UNIFIED PHYSICS

OPTICS

Electromechanical Technology Series

TERC EMT STAFF

DELMAR PUBLISHERS, MOUNTAINVIEW AVENUE, ALBANY, NEW YORK 12205

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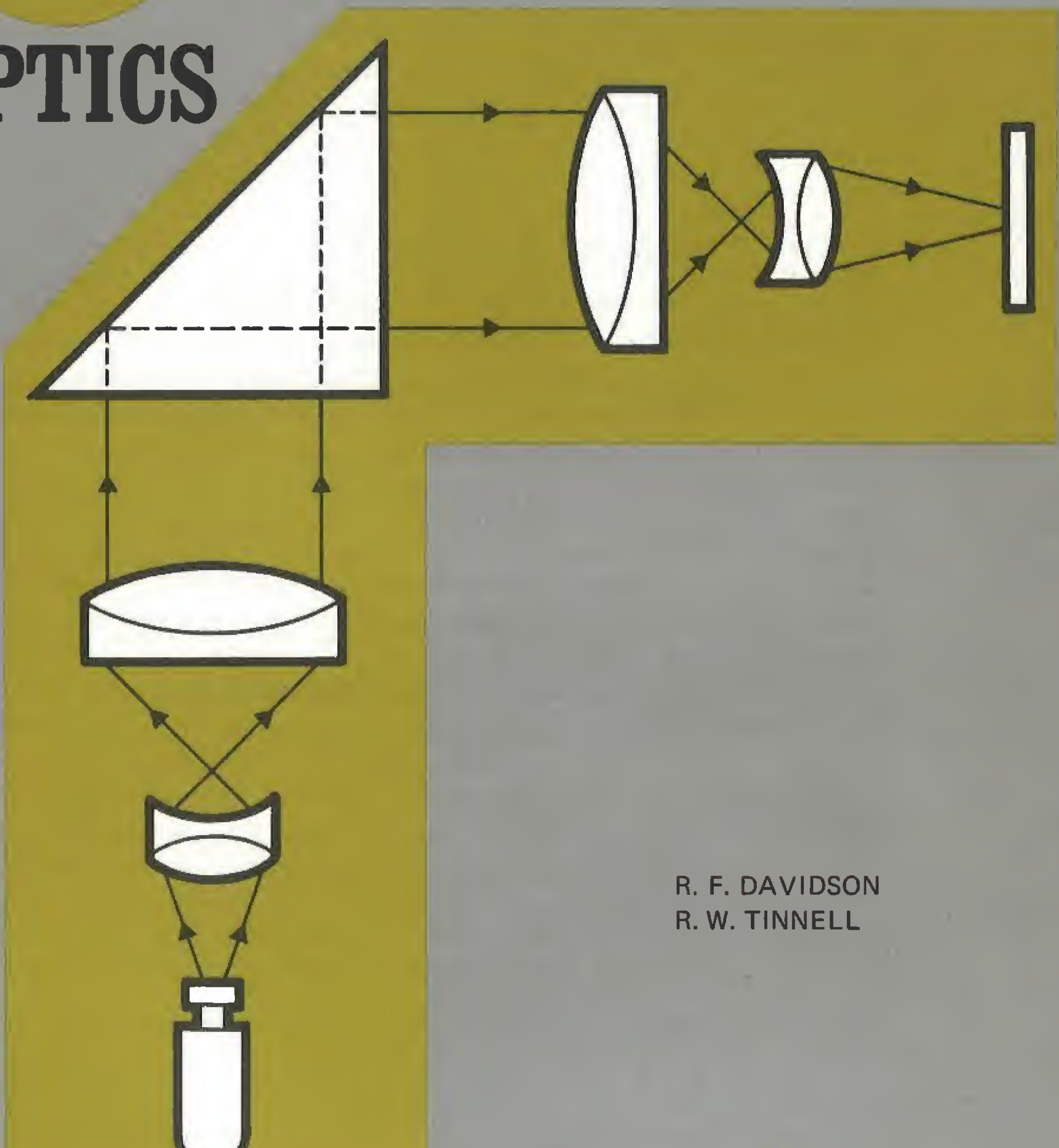

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DELMAR PUBLISHERS

Division of Litton Education Publishing, Inc.

Library of Congress Catalog Card Number:

76 - 170792

PRINTED IN THE UNITED STATES OF AMERICA

Published simultaneously in Canada by
Delmar Publishers, a division of
Van Nostrand Reinhold, Ltd.

The project presented or reported herein was performed pursuant to a grant from the U.S. Office of Education, Department of Health, Education, and Welfare. The opinions expressed herein, however, do not necessarily reflect the position or policy of the U.S. Office of Education, and no official endorsement by the U.S. Office of Education should be inferred.

Foreword

The marriage of electronics and technology is creating new demands for technical personnel in today's industries. New occupations have emerged with combination skill requirements well beyond the capability of many technical specialists. Increasingly, technicians who work with systems and devices of many kinds — mechanical, hydraulic, pneumatic, thermal, and optical — must be competent also in electronics. This need for combination skills is especially significant for the youngster who is preparing for a career in industrial technology.

This manual is one of a series of closely related publications designed for students who want the broadest possible introduction to technical occupations. The most effective use of these manuals is as combination textbook-laboratory guides for a full-time, post-secondary school study program that provides parallel and concurrent courses in electronics, mechanics, physics, mathematics, technical writing, and electromechanical applications.

A unique feature of the manuals in this series is the close correlation of technical laboratory study with mathematics and physics concepts. Each topic is studied by use of practical examples using modern industrial applications. The reinforcement obtained from multiple applications of the concepts has been shown to be extremely effective, especially for students with widely diverse educational backgrounds. Experience has shown that typical junior college or technical school students can make satisfactory progress in a well-coordinated program using these manuals as the primary instructional material.

School administrators will be interested in the potential of these manuals to support a common first-year core of studies for two-year programs in such fields as: instrumentation, automation, mechanical design, or quality assurance. This form of *technical core* program has the advantage of reducing instructional costs without the corresponding decrease in holding power so frequently found in general core programs.

This manual, along with the others in the series, is the result of six years of research and development by the *Technical Education Research Centers, Inc.,* (TERC), a national nonprofit, public service corporation with headquarters in Cambridge, Massachusetts. It has undergone a number of revisions as a direct result of experience gained with students in technical schools and community colleges throughout the country.

Maurice W. Roney

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Preface

The use of optics in science and technology has had a very long history. With the advent of the laser, new applications are appearing daily. Consequently, optics has become even more important to the technician.

These materials are intended to introduce both basic geometric optics and the laser. The student using these materials should have a working knowledge of technical mathematics including algebra and trigonometry.

The sequence of presentation chosen is by no means inflexible. It is expected that individual instructors may choose to use the materials in other than the given sequence.

The particular topics chosen for inclusion in this volume were selected primarily for convenience and economy of materials. Some instructors may wish to omit some of the exercises or to supplement some of them to better meet their local needs.

