delta_{mn} - r_s x^m x^n/r^3, g_{m4} = 0, g_{44} = 1.$  (82a)

The contravariant components of the metric tensor are consequently given, in the same approximation, by

$$g^{00} = 1 + r_s/r, g^{0m} = 0, g^{04} = 0,$$
  
 $g^{mn} = -\delta^{mn} + r_s x^m x^n/r^3, g^{m4} = 0, g^{44} = 1.$  (82b)

We may indeed verify that the relation  $g_{\mu\lambda}g^{\lambda\nu} = \delta^{\nu}_{\mu}$  between the contravariant and covariant components of the above approximate metric tensor is satisfied to orders of magnitude of the square of  $r_s/r$ . A straightforward calculation then gives the following expressions for the Christoffel symbols:

$$\Gamma_{0n}^{0} = -\frac{r_s}{2} \frac{\partial}{\partial x^n} \left(\frac{1}{r}\right),$$

$$\Gamma_{00}^{k} = -\frac{r_s}{2} \left(1 - \frac{r_s}{r}\right) \frac{\partial}{\partial x^k} \left(\frac{1}{r}\right),$$

$$\Gamma_{mn}^{k} = r_s \frac{x^k}{r^3} \delta_{mn} - \frac{3}{2} r_s \frac{x^k x^m x^n}{r^5}.$$
(83)

All other components vanish.

We now use these expressions for the Christoffel symbols in the geodesic equation

$$\ddot{x^k} + \left(\Gamma^k_{\alpha\beta} - \Gamma^0_{\alpha\beta}\dot{x^k}\right)\dot{x^\alpha}\dot{x^\beta} = 0,\tag{84}$$

where a dot denotes differentiation with respect to the time coordinate  $x^0$ . We obtain

$$\Gamma^{0}_{\alpha\beta}\dot{x}^{\dot{\alpha}}\dot{x}^{\dot{\beta}} = \Gamma^{0}_{00} + 2\Gamma^{0}_{0n}\dot{x}^{\dot{n}} + 2\Gamma^{0}_{04}\dot{x}^{\dot{4}} + \Gamma^{0}_{mn}\dot{x}^{\dot{m}}\dot{x}^{\dot{n}} + 2\Gamma^{0}_{m4}\dot{x}^{\dot{m}}\dot{x}^{\dot{4}} + \Gamma^{0}_{44}\dot{x}^{\dot{4}}\dot{x}^{\dot{4}} 
= -r_{s}\dot{x}^{\dot{n}}\frac{\partial}{\partial x^{n}}\left(\frac{1}{r}\right),$$
(85a)

$$\Gamma^{k}_{\alpha\beta}\dot{x}^{\dot{\alpha}}\dot{x}^{\dot{\beta}} = \Gamma^{k}_{00} + 2\Gamma^{k}_{0l}\dot{x}^{\dot{l}} + 2\Gamma^{k}_{04}\dot{x}^{\dot{4}} + \Gamma^{k}_{mn}\dot{x}^{\dot{m}}\dot{x}^{\dot{n}} + 2\Gamma^{k}_{m4}\dot{x}^{\dot{m}}\dot{x}^{\dot{4}} + \Gamma^{k}_{44}\dot{x}^{\dot{4}}\dot{x}^{\dot{4}}$$

$$= -\frac{r_{s}}{2}\frac{\partial}{\partial x^{k}}\left(\frac{1}{r}\right) + r_{s}\left[\frac{r_{s}}{2r}\frac{\partial}{\partial x^{k}}\left(\frac{1}{r}\right) - \left(\dot{x}^{\dot{s}}\dot{x}^{\dot{s}}\right)\frac{\partial}{\partial x^{k}}\left(\frac{1}{r}\right) - \frac{3}{2r^{5}}\left(x^{s}\dot{x}^{\dot{s}}\right)^{2}x^{k}\right].$$
(85b)

Consequently we obtain from the geodesic equation (84) the following equation of motion for the planet:

$$\ddot{x^{k}} - \frac{r_{s}}{2} \frac{\partial}{\partial x^{k}} \left( \frac{1}{r} \right) \\
= r_{s} \left[ \left( \dot{x^{i}} \dot{x^{i}} \dot{x^{j}} \right) \frac{\partial}{\partial x^{k}} \left( \frac{1}{r} \right) - \frac{r_{s}}{2r} \frac{\partial}{\partial x^{k}} \left( \frac{1}{r} \right) - \dot{x^{i}} \frac{\partial}{\partial x^{n}} \left( \frac{1}{r} \right) \dot{x^{k}} + \frac{3}{2r^{5}} \left( x^{s} \dot{x^{i}} \right)^{2} x^{k} \right].$$
(86)

