# Elementary Particles in Physics

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Elementary-particle physics deals with the fundamental constituents of matter and their interactions. In the past several decades an enormous amount of experimental information has been accumulated, and many patterns and systematic features have been observed. Highly successful mathematical theories of the electromagnetic, weak, and strong interactions have been devised and tested. These theories, which are collectively known as the standard model, are almost certainly the correct description of Nature, to first approximation, down to a distance scale 1/1000th the size of the atomic nucleus. There are also speculative but encouraging developments in the attempt to unify these interactions into a simple underlying framework, and even to incorporate quantum gravity in a parameter-free "theory of everything." In this article we shall attempt to highlight the ways in which information has been organized, and to sketch the outlines of the standard model and its possible extensions.

## Classification of Particles

The particles that have been identified in high-energy experiments fall into distinct classes. There are the *leptons* (see *Electron, Leptons, Neutrino, Muonium*), all of which have spin $\frac{1}{2}$. They may be charged or neutral. The charged leptons have electromagnetic as well as weak interactions; the neutral ones only interact weakly. There are three well-defined lepton pairs, the electron ($e^-$) and the electron neutrino ($\nu_e$), the muon ($\mu^-$) and the muon neutrino ($\nu_\mu$), and the (much heavier) charged lepton, the tau ($\tau$), and its tau neutrino ($\nu_\tau$). These particles all have antiparticles, in accordance with the predictions of relativistic quantum mechanics (see *CPT Theorem*). There appear to exist approximate "lepton-type" conservation laws: the number of $e^-$ plus the number of $\nu_e$ minus the number of the corresponding antiparticles $e^+$ and $\bar{\nu}_e$ is conserved in weak reactions, and similarly for the muon and tau-type leptons. These conservation laws would follow automatically in the standard model if the neutrinos are massless. Recently, however, evidence for tiny nonzero neutrino masses and subtle violation of these conservations laws has been observed. There is no understanding of the hierarchy of masses in Table 1 or why the observed neutrinos are so light.

In addition to the leptons there exist *hadrons* (see *Hadrons, Baryons, Hyperons, Mesons, Nucleon*), which have strong interactions as well as the electromagnetic and weak. These particles have a variety of spins, both integral and half-integral, and their masses range from the value of $135\,\mathrm{MeV}/c^2$ for the neutral *pion* $\pi^0$ to $11\,020\,\mathrm{MeV}/c^2$ for one of the upsilon (heavy quark) states. The particles with half-integral spin are called *baryons*, and there is clear evidence for baryon conservation: The number of baryons minus the number of antibaryons is constant in any interaction. The best evidence for this is the stability of the lightest baryon, the proton (if the proton decays, it does so with a lifetime in excess of $10^{33}$ yr). In contrast to charge conservation, there is no

Table 1: The leptons. Charges are in units of the positron ($e^+$) charge $e = 1.602 \times 10^{-19}$ coulomb. In addition to the upper limits, two of the neutrinos have masses larger than $0.05\,\mathrm{eV}/c^2$ and $0.005\,\mathrm{eV}/c^2$, respectively. The $\nu_e$, $\nu_\mu$, and $\nu_\tau$ are mixtures of the states of definite mass.

| Particle | $Q$ | Mass |
|---|---|---|
| $e^-$ | $-1$ | $0.51\,\mathrm{MeV}/c^2$ |
| $\mu^-$ | $-1$ | $105.7\,\mathrm{MeV}/c^2$ |
| $\tau^-$ | $-1$ | $1777\,\mathrm{MeV}/c^2$ |
| $\nu_e$ | $0$ | $< 0.15\,\mathrm{eV}/c^2$ |
| $\nu_\mu$ | $0$ | $< 0.15\,\mathrm{eV}/c^2$ |
| $\nu_\tau$ | $0$ | $< 0.15\,\mathrm{eV}/c^2$ |

Table 2: The quarks (spin-$\frac{1}{2}$ constituents of hadrons). Each quark carries baryon number $B = \frac{1}{3}$, while the antiquarks have $B = -\frac{1}{3}$.

| Particle | $Q$ | Mass |
|---|---|---|
| $u$ (up) | $\frac{2}{3}$ | $1.5 - 5\,\mathrm{MeV}/c^2$ |
| $d$ (down) | $-\frac{1}{3}$ | $5 - 9\,\mathrm{MeV}/c^2$ |
| $s$ (strange) | $-\frac{1}{3}$ | $80 - 155\,\mathrm{MeV}/c^2$ |
| $c$ (charm) | $\frac{2}{3}$ | $1 - 1.4\,\mathrm{GeV}/c^2$ |
| $b$ (bottom) | $-\frac{1}{3}$ | $4 - 4.5\,\mathrm{GeV}/c^2$ |
| $t$ (top) | $\frac{2}{3}$ | $175 - 180\,\mathrm{GeV}/c^2$ |

deep principle that makes baryon conservation compelling, and it may turn out that baryon conservation is only approximate. The particles with integer spin are called *mesons*, and they have baryon number $B = 0$. There are hundreds of different kinds of hadrons, some almost stable and some (known as *resonances*) extremely short-lived. The degree of stability depends mainly on the mass of the hadron. If its mass lies above the threshold for an allowed decay channel, it will decay rapidly; if it does not, the decay will proceed through a channel that may have a strongly suppressed rate, e. g., because it can only be driven by the weak or electromagnetic interactions. The large number of hadrons has led to the universal acceptance of the notion that the hadrons, in contrast to the leptons, are composite. In particular, experiments involving lepton–hadron scattering or $e^+e^-$ annihilation into hadrons have established that hadrons are bound states of point-like spin-$\frac{1}{2}$ particles of fractional charge, known as quarks. Six types of quarks have been identified (Table 2). As with the leptons, there is no understanding of the extreme hierarchy of quark masses. For each type of quark there is a corresponding antiquark. Baryons are bound states of three quarks (e. g., proton = $uud$; neutron = $udd$), while mesons consist of a quark and an antiquark. Matter and decay processes under normal terrestrial conditions involve only the $e^-$, $\nu_e$, $u$, and $d$. However, from Tables 2 and 3 we see that these four types of fundamental particle are replicated in two heavier *families*, ($\mu^-$, $\nu_\mu$, $c$, $s$) and ($\tau^-$, $\nu_\tau$, $t$, $b$). The reason for the existence of these heavier copies is still unclear.

## Classification of Interactions

For reasons that are still unclear, the interactions fall into four types, the *electromagnetic*, *weak*, and *strong*, and the *gravitational* interaction. If we take the proton mass as a standard, the last is $10^{-36}$ times the strength of the electromagnetic interaction, and will mainly be neglected in what follows. (The unification of gravity with the other interactions is one of the major outstanding goals.) The first two interactions were most cleanly explored with the leptons, which do not have strong interactions that mask them. We shall therefore discuss them first in terms of the leptons.

## Electromagnetic Interactions

The electromagnetic interactions of charged leptons (electron, muon, and tau) are best described in terms of equations of motion, derived from a Lagrangian function, which are solved in a power series in the *fine-structure constant* $e^2/4\pi\hbar c = \alpha \simeq 1/137$, a small parameter. The Lagrangian density consists of a term that describes the free-photon field,

$$\mathcal{L}_\gamma = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) \ , \tag{1}$$

where

$$F_{\mu\nu}(x) = \frac{\partial A_\nu(x)}{\partial x^\mu} - \frac{\partial A_\mu(x)}{\partial x^\nu} \tag{2}$$

is the electromagnetic field tensor. $\mathcal{L}_\gamma$ is just $\frac{1}{2}[\mathbf{E}^2(x) - \mathbf{B}^2(x)]$ in more common notation. It is written in terms of the vector potential $A_\mu(x)$ because the terms that involve the lepton and its interaction with the electromagnetic field are simplest when written in terms of $A_\mu(x)$:

$$\mathcal{L}_l = \mathrm{i}\bar{\psi}(x)\gamma^\alpha \left( \frac{\partial}{\partial x^\alpha} - \mathrm{i}eA_\alpha(x) \right) \psi(x) - m\bar{\psi}(x)\psi(x) \ . \tag{3}$$

Here $\psi(x)$ is a four-component spinor representing the electron, muon, or tau, $\bar{\psi}(x) = \psi^\dagger(x)\gamma^0$, the $\gamma^\alpha(\alpha = 0,\ 1,\ 2,\ 3)$ are the Dirac matrices [$4 \times 4$ matrices that satisfy the conditions $(\gamma^1)^2 = (\gamma^2)^2 = (\gamma^3)^2 = -(\gamma^0)^2 = -1$ and $\gamma^\alpha\gamma^\beta = -\gamma^\beta\gamma^\alpha$ for $\beta \neq \alpha$]; $m$ has the dimensions of a mass in the natural units in which $\hbar = c = 1$. If $e$ were zero, the Lagrangian would describe a free lepton; with $e \neq 0$ the interaction has the form

$$-eA^\alpha(x) j_\alpha(x) \ , \tag{4}$$

where the current $j_\alpha(x)$ is given by

$$j_\alpha(x) = -\bar{\psi}(x)\gamma_\alpha\psi(x) \ . \tag{5}$$

The equations of motion show that the current is conserved,

$$\frac{\partial}{\partial x_\alpha} j_\alpha(x) = 0 \ , \tag{6}$$

so that the *charge*

$$Q = \int \mathrm{d}^3\mathbf{r}\, j_0(\mathbf{r}, t) \tag{7}$$

is a constant of the motion.

The form of the interaction is obtained by making the replacement

$$\frac{\partial}{\partial x^\alpha} \to \frac{\partial}{\partial x^\alpha} - \mathrm{i}eA_\alpha(x) \tag{8}$$

in the Lagrangian for a free lepton. This *minimal coupling* follows from a deep principle, *local gauge invariance*. The requirement that $\psi(x)$ can have its phase changed locally without affecting the physics of the lepton, that is, invariance under

$$\psi(x) \to \mathrm{e}^{-\mathrm{i}\theta(x)}\psi(x) \ , \tag{9}$$

can only be implemented through the introduction of a vector field $A_\alpha(x)$, coupled as in (8), and transforming according to

$$A_\alpha(x) \to A_\alpha(x) - \frac{1}{e}\frac{\partial\theta(x)}{\partial x^\alpha} \ . \tag{10}$$

This dictates that the free-photon Lagrangian density contains only the gauge-invariant combination (2), and that terms of the form $M^2 A_\alpha^2(x)$ be absent. Thus local gauge invariance is a very powerful requirement; it implies the existence of a massless vector particle (the photon, $\gamma$), which mediates a long-range force [Fig. 1(a)]. It also fixes the form of the coupling and leads to charge conservation, and implies masslessness of the photon. The resulting theory (see *Quantum Electrodynamics, Compton Effect, Feynman Diagrams, Muonium, Positron*) is in extremely good agreement with experiment, as Table 3 shows. In working out the consequences of the equations of motion that follow from (3), infinities appear, and the theory seems not to make sense. The work of S. Tomonaga, J. Schwinger, R. P. Feynman, and F. J. Dyson in the late 1940s clarified the nature of the problem and showed a way of eliminating the difficulties. In creating *renormalization theory* these authors pointed out that the parameters $e$ and $m$ that appear in (3) can be identified as the charge and the mass of the lepton only in lowest order. When the charge and mass are calculated in higher order, infinite integrals appear. After a rescaling of the lepton fields, it turns out that these are the only infinite integrals in the theory. Thus by absorbing them into the definitions of new quantities, the renormalized (i. e., physically measured) charge and mass, all infinities are removed, and the rest of the theoretically calculated quantities are finite. Gauge invariance ensures that in the renormalized theory the current is still conserved, and the photon remains massless (the experimental upper limit on the photon mass is $6 \times 10^{-17}$ eV/$c^2$).

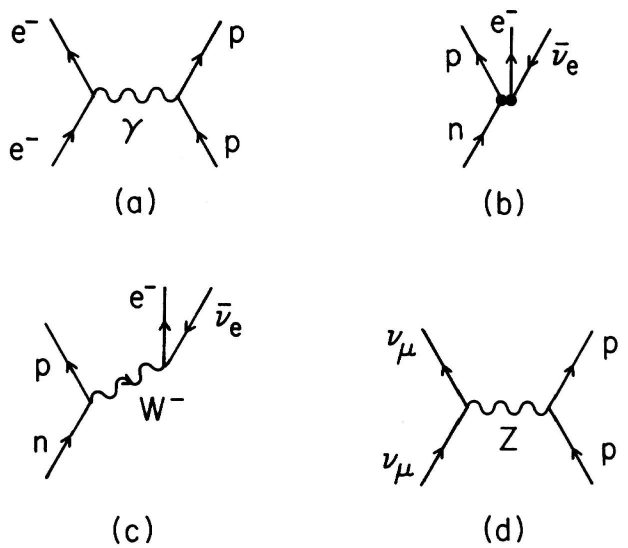


Fig. 1: (a) Long-range force between electron and proton mediated by a photon. (b) Four-fermi (zero-range) description of beta decay ($n \to pe^-\bar{\nu}_e$). (c) Beta decay mediated by a $W^-$. (d) A neutral current process mediated by the $Z$.

Table 3: Extraction of the (inverse) fine structure constant $\alpha^{-1}$ from various experiments, adapted from T. Kinoshita, *J. Phys.* **G 29**, 9 (2003). The consistency of the various determinations tests QED. The numbers in parentheses (square brackets) represent the uncertainty in the last digits (the fractional uncertainty). The last column is the difference from the (most precise) value $\alpha^{-1}(a_e)$ in the first row. A precise measurement of the muon gyromagnetic ratio $a_\mu$ is $\sim 2.4\sigma$ above the theoretical prediction, but that quantity is more sensitive to new (TeV-scale) physics.

| Experiment | Value of $\alpha^{-1}$ | | Difference from $\alpha^{-1}(a_e)$ |
|---|---|---|---|
| Deviation from gyromagnetic ratio, $a_e = (g-2)/2$ for $e^-$ | 137.035 999 58 (52) | $[3.8 \times 10^{-9}]$ | – |
| ac Josephson effect | 137.035 988 0 (51) | $[3.7 \times 10^{-8}]$ | $(0.116 \pm 0.051) \times 10^{-4}$ |
| $h/m_n$ ($m_n$ is the neutron mass) from $n$ beam | 137.036 011 9 (51) | $[3.7 \times 10^{-8}]$ | $(-0.123 \pm 0.051) \times 10^{-4}$ |
| Hyperfine structure in muonium, $\mu^+e^-$ | 137.035 993 2 (83) | $[6.0 \times 10^{-8}]$ | $(0.064 \pm 0.083) \times 10^{-4}$ |
| Cesium $D_1$ line | 137.035 992 4 (41) | $[3.0 \times 10^{-8}]$ | $(0.072 \pm 0.041) \times 10^{-4}$ |

Subsequent work showed that the possibility of absorbing the divergences of a theory in a finite number of renormalizations of physical quantities is limited to a small class of theories, e. g., those involving the coupling of spin-$\frac{1}{2}$ to spin-0 particles with a very restrictive form of the coupling. Theories involving vector (spin-1) fields are only renormalizable when the couplings are minimal and local gauge invariance holds. Thus gauge-invariant couplings like $\bar{\psi}(x)\gamma^\alpha\gamma^\beta\psi(x)F_{\alpha\beta}(x)$, which are known not to be needed in quantum electrodynamics, are eliminated by the requirement of renormalizability. (The apparent infinities for non-renormalizable theories become finite when the theories are viewed as a low energy approximation to a more fundamental theory. In that case, however, the low energy predictions have a very large sensitivity to the energy scale at which the new physics appears.)