The materials are presented in an action-oriented format combining many of the features normally found in a textbook with those usually associated with a laboratory manual. Each experiment contains:

1. An INTRODUCTION which identifies the topic to be examined and often includes a rationale for doing the exercise.
2. A DISCUSSION which presents the background, theory, or techniques needed to carry out the exercise.
3. A MATERIALS list which identifies all of the items needed in the laboratory experiment. (Items usually supplied by the student such as pencil and paper are normally not included in the lists.)
4. A PROCEDURE which presents step-by-step instructions for performing the experiment. In most instances the measurements are done before calculations so that all of the students can at least finish making the measurements before the laboratory period ends.
5. An ANALYSIS GUIDE which offers suggestions as to how the student might approach interpretation of the data in order to draw conclusions from it.
6. PROBLEMS are included for the purpose of reviewing and reinforcing the points covered in the exercise. The problems may be of the numerical solution type or simply questions about the exercise.

Laboratory report writing forms an important part of the learning process employed in this manual. Consequently, students should be encouraged to write at least a brief report for each exercise performed.

This volume is one of a series developed by the Technical Education Research Center at Oklahoma State University under the direction of D.S. Phillips and R.W. Tinnell. The principal authors of this volume were R.F. Davidson and R.W. Tinnell.

An Instructor's Data Book is available for use with this volume. Kenneth Cathy, E.W. Skouby, and Robert D. Bingham were responsible for testing the materials and compiling the instructor's data book for them. Other members of the TERC staff made valuable contributions in the form of criticisms, corrections, and suggestions.

It is sincerely hoped that this approach to optics will make the study of the subject stimulating, interesting and valuable to the student and instructor alike.

The TERC EMT Staff

Contents

experiment 1 *LIGHT SOURCES*

INTRODUCTION. Optics and optical instruments are becoming more and more important to modern technology. In this experiment we will examine some of the important types of light sources and their operating characteristics.

DISCUSSION. Light is radiation that can produce visual stimulation in a human eye. It has a frequency spectrum extending from about 395×10^{12} Hz to about 790×10^{12} Hz.

In a vacuum, light propagates at about 3×10^{6} meters per second. For light (or any other wave), the distance (λ) that it travels during one cycle is called a wavelength and is related to frequency and velocity by

$$\lambda = \frac{v}{f} \tag{1.1}$$

Using this relationship, the range of light wavelength in a vacum varies from 7600×10^{-10} meters to 3800×10^{-10} meters.

The quantity 10^{-10} meters is frequently called an Angstrom unit (Å). Using these units we would say that the range of light wavelengths is about λ = 3800 Å to 7600 Å.

The various wavelengths within the light spectrum represent different colors to the eye. Figure 1-1 shows how the color of light varies with wavelength.

There are several things worth noting about these curves. First notice that the colors vary from violet (short λ, high f) to red (long λ, low f). The region of wavelengths just shorter than violet is called the ultraviolet region while just beyond red is the infrared region.

Also notice that all of the color spectrum can be constructed using only red, blue, and green light. Some optical applications (color TV, for example) use this fact very dramatically.

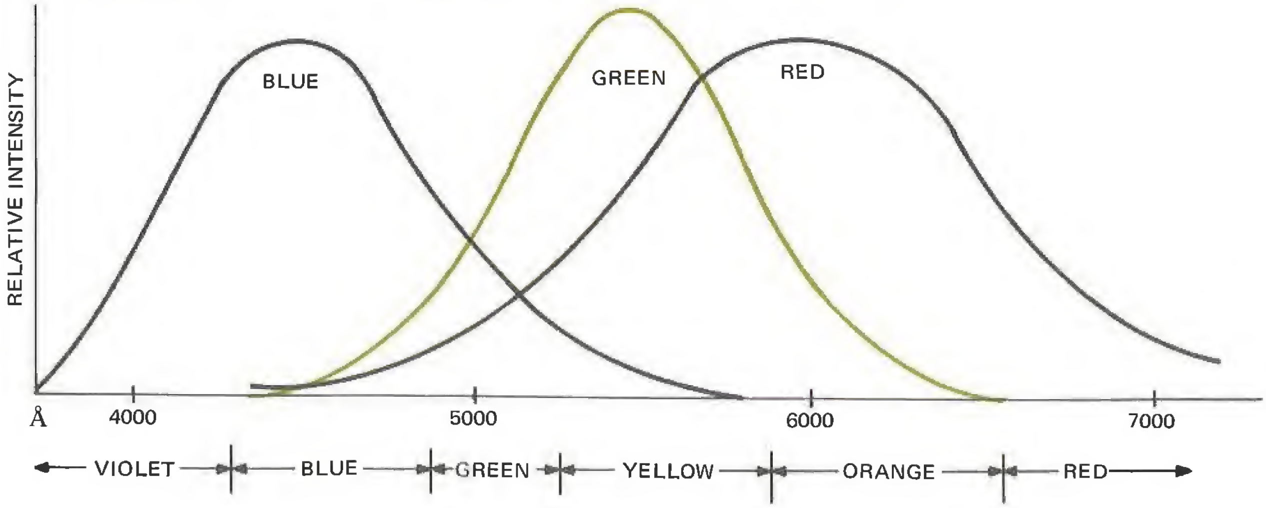

Fig. 1-1 Colors of Various Wavelengths

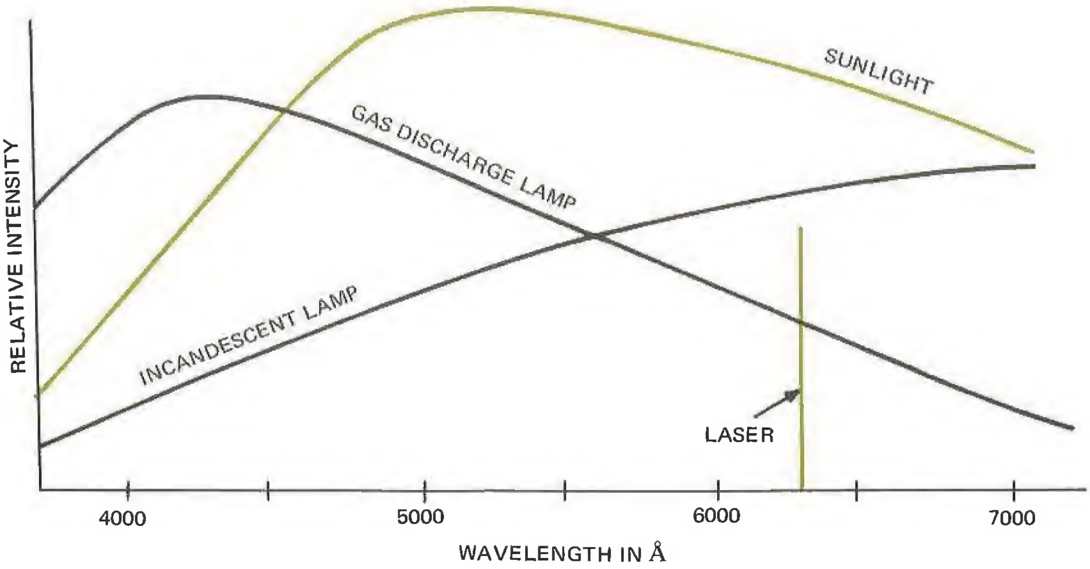

Fig. 1-2 Spectrums of Various Types of Sources

All optical applications require a light source of some type. The source itself may be the sun, an incandescent lamp, a gas discharge lamp or, perhaps, a laser device. In any case the source chosen will have spectral characteristics which influence the application results. Figure 1-2 shows typical spectrums of several light sources.