Replacing now the derivatives with respect to  $x^0$  by those with respect to  $t \equiv x^0/c$  in the latter equation, we obtain

$$\ddot{\mathbf{x}} - GM\nabla\frac{1}{r} = r_s \left[ \left( \dot{\mathbf{x}}^2 \right) \nabla \left( \frac{1}{r} \right) - \frac{GM}{r} \nabla \left( \frac{1}{r} \right) - \left( \dot{\mathbf{x}} \cdot \nabla \frac{1}{r} \right) \dot{\mathbf{x}} + \frac{3}{2r^5} \left( \mathbf{x} \cdot \dot{\mathbf{x}} \right)^2 \mathbf{x} \right], \tag{87}$$

where use has been made of the three-dimensional notation.

Hence the equation of motion of the planet differs from the Newtonian one since the left-hand side of Eq. (87) is proportional to terms of order of magnitude  $r_s$  instead of vanishing identically. This correction leads to a fundamental effect, namely, to a systematically secular change in the perihelion of the orbit of the planet.

To integrate the equation of motion (87) we multiply it vectorially by the radius vector  $\mathbf{x}$ . We obtain

$$\mathbf{x} \times \ddot{\mathbf{x}} = -r_s \left( \dot{\mathbf{x}} \cdot \nabla \left( 1/r \right) \right) \left( \mathbf{x} \times \dot{\mathbf{x}} \right). \tag{88}$$

All other terms in Eq. (87) are proportional to the radius vector  $\mathbf{x}$  and thus contribute nothing. Equation (88) may be integrated to yield the first integral

$$\mathbf{x} \times \dot{\mathbf{x}} = \mathbf{J}e^{-r_s/r}.\tag{89}$$

Here J is a constant vector, the *angular momentum* per mass unit of the planet. One can easily check that the first integral (89) indeed leads back to Eq. (88) by taking the time derivatives of both sides of Eq. (89).

From Eq. (89) we see that the radius vector  $\mathbf{x}$  moves in a plane perpendicular to the constant angular momentum vector  $\mathbf{J}$ , thus the planet moves in a plane similar to the case in Newtonian mechanics. If we now introduce in this plane coordinates r and  $\phi$  to describe the motion of the planet, the equation of motion (87) consequently decomposes into two equations. Introducing now the new variable u = 1/r, we can then rewrite the equations in terms of  $u(\phi)$ , using

$$\dot{r} = -\frac{u'}{u^2}\dot{\phi},$$
 
$$\ddot{r} = \frac{2u'^2}{u^3}\dot{\phi}^2 - \frac{u''}{u^2}\dot{\phi}^2 - \frac{u'}{u^2}\ddot{\phi},$$

where a prime denotes a differentiation with respect to the angle  $\phi$ . We subsequently obtain

$$\ddot{\phi} = 2\frac{u'}{u}\dot{\phi}^2 - \frac{2GM}{c^2}u'\dot{\phi}^2.$$

A straightforward calculation then gives, using the expression for  $\ddot{\phi}$ ,

$$u'' + u - GM \left(\frac{u^2}{\dot{\phi}}\right)^2 = \frac{GM}{c^2} \left[ 2u^2 - u'^2 - 2GMu \left(\frac{u^2}{\dot{\phi}}\right)^2 \right]$$
(90).

The latter equation can be further simplified if we use the first integral

$$r^2\dot{\phi} = Je^{-2GM/c^2r}.$$

We obtain

$$\frac{u^2}{\dot{\phi}} = \frac{1}{J} e^{2GMu/c^2},$$

$$\frac{1}{4GMu/c^2} = \frac{1}{J} \left( -\frac{4GMu}{c^2} \right)$$

$$\left(\frac{u^2}{\dot{\phi}}\right)^2 = \frac{1}{J^2}e^{4GMu/c^2} \approx \frac{1}{J^2}\left(1 + \frac{4GM}{c^2}u\right).$$

Hence, to an accuracy of  $1/c^2$ , Eq. (90) gives

$$u'' + u - \frac{GM}{J^2} = \frac{GM}{c^2} \left( 2u^2 - u'^2 + 2\frac{GM}{J^2} u \right). \tag{91}$$