The electrodynamics of hadrons involves a coupling of the form

$$-eA^\alpha(x)j_\alpha^{\rm had}(x)\ . \tag{11}$$

For one-photon processes, such as photoproduction (e. g., $\gamma p \to \pi^0 p$), matrix elements of the conserved current $j_\alpha^{\rm had}(x)$ are measured to first order in $e$, while for two-photon processes, such as hadronic Compton scattering ($\gamma p \to \gamma p$), matrix elements of products like $j_\alpha^{\rm had}(x)j_\beta^{\rm had}(y)$ enter. Within the quark theory one can write an explicit form for the hadronic current:

$$j_\alpha^{\rm had}(x) = \frac{2}{3}\bar{u}\gamma^\alpha u - \frac{1}{3}\bar{d}\gamma^\alpha d - \frac{1}{3}\bar{s}\gamma^\alpha s \ldots\ , \tag{12}$$

where we use particle labels for the spinor operators (which are evaluated at $x$), and the coefficients are just the charges in units of $e$. The total electromagnetic interaction is therefore $-eA^\alpha j_\alpha^\gamma$, where

$$j_\alpha^\gamma = j_\alpha + j_\alpha^{\rm had} = \sum_i Q_i\bar{\psi}_i\gamma_\alpha\psi_i\ , \tag{13}$$

and the sum extends over all the leptons and quarks ($\psi_i = e$, $\mu$, $\tau$, $\nu_e$, $\nu_\mu$, $\nu_\tau$, $u$, $d$, $c$, $s$, $b$, $t$), and where $Q_i$ is the charge of $\psi_i$.

## Weak Interactions

In contrast to the electromagnetic interaction, whose form was already contained in classical electrodynamics, it took many decades of experimental and theoretical work to arrive at a compact phenomenological Lagrangian density describing the weak interactions. The form

$$\mathcal{L}_W = -\frac{G}{\sqrt{2}}J_\alpha^\dagger(x)J^\alpha(x) \tag{14}$$

involves vectorial quantities, as originally proposed by E. Fermi. The current $J^\alpha(x)$ is known as a charged current since it changes (lowers) the electric charge when it acts on a state. That is, it describes a transition such as $\nu_e \to e^-$ of one particle into another, or the corresponding creation of an $e^-\bar{\nu}_e$ pair. Similarly, $J^\dagger_\alpha$ describes a charge-raising transition such as $n \to p$. Equation (14) describes a zero-range *four-fermi interaction* [Fig. 1(b)], in contrast to electrodynamics, in which the force is transmitted by the exchange of a photon. An additional class of "neutral-current" terms was discovered in 1973 (see *Weak Neutral Currents, Currents in Particle Theory*). These will be discussed in the next section. $J^\alpha(x)$ consists of leptonic and hadronic parts:

$$J^\alpha(x) = J^\alpha_{\rm lept}(x) + J^\alpha_{\rm had}(x) \ . \tag{15}$$

Thus, it describes purely leptonic interactions, such as

$$\begin{aligned} \mu^- &\to e^- + \bar{\nu}_e + \nu_\mu \ , \\ \nu_\mu + e^- &\to \nu_e + \mu^- \ , \end{aligned}$$

through terms quadratic in $J_{\rm lept}$; semileptonic interactions, most exhaustively studied in decay processes such as

$$\begin{aligned} n &\to p + e^- + \bar{\nu}_e \text{ (beta decay)} \ , \\ \pi^+ &\to \mu^+ + \nu_\mu \ , \\ \Lambda^0 &\to p + e^- + \bar{\nu}_e \ , \end{aligned}$$

and more recently in neutrino-scattering reactions such as

$$\begin{aligned} \nu_\mu + n &\to \mu^- + p \text{ (or } \mu^- + \text{hadrons)} \ , \\ \bar{\nu}_\mu + p &\to \mu^+ + n \text{ (or } \mu^+ + \text{hadrons)} \ ; \end{aligned}$$

and, through terms quadratic in $J^\alpha_{\rm had}$, purely nonleptonic interactions, such as

$$\begin{aligned} \Lambda^0 &\to p + \pi^- \ , \\ K^+ &\to \pi^+ + \pi^+ + \pi^- \ , \end{aligned}$$

in which only hadrons appear. The coupling is weak in that the natural dimensionless coupling, with the proton mass as standard, is $Gm_p^2 = 1.01 \times 10^{-5}$, where $G$ is the *Fermi constant*.

The leptonic current consists of the terms

$$J^\alpha_{\rm lept}(x) = \bar{e}\gamma^\alpha(1-\gamma_5)\nu_e + \bar{\mu}\gamma^\alpha(1-\gamma_5)\nu_\mu + \bar{\tau}\gamma^\alpha(1-\gamma_5)\nu_\tau \ . \tag{16}$$

Both polar and axial vector terms appear ($\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ is a pseudoscalar matrix), so that in the quadratic form (14) there will be vector–axial-vector interference terms, indicating *parity nonconservation*. The discovery of this phenomenon, following the suggestion of T. D. Lee and C. N. Yang in 1956 that reflection invariance in the weak interactions could not be taken for granted but had to be tested, played an important role in the determination of the phenomenological Lagrangian (14). The experiments suggested by Lee and Yang all involved looking for a pseudoscalar observable in a weak interaction experiment (see *Parity*), and the first of many experiments (C. S. Wu, E. Ambler, R.

W. Hayward, D. D. Hoppes, and R. F. Hudson) measuring the beta decay of polarized nuclei ($^{60}$Co) showed an angular distribution of the form

$$W(\theta) = A + B\mathbf{p}_e \cdot \langle \mathbf{J} \rangle \ , \tag{17}$$

where $\mathbf{p}_e$ is the electron momentum and $\langle \mathbf{J} \rangle$ the polarization of the nucleus. The distribution $W(\theta)$ is not invariant under mirror inversion ($P$) which changes $\mathbf{J} \to \mathbf{J}$ and $\mathbf{p}_e \to -\mathbf{p}_e$, so the experimental form (17) directly showed parity nonconservation. Experiments showed that both the hadronic and the leptonic currents had vector and axial-vector parts, and that although invariance under particle–antiparticle (*charge*) conjugation $C$ is also violated, the form (14) maintains invariance under the joint symmetry $CP$ (see *Conservation Laws*) when restricted to the light hadrons (those consisting of $u$, $d$, $c$, and $s$). There is evidence that $CP$ itself is violated at a much weaker level, of the order of $10^{-5}$ of the weak interactions. As will be discussed later, this is consistent with second-order weak effects involving the heavy ($b$, $t$) quarks, though it is possible that an otherwise undetected superweak interaction also plays a role. The part of $J^\alpha_{\mathrm{had}}$ relevant to beta decay is $\sim \bar{u}\gamma_\alpha(1-\gamma_5)d$. The detailed form of the hadronic current will be discussed after the description of the strong interactions.

Even at the leptonic level the theory described by (14) is not renormalizable. This manifests itself in the result that the cross section for neutrino absorption grows with energy:

$$\sigma_\nu = (\mathrm{const})G^2 m_p E_\nu \ . \tag{18}$$

While this behavior is in accord with observations up to the highest energies studied so far, it signals a breakdown of the theory at higher energies, so that (14) cannot be fundamental. A number of people suggested over the years that the effective Lagrangian is but a phenomenological description of a theory in which the weak current $J^\alpha(x)$ is coupled to a charged *intermediate vector boson* $W^-_\alpha(x)$, in analogy with quantum electrodynamics. The form (14) emerges from the exchange of a vector meson between the currents (see *Feynman Diagrams*) when the $W$ mass is much larger than the momentum transfer in the process [Fig. 1(c)]. The intermediate vector boson theory leads to a better behaved $\sigma_\nu$ at high energies. However, massive vector theories are still not renormalizable, and the cross section for $e^+e^- \to W^+W^-$ (with longitudinally polarized $W$s) grows with energy. Until 1967 there was no theory of the weak interactions in which higher-order corrections, though extraordinarily small because of the weak coupling, could be calculated.

## Unified Theories of the Weak and Electromagnetic Interactions

In spite of the large differences between the electromagnetic and weak interactions (massless photon versus massive $W$, strength of coupling, behavior under $P$ and $C$), the vectorial form of the interaction hints at a possible common origin. The renormalization barrier seems insurmountable: A theory involving vector bosons is only renormalizable if it is a gauge theory; a theory in which a charged weak current of the form (16) couples to massive charged vector bosons,

$$\mathcal{L}_W = -g_W[J^{\alpha\dagger}(x)W_\alpha^+(x) + J^\alpha(x)W_\alpha^-(x)] \ , \tag{19}$$

does not have that property. Interestingly, a gauge theory involving charged vector mesons, or more generally, vector mesons carrying some internal quantum numbers, had been invented by C. N. Yang and R. L. Mills in 1954. These authors sought to answer the question: Is it possible to construct a theory that is invariant under the transformation

$$\psi(x) \to \exp[\mathrm{i}\mathbf{T}\cdot\boldsymbol{\theta}(x)]\psi(x) \ , \tag{20}$$

where $\psi(x)$ is a column vector of fermion fields related by symmetry, the $T_i$ are matrix representations of a Lie algebra (see *Lie Groups, Gauge Theories*), and the $\theta(x)$ are a set of angles that depend on space and time, generalizing the transformation law (9)? It turns out to be possible to construct such a *non-Abelian gauge theory*. The coupling of the spin-$\frac{1}{2}$ field follows the "minimal" form (8) in that

$$\bar{\psi}\gamma^\alpha\frac{\partial}{\partial x^\alpha}\psi \to \bar{\psi}\gamma^\alpha\left(\frac{\partial}{\partial x^\alpha} + \mathrm{i}gT_iW_\alpha^i(x)\right)\psi \ , \tag{21}$$

where the $W_i$ are vector (gauge) bosons, and the *gauge coupling constant* $g$ is a measure of the strength of the interaction. The vector meson form is again

$$\mathcal{L}_V = -\frac{1}{4}F_{\mu\nu i}(x)F_i^{\mu\nu}(x) \ , \tag{22}$$

but now the structure of the fields is more complicated than in (2):

$$F_{\mu\nu i}(x) = \frac{\partial}{\partial x^\mu}W_\nu^i(x) - \frac{\partial}{\partial x^\nu}W_\mu^i(x) - gf_{ijk}W_\mu^j(x)W_\nu^k(x) \ , \tag{23}$$

because the vector fields $W_\mu^i$ themselves carry the "charges" (denoted by the label $i$); thus, they interact with each other (unlike electrodynamics), and their transformation law is more complicated than (10). The numbers $f_{ijk}$ that appear in the additional nonlinear term in (23) are the *structure constants* of the group under consideration, defined by the commutation rules

$$[T_i, T_j] = \mathrm{i}f_{ijk}T_k \ . \tag{24}$$

There are as many vector bosons as there are generators of the group. The Abelian group $U(1)$ with only one generator (the electric charge) is the local symmetry group of quantum electrodynamics. For the group $SU(2)$ there are three generators and three vector mesons. Gauge invariance is very restrictive. Once the symmetry group and representations are specified, the only arbitrariness is in $g$. The existence of the gauge bosons and the form of their interaction with other particles and with each other is determined. Yang–Mills (gauge) theories are renormalizable because the form of the interactions in (21) and (23) leads to cancellations between different contributions to high-energy amplitudes. However, gauge invariance does not allow mass terms for the vector bosons, and it is this feature that was responsible for the general neglect of the Yang–Mills theory for many years.

S. Weinberg (1967) and independently A. Salam (1968) proposed an extremely ingenious theory unifying the weak and electromagnetic interactions by taking advantage of a theoretical development (see *Symmetry Breaking, Spontaneous*) according to which vector mesons in Yang–Mills theories could acquire a mass without its appearing explicitly in the Lagrangian (the theory without the symmetry breaking mechanism had been proposed earlier by S. Glashow). The basic idea is that even though a theory possesses a symmetry, the solutions need not. A familiar example is a ferromagnet: the equations are rotationally invariant, but the spins in a physical ferromagnet point in a definite direction. A loss of symmetry in the solutions manifests itself in the fact that the ground state, the vacuum, is no longer invariant under the transformations of the symmetry group, e. g., because it is a Bose condensate of scalar fields rather than empty space. According to a theorem first proved by J. Goldstone, this implies the existence of massless spin-0 particles; states consisting of these *Goldstone bosons* are related to the original vacuum state by the (spontaneously broken) symmetry generators. If, however, there are gauge bosons in the theory, then as shown by P. Higgs, F. Englert, and R. Brout, and by G. Guralnik, C. Hagen, and T. Kibble, the massless Goldstone bosons can be eliminated by a gauge transformation. They reemerge as the longitudinal (helicity-zero) components of the vector mesons, which have acquired an effective mass by their interaction with the groundstate condensate (the *Higgs mechanism*). Renormalizability depends on the symmetries of the Lagrangian, which is not affected by the symmetry-violating solutions, as was elucidated through the work of B. W. Lee and K. Symanzik and first applied to the gauge theories by G. 't Hooft.

The simplest theory must contain a $W^+$ and a $W^-$; since their generators do not commute there must also be at least one neutral vector boson $W^0$. A scalar (*Higgs*) particle associated with the breaking of the symmetry of the solution is also required. The simplest realistic theory also contains a photon-like object with its own coupling constant [hence the description as $SU(2) \times U(1)$]. The resulting theory incorporates the Fermi theory of charged-current weak interactions and quantum electrodynamics. In particular, the vector boson extension of the Fermi theory in (19) is reproduced with $g_w = g/2\sqrt{2}$, where $g$ is the $SU(2)$ coupling, and $G \approx \sqrt{2}g^2/8M_W^2$. There are two neutral bosons, the $W^0$ of $SU(2)$ and $B$ associated with the $U(1)$ group. One combination,

$$A = \cos\theta_W B + \sin\theta_W W^0 \ , \tag{25}$$

is just the photon of electrodynamics, with $e = g \sin\theta_W$. The weak (or Weinberg) angle $\theta_W$ which describes the mixing is defined by $\theta_W \equiv \tan^{-1}(g'/g)$, where $g'$ is the $U(1)$ gauge coupling. In addition, the theory makes the dramatic prediction of the existence of a second (massive) neutral boson orthogonal to $A$:

$$Z = -\sin\theta_W B + \cos\theta_W W^0 \ , \tag{26}$$

which couples to the neutral current

$$J_\alpha^Z = \sum_i T_3(i)\bar{\psi}\gamma_\alpha(1-\gamma_5)\psi_i - 2\sin^2\theta_W j_\alpha^\gamma \ , \tag{27}$$

where $j_\alpha^\gamma$ is the electromagnetic current in (13) and $T_3(i)$ [$+\frac{1}{2}$ for $u$, $\nu$; $-\frac{1}{2}$ for $e^-$, $d$] is the eigenvalue of the third generator of $SU(2)$. The $Z$ mediates a new class of weak interactions (see *Weak Neutral Currents*),

$$\begin{aligned}
(\nu/\bar{\nu}) + p,\ n &\rightarrow (\nu/\bar{\nu}) + \text{hadrons} \ ,\\
(\nu/\bar{\nu}) + \text{nucleon} &\rightarrow (\nu/\bar{\nu}) + \text{nucleon} \ ,\\
\nu_\mu + e^- &\rightarrow \nu_\mu + e^- \ ,
\end{aligned}$$

characterized by a strength comparable to the charged-current interactions [Fig. 1(d)]. Another prediction is that of the existence, in electromagnetic interactions such as

$$e^- + p \rightarrow e^- + \text{hadrons} \ ,$$

of tiny parity-nonconservation effects that arise from the exchange of the $Z$ between the electron and the hadronic system. Neutral current-induced neutrino processes were observed in 1973, and since then all of the reactions have been studied in detail. In addition, parity violation (and other axial current effects) due to the weak neutral current has been observed in polarized Möller ($e^-e^-$) scattering and in asymmetries in the scattering of polarized electrons from deuterons, in the induced mixing between $S$ and $P$ states in heavy atoms (*atomic parity violation*), and in asymmetries in electron–positron annihilation into $\mu^+\mu^-$, $\tau^+\tau^-$, and heavy quark pairs. All of the observations are in excellent agreement with the predictions of the standard $SU(2)\times U(1)$ model and yield values of $\sin^2\theta_W$ consistent with each other. Another prediction is the existence of massive $W^\pm$ and $Z$ bosons (the photon remains massless because the condensate is neutral), with masses

$$M_W^2 = \frac{A^2}{\sin^2\theta_W} \ , \qquad M_Z^2 = \frac{M_W^2}{\cos^2\theta_W} \ . \tag{28}$$

where $A \sim \pi\alpha/\sqrt{2}G \sim (37\,\text{GeV})^2$. (In practice, a significant, 7%, higher-order correction must be included.) Using $\sin^2\theta_W$ obtained from neutral current processes, one predicted $M_W = 80.2\pm1.1\,\text{GeV}/c^2$ and $M_Z = 91.6\pm0.9\,\text{GeV}/c^2$'. In 1983 the $W$ and $Z$ were discovered at the new $\bar{p}p$ collider at CERN. The current values of their masses, $M_W = 80.425\pm0.038\,\text{GeV}/c^2$, $M_Z = 91.1876\pm 0.0021\,\text{GeV}/c^2$, dramatically confirm the standard model (SM) predictions.