Notice that gas discharge lamps tend to have more ultraviolet while sunlight and incandescent lamps tend to have more infrared. Laser light is normally single frequency (monochromatic) energy.

In analyzing optical problems we frequently assume that light from our source comes from a single point. We call such an idealized source a point source. Let's look at the light from such a source for a moment. Figure 1-3 shows a diagramatic representation of a point source and the light coming from it. Let's suppose that the source emits light equally in all directions. If this is the case and the light travels at the same velocity in all directions, then the light would spread out in a spherical pattern.

The light intensity (I_o) of the source would be evenly distributed over the surface of this spherical pattern as it moved away from the source. Now suppose we intercept the light in one unit of wavefront area (A) a distance (r) from the source. The intensity of the light in this unit of area will be

$$I = I_o \frac{A}{A_T}$$

where A_T is the total surface area of the sphere having a radius of r. Since the surface area of this sphere is

$$A_T = 4\pi r^2$$

we have, for the intensity intercepted,

$$I = I_o \frac{A}{4\pi r^2} \qquad \textbf{(1.2)}$$

In other words, for a given interception area the intensity received from a given point source is proportional to $1/r^2$. Because of this, point source radiation is sometimes called inverse square law radiation.

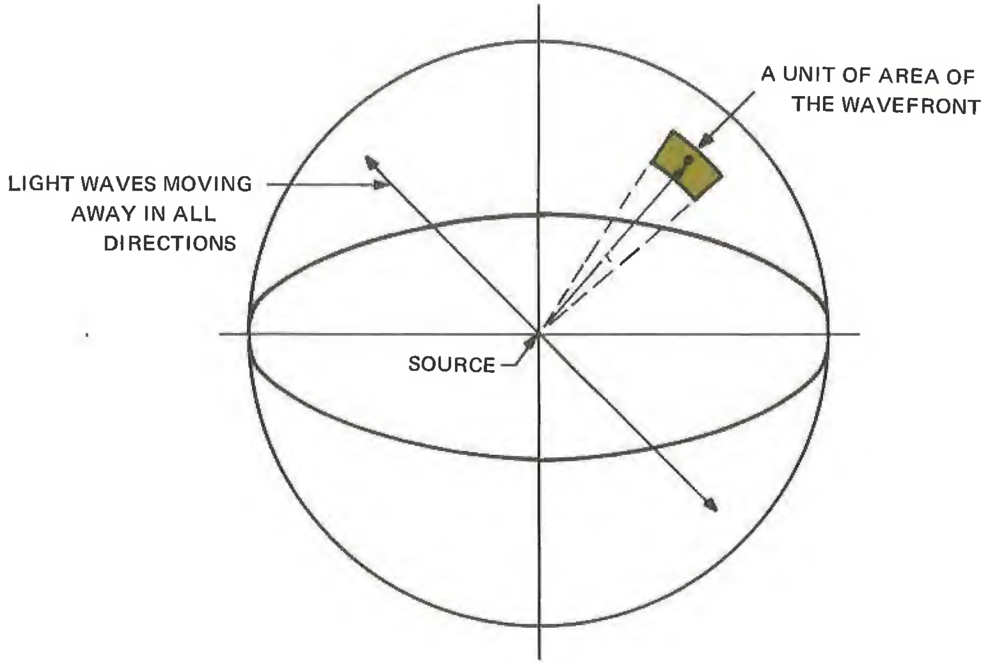

Fig. 1-3 A Point Source and Spherical Wavefront

If we use mirrors to concentrate the light from the source, equation 1.2 becomes

$$I = I_o \frac{KA}{4\pi r^2} \quad \textbf{(1.3)}$$

where K is the concentration factor. If the light from the source is all concentrated into one-half of a sphere, then K would be 2. Or if the light covered only 1/8 of the sphere, K would be 8.

Notice that we have not changed the inverse square relationship by simply concentrating the light. In fact, if the light is diverging from the source at all (that is, if the beam is cone-shaped, no matter how small the cone angle) the inverse square law applies.

Actually, most practical sources are not point sources; however, it is usually possible to keep the effective source size small enough that it can be considered a point source.

MATERIALS

1 Optical bench

1 Collimated white light source

1 Carriage

1 Screen holder

1 White screen

1 3 x 5 file card

1 Steel rule approx. 15 cm long

1 Sheet of linear graph paper

PROCEDURE

1. Set up the optical bench as indicated in figure 1-4. You may wish to use nontoxic smoke in this experiment. It will allow you to see the light distribution better.

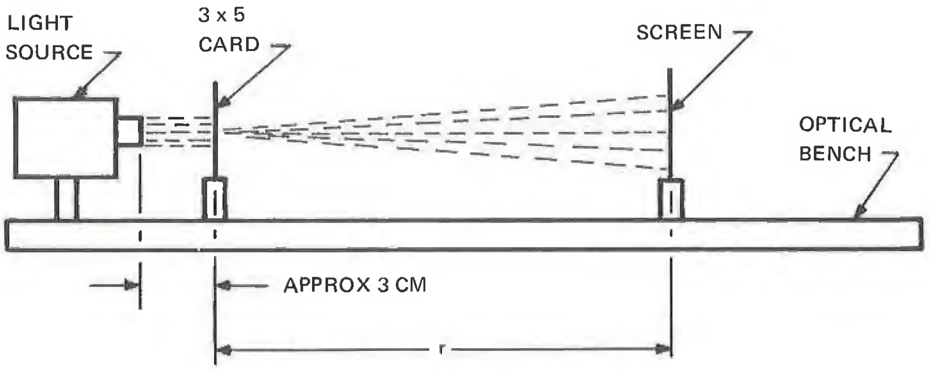

Fig. 1-4 The Experimental Setup

2. Using a sharp pencil point, punch a hole about 1 mm in diameter in the center of the 3 x 5 file card.

3. Position the card so that the hole is in the center of the beam from the light source.

4. Move the screen up until it is about 3 cm from the 3 x 5 card.

5. You should be able to see a circle of light on the screen.

6. As carefully as you can, measure the diameter (d) of the circle of light on the screen in millimeters.

7. Record the spacing (r) between the 3 x 5 card and the screen.

8. Increase the spacing between the 3 x 5 card and the screen by 3 cm.

9. Repeat steps 6 and 7.

10. Repeat steps 8 and 9 until you have at least ten sets of data.

11. Compute the area (A) of the circle of light for each set of data using

$$A = \frac{\pi d^2}{4}$$

12. Square each of your values of the spacing between the 3 x 5 card and the screen. Record the results (r^2).

13. Plot a curve of area (A) versus the square of the spacing (r^2).

d (mm)	A (cm^2)	r (cm)	r^2 (cm^2)

Fig. 1-5 The Data Table

ANALYSIS GUIDE. Explain how your results tend to confirm the inverse square law. That is, refer to equation 1.3 and notice that we were holding everything constant except A and r^2. What is the relationship between A and r^2 under these conditions? Tell why you think the system you used should act like a point source.