Equation (91) can be used to determine the motion of the planet. The Newtonian equation of motion that corresponds to Eq. (91) is one whose left-hand side is identical to the above equation, but is equal to zero rather than to the terms on the right-hand side. This fact can easily be seen if one lets  $GM/c^2$  go to zero in Eq. (91). Therefore in the Newtonian limit we have

$$u'' + u - \frac{GM}{J^2} \approx 0, (92)$$

whose solution can be written as

$$u \approx u_0 \left( 1 + \epsilon \cos \phi \right). \tag{93}$$

Here  $u_0$  is a constant, and  $\epsilon$  is the eccentricity of the ellipse,  $\epsilon = (1 - b^2/a^2)^{1/2}$ , where a and b are the semimajor and semiminor axes of the ellipse. Using the solution (93) in the Newtonian limit of the equation of motion (92) then determines the value of the constant  $u_0$ , as  $u_0 = GM/J^2$ .

To solve the equation of motion (91), we therefore assume a solution of the form

$$u = u_0 \left( 1 + \epsilon \cos \alpha \phi \right), \tag{94}$$

where  $\alpha$  is some parameter to be determined, and whose value in the usual nonrelativistic mechanics is unity. The appearance of the parameter  $\alpha \neq 1$  in

our solution is an indication that the motion of the planet will no longer be a closed ellipse.

Using the above solution in Eq. (91), and equating coefficients of  $\cos \alpha \phi$ , then gives

$$\alpha^2 = 1 - \frac{2GM}{c^2} \left( 2u_0 + \frac{GM}{J^2} \right).$$

If we substitute for  $GM/J^2$  in the above equation its nonrelativistic value  $u_0$ , then the error will be of a higher order. Hence the latter equation can be written as

$$\alpha^2 = 1 - \frac{6GM}{c^2} u_0$$

or

$$\alpha = 1 - \frac{3GM}{c^2} u_0. \tag{95}$$

Successive perihelia occur at two angles  $\phi_1$  and  $\phi_2$  when  $\alpha\phi_2 - \alpha\phi_1 = 2\pi$ . Since the parameter  $\alpha$  is smaller than unity, we have  $\phi_2 - \phi_1 = 2\pi/\alpha > 2\pi$ . Hence we can write  $\phi_2 - \phi_1 = 2\pi + \Delta\phi$ , with  $\Delta\phi > 0$ , or

$$\alpha (\phi_2 - \phi_1) = \alpha (2\pi + \Delta \phi) = \left(1 - \frac{3GM}{c^2} u_0\right) (2\pi + \Delta \phi) = 2\pi.$$
 (96)

As a result there will be an *advance* in the perihelion of the orbit of the planet per revolution given by Eq. (96) or, to first order, by

$$\Delta \phi = 6\pi G M u_0 / c^2. \tag{97}$$

The constant  $u_0$  can also be expressed in terms of the eccentricity, using the Newtonian approximation. Denoting the radial distances of the orbit, which correspond to the angles  $\phi_2 = 0$  and  $\phi_1 = \pi$ , by  $r_2$  and  $r_1$ , respectively, we have from Eq. (93),

$$1/r_2 = u_0 (1 + \epsilon), \quad 1/r_1 = u_0 (1 - \epsilon).$$

Hence since  $r_1 + r_2 = 2a$ , we obtain (see Fig. 9)

$$2a = r_1 + r_2 = 2/u_0 (1 - \epsilon^2)$$
,

where a is the semimajor axis of the orbit, and therefore

$$u_0 = 1/a \left(1 - \epsilon^2\right).$$

Using this value for  $u_0$  in the expression (97) for  $\Delta \phi$ , we obtain for the perihelion advance the expression

$$\Delta \phi = \frac{6\pi GM}{c^2 a \left(1 - \epsilon^2\right)} \tag{98}$$

in radians per revolution (see Fig. 10). This is the standard general relativistic formula for the advance of the perihelion.

In the next section we discuss the deflection of a light ray moving in a gravitational field.

## 13 Deflection of light in a gravitational field

To discuss the effect of gravitation on the propagation of light signals we may use the geodesic equation, along with the null condition ds = 0 at a fixed velocity. A light signal propagating in the gravitational field of the Sun, for instance, will thus be described by the null geodesics in the cosmological Schwarzschild field at dv = 0.

Using the approximate solution for the cosmological Schwarzschild metric, given by Eq. (82a), we obtain

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = \left(1 - \frac{2GM}{c^2r}\right)c^2dt^2 - \left[dx^sdx^s + \frac{2GM}{c^2}\frac{(x^sdx^s)^2}{r^3}\right] = 0.$$
 (99)

Hence we have, to the first approximation in  $GM/c^2$ , the following equation of motion for the propagation of light in a gravitational field:

$$\left(1 + \frac{2GM}{c^2 r}\right) \left[ \left(\dot{x}^s \dot{x}^s\right) + \frac{2GM}{c^2} \frac{\left(x^s \dot{x}^s\right)^2}{r^3} \right] = c^2,$$
(100)

where a dot denotes differentiation with respect to the time coordinate  $t \equiv x^0/c$ .