The $Z$ factories LEP and SLC, located respectively at CERN (Switzerland) and SLAC (USA), allowed tests of the standard model at a precision of $\sim 10^{-3}$,

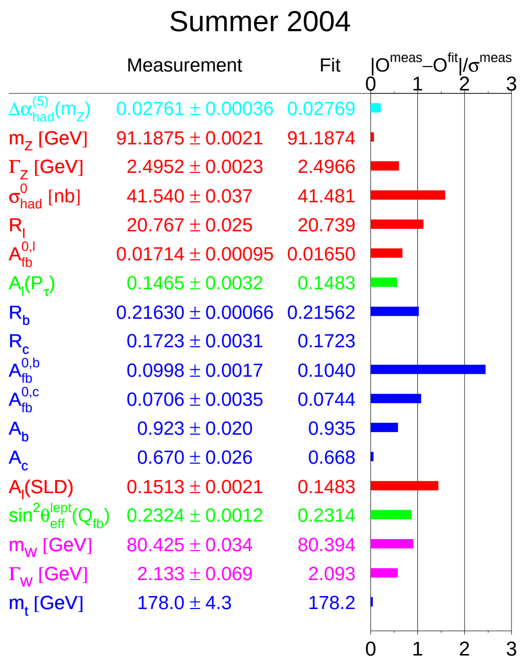


Fig. 2: Precision observables, compared with their expectations from the best SM fit, from *The LEP Collaborations*, hep-ex/0412015.

much greater than had previously been possible at high energies. The four LEP experiments accumulated some $2 \times 10^7 Z's$ at the $Z$-pole in the reactions $e^+e^- \to Z \to \ell^+\ell^-$ and $q\bar{q}$. The SLC experiment had a smaller number of events, $\sim 5 \times 10^5$, but had the significant advantage of a highly polarized ($\sim 75\%$) $e^-$ beam. The $Z$ pole observables included the $Z$ mass (quoted above), decay rate, and cross section to produce hadrons; and the branching ratios into $e^+e^-, \mu^+\mu^-, \tau^+\tau^-$ as well as into $q\bar{q}$, $c\bar{c}$, and $b\bar{b}$. These could be combined to obtain the stringent constraint $N_\nu = 2.9841 \pm 0.0083$ on the number of ordinary neutrinos with $m_\nu < M_Z/2$ (i. e., on the number of families with a light $\nu$). The $Z$-pole experiments also measured a number of asymmetries, including forward-backward (FB), polarization, the $\tau$ polarization, and mixed FB-polarization, which were especially useful in determining $\sin^2\theta_W$. The leptonic branching ratios and asymmetries confirmed the lepton family universality predicted by the SM. The results of many of these observations, as well as some weak neutral current and high energy collider data, are shown in Figure 2.

The LEP II program above the $Z$-pole provided a precise determination of

$M_W$ (as did experiments at the Fermilab Tevatron $\bar{p}p$ collider (USA)), measured the four-fermion cross sections $e^+e^- \to f\bar{f}$, and tested the (gauge invariance) predictions of the SM for the gauge boson self-interactions.

The $Z$-pole, neutral current, and boson mass data together establish that the standard (Weinberg–Salam) electroweak model is correct to first approximation down to a distance scale of $10^{-16}$ cm (1/1000th the size of the nucleus). In particular, this confirms the concepts of renormalizable field theory and gauge invariance, as well as the SM group and representations. The results yield the precise world average $\sin^2\theta_W = 0.23149 \pm 0.00015$. (It is hoped that the value of this one arbitrary parameter may emerge from a future unification of the strong and electromagnetic interactions.) The data were precise enough to allow a successful prediction of the top quark mass (which affected higher order corrections) before the $t$ was observed directly, and to strongly constrain the possibilities for new physics that could supersede the SM at shorter distance scales. The major outstanding ingredient is the Higgs boson, which is hard to produce and detect. The precision experiments place an upper limit of around $250\,\mathrm{GeV}/c^2$ on the Higgs mass (which is not predicted by the SM), while direct searches at LEP II imply a lower limit of $114.4\,\mathrm{GeV}/c^2$. Some physicists suspect that the elementary Higgs field may be replaced by a dynamical or bound-state symmetry-breaking mechanism, but the possibilities are strongly constrained by the precision data. Unified theories, such as superstring theories, generally imply an elementary Higgs. It is hoped that the situation will be clarified by the next generation of high energy colliders.

## The Strong Interactions

The strength of the coupling that manifests itself in nuclear forces and in the interaction of pions with nucleons is such that perturbation theory, so useful in the electromagnetic interaction, cannot be applied to any field theory of the strong interactions in which the mesons and baryons are the fundamental fields. The large number of hadronic states strongly suggests a composite structure that cannot be viewed as a perturbation about noninteracting systems. In fact, it is now generally believed that the strong interactions are described by a gauge theory, quantum chromodynamics (QCD), in which the basic entities are quarks rather than hadrons. Nevertheless, prior and parallel to the development of the quark theory a wealth of experimental information concerning the hadrons and their interactions was accumulated. In spite of the absence of guidance from field theory, and in spite of the fact that each jump in available accelerator energy brought a shift in the focus of attention, certain simple patterns were identified.

## Internal Symmetries

The first hint of a new symmetry can be seen in the remarkable resemblance between neutron and proton. They differ in electromagnetic properties, and, other than that, by effects that are very small; for example, they differ in mass by 1 part in 700. W. Heisenberg conjectured that the neutron and proton are two states of a single entity, the *nucleon* (see *Nucleon*), just as an electron with spin up and an electron with spin down are two states of a single entity, even though in an external magnetic field they have slightly different energies. Pursuing this analogy, Heisenberg and E. U. Condon proposed that the strong interactions are invariant under transformations in an internal space, in which the nucleon is a spinor (see *Isospin*). Thus, the nucleon is an isospin doublet, with $I_z(p) = \frac{1}{2}$ and $I_z(n) = -\frac{1}{2}$, and isospin (in analogy with angular momentum) is conserved. In the language of group theory, the assertion is that the strong interactions are invariant under the transformations of the group $SU(2)$, and that particles transform as irreducible representations. The electromagnetic and weak interactions violate this invariance. The expression for the charge of the nucleons and antinucleons,

$$Q = I_z + B/2 \ , \tag{29}$$

shows that the charge picks out a preferred direction in the internal space. (It is now believed that the strong interactions themselves have a small piece which breaks isospin symmetry, in addition to electroweak interactions.)

With the discovery of the three pions ($\pi^+$, $\pi^0$ , $\pi^-$) with mass remarkably close to that predicted by H. Yukawa (1935) in his seminal work explaining nuclear forces in terms of an exchange of massive quanta of a mesonic field, the notion of isospin acquired a new significance. It was natural, in view of the small $\pi^\pm - \pi^0$ mass difference, to assign the pion to the $I = 1$ representation of $SU(2)$. The invariance of the pion–nucleon interaction under isospin transformations led to a number of predictions, all of which were confirmed. In particular, states initiated in pion-nucleon collisions could only have isospin $\frac{1}{2}$ or $\frac{3}{2}$. Early work on pion–nucleon scattering led to the discovery of a resonance with rest mass $1236\,\mathrm{MeV}/c^2$, width $115\,\mathrm{MeV}/c^2$, and angular momentum and parity $J^P = \frac{3}{2}^+$. This resonance occurred in $\pi^+ p$ scattering, so that it had to have $I = \frac{3}{2}$, and its effects seen in $\pi^- p \to \pi^- p$ and $\pi^- p \to \pi^0 n$ should be the same as those in $\pi^+ p \to \pi^+ p$. This prediction was borne out by experiment.

Formally, $SU(2)$ invariance is described by defining generators $I_i$; $(i = 1, 2, 3)$ obeying the Lie algebra

$$[I_i,\, I_j] = \mathrm{i} e_{ijk} I_k \ , \tag{30}$$

where $e_{ijk}$ is totally antisymmetric in the indices and $e_{123} = 1$. The statement that a pion is an $I = 1$ state then means that the pion field $\Pi_a$ transforms according to

$$[I_i, \Pi_a] = -(\mathcal{I}_i)_{ab} \Pi_b \ , a = 1,\, 2,\, 3 \ , \tag{31}$$

where the $\mathcal{I}_i$ are $3 \times 3$ matrices satisfying (30). In relativistic quantum mechanics conservation laws must be local, so the conservation law

$$\frac{\mathrm{d}I_i}{\mathrm{d}t} = 0 \tag{32}$$

really follows from the local conservation law

$$\frac{\partial}{\partial x^\mu} \mathcal{I}_i^\mu(x) = 0 \tag{33}$$

for the isospin-generating currents, for which

$$I_i = \int d^3\mathbf{r}\, \mathcal{I}_i^0(\mathbf{r}, t) \ . \tag{34}$$

Isospin [and $SU(3)$] are global symmetries: The symmetry transformations are the same at all space-time points, as opposed to the local (gauge) transformations in (20). Hence, they are not associated with gauge bosons or a force.

In the early 1950s a number of new particles were discovered. The great confusion generated by the widely differing rates of production and decay was cleared up by M. Gell-Mann and K. Nishijima, who extended the notion of isospin conservation to the strong interactions of the new particles, classified them (and along the way noted "missing" particles that had to exist, and were subsequently found), and discovered that the observed patterns of reactions could be explained by assigning a new quantum number $S$ (strangeness) to each isospin multiplet.

The selection rules were

$$\Delta S = 0 \tag{35}$$

for the strong and electromagnetic interactions, and

$$\Delta S = 0, \ \pm 1 \tag{36}$$

for the weak interactions. Relation (29) now takes the form

$$Q = I_z + (B + S)/2 \ . \tag{37}$$

[Equation (37) holds for all hadrons except for those involving heavy ($c$, $b$, and $t$) quarks, discovered in the 1970s and later.]

The success of the strangeness scheme immediately started a search for a higher symmetry that would include isospin and strangeness (or hypercharge, $Y = B + S$), and that would, in some limit, include the nucleons and the newly discovered strange baryons in a supermultiplet. The search ended when M. Gell-Mann and Y. Ne'eman discovered that the Lie group $SU(3)$ was the appropriate (global) symmetry. The group is generated by eight operators $F_i$ ($i = 1,\ 2,\ 3,\ \ldots,\ 8$), of which the first three may be identified with the isospin generators $I_i$, and (by convention) $F_8$ is related to hypercharge. The other four change isospin and strangeness. The nucleons and six other baryons discovered in the 1950s fit into an eight-dimensional (octet) representation containing doublets with $I = \frac{1}{2}$ and $Y = \pm 1$, and $I = 1, 0$ states with $Y = 0$. Similarly, the $I = 1$ pions, the $(K^+, K^0)$ with $I = \frac{1}{2}$, $Y = 1$, and $(\bar{K}^0, K^-)$ with $I = \frac{1}{2}$, $Y = -1$, could be fitted into an octet that was soon completed with the discovery of an $I = Y = 0$ pseudoscalar meson, the $\eta$ (see Table 4). $SU(3)$ is only an approximate symmetry of the strong interactions. Mass splittings within $SU(3)$ multiplets and other breaking effects are typically 20-30%.

Most interesting is that the search for partners of the *resonance* $\Delta(1236)$ with $I = \frac{3}{2}$ led to a dramatic confirmation of $SU(3)$. The simplest representation containing an ($I = \frac{3}{2}$, $Y = 1$) state is the 10-dimensional representation, which

Table 4: Table of low-lying mesons and baryons, grouped according to $SU(3)$ multiplets. There may be considerable mixing between the $SU(3)$ singlets $\eta'$, $\varphi$, and $f'$ and the corresponding octet states $\eta$, $\omega$, $f$.

| Particle | $B$ | $Q$ | $Y$ | $I$ | $J^P$ | Mass (GeV/$c^2$) | Quark content |
|---|---|---|---|---|---|---|---|
| $\pi$ | 0 | 1, 0, −1 | 0 | 1 | $0^-$ | 0.14 | $u\bar{d},\ u\bar{u}-d\bar{d},\ d\bar{u}$ |
| $K$ | 0 | 1, 0 | 1 | $\frac{1}{2}$ | $0^-$ | 0.49 | $u\bar{s},\ d\bar{s}$ |
| $\bar{K}$ | 0 | 0, −1 | −1 | $\frac{1}{2}$ | $0^-$ | 0.49 | $s\bar{d},\ s\bar{u}$ |
| $\eta$ | 0 | 0 | 0 | 0 | $0^-$ | 0.55 | $u\bar{u}+d\bar{d}-2s\bar{s}$ |
| | | | | | | | |
| $\eta'$ | 0 | 0 | 0 | 0 | $0^-$ | 0.96 | $u\bar{u}+d\bar{d}+s\bar{s}$ |
| | | | | | | | |
| $\rho$ | 0 | 1, 0, −1 | 0 | 1 | $1^-$ | 0.77 | $u\bar{d},\ u\bar{u}-d\bar{d},\ d\bar{u}$ |
| $K^*$ | 0 | 1, 0 | 1 | $\frac{1}{2}$ | $1^-$ | 0.89 | $u\bar{s},\ d\bar{s}$ |
| $\bar{K}^*$ | 0 | 0, −1 | −1 | $\frac{1}{2}$ | $1^-$ | 0.89 | $s\bar{d},\ s\bar{u}$ |
| $\omega$ | 0 | 0 | 0 | 0 | $1^-$ | 0.78 | $u\bar{u}+d\bar{d}$ |
| | | | | | | | |
| $\phi$ | 0 | 0 | 0 | 0 | $1^-$ | 1.02 | $s\bar{s}$ |
| | | | | | | | |
| $A_2$ | 0 | 1, 0, −1 | 0 | 1 | $2^+$ | 1.32 | $u\bar{d},\ u\bar{u}-d\bar{d}\,d\bar{u}$ |
| $K^*(1430)$ | 0 | 1, 0 | 1 | $\frac{1}{2}$ | $2^+$ | 1.43 | $u\bar{s},\ d\bar{s}$ |
| $\bar{K}^*(1430)$ | 0 | 0, −1 | −1 | $\frac{1}{2}$ | $2^+$ | 1.43 | $s\bar{d},\ s\bar{u}$ |
| $f$ | 0 | 0 | 0 | 0 | $2^+$ | 1.28 | $u\bar{u}+d\bar{d}$ |
| | | | | | | | |
| $f'$ | 0 | 0 | 0 | 0 | $2^+$ | 1.53 | $s\bar{s}$ |
| | | | | | | | |
| $N$ | 1 | 1, 0 | 1 | $\frac{1}{2}$ | $\frac{1}{2}^+$ | 0.94 | $uud,\ udd$ |
| $\Lambda$ | 1 | 0 | 0 | 0 | $\frac{1}{2}^+$ | 1.12 | $uds-dus$ |
| $\Sigma$ | 1 | 1, 0, −1 | 0 | 1 | $\frac{1}{2}^+$ | 1.19 | $uus,\ uds+dus,\ dds$ |
| $\Xi$ | 1 | 0, −1 | −1 | $\frac{1}{2}$ | $\frac{1}{2}^+$ | 1.32 | $uss,\ dss$ |
| | | | | | | | |
| $\Delta$ | 1 | 2, 1, 0, −1 | 1 | $\frac{3}{2}$ | $\frac{3}{2}^+$ | 1.23 | $uuu,\ uud,\ udd,\ ddd$ |
| $\Sigma(1385)$ | 1 | 1, 0, −1 | 0 | 1 | $\frac{3}{2}^+$ | 1.39 | $uus,\ uds,\ dds$ |
| $\Xi^*(1530)$ | 1 | 0, −1 | −1 | $\frac{1}{2}$ | $\frac{3}{2}^+$ | 1.53 | $uss,\ dss$ |
| $\Omega^-$ | 1 | −1 | −2 | 0 | $\frac{3}{2}^+$ | 1.67 | $sss$ |

also contains ($I = 1$, $Y = 0$) and ($I = \frac{1}{2}$, $Y = -1$) states and an isosinglet $Y = -2$ particle. The symmetry-breaking pattern that explained the mass splittings among the isospin multiplets in the octet predicted equal mass splittings. Thus, when the $I = 1$ $\Sigma(1385)$ was discovered, predictions could be made about the $I = \frac{1}{2}$ $\Xi^*$, found at mass $1530\,\mathrm{MeV}/c^2$, and the $\Omega^-$, predicted at $1675\,\mathrm{MeV}/c^2$. The latter mass is too low to permit a strangeness-conserving decay to $\Xi^0 K^-$, so the $\Omega^-$ had to be long-lived, only decaying by a chain of $\Delta S = 1$ weak interactions with a very clear signature. The dramatic discovery in 1964 of the $\Omega^-$ with all the right properties convinced all doubters. [see *SU(3) and Higher Symmetries, Hyperons, Hypernuclear Physics and Hypernuclear Interactions*].