PROBLEMS

1. What colors correspond to wavelengths of 4500 Å, 5000 Å, 6000 Å, and 7000 Å?

2. What are the frequencies in Hz of the wavelengths in problem 1?

3. What color do you think the laser source in figure 1-2 would appear to be?

4. Based on figure 1-2, explain how a combination of incandescent and arc discharge lamps might be used to approximate sunlight.

5. A point source produces a light intensity of 10 candlepower/m^2 when it is two meters from the screen. How far away will it be when the intensity is 3 candlepower/m^2?

6. Two lamps produce the same light intensity on a screen. One of the lights is rated at 8 candlepower and is 4 meters from the screen. The other lamp is 6 meters from the screen. What is the candlepower of the second lamp?

experiment 2 *LASER SOURCES*

INTRODUCTION. With the advent of the laser, a most useful source of monochromatic coherent radiations was achieved. This type of light radiation is most useful in industrial applications and research. It is the purpose of this experiment to look at some of the uses and effects of this type of radiation.

DISCUSSION. Before things get too confusing, terminology-wise, let's attempt to clear up a few points. In optical discussions we often use the term "ray". Actually, a practically zero-width beam of light makes most of the constructions and derivations of geometrical optics quite simple. Although used in some tests, *wave* constructions illustrating reflection and refraction become quite complex. The most popular technique is that of considering light to travel in a straight line, in other words, "ray" tracing. However, with devices like slits and polarizers, the "ray" concept is of little use. In such cases we must often look a little more closely at the nature of light and consider it to be electromagnetic waves. This is really a little closer to the actual character of light.

Light, depending upon the circumstances, can behave either as particles or as waves. For instance, in the phenomenan of interference or of diffraction, we usually regard light as being a wave for reasonable simplicity in analysis. However, when dealing with quantum effects (lasers, spectra, photo-electricity) we frequently consider light as being composed of particles termed photons. This does not mean that light behaves one way one time and another way the next. All it means is that the *behavior* of light under one set of conditions may be more easily described by a wave, while under another set of conditions the easiest description is from a particle viewpoint.

Perhaps the most widely held view of the nature of light is that of the "wave packet" or wave trains. In other words, instead of a continuous wave, such as encountered in radio, light is considered to consist of bundles or bursts of electromagnetic energy. You may wonder why we do not see these bundles or wave packets. One reason is that from an ordinary light source there are many millions of these wave packets emitted each second so that to the ordinary eye, the light appears to be continuous. A wave packet is illustrated in figure 2-1.

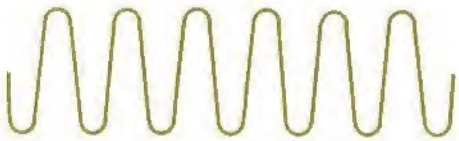

Fig. 2-1 Characterization of Light as a Wave Packet

Whether you use the continuous wave concept or the wave packet concept to explain an observed effect depends upon the particular experiment or situation. However, the wave packet or bundle of electromagnetic energy view is by far the most popular among physicists.

Ordinary white light consists of many different wavelength components (red, blue, green, etc.). Thus, we say that white light is polychromatic. By employing suitable filters or by using certain gases, such as in the case of a neon sign, we could achieve almost monochromatic (single frequency) operation. However, a monochromatic source of this type would still not be coherent. The main reason is that the light is emitted due to transitions between energy levels of different atoms. That is, electrons in a material's atoms are constantly rising to and dropping from different energy states. When an electron drops from a higher to a lower energy state, a photon or wave packet of light is emitted. The relation for an emitted photon caused by an electron dropping from a higher to lower state is

$$E_2 - E_1 = \frac{hc}{\lambda} \qquad (2.1)$$

where E_2 - E_1 is the upper minus the lower energy level, h is Planck's constant, λ is the wavelength of the emitted photon and c is the velocity.

Now, with ordinary monochromatic sources, these photons are emitted at random many millions of times per second. That is, there is no synchronization between the emission of one photon and the emission of another. Thus, because of this randomness of photons, we say that the source is incoherent.

The laser, on the other hand, emits monochromatic coherent radiation. The photons emitted by this source are single frequency and are all in phase with one another. This is illustrated in figure 2-2 where the top three waves, while of the same frequency, are not coherent. The lower three are both single frequency and coherent.

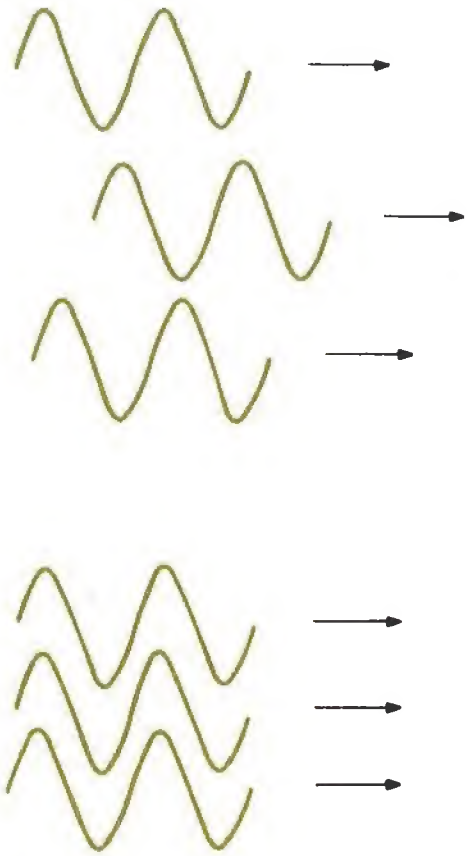

Fig. 2-2 Incoherent and Coherent Wavetrains

The details of laser operation can be found in a wide variety of books and will not be presented here. All we are concerned about is the type of light emitted, monochromatic and coherent.

The laser, by virtue of its type of emitted light, offers an interesting variety of optical effects which are normally difficult to observe with ordinary sources. Due to the completely random emission of wave packets by normal sources, some interference effects are not noticeable, whereas with the "in phase" emissions of the laser, they are. Some of these coherent interference effects can be quite troublesome.

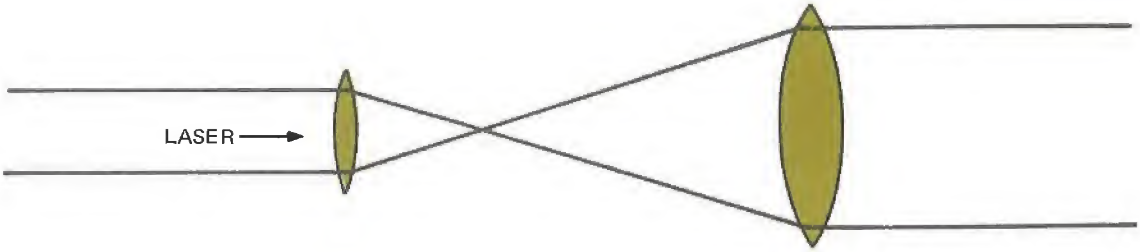

Fig. 2-3 Collimator Using Converging Lenses

You probably have noticed that the laser emits a very narrow and highly directional beam. The narrowness is due to several power and efficiency design considerations, while the direction is a property of coherent radiation. For all intents and purposes, the beam is usually parallel (no divergence). Some spreading or divergence of the beam can be noticed at very large distances. However, we will assume the beam to be nearly perfectly parallel or *collimated*.