Just as in the case of planetary motion (see previous section), the motion here also takes place in a plane. Hence in this plane we may introduce the polar coordinates r and  $\phi$ . The equation of motion (100) then yields, to the first approximation in  $GM/c^2$ , the following equation in the polar coordinates:

$$\left(\dot{r}^2 + r^2\dot{\phi}^2\right) + \frac{4GM}{c^2}\frac{\dot{r}^2}{r} + \frac{2GM}{c^2}r\dot{\phi}^2 = c^2.$$
 (101)

Changing now variables from r to  $u(\phi) \equiv 1/r$ , we obtain

$$\[ u'^2 + u^2 + \frac{2GMu}{c^2} \left( 2u'^2 + u^2 \right) \] \left( \frac{\dot{\phi}}{u^2} \right)^2 = c^2, \tag{102}$$

where a prime denotes differentiation with respect to the angle  $\phi$ . Moreover we may use the first integral of the motion,

$$r^2\dot{\phi} = Je^{-2GM/c^2r},\tag{103}$$

in Eq. (102), thus getting

$$u'^{2} + u^{2} + \frac{2GMu}{c^{2}} \left(2u'^{2} + u^{2}\right) = \left(\frac{c}{J}\right)^{2} e^{4GMu/c^{2}}.$$
 (104)

Differentiation of this equation with respect to  $\phi$  then gives

$$u'' + u + \frac{GM}{c^2} \left( 2u'^2 + 4uu'' + 3u^2 \right) = \frac{2GM}{J^2}.$$
 (105)

In Eq. (105) terms have been kept to the first approximation in  $GM/c^2$  only.

To solve Eq. (105) we notice that, in the lowest approximation, we have, from Eq. (104),

$$u^{\prime 2} \approx \left(\frac{c}{I}\right)^2 - u^2,\tag{106}$$

$$u'' \approx -u. \tag{107}$$

Hence using these approximate expressions in Eq. (105) gives

$$u'' + u = \frac{3GM}{c^2}u^2 \tag{108}$$

for the equation of motion of the orbit of the light ray propagating in a spherically symmetric gravitational field.

In the lowest approximation, namely, when the gravitational field of the central body is completely neglected, the right-hand side of Eq. (108) can be taken as zero, and therefore u satisfies the equation u'' + u = 0. The solution of this equation is a straight line given by

$$u = \frac{1}{R}\sin\phi,\tag{109}$$

where R is a constant. This equation for the straight line shows that  $r \equiv 1/u$  has a minimum value R at the angle  $\phi = \pi/2$ . If we denote  $y = r \sin \phi$ , the straight line (109) can then be described by

$$y = r\sin\phi = R = \text{constant} \tag{110}$$

(see Fig. 11).

We now use the approximate value for u, Eq. (109), in the right-hand side of Eq. (108), since the error introduced in doing so is of higher order. We therefore obtain the following for the equation of motion of the orbit of the light ray:

$$u'' + u = \frac{3GM}{c^2 R^2} \sin^2 \phi. \tag{111}$$

The solution of this equation is then given by

$$u = \frac{1}{R}\sin\phi + \frac{GM}{c^2R^2}(1+\cos^2\phi)$$
. (112)

Introducing now the Cartesian coordinates  $x=r\cos\phi$  and  $y=r\sin\phi$ , the above solution can then be written as

$$y = R - \frac{GM}{c^2 R} \frac{2x^2 + y^2}{(x^2 + y^2)^{1/2}}.$$
 (113)

We thus see that for large values of |x| the above solution asymptotically approaches the following expression:

$$y \approx R - \frac{2GM}{c^2R}|x|. \tag{114}$$

As seen from Eq. (114), asymptotically, the orbit of the light ray is described by two straight lines in the spacetime. These straight lines make angles with respect to the x axis given by  $\tan \phi = \pm \left(2GM/c^2R\right)$  (see Fig. 12). The angle of deflection  $\Delta \phi$  between the two asymptotes is therefore given by

$$\Delta \phi = \frac{4GM}{c^2 R}.\tag{115}$$

This is the angle of *deflection* of a light ray in passing through the gravitational field of a central body, described by the cosmological Schwarzschild metric. For a light ray just grazing the Sun, Eq. (115) gives the value

$$\Delta \phi = \frac{4GM_{\text{Sun}}}{c^2 R_{\text{Sun}}} = 1.75 \text{seconds.}$$

This is the standard general-relativistic formula. Observations indeed confirm this result. One of the latest measurements gives  $1.75\pm0.10$  seconds. It is worth mentioning that only general relativity theory and the present theory predict the correct factor of the deflection of light in the gravitational field.