## S-Matrix Theory

The construction of higher-energy accelerators, the invention of the bubble chamber by D. Glaser, and the combination of large hydrogen bubble chambers, rapid scanning facilities, and high-speed computers into a massive data production and analysis technology, pioneered by L. Alvarez and collaborators, led to the discovery of many new resonances during the 1950s and 1960s. The basic procedure was to measure charged tracks in bubble-chamber pictures, taken in strong magnetic fields, and to calculate the invariant masses $(\sum E_i)^2 - (\sum \mathbf{p}_i c)^2$ for various particle combinations. Resonances manifest themselves as peaks in mass distributions, and the events in the resonance region may be further analyzed to find out the spin and parity of the resonance. Baryonic resonances were also discovered in phase-shift analyses of angular distributions in pion–nucleon and $K$–nucleon scattering reactions. The patterns of masses and quantum numbers of the resonances showed that all the mesonic resonances came in $SU(3)$ octets and singlets, and the baryonic ones in $SU(3)$ decuplets, octets, and singlets.

There was good evidence that there was no fundamental distinction between the stable particles and the highly unstable resonances: The $\Delta$ and the $\Omega^-$, discussed above, are good examples, and theoretically it was found that both stable (bound) states and resonant ones appeared in scattering amplitudes as pole singularities, differing only in their location. Furthermore, the role assigned by Yukawa to the pion as the nuclear "glue" – it was the particle whose exchange was largely responsible for the nuclear forces – had to be shared with other particles: Various vector and scalar mesons were seen to contribute to the nuclear forces, and G. F. Chew and F. E. Low explained much of low-energy pion physics in terms of nucleon exchange. Chew, in collaboration with S. Mandelstam and S. Frautschi, proposed to do away with the notion of any particles being "fundamental." They hypothesized that the collection of all scattering amplitudes, the scattering matrix, be determined by a set of self-consistency conditions, the *bootstrap* conditions (see *S-Matrix Theory*), according to which, crudely stated, the exchange of all possible particles should yield a "potential" whose bound states and resonances should be identical with the particles inserted into the exchange term.

Much effort was devoted to bootstrap and S-matrix theory during the 1960s and early 1970s. The program had its greatest success in developing phenomenological models for strong interaction scattering amplitudes at high energies and low-momentum transfers, such as elastic scattering and total cross sections. In particular, Mandelstam applied an idea due to T. Regge to relativistic quantum mechanics, which related a number (perhaps infinite) of particles and resonances with the same $SU(3)$ and other internal quantum numbers, but different masses and spins, into a family or *Regge trajectory*. The exchange of this trajectory of particles led to much better behaved high-energy amplitudes than the exchange of one or a small number, in agreement with experiment (see *Regge Poles*). Related models had some success in describing inclusive processes (in which one or a few final particles are observed, with the others summed over) and other highly inelastic processes (see *Inclusive Reactions*).

The more ambitious goal of understanding the strong interactions as a bootstrap (self-consistency) principle met with less success, although a number of models and approximation schemes enjoyed some measure in limited domains. The most successful was the dual resonance model pioneered by G. Veneziano. The dual model was an explicit closed-form expression for strong-interaction scattering amplitudes which properly incorporated poles for the Regge trajectories of bound states and resonances that could be formed in the reaction, Regge asymptotic behavior, and duality (the property that an amplitude could be described *either* as a sum of resonances in the direct channel *or* as a sum of Regge exchanges). However, the original simple form did not incorporate unitarity, i. e., the amplitudes did not have branch cuts corresponding to multiparticle intermediate states, and the resonances in the model had zero width (their poles occurred on the real axis in the complex energy plane instead of being displaced by an imaginary term corresponding to the resonance width). Perhaps the most important consequence of dual models was that they were later formulated as *string theories*, in which an infinite trajectory of "elementary particles" could be viewed as different modes of vibration of a one-dimensional string-like object (see *String Theory*). String theories never quite worked out as a model of the strong interactions, but the same mathematical structure reemerged later in "theories of everything."

Many of the S-matrix results are still valid as phenomenological models. However, the bootstrap idea has been superseded by the success of the quark theory and the development of QCD as the probable field theory of the strong interactions.

## Quarks as Fundamental Particles

The discovery of $SU(3)$ as the underlying internal symmetry of the hadrons and the classification of the many resonances led to the recognition of two puzzles: Why did mesons come only in octet and singlet states? Why were there no particles that corresponded to the simplest representations of $SU(3)$, the triplet 3 and its antiparticle $3^*$? M. Gell-Mann and G. Zweig in 1964 independently proposed that such representations do have particles associated with them (Gell-Mann named them quarks), and that all observed hadrons are made of $(q\bar{q})$

Table 5: The u, d, and s quarks.

| | $B$ | $Q$ | $Y$ | $I$ | $I_z$ |
|---|---|---|---|---|---|
| $u$ | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $d$ | $\frac{1}{3}$ | $-\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ |
| $s$ | $\frac{1}{3}$ | $-\frac{1}{3}$ | $-\frac{2}{3}$ | 0 | 0 |

(quark + antiquark) if they have baryon number $B = 0$ and of $(qqq)$ (three quarks) if they have baryon number $B = 1$. They proposed that there exist three different kinds of quarks, labeled $u$, $d$, and $s$. These were assumed to have spin $\frac{1}{2}$ and the internal quantum numbers listed in Table 5. The quark contents of the low-lying hadrons are given in Table 4. The vector meson octet ($\rho$, $K^*$, $\omega$) differs from the pseudoscalars ($\pi$, $K$, $\eta$) in that the total quark spin is 1 in the former case and zero in the latter. The ($A_2$, $K^*(1490)$, $f$) octet are interpreted as an orbital excitation ($^3P_2$). All of the known particles and resonances can be interpreted in terms of quark states, including radial and orbital excitations and spin.

The first question was answered automatically, since products of the simplest representations decompose according to the rules

$$\begin{aligned} \mathbf{3} \times \mathbf{3}^* &= \mathbf{1} + \mathbf{8}\ , \\ \mathbf{3} \times \mathbf{3} \times \mathbf{3} &= \mathbf{1} + \mathbf{8} + \mathbf{8} + \mathbf{10}\ . \end{aligned} \tag{38}$$

A problem immediately arose in that the decuplet to which $\Sigma(1236)$ belongs, being the lowest-energy decuplet, should have its three quarks in relative $S$ states. Thus the $\Delta^{++}$, whose composition is $uuu$, could not exist, since the spin-statistics connection requires that the wave function be totally antisymmetric, which it manifestly is not when the $\Delta^{++}$ is in a $J_z = \frac{3}{2}$ state, with all spins up, for example. The solution to this problem, proposed by O. W. Greenberg, M. Han, and Y. Nambu and further developed by W. A. Bardeen, H. Fritzsch, and M. Gell-Mann, was the suggestion that in addition to having an $SU(3)$ label such as $(u, d, s)$ – named *flavor* by Gell-Mann – and a spin label (up, down), quarks should have an additional three-valued label, named *color*. Thus according to this proposal there are really nine light quarks:

$$\begin{matrix} u_{\mathrm{R}} & u_{\mathrm{B}} & u_{\mathrm{Y}} \\ d_{\mathrm{R}} & d_{\mathrm{B}} & d_{\mathrm{Y}} \\ s_{\mathrm{R}} & s_{\mathrm{B}} & s_{\mathrm{Y}} \end{matrix}$$

Hence, the low-lying $(qqq)$ state could be symmetric in the flavor and spin labels, provided it were totally antisymmetric in the color (red, blue, yellow) labels. More colors could be imagined but at least three are needed. Transformations among the color labels lead to another symmetry, $SU(3)_{\mathrm{color}}$. The totally antisymmetric state is a color singlet. The mesons can also be constructed as color singlets, for example,

$$\pi^+ = \frac{1}{\sqrt{3}}(u_\mathrm{R}\bar{d}_\mathrm{R} + u_\mathrm{B}\bar{d}_\mathrm{B} + u_\mathrm{Y}\bar{d}_\mathrm{Y}) \ .$$

The existing hadronic spectrum shows no evidence for states that could be color octets, for example, so the present attitude is that either color nonsinglet states are very massive compared with the low-lying hadrons or that it is an intrinsic part of hadron dynamics that *only color singlet states are observable.*

The first evidence that there are *three* (and not more) colors came from the study of $\pi^0 \to 2\gamma$ decay. Using general properties of currents, S. Adler and W. A. Bardeen were able to prove that the $\pi^0$ decay rate was uniquely determined by the process in which the $\pi^0$ first decays into a $u\bar{u}$ or a $d\bar{d}$ pair, which then annihilates with the emission of two photons. The matrix element depends on the charges of the quarks, and a calculation of the width yields 0.81 eV. With $n$ colors, this is multiplied by $n^2$, and the observed width of $7.8 \pm 0.6$ eV supports the choice of $n = 3$. Subsequent evidence for three colors was provided by the total cross section for $e^+e^-$ annihilation into hadrons (see below), and by the elevation of the $SU(3)_\mathrm{color}$ symmetry to a gauge theory of the strong interactions.

The quark model has been extremely successful in the classification of observed resonances, and even predictions of decay widths work very well, with much data being correlated in terms of a few parameters. The ingredients that go into the calculation are (a) that quarks are light, with the $(u, d)$ doublet almost degenerate, with mass in the 300-MeV/$c^2$ range (one-third of a nucleon), (b) that the s quark is about 150 MeV/$c^2$ more massive – this explains the pattern of $SU(3)$ symmetry breaking – and (c) that the low-lying hadrons have the simple $q\bar{q}$ or $qqq$ content, without additional $q\bar{q}$ pairs. However, nobody has ever observed an isolated quark (free quarks should be easy to identify because of their fractional charge). It is now generally believed that quarks are confined, i. e., that it is impossible, even in principle, for them to exist as isolated states. However, in the 1960s this led most physicists to doubt the existence of quarks as real particles. That view was shattered by the deep inelastic electron scattering experiments in the late 1960s.

## Deep Inelastic Reactions and Asymptotic Freedom

In 1968 the first results of the inelastic electron-scattering experiments (Fig. 3),

$$e + p \to e' + \text{hadrons}$$

measured at the Stanford Linear Accelerator Center (SLAC), were announced. The experiments were done in a kinematic region that was new. Both the momentum transfer squared (that is, the negative mass squared of the virtual photon exchanged) and the "mass" of the hadronic state produced were large. The cross section could be written as

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}E'\mathrm{d}\Omega} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_\mathrm{point} \left(W_2(x, Q^2) + 2W_1(x, Q^2)\tan^2\frac{\theta}{2}\right) \ , \tag{39}$$

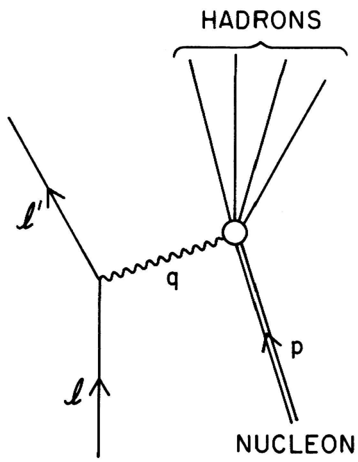


Fig. 3: Kinematics of deep inelastic lepton scattering.

where $(\mathrm{d}\sigma/\mathrm{d}\Omega)_{\mathrm{point}}$ is essentially the cross section for a collision with a free point particle, and the hadronic part of the process was expressed in terms of certain structure functions $W_1$ and $W_2$. In (39), $E'$ is the energy of the final electron, $\theta$ the scattering angle, $Q^2 = -(p_e - p_{e'})^2$, and the quantity $x$ is $Q^2/2m_p\nu$, where $\nu = p \cdot q/m_p$ is the electron energy loss. No one knew what to expect for the behavior of $W_1$ and $W_2$. On the one hand, cross sections for production of definite resonances (*exclusive* reactions rather than *inclusive* ones) fell as powers of $1/Q^2$; on the other hand, J. D. Bjorken had predicted, on simple grounds of the irrelevance of masses when all the variables were large, that the dimensionless functions $F_1(x, Q^2) \equiv m_p W_1$ and $F_2(x, Q^2) \equiv \nu W_2$ should depend on $x$ alone.

The results spectacularly confirmed Bjorken's conjecture of *scaling*. R. P. Feynman interpreted the detailed shapes of the distributions with his parton model, in which the proton, in a frame in which it is moving rapidly, looks like a swarm of independently moving point "parts" without any structure. The shape of $F_2$ can be interpreted as $\Sigma_p q_p^2 x f_p(x)$, where $f_p(x)$ is the probability distribution for a parton to carry a fraction $x$ of the proton's momentum and $q_p$ is the parton's charge, while the relation between $F_1$ and $F_2$ depends on the parton spin. The observed relation $F_2 \simeq 2xF_1$ establishes that the partons have spin $\frac{1}{2}$. Comparing the structure functions obtained from $e$ and $\mu$ scattering (from proton and nuclear targets) with those obtained by weak reactions such as

$$\nu_\mu(\bar{\nu}_\mu)p \to \mu^-(\mu^+) + \text{hadrons} \ ,$$

one can constrain the parton quantum numbers. They are consistent with the assumption that the partons are quarks, and that the proton consists of the three *valence* quarks assigned to it by the naive quark model, supplemented with a *sea* of quark–antiquark pairs. The relative amount of $q\bar{q}$ sea and its composition

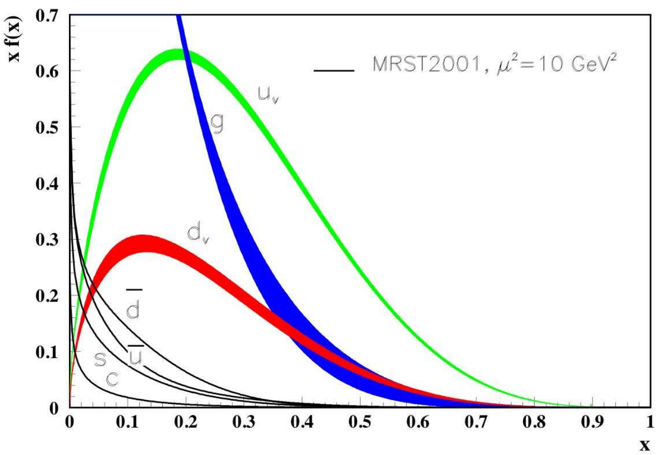


Fig. 4: Distributions $f(x)$ times the fraction $x$ of the proton's momentum carried by valence quarks $u_v$, $d_v$; gluons $g$; and sea quarks $\bar{d}$, $\bar{u}$, $s = \bar{s}$, and $c = \bar{c}$, from S. Eidelman *et al.* [Particle Data Group], *Phys. Lett.* **B592**, 1 (2004).

(e. g., amount of $s\bar{s}$ relative to $u\bar{u}$) are also determined. The mechanism for deep inelastic scattering is the ejection of a single quark by the virtual photon, or by the weak current in the neutrino reactions. The model assumes that the quarks that make up the proton do not interact, and that seems somewhat mysterious. Furthermore, the model of the mechanism suggests that one quark is strongly deflected from the original path. If that is so, where is it?