In some cases, the narrowness of the beam is undesirable, so we must widen and re-collimate the beam. The general technique is to use one lens to diverge the beam and another to collimate it with a wider diameter. This is illustrated, using two converging lenses, in figure 2-3. A more popular technique is shown in figure 2-4.

In practice, however, the lenses are not simple lenses but are compound. One reason for this is to reduce spherical aberration. Some aberration effects will be produced in this experiment, mainly due to lens quality. If you use high quality corrected lenses, you should be able to observe a simple Gaussian intensity distribution if the widened and collimated beam is observed on a screen. A Gaussian intensity distribution is one in which the pattern is circular with the intensity dropping off as $1/r^2$ from the center. This, by the way, is the main oscillatory mode of the laser so that by widening the beam, the cross-sectional intensity distribution remains the same.

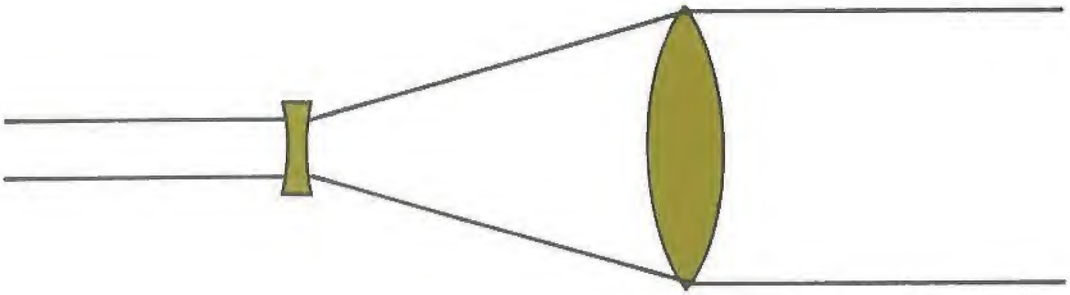

Fig. 2-4 Collimator Using Converging and Diverging Lenses

A couple of other points should be mentioned. **Above all else, do not, under any circumstances, look directly into the laser!** Even though the laser used here is a low output type, the intensity is still enough to cause retinal burns. Also, do not look at mirrored reflection of the laser beam, that is, a reflection from a mirror or polished metal surface. You may observe the laser by watching the reflection from a piece of dull-white paper, the meter stick, or a ground glass screen.

MATERIALS

1 He-Ne laser

3 10mm diameter negative lenses of different types

3 Positive lenses of different types

1 Ground glass screen

1 White screen

1 Screen holder

2 Lens holders

3 Carriages

1 Optical bench

PROCEDURE

1. Turn the laser on and aim it at a blank wall, preferably with a slightly rough surface. Note the observed pattern. **Do not under any circumstances look directly into the laser!**
2. Move your head toward, away, and from side to side with respect to the spot on the wall. Note any observed changes in the pattern as you move.
3. Turn the laser off and set up the optical bench with the laser at one end, the ground glass screen at the other.
4. Turn the laser on and note the pattern on the screen as to any diffusion present.
5. Repeat steps 3 and 4 with the white screen.
6. Remove the screen and align the optical bench axis parallel with the laser beam.
7. Place the white screen at the end of the bench opposite the laser.
8. Using the assorted lenses *design* experimentally at least three collimators, one of which uses two converging lenses. Read the following steps before designing the collimators.

THE FIRST COLLIMATOR

THE SECOND COLLIMATOR

THE THIRD COLLIMATOR

Fig. 2-5 Sketches of the Results

9. Check the collimation of each setup in step 8 by noting whether the enlarged beam changes diameter as the screen is moved toward and away from the collimating lenses. If the widened beam does change diameter it is not properly collimated. You may wish to use nontoxic smoke in this experiment so that you can see the light better.

10. Of the three collimators you design, note which gives the best intensity distribution on the screen. Note also the resultant beam diameter of each.

11. Draw each of your collimators to scale. For each, list the lens separation and the final enlarged beam width.

12. The best collimator should give a Gaussian intensity distribution and have no observable beam divergence.

ANALYSIS GUIDE. In the analysis of this experiment you should discuss the appearance of the patterns in steps 1 and 2. Along with the sketches of the three collimators, you should discuss any anomalies in the observed intensity distribution and their probable causes. Discuss also which collimator produced the largest beam width and why.

PROBLEMS

1. Is it true that light acts like a wave sometimes and like particles at other times?

2. Describe a "wave packet."

3. Is white light polychromatic or monochromatic?

4. What is the meaning of "coherent" as applied to light?

5. What is a collimator?

6. What is a Gaussian intensity distribution?

experiment 3 REFLECTION I (Plane Surfaces)

INTRODUCTION. Many industrial optical systems involve the reflection of light at plane surfaces. The purpose of this experiment is to investigate the phenomenon of reflection at a plane surface and some of its practical applications.

DISCUSSION. A *ray* of light is said to be *reflected* when it is sent back into the same medium from which it came.

There are two kinds of reflection, regular and irregular. If a beam of light falls upon a perfectly flat and highly polished piece of metal, a plane mirror or the surface of still water, it will undergo regular reflection. That is, the beam will take on a definite course. However, if the beam falls on a surface which is not perfectly plane, such as paper, unpolished metal, or powdered glass, the beam will undergo irregular reflection and each ray contained in the beam will proceed in a different direction. The beam undergoing irregular reflection is said to be scattered or diffused. In both cases, each individual ray contained in the beam obeys the laws of reflection which will be discussed later.

As an example of regular reflection, if a rubber ball is dropped vertically onto a polished horizontal surface, the ball will return along its original path. If the ball is thrown down at an angle (assuming a perfectly elastic collision) it will rise along a path having the same slope as before impact and will not deviate left or right.

Irregular reflection may be seen in the same way but instead of a smooth floor, suppose the surface is angular stones. If the ball is again thrown down, it is no longer possible to tell in what direction it will rebound. The direction will depend upon the slope of the point of contact.

If an unpolished surface such as a sheet of paper is sufficiently magnified, it will resemble a field of boulders. Consequently, light striking the paper is irregularly reflected or scattered.

There are two basic laws of reflection. These are:

(1) The incident and reflected rays will lie on opposite sides of the normal at the point of incidence, and all three are in the same plane.

(2) The angles of incidence and reflection will be equal.

These laws are shown in figure 3-1.

Consider the situation shown in figure 3-2. If A is a luminous point or object, rays of light proceed outward from it in all directions. Two of these rays, B and C, strike the reflecting surface JK and after reflection will continue to diverge. Lines $\overline{MN}$ and $\overline{QP}$ are normals to the points of incidence while L and S are the points of incidence for rays B and C, respectively.