In the next section the gravitational radiation prediction is considered.

#### 14 Gravitational radiation

In the following we show that the present theory also predicts gravitational radiation, a distinguished result of classical general relativity theory. We will not develop a complete theory of gravitational radiation. Rather we will confine ourselves in showing that the present theory does predict the phenomenon. This is done in the weak field approximation, as is usually done in standard general relativity theory.

#### 14.1 Linear approximation

For convenience, the coordinate system to be used in the linearized theory will be Cartesian, and hence the Minkowskian metric will have the form

$$\eta_{\mu\nu} = \eta^{\mu\nu} = (1, -1, -1, -1, 1),$$
(116)

when c and  $\tau$  are taken as unity. The gravitational field described by the metric tensor  $g_{\mu\nu}$  is now called weak if it differs from the Minkowskian metric tensor by terms which are much smaller than unity,

$$|g_{\mu\nu} - \eta_{\mu\nu}| \ll 1. \tag{117}$$

The above condition need not be satisfied in the entire spacetime, and it could be valid at a region of it.

We now assume that the metric tensor can be expanded as an infinite series,

$$g_{\mu\nu} = \eta_{\mu\nu} + \lambda_1 g_{\mu\nu} + \lambda_2^2 g_{\mu\nu} + \cdots,$$
 (118)

where  $\lambda$  is some small parameter, and we limit ourselves to the first-order term  $_1g_{\mu\nu}$  alone. Hence we can write

$$g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu},\tag{119a}$$

where  $h_{\mu\nu} = \lambda_1 g_{\mu\nu}$ . We also expand the contravariant components of the metric tensor.

$$g^{\mu\nu} \approx \eta^{\mu\nu} + h^{\mu\nu}.\tag{119b}$$

From the condition  $g_{\mu\lambda}g^{\lambda\nu} = \delta^{\nu}_{\mu}$  one then is able to relate  $h^{\mu\nu}$  to  $h_{\mu\nu}$  (neglecting nonlinear terms),

$$h^{\mu\nu} = -\eta^{\mu\rho}\eta^{\nu\sigma}h_{\sigma\rho}.\tag{120}$$

#### 14.2 The linearized Einstein equations

We can now derive the linearized Einstein equations. To this end we have to find the first approximate value of the Einstein tensor, the Ricci tensor, the Ricci scalar, and the Christoffel symbols. A simple calculation then gives

$$\Gamma^{\mu}_{\alpha\beta} \approx \frac{1}{2} \eta^{\mu\lambda} \left( h_{\lambda\alpha,\beta} + h_{\lambda\beta,\alpha} - h_{\alpha\beta,\lambda} \right)$$
 (121)

for the Christoffel symbols and

$$R_{\alpha\beta\gamma\delta} \approx \frac{1}{2} \left( h_{\alpha\delta,\beta\gamma} + h_{\beta\gamma,\alpha\delta} - h_{\beta\delta,\alpha\gamma} - h_{\alpha\gamma,\beta\delta} \right) \tag{122}$$

for the Riemann tensor. Accordingly we have the following expressions for the Ricci tensor, the Ricci scalar, and the Einstein tensor, respectively:

$$R_{\beta\delta} \approx \frac{1}{2} \eta^{\alpha\gamma} \left( h_{\alpha\delta,\beta\gamma} + h_{\beta\gamma,\alpha\delta} - h_{\beta\delta,\alpha\gamma} - h_{\alpha\gamma,\beta\delta} \right)$$
 (123)

$$R \approx \eta^{\alpha\gamma} \eta^{\beta\delta} \left( h_{\alpha\delta,\beta\gamma} - h_{\beta\delta,\alpha\gamma} \right) \tag{124}$$

$$G_{\mu\nu} \approx -\frac{1}{2} \left[ h_{,\mu\nu} + \eta^{\rho\sigma} \left( h_{\mu\nu,\rho\sigma} - h_{\mu\rho,\nu\sigma} - h_{\nu\rho,\mu\sigma} \right) - \eta_{\mu\nu} \eta^{\rho\sigma} \left( h_{,\rho\sigma} - \eta^{\alpha\beta} h_{\rho\sigma,\alpha\beta} \right) \right], \tag{125}$$

where  $h = \eta^{\alpha\beta} h_{\alpha\beta}$ .