The problem of how the quarks appear to be noninteracting is answered by quantum field theory. There we find that the coupling strength is really momentum dependent. For example, in quantum electrodynamics, because of the polarizability of the vacuum, the net charge of an electron seen from afar (low momentum transfer) is smaller than the charge as seen close in (large momentum transfer) where it is not shielded by the positrons produced virtually in the vacuum. Quantum electrodynamics is not the right kind of theory for quarks, since the coupling (charge) increases with momentum transfer. It was pointed out by D. Gross and F. Wilczek, by D. Politzer, and by G. 't Hooft that a theory of quarks coupled via Yang–Mills vector mesons will have the property desired for the quarks probed with high-momentum-transfer currents. The requirement of such a high-momentum-transfer decoupling, named *asymptotic freedom*, thus suggests that the "glue" that binds the quarks together is generated by a non-Abelian gauge theory, which has the attraction of being renormalizable, universal (only one coupling constant), and unique, once the number of "colors," that is, the group structure, is determined. The high-energy lepton scattering experiments provide evidence for the existence of some kind of flavor-neutral glue, in that the data are well fitted in terms of quarks, except that only about 50% of the momentum of the initial proton is attributable to quarks. It is now

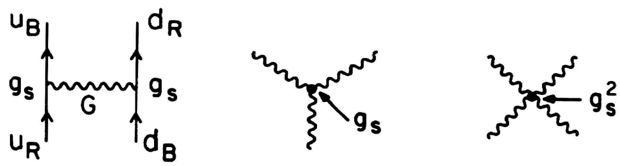


Fig. 5: Fundamental QCD interactions. $g_s$ is the $SU(3)_{\rm color}$ gauge coupling, which becomes small at large momentum transfers, and $G$ is a gluon.

understood that the proton momentum is shared by the valence and sea quarks and electrically neutral *gluons*. Fig. 4 shows the current experimental situation.

## Quantum Chromodynamics

Quantum chromodynamics (QCD), the modern theory of the strong interactions, is a non-Abelian gauge theory based on the $SU(3)_{\rm color}$ group of transformations which relate quarks of different colors. (The transformations are carried out simultaneously for each flavor, which does not change. The number of flavors is arbitrary.) The gauge bosons associated with the eight group generators, known as *gluons*, can be emitted or absorbed by quarks in transitions in which the color (but not flavor) can change. Since the gluons themselves carry color they can interact with each other as well (Fig. 5). As long as there are no more than 16 flavors, QCD is weakly coupled at large momentum transfers (asymptotic freedom) and strongly coupled at small momentum transfers, in agreement with observations.

For the theory to be renormalizable the gluons must either acquire mass through a Higgs mechanism (spontaneous symmetry breaking) or remain massless. The first type of theory destroys asymptotic freedom (its *raison d'être*), and the second has the difficulty that no massless vector mesons, aside from the photon, have ever been observed. It has been proposed that the theory has a structure such that only color singlets are observable, so that the vector mesons, like the quarks, are somehow *confined*. It has been speculated that the non-Abelian field lines, in contrast to electric and magnetic field lines, do not fan out all over space, but remain confined to a narrow cylindrical region, which leads to an interaction energy that is proportional to the separation of the sources of the field lines, and thus confinement. The linearly extended structure so envisaged is reminiscent of the string models suggested by duality, and thus may yield the spectrum characteristics of linear Regge trajectories and the associated high-energy behavior. At large separations the potential presumably breaks down, with energy converted into $(q\bar{q})$ pairs, that is, hadrons. This would explain why quarks are never seen in deep inelastic scattering or other processes. This picture of quark and gluon confinement has not been rigorously established in QCD, but is strongly supported by calculations in the most promising approximation scheme for strongly coupled theories: viz., *lattice calculations*, in which the space-time continuum is replaced by a discrete four-dimensional lattice (see

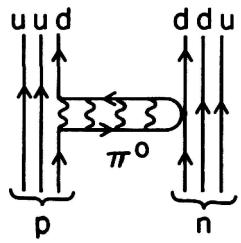


Fig. 6: One-pion exchange in QCD.

*Lattice Gauge Theory*).

QCD is very successful qualitatively, but is hard to test quantitatively. This is partly because the coupling is large for most hadronic processes. Also, QCD brings a subtle change in perspective. The "strong interactions" are those mediated by the color gluons between quarks, and they give rise to the color singlet hadronic bound states. The interaction between these states need not be simple, any more than the interactions between molecules (the van der Waals forces) manifest the simplicity of the underlying Coulomb force in electromagnetism. It is hoped, but not conclusively proved, that successful phenomenological models such as Regge theory or the one-boson-exchange potential emerge as complicated higher-order effects (Fig. 6). Similarly, it has not been possible to fully calculate the hadron spectrum (because of strong coupling, relativistic, and many-body effects), but lattice attempts are promising. *Glueballs* (bound states of gluons) and other nonstandard color singlet states are expected. Candidate mesons exist but have not been unambiguously interpreted. Similar statements apply to *pentaquark* states, such as $uudd\bar{s}$. QCD fairly naturally explains the observed hadronic symmetries. Parity and $CP$ invariance (except for possible subtle nonperturbative effects) and the conservation of strangeness and baryon number are automatic, while approximate symmetries such as isospin, $SU(3)_{\text{flavor}}$, and chiral symmetry (see below) can be broken only by quark mass terms.

More quantitative tests of QCD are possible in high-momentum-transfer processes, in which one glimpses the underlying quarks and gluons. To zeroth order in the strong coupling $g_s$, QCD reproduces the quark–parton model. Higher-order corrections lead to calculable logarithmic variations of $F_1$ and $F_2$ with $Q^2$, in agreement with the data. These experiments have been pushed to much higher $Q^2$ at the $e+p$ collider HERA at DESY in Hamburg, and the results are in excellent agreement with QCD.

Another consequence of the quark–parton picture is the prediction that at high energies the cross section for $e^+ + e^- \rightarrow$ hadrons should proceed through the creation (via a virtual photon) of a $q\bar{q}$ pair, which subsequently converts into hadrons through the breakdown mechanism (Fig. 7). Thus the cross section is expected to be point-like, with the modification that the quark charges appear

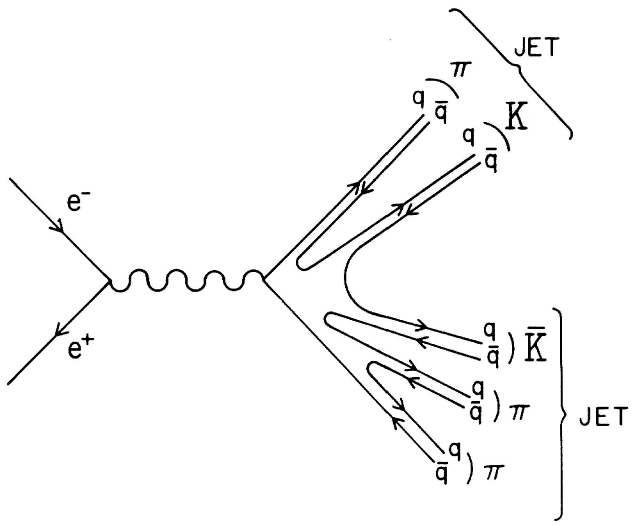


Fig. 7: Schematic picture of hadron production in $e^+ + e^-$ annihilation.

at the production end, so that

$$R \equiv \frac{\sigma(e^+ + e^- \to \text{hadrons})}{\sigma(e^+ + e^- \to \mu^+ + \mu^-)} \Rightarrow \sum_{\text{all quarks}} Q_i^2 \ , \tag{40}$$

where the $Q_i$ are the quark charges. At relatively low energies the $(u, d, s)$ contribution is 2/3 per color, that is, 2. Above the energy (3–4 GeV) needed to produce a charm quark pair ($c\bar{c}$) one expects $R = 10/3$, while above the bottom ($b\bar{b}$) threshold ($\sim$ 10 GeV), $R \sim 11/3$. Calculable higher-order corrections in QCD increase these predictions slightly. The new contribution of a virtual $Z$ boson becomes apparent above $\sim$ 30 GeV, with the $Z$ peak dominating at $\sim$ 90 GeV. These predictions are in excellent agreement with the data (Fig. 8), strongly supporting QCD and the existence of color.

In large-$Q^2$ processes at sufficiently high energies it is expected (and observed) that the produced hadrons tend to cluster in reasonably well-collimated *jets* of particles following approximately the direction of the final quarks. For example, the angular distribution of the jets observed in $e^+e^-$ annihilation at SLAC and DESY confirms that the quarks have spin $\frac{1}{2}$, as well as the existence of the intermediate quark–antiquark state. Similarly, experiments at DESY have shown the existence of three-jet events whose characteristics are consistent with the hadronization of a $q\bar{q}$ pair as well as a gluon (gluon bremsstrahlung, analogous with electron bremsstrahlung). These results give fairly convincing evidence for the existence of gluons, and in particular establish their spin as 1. Finally, jets produced in hadronic processes, especially at high-energy proton–antiproton colliders at CERN and Fermilab, probe the strong interactions at extremely high $Q^2$ (e. g., $10^4\,\text{GeV}^2$). The observations are all consistent with

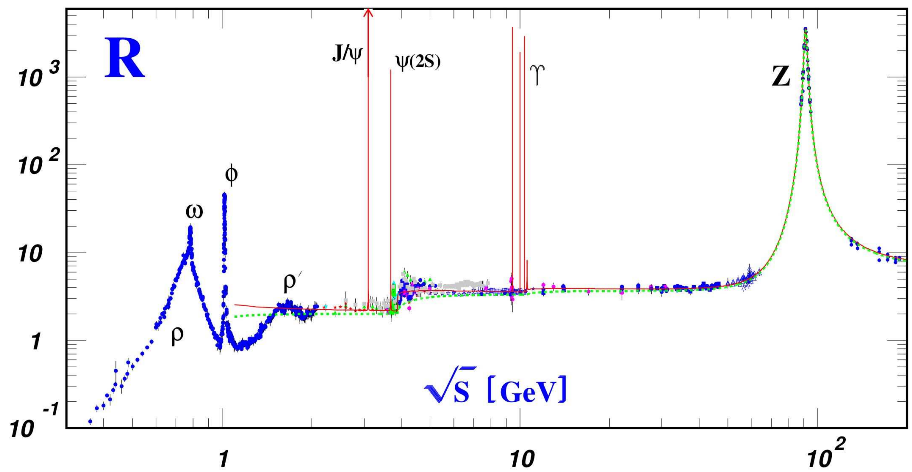


Fig. 8: Data on $R = \sigma(e^+ + e^- \to \text{hadrons})/\sigma(e^+ + e^- \to \mu^+ + \mu^-)$ as a function of the center-of-mass energy $W = \sqrt{s}$, from S. Eidelman *et al.* [Particle Data Group], *Phys. Lett.* **B592**, 1 (2004). The predictions of the quark parton model and a fit to QCD are also shown.

the QCD predictions of underlying hard quark and gluon-scattering processes.

One test of QCD is to extract the coupling $g_s$ at various scales to see whether it decreases at higher energy scales as predicted. The results, shown in Fig. 9, are in spectacular agreement with the QCD expectations.

It is believed that at high temperatures and densities confinement would no longer be relevant and that a plasma of quarks and gluons should be possible. Presumably, quarks and gluons were unconfined in the very early Universe when the temperature exceeded $\sim 1\,\text{GeV}$. Experiments at the high energy heavy-ion collider RHIC at the Brookhaven National Laboratory are attempting to recreate such a state. They show possible indications, but the signatures are not completely clear.

In view of these various successes, QCD is almost certainly the “correct” theory of the strong interactions, even though there has been no single “gold-plated” test. In fact, QCD is the only realistic candidate within the framework of renormalizable field theories.

## Hadronic Weak Interactions, Current Algebra, and Heavy Quarks

The weak interactions, whether in the phenomenological local current–current form or in the Weinberg–Salam $[SU(2) \times U(1)]$ form, need a specification of the hadronic currents, both vector and axial. The experimental evidence showed that the phenomenological form for the part of the current involving the proton

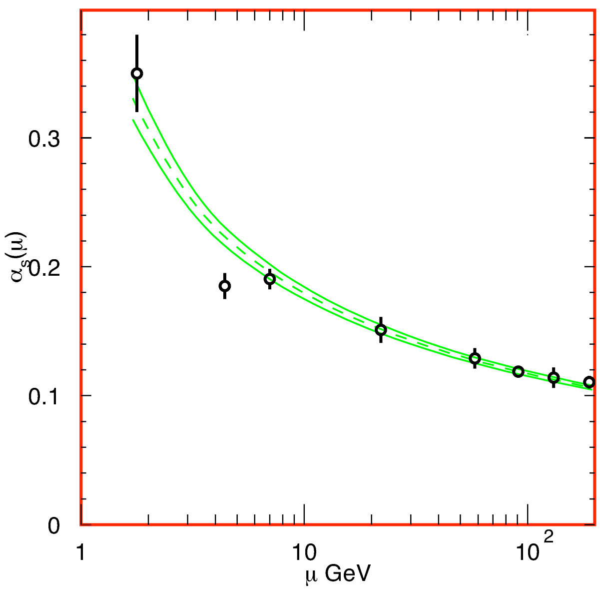


Fig. 9: Values of the strong coupling $\alpha_s = g_s^2/4\pi$ from a variety of determinations as a function of the scale $\mu$ at which they are measured, from S. Eidelman *et al.* [Particle Data Group], *Phys. Lett.* **B592**, 1 (2004). The predictions of QCD, with an overall scale determined from the data, are also shown.

and neutron was

$$\bar{n}(x)[(0.9744 \pm 0.0010)\gamma^\alpha - (1.227 \pm 0.004)\gamma^\alpha\gamma_5]\, p(x) \ . \tag{41}$$

This form bears a strong resemblance to the leptonic form in (16), and that, paradoxically, is surprising, since, in general, *form factors* due to strong-interaction corrections should appear. For the vector part, for example, the general expectation would be that for low momentum transfers from proton to neutron (as in beta decay)

$$\langle n|V_\alpha|p\rangle = G_V(q^2)\bar{n}\gamma_\alpha p \tag{42}$$

and the question is, Why should $G_V(0)$ be equal to unity? (The small deviation of the vector coefficient in (41) from unity is due to the Cabibbo angle factor discussed below.) R. P. Feynman and M. Gell-Mann, as well as S. Gershtein and Y. B. Zel'dovich, pointed out that if the vector current were conserved, and if $\int V_0(x)\, \mathrm{d}^3x$ is normalized like the generator of a symmetry group, then the observed result would follow. Feynman and Gell-Mann identified the vector current with the current generating the isospin transformations. This satisfied the conditions that led to $G_V(0) = 1$, and it led to a model-independent characterization of the weak vector current. From this it was possible to predict unambiguously the rate for the decay $\pi^+ \to \pi^0 + e^+ + \nu_e$, and certain "magnetic" corrections to beta-decay spectra, all in excellent agreement with experiment.