If the two rays are extended behind the reflecting surface they will intersect at A′. This point is called the *image* of A and is the position that A would appear to occupy to an observer at point B or C who is looking towards the surface. Since rays B and C only appear to come from A′, A′ is said to be a *virtual* image. This type of image is always

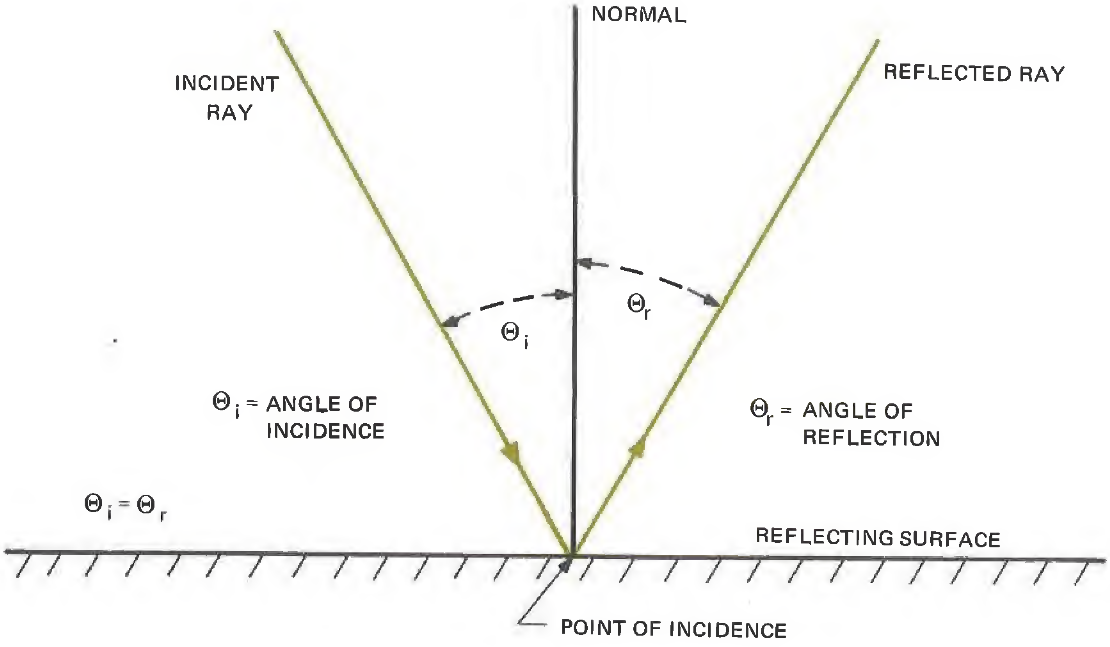

Fig. 3-1 Laws of Reflection

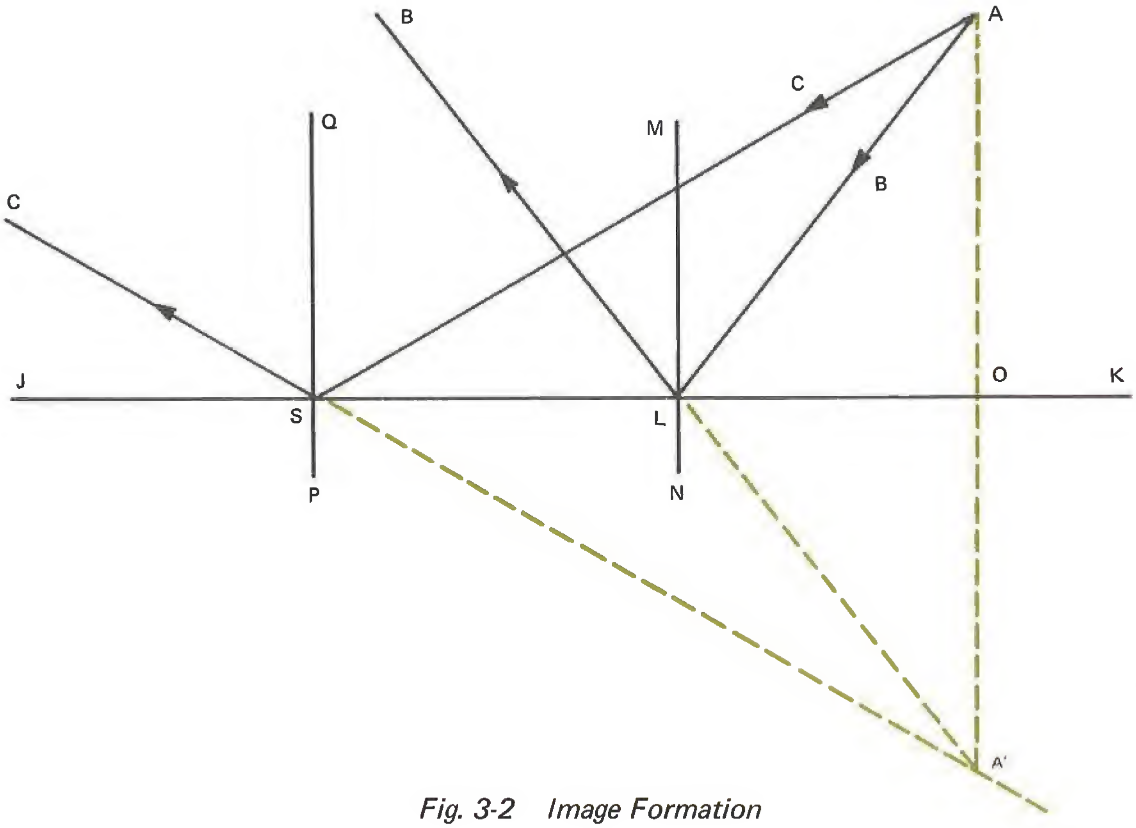

Fig. 3-2 Image Formation

produced when a divergent beam is reflected. The angle SLA is equal to the angle SLA′ and the angle LSA is equal to LSA′. Also since $\overline{SL}$ is common to both triangles, we see that

$$\overline{LA} = \overline{LA'}$$

In the same way

$$\angle ALO = \angle A'LO$$

and $\angle AOL = \angle A'OL$

but $\overline{LO}$ is common

so $\overline{AO} = \overline{A'O}$

Since $\overline{AO} = \overline{A'O}$, the image is as far behind the reflecting surface as the object is in front of it.

From the second law of reflection, we see that if a mirror is turned through an angle of 10°, for example, the angle of incidence of a light of fixed direction must change by 10°. The angle of reflection must also change by 10° and thus the angle between the incident and the reflected rays will change by 20°. Therefore, if the mirror moves with some angular velocity ω, the angular velocity of the reflected ray will be 2ω.

For example, if a mirror is held perpendicularly to an incident ray, the ray will be reflected back along its own path. If the mirror is turned an angle of 45°, the reflected ray will make an angle of 90° with the incident ray. In other words, the ray rotates through an angle which is twice the rotation angle of the mirror.

The laws of reflection may be usefully applied in measuring very small angular movements such as:

a) Sub-dividing degrees into minutes and seconds.

b) Detecting very small electric currents.

c) Observing the expansion of metals.