A simplification in the linearized field equations occurs if we introduce the new variables

$$\gamma_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h,\tag{126}$$

from which one obtains

$$h_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\gamma,\tag{127}$$

with  $\gamma = \eta^{\alpha\beta}\gamma_{\alpha\beta}$ . Introducing the above expressions into the Einstein field equations we obtain

$$\bigcirc \gamma_{\mu\nu} - \eta^{\alpha\beta} \left( \gamma_{\alpha\mu,\beta\nu} + \gamma_{\alpha\nu,\beta\mu} \right) + \eta_{\mu\nu} \eta^{\lambda\rho} \eta^{\alpha\beta} \gamma_{\lambda\alpha,\rho\beta} = -2\kappa T_{\mu\nu}$$
 (128)

for the linearized gravitational field equations. In Eq. (128) the symbol  $\bigcirc$  is the operator in flat space,

$$\bigcirc f = \eta^{\alpha\beta} f_{,\alpha\beta} = \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{1}{\tau^2} \frac{\partial^2}{\partial v^2} \right) f. \tag{129}$$

We can simplify still further the above field equations by choosing coordinates in which

$$\gamma_{\mu} = \eta^{\rho\sigma} \gamma_{\mu\rho,\sigma} = 0. \tag{130}$$

This is similar to choosing a gauge in solving the wave equation in electrodynamics. As a result we finally obtain for the linearized Einstein equations the following:

$$\bigcirc \gamma_{\mu\nu} = -2\kappa T_{\mu\nu},\tag{131}$$

along with the supplementary condition

$$\eta^{\rho\sigma}\gamma_{\mu\rho,\sigma} = 0, \tag{132}$$

which solutions  $\gamma_{\mu\nu}$  of Eq. (131) should satisfy. Finally we see from Eq. (131) that a necessary condition for Eq. (132) to be satisfied is that

$$\eta^{\alpha\beta}T_{\mu\alpha,\beta} = 0, \tag{133}$$

which is an expression for the conservation of the energy and momentum without including gravitation.

#### 14.3 Gravitational waves

In vacuum, Eq. (131) reduces to

$$\bigcirc \gamma_{\mu\nu} = 0,$$
(134)

or

$$\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}\right) \gamma_{\mu\nu} = \frac{1}{\tau^2} \frac{\partial^2 \gamma_{\mu\nu}}{\partial v^2}.$$
 (135)

Thus the gravitational field, like the electromagnetic field, propagates in vacuum with the speed of light. The above analysis also shows the existence of gravitational waves.

## References

- 1. M. Carmeli, Cosmological Special Relativity: The Large-Scale Structure of Space, Time and Velocity (World Scientific, River Edge, N.J., 1997).
- 2. M. Carmeli, Commun. Theor. Phys. 5, 159 (1996).
- 3. S. Behar and M. Carmeli, *Intern. J. Theor. Phys.* 39, 1375 (2000). (astro-ph/0008352)
- 4. M. Carmeli and S. Behar, Cosmological general relativity, pp. 5–26, in: *Quest for Mathematical Physics*, T.M. Karade, *et al.* Editors, New Delhi (2000).
- 5. M. Carmeli and S. Behar, Cosmological relativity: a general relativistic theory for the accelerating universe, Talk given at Dark Matter 2000, Los Angeles, February 2000, pp.182–191, in: Sources and Detection of Dark Matter/Energy in the Universe, D. Cline, Ed., Springer (2001).
- 6. A. Einstein, *The Meaning of Relativity*, 5th Edition (Princeton Univ. Press, Princeton, 1955).
- 7. I.S. Gradshteyn and I.M. Ryshik, *Table of Integrals, Series and Products* (Academic Press, New York; 1980).
- 8. P.J.E. Peebles, Status of the big bang cosmology, in:
- Texas/Pascos 92: Relativistic Astrophysics and Particle Cosmology, C.W. Akerlof and M.A. Srednicki, Editors (New York Academy of Sciences, New York, 1993), p. 84.
- 9. M. Carmeli, Classical Fields: General Relativity and Gauge Theory (Wiley, New York, 1982).
- 10. P.M. Garnavich *et al.*, Astrophys. J. **493**, L53 (1998). [Hi-Z Supernova Team Collaboration (astro-ph/9710123)].
- 11. B.P. Schmidt *et al.*, *Astrophys. J.* **507**, 46 (1998). [Hi-Z Supernova Team Collaboration (astro-ph/9805200)].
- 12. A.G. Riess *et al.*, *Astronom. J.* **116**, 1009 (1998). [Hi-Z Supernova Team Collaboration (astro-ph/9805201)].
- 13. P.M. Garnavich *et al.*, *Astrophys. J.* **509**, 74 (1998). [Hi-Z Supernova Team Collaboration (astro-ph/9806396)].
- 14. S. Perlmutter *et al.*, *Astrophys. J.* **483**, 565 (1997). [Supernova Cosmology Project Collaboration (astro-ph/9608192)].
- 15. S. Perlmutter *et al.*, *Nature* **391**, 51 (1998). [Supernova Cosmology Project Collaboration (astro-ph/9712212)].
- 16. S. Perlmutter *et al.*, *Astrophys. J.* **517**, 565 (1999). [Supernova Cosmology Project Collaboration (astro-ph/9812133)].
- 17. W.L. Freedman *et al.*, Final results from the Hubble Space Telescope, Talk given at the 20th Texas Symposium on Relativistic Astrophysics, Austin, Texas 10-15 December 2000. (astro-ph/0012376)
- 18. P. de Bernardis et al., Nature 404, 955 (2000). (astro-ph/0004404)
- 19. J.R. Bond et~al., in Proc.~IAU~Symposium~201~(2000). (astro-ph/0011378)
- 20. A.V. Filippenko and A.G. Riess, p.227 in: Particle Physics and Cos-