In 1963, after the discovery of flavor $SU(3)$, N. Cabibbo generalized this assignment and proposed that the weak current, now also describing the weak interactions of the strange particles, has a form involving the $SU(3)$-generating currents. The next important step was to give a general characterization of the axial current. M. Gell-Mann proposed that there exists an additional global symmetry of the strong interactions, also of the $SU(3)$ type, but generated by eight pseudoscalar charges whose associated currents are axial currents, in fact, the weak axial currents. The difficulty that in the symmetric limit a pseudoscalar charge acting on a proton state, for example, yields another "odd" proton, for which there is no experimental evidence, was resolved by Y. Nambu, who made use of the Goldstone mechanism discussed earlier. The proposal was that in the symmetry limit the axial current is conserved, but that the solutions do not obey the symmetry. This leads to the prediction of massless pseudoscalar particles, which would approximately describe the pions and their $SU(3)$ partners. The symmetry generated by the $SU(3)$ generators $F_i$ and the pseudoscalar $F_{5i}$ is called a *chiral* $[SU(3) \times SU(3)]$ symmetry, and through the study of weak interactions it has been established that it is a good symmetry, leading to a number of experimentally verified relations involving matrix elements of the weak hadronic currents. In particular, in 1965 S. Adler and W. Weisberger independently derived a relation between the coefficient $G_A(0)$ of the axial current in (41) in terms of other observables, such as the $\pi p$ cross section. They obtained $G_A(0) \simeq 1.21$, in excellent agreement with experiment. The algebra $SU(3) \times SU(3)$ emerges quite naturally in a quark model. With minimal couplings, the currents

$$\bar{q}(x)\gamma^\alpha \lambda_i q(x) \ , \qquad \bar{q}(x)\gamma^\alpha \gamma_5 \lambda_i q(x) \ , \tag{43}$$

where the $\lambda_i$ are the $SU(3)$ generalizations of the $SU(2)$ Pauli matrices, are conserved in the limit that the quark masses vanish. The *current* quark masses, which are the masses appearing in the QCD Lagrangian (they may actually be generated in the electroweak sector by the same Higgs mechanism which yields the $W$ and $Z$ masses), break the symmetries associated with the axial currents explicitly and generate small masses for the $\pi$, $K$, and $\eta$. Quark mass differences break the vector symmetries and lead to multiplet mass differences. From these effects one can estimate the current masses given in Table 2. The $u$ and $d$ masses are extremely small. (The much larger *constituent* masses of order $300\,\mathrm{MeV}/c^2$ in the naive quark model are dynamical masses associated with the spontaneous breaking of chiral symmetry.) Experiments indicate $m_u \neq m_d$, implying a breaking of isospin in the strong interactions, which is however no larger than the (separate) electromagnetic breaking because of the small scale of the masses. The much larger $m_s$ leads to a substantial breaking of $SU(3)$.

The quark model gives a simple expression for the electromagnetic current of the hadrons [Eq. (12)], with the coefficients determined by the quark charges. Similarly, the weak neutral current coupling to the $Z$ is given in (27). The weak current that couples to the charged $W$ is given (for three quarks) by

$$\begin{aligned}
J_W^\alpha &= (\bar{u}\bar{d}\bar{s})Q_W\gamma^\alpha(1-\gamma_5)\begin{pmatrix} u \\ d \\ s \end{pmatrix} \\
&= (\bar{u}\bar{d}\bar{s})\begin{pmatrix} 0 & 0 & 0 \\ \cos\theta_C & 0 & 0 \\ \sin\theta_C & 0 & 0 \end{pmatrix}\gamma^\alpha(1-\gamma_5)\begin{pmatrix} u \\ d \\ s \end{pmatrix} \\
&= (\bar{d}\cos\theta_C + \bar{s}\sin\theta_C)\gamma^\alpha(1-\gamma_5)u \ .
\end{aligned} \tag{44}$$

This form leads to the Cabibbo theory, and the angle $\theta_C$, the so-called Cabibbo angle, is of magnitude 0.22 rad. Its origin lies in the difference in the ways in which the strong and weak interactions break $SU(3)$ symmetry. This is associated with the quark masses, which are generated by the Higgs mechanism in the standard model. Their values, as well as $\theta_C$ and the other *mixing angles* introduced later, are free parameters that are not understood at present.

The part of the weak current involving $s$ in (44) has the property that the change in strangeness and the change in charge are equal ($\Delta S = \Delta Q$), in agreement with experiment. The Cabibbo theory explains a large number of strange particle decays with a universal choice of $\theta_C$. It has one difficulty: If this current is incorporated into a gauge theory of the weak interactions (e. g., the $SU(2) \times U(1)$ model) in a manner analogous to the leptonic current, then the neutral intermediate $W^0$ vector meson couples naturally to a neutral current obtained by commuting $Q_W$ with its adjoint,

$$[Q_W, Q_W^\dagger] = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \cos^2\theta_C & \sin\theta_C\cos\theta_C \\ 0 & \sin\theta_C\cos\theta_C & \sin^2\theta_C \end{pmatrix} \ . \tag{45}$$

Among the neutral currents there will be *strangeness-changing neutral currents* of the type $(\bar{d}s + \bar{s}d)\sin\theta_C\cos\theta_C$ (we ignore the $\gamma$ matrices for brevity), which give rise to processes such as

$$\begin{aligned}
K^+ &\rightarrow \pi^+ + \nu + \bar{\nu} \ , \\
K_L^0 &\rightarrow \mu^+ + \mu^-
\end{aligned} \tag{46}$$

at rates much larger than experimental limits or observations. Thus, a major modification is needed. The solution had actually been proposed before the Weinberg–Salam theory became popular. In 1970, S. Glashow, J. Iliopoulos, and L. Maiani, building on some earlier work of Glashow and J. D. Bjorken, proposed that the number of quark flavors be extended, with a fourth quark $c$ carrying a new conserved quantum number called charm. The $c$ quark is taken to have charge $\frac{2}{3}$, hypercharge $\frac{1}{3}$, and baryon number $B = \frac{1}{3}$. The weak current,

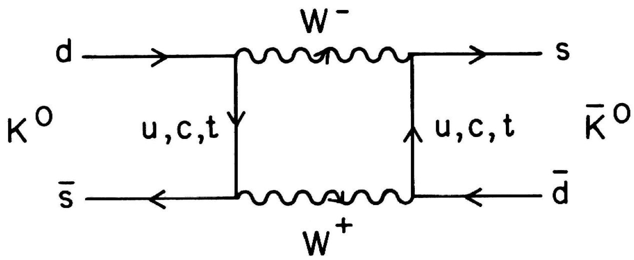


Fig. 10: Higher-order weak diagram leading to the mass difference between $K_s \sim K^0 + \bar{K}^0$ and $K_L \sim K^0 - \bar{K}^0$. Inclusion of the diagram with intermediate $t$ quarks can also account for the observed $CP$ violation.

constructed to have the form

$$(\bar{u}\bar{d}\bar{s}\bar{c})Q_W\gamma^\alpha(1-\gamma_5)\begin{pmatrix} u \\ d \\ s \\ c \end{pmatrix} \tag{47}$$

with

$$Q_W = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \cos\theta_C & 0 & 0 & -\sin\theta_C \\ \sin\theta_C & 0 & 0 & \cos\theta_C \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{48}$$

treats the $s$ quark symmetrically with the $d$. This implies that the neutral current constructed as in (45) does not have any strangeness-changing (or charm-changing) terms. The smallness of the Cabibbo angle implies that the dominant charged-current transitions are $u \to d$ and $c \to s$, which means that mesons involving $c$ quarks are predicted to usually decay into final states with strangeness (e. g., a $\bar{K}$) rather than into nonstrange (e. g., pions) final states.

Notice that if we set $\theta_C = 0$ and replace $(u, d, s, c)$ by $(e^-, \nu_e, \mu^-, \nu_\mu)$, we get the leptonic current. It was this analogy that originally led Bjorken and Glashow to generalize the $SU(3)$ flavor symmetry to the four-flavor $SU(4)$ symmetry. The analogy goes deep. It turns out that renormalizability demands that each lepton pair [e. g., $(e, \nu_e)$] must have a compensating quark pair [e. g., $(u, d)$], so that the existence of the charmed quark is compelling in gauge theories. In the early 1970s it was also realized that the observed mass difference between the two neutral kaons could be accounted for by a calculable higher-order weak effect (Fig. 10) if the $c$ quark existed and had a mass around $1.5\,\mathrm{GeV}/c^2$.

In 1974 in the experimental study of the ratio $R$ in (40), B. Richter and collaborators at the Stanford Linear Accelerator Center, simultaneously with S. Ting and collaborators at Brookhaven National Laboratory, who were studying the reaction

$$p + \mathrm{Be} \to e^+ + e^- + \mathrm{hadrons},$$

found an extremely sharp resonance at 3097 MeV and, soon after that, another one at 3685 MeV (Fig. 8). It is interesting that these had been anticipated theoretically by T. Appelquist and D. Politzer, who advanced reasons why the $J = 1$ $c\bar{c}$ (charmed quark–antiquark) states should be quite long-lived. This interpretation of the $J/\psi(3097)$ and $\psi(3685)$ as $1\,^3S_1$ and $2\,^3S_1$ *charmonium* states was confirmed by the later discovery of $^3P_0$, $^3P_1$, $^3P_2$, and $^2S_0$ states as well as additional $^3S_1$ radial excitations in the vicinity of the ones already discovered. There now exists a complete and well-studied charmonium spectroscopy. The arrangement of levels and the spacing cannot be understood in terms of a simple $1/r$ potential, as for positronium. Rather, a better fit is obtained with a potential of the form $V(r) = A/r + Br$, where the first term represents the short-range (asymptotically weakly coupled) contribution and the second term represents the long-range confining potential. The extreme narrowness of the $^3S_1$ resonances can be understood qualitatively by arguing that a $c\bar{c} \to u\bar{u}$ (say) transition can only take place through the mediation of three color-carrying gluons, and these couple weakly for large momentum transfers.

Numerous hadrons consisting of a single $c$ quark bound to ordinary quarks or antiquarks have also been identified and their decay modes studied. These include the spin-0 ($c\bar{u}$) $D$ meson, with mass 1865 MeV/$c^2$, the corresponding spin-1 $D^*$ meson of mass 2007 MeV/$c^2$, their charged ($D^+ = c\bar{d}$) partners, and the $D_s^+$ ($c\bar{s}$) at 1968 MeV/$c^2$, as well as charmed baryons with isospin 0, 1, and $\frac{1}{2}$ at 2285, 2455, and 2470 MeV/$c^2$, respectively. The spectroscopy of the charmed hadrons, as well as additional evidence from deep inelastic neutrino scattering, have thus clearly established the existence of the $c$ quark as needed for a consistent and realistic gauge theory of the weak interactions.

Similarly, a third (heavy) $\tau$ lepton was discovered at SLAC in 1976. The observed weak interactions of the $\tau$ left little doubt of the existence of a third (approximately) massless neutrino partner, but the interactions of the $\nu_\tau$ were not observed directly until 2000 at Fermilab. Renormalizability then implied that another quark pair, which we label $(t, b)$ for *top* and *bottom*, respectively, with charges $\frac{2}{3}$ and $-\frac{1}{3}$, should exist. This notion of lepton–quark symmetry was confirmed in 1977 by the discovery by L. Lederman and collaborators at the Fermi National Accelerator Laboratory of two new narrow resonances, the $\Upsilon(9\,460\,\text{MeV}/c^2)$ and $\Upsilon'(10\,023\,\text{MeV}/c^2)$, in the reaction

$$p + \text{Be} \to \Upsilon + \text{hadrons} \to \mu^+ + \mu^- + \text{hadrons}.$$

The interpretation of these states as $^3S_1$ states of a new *quarkonium* composed of $b\bar{b}$ quarks with charge $\frac{1}{3}$ was supported by their production in $e^+e^-$ collisions at DESY and at the Cornell Electron Storage Ring (CESR), where additional $^3S_1$ as well as $P$ wave states were also discovered. The bottom mesons $B^+ = u\bar{b}$ and $B^0 = d\bar{b}$, with masses $\sim$5279 MeV/$c^2$, as well as their antiparticles and many other bottom mesons and baryons, have also been identified and studied in detail at a number of laboratories.

The observed weak interactions of the $b$ quark as well as renormalizability and lepton–quark symmetry indicated that a top quark ($t$) should exist. Furthermore, by the early 1990's indirect arguments based on the consistency of weak neutral current and $Z$-pole data, which are sensitive to a heavy $t$ quark mass through higher-order corrections, predicted a mass $m_t = 150 \pm 30\,\mathrm{GeV}/c^2$. The $t$ was finally discovered at the Tevatron in 1994 in the expected range. It is too short-lived to form observable bound states. Rather, pairs of $t$ and $\bar{t}$ are produced through the strong interactions, and the direct decays into $b$ ($\bar{b}$) and additional quark jets or leptons are observed. The current indirect prediction $m_t = 172^{+10}_{-7}\,\mathrm{GeV}/c^2$ is in excellent agreement with the observed value of $178.0 \pm 4.3\,\mathrm{GeV}/c^2$, dramatically confirming the standard electroweak model. Why the $t$ is so much heavier than the other fermions is not understood.

In the four-quark model, $Q_W$ in (47) involves a single mixing angle $\theta_C \simeq 0.22$, and the higher-order diagram in Fig. 10 can account for the $K_L - K_S$ mass difference. The generalization of $Q_W$ to six quarks involves three mixing angles and one complex phase. In addition to $\theta_C$, the dominant $b$ quark transition $b \to c$ is described by a mixing angle $\theta_{bc} \simeq 0.04$ (determined from the relatively long $b$ lifetime). The much weaker $b \to u$ transition is characterized by the angle $\theta_{bu} \simeq 0.004$. These angles suffice to account for the large mixing observed at DESY between the $B^0 = d\bar{b}$ and $\bar{B}^0 = d\bar{b}$ states, by a second-order diagram analogous to Fig. 10 (with the external $s$ replaced by $b$). Large mixing between the $B^0_s = s\bar{b}$ and $\bar{B}^0_s = \bar{s}b$ is predicted.

Even after the discovery that the weak interactions violated parity ($P$) and charge conjugation ($C$) invariance, it was generally believed that $CP$ invariance was maintained. In particular, the $CP = +1$ state $K_S \sim K^0 + \bar{K}^0$ should decay rapidly ($\tau \sim 0.89 \times 10^{-10}\,\mathrm{s}$) into $\pi^+\pi^-$ or $\pi^0\pi^0$, while the $CP = -1$ state $K_L \sim K^0 - \bar{K}^0$ should decay into $3\pi$ in $\sim 5.2 \times 10^{-8}\,\mathrm{s}$. However, in 1964 J. Cronin, V. Fitch, and collaborators working at Brookhaven observed the $CP$ violating decays $K_L \to 2\pi$ with branching ratios of $\sim 10^{-3}$. The results could be accounted for by a small $CP$ violating mixing between the $K_L$ and $K_S$ states. One possibility was that this mixing is generated by an entirely new *superweak* $\Delta S = 2$ interaction. In 1973, M. Kobayashi and M. Maskawa suggested that $CP$ breaking could be generated by the higher-order diagram in Fig. 10 if there were three (or more) fermion families, implying observable phases in $Q_W$. The observed $K_L - K_S$ mixing could thus be generated either by the three-family standard model or by the superweak model. However, in the 1990's experiments at CERN and Fermilab observed small differences in the $CP$ violating $K_L \to \pi^0\pi^0$ and $\pi^+\pi^-$ rates (relative to $K_S$,) that required direct $CP$ breaking in the decay amplitude, not just in the state mixing, strongly confirming the standard $SU(2) \times U(1)$ model interpretation.

Subsequently, there have been very detailed studies of $CP$-violating asymmetries, rare decays, and other aspects of $B$ meson decays at a number of laboratories, especially the "$B$ factories" at CESR, SLAC and KEK (Japan). All data is consistent with the standard model, in particular with the interpretation that the patterns of decays and $CP$-violation can be accounted for by the unitary Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix $Q_W$. This is illustrated by the *unitarity triangle* shown in Fig. 11. The charged current sector will be tested even more stringently in the future following the (probable) measurement of $B^0_s\bar{B}^0_s$ mixing and more precise measurements of rare decays and $CP$-violating asymmetries in $B$ meson decays.

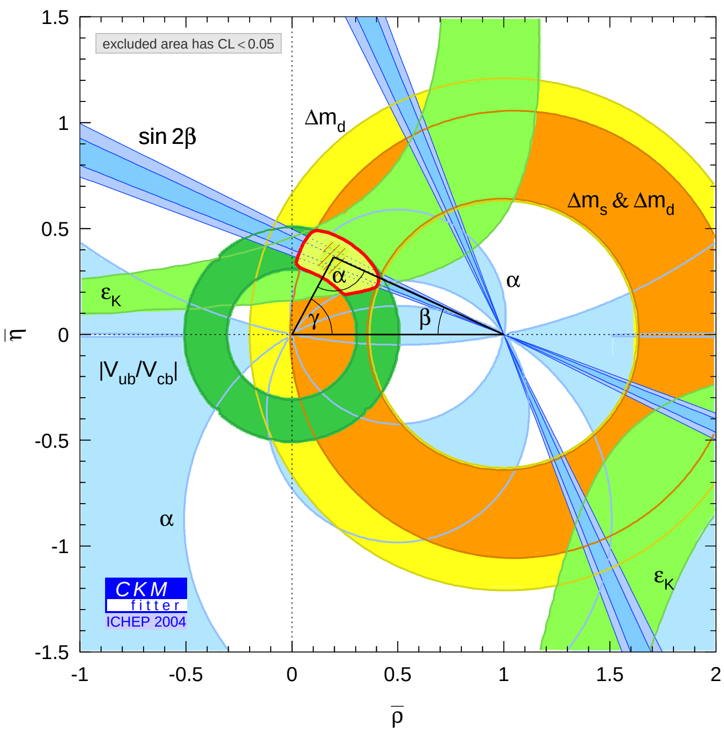


Fig. 11: The current status of the CKM matrix, from J. Charles *et al.* [CKMfitter Group Collaboration], hep-ph/0406184 and ckmfitter.in2p3.fr. $\bar{\rho}$ and $\bar{\eta}$ parametrize elements of the matrix. Various observations from $K$ and $B$ decays determine the shaded bands. They are all consistent with each other within the region surrounding the vertex (labeled by angle $\alpha$) of the unitarity triangle.