Let's now look at a couple of practical uses for reflection. Suppose we have a small magnet which is free to rotate in the vertical plane. Attached to the magnet at a point M is a small, lightweight mirror, as illustrated in figure 3-3.

Let $\overline{st}$ be the position of the magnet-mirror assembly at rest, and $\overline{OM}$, a ray of light falling normally on the mirror at point M. In this position the reflected ray is also along $\overline{OM}$.

If the magnet moves through an angle Θ to position $\overline{s't'}$, the normal to the mirror, $\overline{MN}$, must move through Θ. The angle of incidence is now $\angle OMN = \Theta$, so the angle of reflection must also equal Θ. The reflected ray travels along $\overline{MX}$ making an angle of 2Θ with the incident ray $\overline{OM}$.

The reflected ray thus acts as a weightless lever arm which rotates through twice the angle of the magnet rotation. Its position can be observed by allowing it to fall upon a graduated scale and watching the motion of the spot of light. By placing the scale at a sufficient distance from the mirror, very small movements of the magnet may be observed by the amplified movements of the spot of light on the scale. A device which utilizes this principle is an optical galvanometer.

Let's now consider another application depicted by figure 3-4.

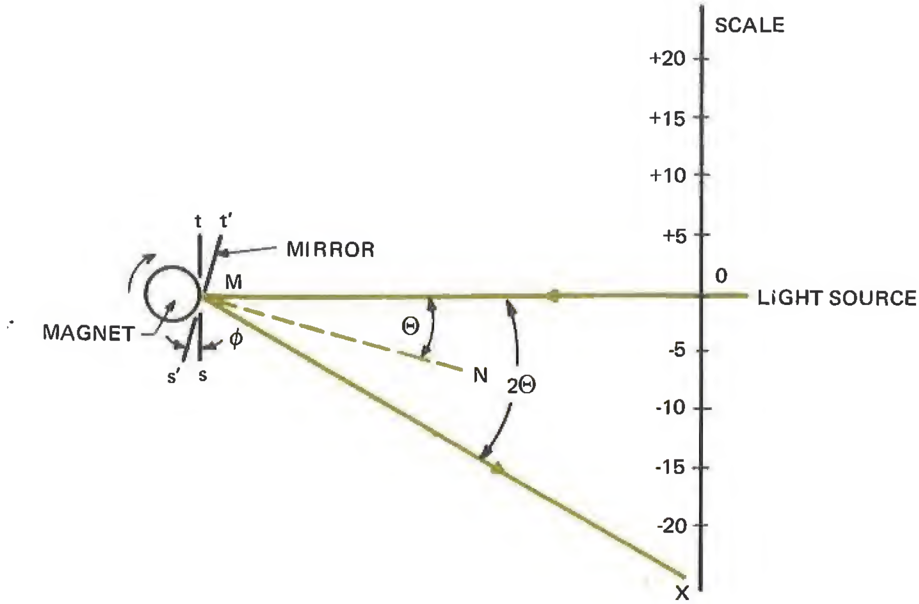

Fig. 3-3 Optical Lever

In this apparatus a rod of metal is placed in a horizontal position and rests on rollers in a trough. The trough contains oil or water which is heated. The lever BD is vertical and pivoted at D. One end of the lever is against the rod at B, a mirror is attached to the pivot point at D. Light from a source at 0 falls on the mirror and is reflected back onto the scale. Points S and O coincide prior to heating. As the rod heats up, it expands and DB is turned into position DB′, the mirror will rotate counter-clockwise through some angle Θ, and the reflected beam will rotate through 2Θ as shown.

In either of the previous examples, it would seem logical to increase the sensitivity by moving the scale farther away from the mirror so that the linear distance from O to the spot would increase for a constant angle. While this can be done in practical situations, the intensity of the source, the dispersion or spreading of the beam and the random small vibrations of the mirror limit the extent to which this can be done.

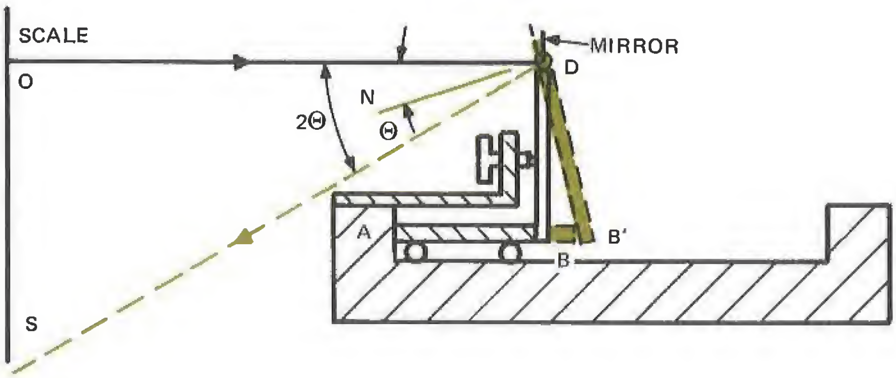

Fig. 3-4 Optical Lever on Metal Expansion Apparatus

MATERIALS

1 Front-surfaced plane mirror

4 Straight pins

1 Straightedge

1 Protractor

PROCEDURE

1. On a sheet of white paper, construct line $\overline{MZ}$ and set the reflecting surface of the mirror along this line. Refer to figure 3-5 as you perform these procedural steps.

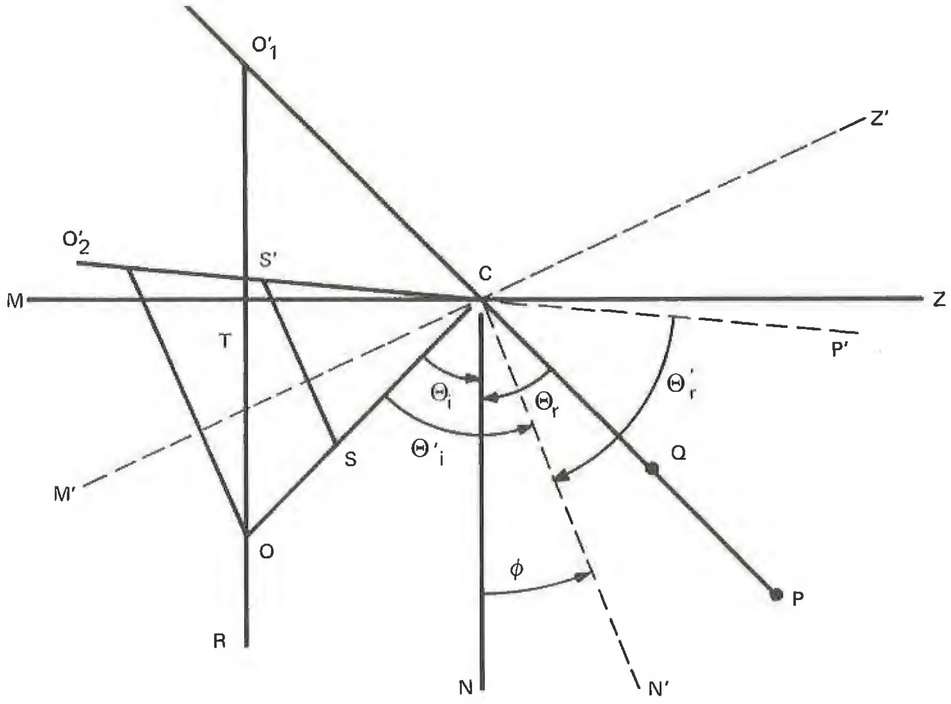

Fig. 3-5 Geometric Construction with the Plane Mirror

2. At some distance (say two inches) from the silvered surface, place a pin. This will be the object. Label the location of the pin, point O.