 $mology:\ Second\ Tropical\ Workshop,\ J.F.$  Nieves, Editor (AIP, New York, 2000).

21. A.G. Riess et al., Astrophys. J., in press. (astro-ph/0104455)

Table 1: The Cosmic Times with respect to the Big Bang, the Cosmic Temperature and the Cosmic Pressure for each of the Curves in Fig. 1.

Curve No*.	$\Omega_m$	Time in Units of $\tau$	$\begin{array}{c} \text{Time} \\ \text{(Gyr)} \end{array}$	Temperature (K)	Pressure $(g/cm^2)$
DECELERATING EXPANSION					
-1	100				4 400
1	100	$3.1 \times 10^{-6}$	$3.87 \times 10^{-5}$	1096	-4.499
2	25	$9.8 \times 10^{-5}$	$1.22 \times 10^{-3}$	195.0	-1.091
3	10	$3.0 \times 10^{-4}$	$3.75 \times 10^{-3}$	111.5	-0.409
4	5	$1.2 \times 10^{-3}$	$1.50 \times 10^{-2}$	58.20	-0.182
5	1.5	$1.3 \times 10^{-2}$	$1.62 \times 10^{-1}$	16.43	-0.023
CONSTANT EXPANSION					
6	1	$3.0 \times 10^{-2}$	$3.75 \times 10^{-1}$	11.15	0
ACCELERATING EXPANSION					
7	0.5	$1.3 \times 10^{-1}$	1.62	5.538	+0.023
8	0.245	1.0	12.50	2.730	+0.034

\*The calculations are made using Carmeli's cosmological transformation, Eq. (2), that relates physical quantities at different cosmic times when gravity is extremely weak.

For example, we denote the temperature by  $\theta$ , and the temperature at the present time by  $\theta_0$ , we then have

$$\theta = \frac{\theta_0}{\sqrt{1 - \frac{t^2}{\tau^2}}} = \frac{\theta_0}{\sqrt{1 - \frac{(\tau - T)^2}{\tau^2}}} = \frac{2.73K}{\sqrt{\frac{2\tau T - T^2}{\tau^2}}} = \frac{2.73K}{\sqrt{\frac{T}{\tau}\left(2 - \frac{T}{\tau}\right)}},$$

where T is the time with respect to B.B.

The formula for the pressure is given by Eq. (30),  $p=c(1-\Omega)/8\pi G\tau$ . Using  $c=3\times 10^{10}cm/s$ ,  $\tau=3.938\times 10^{17}s$  and  $G=6.67\times 10^{-8}cm^3/gs^2$ , we obtain

$$p = 4.544 \times 10^{-2} (1 - \Omega) g/cm^2$$
.