## Neutrino Mass and Oscillations

For many years all experimental searches for nonzero neutrino masses yielded negative results, implying that they are either massless or much lighter than the charged leptons and quarks. The original $SU(2) \times U(1)$ standard model was constructed to yield massless $\nu$'s, but most theoretical ideas for more complete theories predicted that $m_\nu$ would be nonzero at some level. The first experimental hints of nonzero mass were from the Homestake Solar Neutrino Experiment in the late 1960's. R. Davis and collaborators observed events in a large underground chlorine experiment that were later confirmed to be induced by electron neutrinos ($\nu_e$) produced by nuclear reactions in the core of the Sun. However, they only observed about one third of the rate predicted by J. Bahcall and collaborators based on the observed Solar luminosity and theoretical models of the Sun. This *Solar neutrino problem* was later confirmed by other experiments using gallium and water-based detectors. One explanation was that the $\nu_e$ were oscillating (converting) into other types of neutrino ($\nu_\mu$ or $\nu_\tau$), to which the experiments were insensitive, through effects associated with (tiny) neutrino masses and mixings (analogous to the mixings observed for the quarks). However, there was the alternative possibility that the astrophysical uncertainties in the theoretical calculation had been underestimated. The situation was resolved in 2002 when the Sudbury Neutrino Observatory (Canada) was able to measure the $\nu_e$ and total fluxes independently using a heavy water detector, with the result that the $\nu_e$ really were converting to $\nu_\mu$ or $\nu_\tau$. This was later confirmed by the observation by the KamLAND experiment (Japan) of the disappearance of $\bar{\nu}_e$'s produced in power reactors. Prior to the Sudbury results, a different type of *atmospheric neutrino* oscillation, of $\nu_\mu$'s produced by cosmic ray interactions in the atmosphere into (presumably) $\nu_\tau$'s, was established by the Super-Kamiokande experiment (Japan).

There are three types of neutrinos, and *neutrino oscillations* presumably involve three mixing angles and one or more leptonic $CP$-violating phases, similar to the CKM quark mixing. However, a simplified description assumes that only two states are relevant for a given type of experiment. For example, suppose that the $\nu_e$ and $\nu_\mu$, the states which are associated with the $e^-$ and $\mu^-$, are superpositions of states $\nu_{1,2}$ of definite masses $m_{1,2}$,

$$
\begin{aligned}
\nu_e &= \nu_1 \cos\theta + \nu_2 \sin\theta \ , \\
\nu_\mu &= -\nu_1 \sin\theta + \nu_2 \cos\theta \ . \qquad (49)
\end{aligned}
$$

Then if an initially produced $\nu_e$ has energy $E$ large compared to $m_{1,2}$ the mixing effect may cause it to transform into $\nu_\mu$ after travelling a distance $L$, with probability

$$P(\nu_e \to \nu_\mu; L) = \sin^2 2\theta \sin^2 \frac{1.27 \Delta m^2 L}{E} \ , \qquad (50)$$

where $\Delta m^2 = m_2^2 - m_1^2$ is expressed in eV/c$^2$, $L$ in m, and $E$ in MeV. The observed Solar and atmospheric oscillation parameters (which correspond to different $\Delta m^2$ and $\theta$) are shown in Fig. 12. Note that the mass scales are tiny compared to the other fermions, but that the mixing angles are large, unlike the small quark mixings (there is a third neutrino mixing angle that is consistent with zero).

The neutrino masses and mixings are interesting because they are so different, and they may be an indication of new physics underlying the standard model at much shorter distance scales (i. e., much larger mass scales). For example, there are various *seesaw* models which predict very small neutrino masses $m_\nu \sim m_D^2/M \ll m_D$, where $m_D$ is comparable to the quark and charged lepton masses and $M \gg m_D$ is the new physics scale (e. g., $10^{14}$ GeV/c$^2$). Another interesting possibility is that the neutrinos are *Majorana*, which means that the mass effects can convert neutrinos into antineutrinos so that the total *lepton number* (number of leptons minus the number of antileptons) is not conserved.

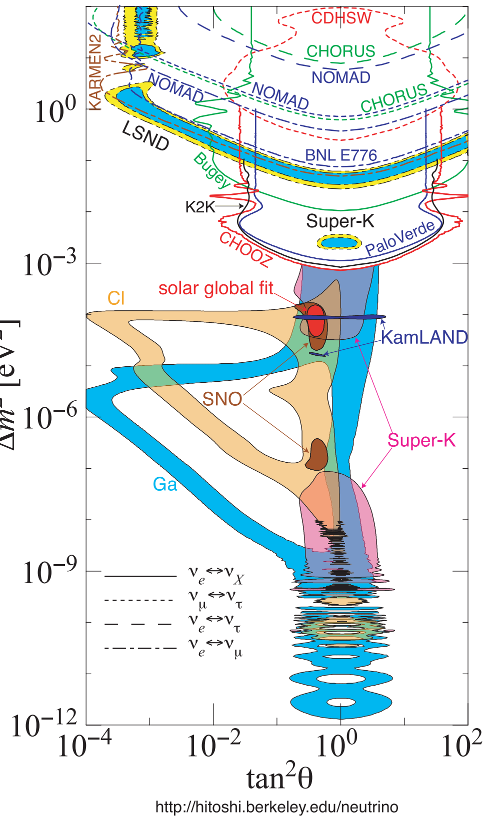


Fig. 12: Current status of neutrino oscillations, from H. Murayama, http://hitoshi.berkeley.edu/neutrino. The regions for $\Delta m^2 \sim 2 \times 10^{-3}$ eV$^2$ and $\tan^2\theta \sim 1$, and for $\Delta m^2 \sim 8 \times 10^{-5}$ eV$^2$ and $\tan^2\theta \sim 0.4$ are indicated by the atmospheric and Solar neutrino oscillations, respectively. (The two regions represent different combinations of neutrino states.)

This will be probed in *neutrinoless double beta decay* ($\beta\beta_{0\nu}$) experiments searching for rare nuclear decays $(Z,N) \to (Z+2, N-2) + e^- + e^-$, (i. e., without the two antineutrinos expected from two successive $\beta$ decays). Another outstanding issue is the absolute neutrino mass scale, since oscillations only probe mass-square differences. There is an upper limit of $\sim 0.2\,\mathrm{eV/c^2}$ from cosmology (otherwise, relic neutrinos left over from the big bang would modify the formation of structure in the Universe). This will be refined in the future, and $\beta\beta_{0\nu}$ and kinematic effects in ordinary $\beta$ decay may also be important. More detailed information on the spectrum, mixings, and possible leptonic $CP$ violation are also anticipated, as is clarification of possible indications of additional types of oscillations that cannot be accommodated in the three-neutrino framework.

## Problems with the Standard Model

The standard model (QCD plus the Weinberg–Salam electroweak model and general relativity) has been spectacularly successful. Although the elementary Higgs mechanism for symmetry breaking has not yet been tested and may possibly be replaced by a dynamical mechanism, the basic structure of the standard model is almost certainly correct at some level. However, it contains far too much arbitrariness to be the final story of Nature. One way of seeing this is that the minimal version has 27 free parameters (29 for Majorana neutrinos), *not* including electric charges. In addition, the standard model suffers from:

(a) *The Gauge Problem:* The standard model gauge group is a complicated direct product of three factors with three independent coupling constants. Charge quantization, which refers to the fact that the magnitudes of the proton and of the electron electric charges are the same, is not explained.

(b) *Fermion Problem:* The standard model involves a very complicated reducible representation for the fermions. Ordinary matter can be constructed out of the fermions of the first family $(\nu_e, e^-, u, d)$. We have no fundamental understanding of why the additional families $(\nu_\mu, \mu^-, c, s)$, $(\nu_\tau, \tau, t, b)$, which appear to be identical with the first except that they are heavier, exist. In addition, the standard model does not explain or predict the pattern of fermion masses, which are observed to vary over many orders of magnitude, or mixings. Also, the $CP$ violation associated with quark mixing is not sufficient to explain the generation of the excess of matter compared to antimatter in the universe (the *baryon asymmetry*), therefore requiring the existence of some additional form of $CP$ violation.

(c) *Higgs/Hierarchy Problem:* The spontaneous breakdown of the $SU(2) \times U(1)$ symmetry in the standard model is accomplished by the introduction of a Higgs field. Consistency requires that the mass of the Higgs boson be not too much different from the weak interaction scale; that is, it should be equal to the $W$ mass within one or two orders of magnitude. However, there are higher-order corrections which change (renormalize) the value of the square of the Higgs mass by $\delta m_{\mathrm{H}}^2 \sim m_p^2$, where $m_p = (G_N/\hbar c)^{-1/2} \simeq 10^{19}\,\mathrm{GeV}/c^2$ is the *Planck* (gravity) scale. Therefore, $\delta m_{\mathrm{H}}^2/M_W^2 \geq 10^{34}$, and the bare value of $m_{\mathrm{H}}^2$ in the original Lagrangian must be adjusted or fine-tuned to 34 decimal places. Such a fine-tuning is possible, but extremely unattractive.

(d) *Strong CP Problem:* It is possible to add an additional term, characterized by a dimensionless parameter $\theta$, to the QCD Lagrangian which breaks $P$, $T$, and $CP$ invariance. Limits on the electric dipole moment of the neutron require $\theta$ to be less than $10^{-9}$. However, weak interaction corrections change or renormalize the lowest-order value of $\theta$ by about $10^{-3}$ – that is, $10^6$ times more than the total value is allowed to be. Again, one must fine-tune the bare value against the correction to a high degree of precision.

(e) *Graviton Problem:* The graviton problem has several aspects. First, gravity is not unified with the other interactions in a fundamental way. Second, even though general relativity can be incorporated into the model by hand, we have no idea how to achieve a mathematically consistent theory of quantum gravity: attempts to quantize gravity within the standard model framework lead to horrible divergences and a nonrenormalizable theory. Finally, there is yet another fine-tuning problem associated with the cosmological constant. The vacuum energy density $\langle V \rangle$ associated with the spontaneous symmetry breaking of the $SU(2) \times U(1)$ model generates an effective renormalization of the cosmological constant $\delta\Lambda = 8\pi G_n \langle V \rangle$, which is about 50 orders of magnitude larger than the observed limit (or value, if the observed *dark energy* or *acceleration* of the universe is due to a cosmological constant). One must fine-tune the bare cosmological constant against the correction to this incredible degree of precision.

## Extensions of the Standard Model

There must almost certainly be new physics beyond the standard model. Some of the possible types of new physics that have been discussed extensively in recent years are shown in Table 6. Additional gauge bosons, fermions, or Higgs bosons do not by themselves solve any problems, but may exist at accessible energies as remnants of underlying physics such as unified gauge groups or superstrings. The $Z$-pole data from CERN and SLAC, as well as the experimental evidence from the cosmological abundance of helium (interpreted within the framework of the standard "big-bang" cosmological model), implies that there are no additional fermion families beyond the three already known, unless the associated neutrinos are very heavy (greater than $\simeq 45\,\mathrm{GeV}/c^2$). More likely are *exotic* heavy fermions, predicted by many theories, which are not simply repetitions of the known families. New gauge interactions, especially heavy electrically neutral $Z'$ bosons analogous to the $Z$ in (26) are predicted by many extensions, as are extended Higgs sectors.

*Family symmetries* are new global or gauge symmetries which relate the fermion families. Another approach to understanding the fermion spectrum is to assume that the quarks and leptons are bound states (*composites*) of still smaller constituents. However, experimental searches for substructure suggest that the underlying particles must be extremely massive ($> 1000\,\mathrm{GeV}/c^2$). Hence, unlike all previous layers of matter, extremely strong binding would be required. Neither of these ideas has led to a particularly attractive model.

The fine-tuning problem associated with the Higgs mechanism could be solved if the elementary Higgs fields were replaced by some sort of bound-state mechanism. However, models which generate fermion as well as $W, Z$ masses are complicated and tend to predict certain unobserved decays at too rapid a rate and lead to unacceptably large corrections to precision electroweak data. *Little Higgs* models are a variant in which the elementary Higgs is replaced by the approximately massless Goldstone boson of a new global symmetry, which ensures cancellation between different higher-order corrections to the square of

Table 6: Some possible extensions of the standard model.

| Model | Typical scale (GeV) | Motivation |
|---|---|---|
| New $W$s, $Z$s, fermions, Higgs | $10^2$–$10^{19}$ | Remnant of something else |
| Family symmetry | $10^2$–$10^{19}$ | Fermion (No compelling models) |
| Composite fermions | $10^2$–$10^{19}$ | Fermion (No compelling models) |
| Composite Higgs | $10^3$–$10^4$ | Higgs (No compelling models) |
| Composite $W$, $Z$ ($G$, $\gamma$?) | $10^3$–$10^4$ | Higgs (No compelling models) |
| Little Higgs | $10^3$–$10^4$ | Higgs |
| Large extra dimensions ($d > 4$) | $10^3$–$10^6$ | Higgs, graviton |
| New global symmetry | $10^8$–$10^{12}$ | Strong CP |
| Kaluza–Klein<br>Higgs (0) $\Leftrightarrow$ gauge (1) $\Leftrightarrow$<br>Graviton (2) ($d > 4$) | $10^{19}$ | Graviton |
| Grand unification<br>Strong $\Leftrightarrow$ electroweak | $10^{14}$–$10^{19}$ | Gauge |
| Supersymmetry/supergravity<br>Fermion $\Leftrightarrow$ boson | $10^2$–$10^4$ | Higgs, graviton |
| Superstrings<br>Strong $\Leftrightarrow$ electroweak $\Leftrightarrow$<br>gravity<br>Fermion $\Leftrightarrow$ boson ($d > 4$) | $10^{19}$ | All problems!? |

the Higgs mass. Composite $W$ and $Z$ bosons could be an alternative to spontaneous symmetry breaking, but such theories abandon most of the advantages of gauge theories. The strong $CP$ problem could be resolved by the addition of a new global symmetry which ensures that $\theta$ is a dynamical variable which is zero in the lowest energy state. Such models imply a weakly coupled Goldstone boson (the *axion*) associated with the symmetry breaking. Constraints from astrophysics and cosmology limit the symmetry breaking scale to the range $10^8$–$10^{12}$ GeV.

*Kaluza–Klein* theories are gravity theories in $d > 4$ dimensions of space and time. It is assumed that four dimensions remain flat while the other $d - 4$ are *compactified* or curled up into a compact manifold with a typical radius of $\hbar/m_p c \sim 10^{-33}$ cm. Gravitational interactions associated with these unobserved compact dimensions would appear as effective gauge interactions and Higgs

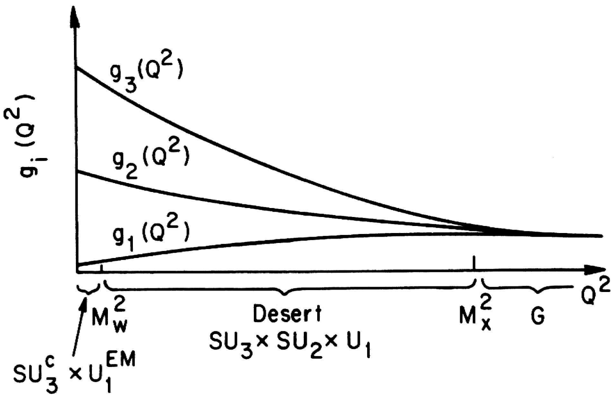


Fig. 13: The momentum dependent normalized coupling constants $g_3 \equiv g_s$, $g_2 \equiv g$,and $g_1 \equiv \sqrt{5/3}g'$.

particles in our four-dimensional world. Although such ideas are extremely attractive for unifying the interactions they have great difficulty in incorporating parity violation and also in achieving a stable configuration for the compact manifold. Many of the aspects have reemerged more successfully within the framework of *superstrings*.

Similarly, theories of *large extra dimensions* postulate the existence of one or more extra dimensions of space that could be as large as a fraction of a mm! There are many variants on these ideas, but one promising version assumes that *gravitons* (the spin-2 quanta of the gravitational field) can propagate freely in the *bulk* of the extra dimensions, while the ordinary particles are somehow confined to the ordinary 3-dimensional space (the *brane*). This eliminates the Higgs/hierarchy problem because the fundamental (largest) mass scale of nature $m_f$ can be far smaller (e. g., $10^5$ GeV/c$^2$) than the Planck scale $m_p \sim 10^{19}$ GeV/c$^2$, with the apparent weakness of gravity due to the fact that the $1/r^2$ force law is modified for distances smaller than the size of the extra dimension. Such ideas may emerge as a limiting case of superstring theories (which involve extra dimensions), but introduce a new hierarchy problem, viz., why is the large dimension much larger than the expected $\hbar/m_f c$?

Much work has been devoted to *grand unified gauge theories* (GUTs), in which it is proposed that at very high energies (or very short distances) the underlying symmetry is a single gauge group that contains as its subgroups both $SU(3)$ for color and $SU(2) \times U(1)$ for the weak and electromagnetic interactions. The larger symmetry is manifest above a unification scale $M_X$ at which it is spontaneously broken. At lower energies, the symmetry is hidden and the strong and electroweak interactions appear different. The unification scale can be estimated from the observed low-energy coupling constants. The energy-dependent couplings are expected to meet at $M_X$, as in Fig. 13. Since they vary only logarithmically, $M_X$ is predicted to be extremely large. Two versions of

GUTs are typically considered, one in which only the standard model particles exist below $M_X$ and contribute to the variation of the couplings, and one in which a new class of *superpartners* and an extra Higgs multiplet expected in *supersymmetry* (discussed below) are included. The predicted values of $M_X$ are typically around $10^{14}\,\mathrm{GeV}/c^2$ and $3 \times 10^{16}\,\mathrm{GeV/c^2}$ for these two cases, respectively. (These heady speculations rest on the assumption that nothing fundamentally different occurs in a leap in energy scale from 1 to $M_X$. This is extremely unlikely, but then who would have thought that Maxwell's equations are good over distances ranging from astronomical down to $10^{-15}\,\mathrm{cm}$?) From the strong and electromagnetic couplings one can predict the weak angle $\sin^2\theta_W$. The simplest non-supersymmetric GUTs predict a value $\sim 0.20$, well below the present precisely known value, while the supersymmetric ones are in reasonable agreement with experiment, especially when uncertainties from mass differences at low energy and $M_X$ are taken into account.

In addition to unifying the interactions, the additional symmetries within a grand unified theory relate quarks, antiquarks, leptons, and antileptons. For example, the simplest grand unified theory – the 1974 $SU(5)$ model of H. Georgi and S. Glashow – assigns the left-helicity (i. e., spin oriented opposite to momentum) fermions of the first family to a reducible 5*+ 10-dimensional representation:

$$W^{\pm} \updownarrow \quad \begin{array}{cc} 5^* & 10 \\ \begin{pmatrix} \nu_e & & \\ & & \bar{d} \\ e^- & & \end{pmatrix} & \begin{pmatrix} & u & \\ e^+ & & \bar{u} \\ & d & \end{pmatrix} \\ \leftarrow X, Y \rightarrow & \leftrightarrow X, Y \leftrightarrow \end{array} \tag{51}$$

The neutrino and electron can be rotated into the anti-down quark by the new symmetry generators, as can the positron and the up, down, and anti-up quarks. Because of this relation between the different types of fermions, the grand unified theories naturally explain charge quantization. In addition, there are new gauge bosons associated with these extra symmetries. The $SU(5)$ model contains new bosons which carry both color and electric charge, known as the $X$ and $Y$ bosons, with masses $M_X \sim M_Y \sim 10^{14}\,\mathrm{GeV}/c^2$ $(3 \times 10^{16}\,\mathrm{GeV/c^2})$ for the non-supersymmetric (supersymmetric) case. These can mediate proton decay (and also the decay of otherwise stable bound neutrons). A diagram for $p \to e^+\pi^0$ is shown in Fig. 14. Motivated by such predictions, a number of experiments have searched for proton decay. No events have been observed, and one finds $\tau(p \to e^+\pi^0) > 5 \times 10^{33}\,\mathrm{yr}$, in conflict with the prediction $4 \times 10^{29\pm 2}\,\mathrm{yr}$ of the simplest non-supersymmetric GUTs. The predicted lifetime is much longer in the supersymmetric case because of the larger $M_X$. However, there are other diagrams involving the superpartners in those models that lead to such modes as $p \to K^+\bar{\nu}$, with typical lifetimes around $10^{33}\,\mathrm{yr}$. The simplest such models are excluded by the corresponding experimental bounds, but some extended or nonminimal models are still viable.

One of the mysteries of nature is the *cosmological baryon asymmetry.* It is an observational fact that our part of the Universe consists of matter and not antimatter: there is approximately one baryon for every $10^{10}$ microwave photons,

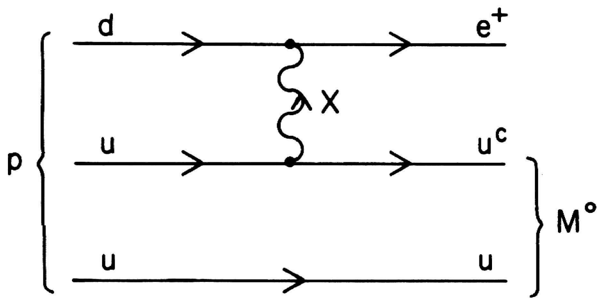


Fig. 14: A typical proton decay diagram.

but essentially no antibaryons. This could not be due to initial conditions if the Universe underwent a period of extremely rapid expansion (*inflation*), which many cosmologists believe is the most likely explanation of the almost exact flatness and homogeneity of the Universe. At one time it was believed that in GUTs this asymmetry could be generated dynamically in the first instant (the first $10^{-35}$ s or so) after the big bang, by baryon number violating interactions related to those which lead to proton decay. This now appears unlikely because the simplest GUTs predict that equal baryon and lepton asymmetries would have been generated, and these would later have been erased due to non-perturbative baryon and lepton number violating processes associated with the electroweak theory prior to the *electroweak phase transition*, i. e., when the temperature was higher than the electroweak scale. (A subsequent period of inflation would also have wiped out the asymmetry.) However, alternative ideas have emerged for dynamically generating the asymmetry. These include the possibility of first generating the lepton asymmetry by heavy particle decays in seesaw models of Majorana neutrino mass (*leptogenesis*), with some of the lepton asymmetry converted to a baryon asymmetry by the nonperturbative effects; or the possibility of actually generating the asymmetry during the electroweak transition, especially in extensions of the supersymmetric standard model.

Grand unified theories predict the existence of superheavy *magnetic monopole* states with masses $M_M \sim M_X/\alpha_G \sim 10^{16} - 10^{18}\,\mathrm{GeV}/c^2$. These may have been produced during phase transitions during the early Universe. If they were still left over as relics today they would contribute far more mass to the present Universe than is allowed by observations. However, they would have been diluted to a negligible density if the Universe underwent a period of inflation.

Grand unified theories offer little or no help with the fermion, Higgs, strong $CP$, or graviton problems, and the simplest non-supersymmetric versions are ruled out by $\sin^2\theta_W$ and proton decay. GUTs are probably not ambitious enough, but it is likely that some ingredients (e. g., related to charge quantization) will survive. It is also possible that there is an underlying grand unification in a theory with extra space dimensions, such as a superstring theory, but the full GUT symmetry is not manifest in the three large dimensions of space.

*Supersymmetry* (see *Supersymmetry and Supergravity*) is a new kind of symmetry in which fermions are related to bosons. Realistic models are complicated in that they require more than a doubling of the number of fundamental particles. For example, one must introduce a spin-0 superpartner of each known fermion, a spin-$\frac{1}{2}$ partner of each gauge boson or Higgs field, etc. The primary motivation is the Higgs problem: higher-order corrections associated with the new particles cancel the unacceptable renormalization of the Higgs mass that occurs in the standard model. If supersymmetry were exact, the new particles would be degenerate with the ordinary particles. However, there exist experimental lower limits of order $100 - 200\,\mathrm{GeV}/c^2$ on the masses of most superpartners, so that supersymmetry (if it exists) must be broken. Some models assume that supersymmetry is broken at some very large scale (e. g., $10^{11}\,\mathrm{GeV}$) in a sector which is only coupled very weakly (by gravitational-strength interactions) to the ordinary particles and their partners, implying superpartner masses in the range $10^2$–$10^4\,\mathrm{GeV}/c^2$. (Larger masses would fail to solve the Higgs problem.) Variant versions break the supersymmetry at a lower scale (e. g., $10^5\,\mathrm{GeV}$) in a sector that is somewhat more strongly coupled (by gauge interactions) to the ordinary sector, again leading to $10^2$–$10^4\,\mathrm{GeV}/c^2$ superpartner masses. In both cases, the superpartner mass scale actually determines the scale of electroweak symmetry breaking.

Many supersymmetric theories involve a conserved *R-parity*, which distinguishes the ordinary particles from their superpartners. This implies that the lightest superpartner (the LSP) is stable. If it is electrically neutral, such as the spin-$\frac{1}{2}$ partner of a neutral gauge or Higgs field, it would be an excellent candidate for the *cold dark matter* of the Universe. (An alternate possibility if *R-parity* is not conserved would be the axion described above. Massive neutrinos in the $\mathrm{eV/c^2}$ range are *not* good candidates because they are too light, and would prevent mass from clustering on small scales.) As described in the GUT section, supersymmetric grand unified theories lead to larger values for $\sin^2\theta_W$ (in better agreement with experiment), and a larger unification scale, reducing but not necessarily solving the problem that proton decay has not been observed.

Just as the promotion of an ordinary internal symmetry to a gauge symmetry implies the existence of spin-1 gauge bosons, the requirement that supersymmetry transformations can be performed independently at different space-time points implies the existence of spin-2 gravitons; i. e., gauged supersymmetry automatically unifies gravity. However, *supergravity* theories do not solve the problems of quantum gravity. Higher-order corrections are still divergent and nonrenormalizable. There is no experimental evidence for supersymmetry, but if it exists at the scales suggested by the Higgs problem it should be observed in the near future at the LHC *pp* collider being constructed at CERN. Whether Nature chooses to make use of supersymmetry remains to be seen.

*Superstrings* are a very exciting development which may yield finite theories of all interactions with no free parameters. Superstring theories introduce new structure around the Planck scale. The basic idea is that instead of working with (zero-dimensional) point particles as the basic quantities, one considers one-dimensional objects known as strings, which may be open (e. g., ending on membrane-like objects known as *branes*) or closed. The quantized vibrations of these strings lead to an infinite set of states, the spectrum of which is controlled by the string tension, given by the square of the Planck scale $\tau \sim m_p^2$. When one probes or observes a string at energies much less than the Planck scale, one sees only the "massless" modes, and these represent ordinary particles. The physical size of the string is given by the inverse of the Planck scale $\simeq 10^{-33}$ cm, and at larger scales a string looks like a point particle. There are actually 5 types of consistent string theories, but it is now believed that these (and one 11-dimensional supergravity theory) are limiting cases of an even more fundamental (and mysterious) *M theory*.

The mathematical consistency of superstring theories requires (in most versions) that there are nine dimensions of space and one of time. Presumably, the extra six space dimensions are curled into a compact manifold of radius $\sim 10^{-33}$ cm, reminiscent of Kaluza–Klein theories. (It is possible that one or more or the dimensions have much larger sizes, leading to a realization of the large extra dimension theories described earlier.) In the most realistic closed string case the interactions are due to gauge interactions in the ten-dimensional space. The absence of mathematical pathologies requires an essentially unique gauge group called $E_8 \times E_8$. At energy scales less than the Planck scale an effective particle field theory in four dimensions emerges, with a supersymmetric gauge symmetry based on a subgroup of $E_8 \times E_8$. The effective group, the number of fermions, and their masses, mixings, etc., are all determined by the way in which the extra dimensions are compactified. In the open string theories the gauge and other interactions are determined by the configurations of the branes in the nine space dimensions.

In principle, superstring theories have no arbitrary parameters or other features, and most likely they yield completely finite (not just renormalizable) quantum theories of gravity and all the other interactions. That is, they are candidates for the ultimate "theory of everything." However, there are enormous numbers of possible ways in which the extra dimensions can compactify, and at present we do not possess the principles or mathematical tools to determine which is chosen. It is therefore not clear what the predictions of superstring theories really are (e. g., whether they lead to an effective supergravity GUT) or whether they correspond to the real world. In fact, there have been recent speculations that there is no selection principle, and that the choice of possible compactifications is essentially random out of an enormously large *landscape* of possible vacua. There may be different physics occurring in different regions of a very large chaotic Universe. In that case it is possible that supersymmetry is broken at a high scale, and the Higgs hierarchy problem is apparently solved by a fine-tuning, perhaps related to *anthropic* selection principles, i. e., that life could only develop in the subset of worlds for which the "fine-tuning" occurred.

## The Future

The standard model is extremely successful, but there is almost certainly new physics underlying it. There are many theoretical ideas concerning this new physics. Some of the most promising involve energy scales much larger than will ever be probed by direct experimentation, though they may still lead to testable low-energy predictions. Apart from theoretical ideas and new computational techniques (e. g., lattice calculations or efficient ways to calculate Feynman diagrams in QCD emerging from string techniques), many types of experiments will improve the tests of the standard model and search for manifestations of new physics. These include direct searches for new particles at high-energy accelerators, especially the high energy $pp$ LHC collider under construction at CERN, which will have enormous discovery reach, and a proposed $e^+e^-$ International Linear Collider (ILC), which would be complementary to the LHC by being able to carry out more precise measurements. There are also plans for a new generation of neutrino experiments to elucidate the details of the neutrino masses and mixings, and possibilities for searches for rare decays or interactions of muons, kaons, $B$ mesons, etc., that are forbidden or strongly suppressed in the standard model. Other probes include future high precision electroweak measurements and searches for electric dipole moments, magnetic monopoles, and proton decay. Finally, there has been an increasingly close connection between particle physics and cosmology. The dynamics of the early Universe was controlled by the elementary particles and their interactions, and refined studies of the large-scale structure of the Universe and of other relics from the big bang place severe constraints on new physics.

It is impossible to do justice to the subtlety and richness of this field in a brief survey, or to give their due to the thousands of researchers who have painstakingly uncovered the beautiful structure that is emerging. The interplay of imaginative experimentation and daring conjectures has been a source of wonder to all who have witnessed the growth and maturing of elementary-particle physics.

**See also:** Baryons; Compton Effect; Conservation Laws; Currents in Particle Theory; Electron; Feynman Diagrams; Field Theory, Axiomatic; Gauge Theories; Grand Unified Theories; Gravitation; Hadrons; Hypernuclear Physics and Hypernuclear Interactions; Hyperons; Inclusive Reactions; Isospin; Lattice Gauge Theory; Leptons; Lie Groups; Mesons; Muonium; Neutrino; Nucleon; Partons; Positron–Electron Colliding Beams; Positronium; Quantum Electrodynamics; Quantum Field Theory; Quarks; Regge Poles; Renormalization; S-Matrix Theory; Symmetry Breaking, Spontaneous; String Theory; Strong Interactions; *SU(3)* and Higher Symmetries; Supersymmetry and Supergravity; Weak Interactions; Weak Neutral Currents.

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Every other April the Particle Data Group’s updated “Review of Particle Properties”, which has many useful review articles, appears in *Physical Review D* or *Physics Letters B* and online at http://pdg.lbl.gov/.