 $\hbox{ Table 2: Cosmological parameters in cosmological general relativity and in standard theory } \\$ 

	COSMOLOGICAL RELATIVITY	STANDARD THEORY
Th	C1:4	C
Theory type	Spacevelocity	Spacetime
Expansion	Tri-phase:	One phase
type	decelerating, constant,	
· -	accelerating	
Present expansion	Accelerating	One of three
•	(predicted)	possibilities
Pressure	$0.034g/cm^2$	Negative
Cosmological constant	$1.934 \times 10^{-35} s^{-2}$	Depends
	(predicted)	- ·P · ·
$\Omega_T = \Omega_m + \Omega_\Lambda$	1.009	Depends
Constant-expansion	8.5Gyr ago	No prediction
occurs at	,	r
Constant-expansion	Fraction of	Not known
duration	second	
Temperature at	146K	No prediction
constant expansion		

#### FIGURE CAPTIONS

- Fig. 1 Hubble's diagram describing the three-phase evolution of the Universe according to cosmological general relativity theory. Curves (1) to (5) represent the stages of decelerating expansion according to  $r(v) = (c\tau/\alpha) \sin \alpha v/c$ , where  $\alpha^2 = (\Omega 1)/2$ ,  $\Omega = \rho/\rho_c$ , with  $\rho_c$  a constant,  $\rho_c = 3/8\pi G\tau^2$ , and c and  $\tau$  are the speed of light and the Hubble time in vacuum (both universal constants). As the density of matter  $\rho$  decreases, the Universe goes over from the lower curves to the upper ones, but it does not have enough time to close up to a big crunch. The Universe subsequently goes to curve (6) with  $\Omega = 1$ , at which time it has a constant expansion for a fraction of a second. This then followed by going to the upper curves (7), (8) with  $\Omega < 1$  where the Universe expands with acceleration according to  $r(v) = (c\tau/\beta) \sinh \beta v/c$ , where  $\beta^2 = (1 \Omega)/2$ . Curve no. 8 fits the present situation of the Universe. (Source: S. Behar and M. Carmeli, Ref. 3)
- Fig. 2 Hubble's diagram of the Universe at the present phase of evolution with accelerating expansion. (Source: S. Behar and M. Carmeli, Ref. 3)
- Fig. 3 Hubble's diagram describing decelerating, constant and accelerating expansions in a logarithmic scale. (Source: S. Behar and M. Carmeli, Ref. 3)
- Fig. 4 Distance vs. redshift diagram showing the deviation from a constant toward an accelerating expansion. (Source: A. Riess *et al.*, Ref. 12)
- Fig. 5 Relative intensity of light and relative distance vs. redshift. (Source: A. Riess *et al.*, Ref. 12)
- Fig. 6 Hubble diagram of SNe Ia minus an empty (i.e., "empty"  $\Omega=0$ ) Universe compared to cosmological and astrophysical models. The points are the redshift-binned data from the HZT (Riess et~al.~1998) and the SCP (Perlmutter et~al.~1999). The measurements of SN 1997ff are inconsistent with astrophysical effects which could mimic previous evidence for an accelerating Universe from SNe Ia at  $z\approx0.5$ . (Source: A. Riess et~al., Ref. 21)
- Fig. 7 Same as Fig. 6 with the inclusion of a family of plausible, flat  $\Omega_{\Lambda}$  cosmologies. The transition redshift (i.e., the coasting point) between the accelerating and decelerating phases is indicated and is given as  $[2\Omega_{\Lambda}/\Omega_M]^{1/3} 1$ . SN 1997ff is seen to lie within the epoch of deceleration. This conclusion is drawn from the result that the apparent brightness of SN 1997ff is inconsistent with values of  $\Omega_{\Lambda} \geq 0.9$  and hence a transition redshift greater than that of SN 1997ff. (Source: A. Riess *et al.*, Ref. 21)
- Fig. 8 Propagation of light in curved spacetime.
- Fig. 9 Newtonian limit of planetary motion. The motion is described by a closed ellipse if the effect of other planets is completely neglected.
- Fig. 10 Planetary elliptic orbit with perihelion advance. The effect is a general

relativistic one. The advance of the perihelion is given by  $\Delta \phi$  in radians per revolution, where  $\Delta \phi = 6\pi GM/c^2a(1-\epsilon^2)$ , with M being the mass of the Sun, a the semimajor axis, and  $\epsilon$  the eccentricity of the orbit of the planet.

Fig. 11 Light ray when the effect of the central body's gravitational field is completely neglected. The light ray then moves along the straight line  $y=r\sin\phi=R={\rm constant},$  namely,  $u=1/r=(1/R)\sin\phi$ 

Fig. 12 Bending of a light ray in the gravitational field of a spherically symmetric body. The angle of deflection  $\Delta \phi = 4GM/c^2R$ , where M is the mass of the central body and R is the closest distance of the light ray from the center of the body.