3. At a convenient distance to one side of O, place pins (points P and Q) so that P, Q, and the image of O are in a straight line.

4. Place another pin (R) in front of O such that O is between R and the mirror and that O, R, and the image of O are in a straight line.

5. Remove the mirror and pins. Construct a straight line through P and Q and extend this line behind the mirror.

6. Construct a straight line through R and O and extend this line behind the mirror until it intersects the line drawn in step 5. Label the intersection point O'_1 and the intersection points to line $\overline{MZ}$ as C and T as shown in figure 3-5.

7. Construct line $\overline{OC}$ and the normal to the mirror at C, line $\overline{CN}$.

8. Measure and record the angles of incidence (Θ_i) reflection (Θ_r) and the lengths of $\overline{OT}$ and $\overline{O'_1T}$.

9. Rotate the mirror about point C by some angle ϕ (line $\overline{M'Z'}$).

10. Place the object pin at the original point O and another pin S along the line OC drawn in step 7.

11. Place a pin at P′ such that the new image of O (O'_2), the image of S (S′), and pin P′ are in a straight line.

12. Construct line $\overline{P'C}$ and the normal to $\overline{M'Z'}$, $\overline{CN'}$.

13. Measure and record the new angles of incidence and reflection, Θ'_i and Θ'_r and the rotation angle ϕ.

Θ_i	Θ_r	$\overline{OT}$	$\overline{O'_1T}$	Θ'_i	Θ'_r	ϕ

Fig. 3-6 The Data Table

ANALYSIS GUIDE. In the analysis of these data you should compare the angles of incidence and reflection and the object and image distances before mirror rotation. You should also compare the angles of incidence before and after rotation and relate these comparisons to the angle of rotation. Compare the angle between the incident and reflected rays before and after rotation to the mirror rotation angle. Explain this relationship.

PROBLEMS

1. What is meant by the statement, "An image is just as far behind a mirror as the object is in front?"

2. Explain why pin S was used and also why P′ must be aligned with O'_2 and S′.

3. Would the theory be justified if the mirror had been rotated about some other point than C? Explain.

4. If the mirror had rotated at ω = 0.5 radians per second, the reflected ray would have rotated with respect to the incident ray at ____________.

5. The angular amplification of the optical lever is always what value?

experiment 4 REFLECTION II (Curved Surfaces)

INTRODUCTION. It is often not only desirable to reflect or change the direction of a light beam, but also to either converge or diverge it. In this way we can use reflecting surfaces as image or intensity amplifiers. The purpose of this experiment is to investigate the reflection of light at a curved surface and some of its practical implications.

DISCUSSION. Figure 4-1 illustrates a concave mirror with a constant radius of curvature R. The concave side is to the left, the center of curvature is at C.

Before proceeding further, it is well to adopt some sign conventions. In general, an object or image point may lie on either side of a reflecting surface. Also, objects and images of finite size may extend above or below the axis of the surface. By adopting and adhering to a sign convention, we need not develop special formulas for a large number of cases. One formula can cover all the cases with the

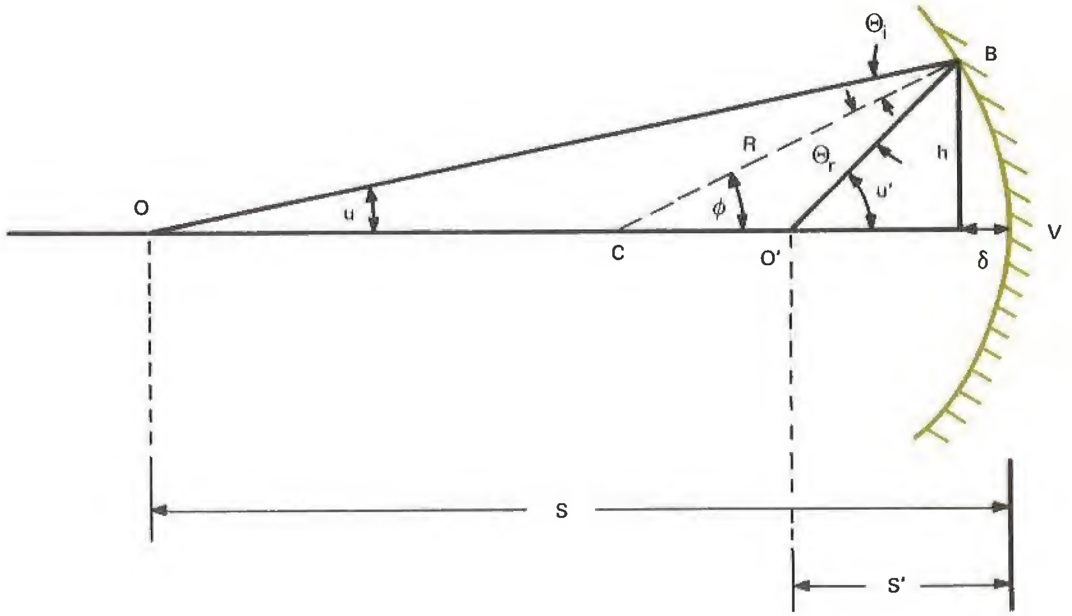

Fig. 4-1 Concave Mirror with Object and Image Points

algebraic sign telling us relative directions (left or right, up or down). The sign conventions we shall use are:

1. All distances are measured along the axis *from* the surface *to* the point in question.
2. An object distance S is positive if the direction from the surface to the object is *opposite* to that of the *oncoming* light.
3. An image distance S′ is positive if the direction from the surface to the image is the *same* as that of the *ongoing* light.
4. A radius of curvature R is positive if the direction from the vertex of the surface to the center of curvature is the *same* as that of the *ongoing* light.
5. An object or image lateral dimension *above* the axis is positive.

Let us now refer back to figure 4-1. For the triangles OBC and O′BC, we can see that

$$\phi = u + \Theta_i$$

and

$$u' = \phi + \Theta_r$$

Therefore since

$$\Theta_i = \Theta_r.$$

$$\phi - u = u' - \phi$$

or

$$u' + u = 2\phi$$

Now h is the height of B above the axis while δ is the shortest distance from the vertex V to the foot of h. We then have

$$\tan u = \frac{h}{S - \delta} \quad \text{and} \quad \tan u' = \frac{h}{S' - \delta}$$

while

$$\tan \phi = \frac{h}{R - \delta}$$

These equations will lead to complicated relations so let's make some approximations. If the angle u is small, then the angles u′ and ϕ will also be small. The tangent of a small angle is very nearly equal to the angle itself expressed in radians. Also if u is small and the radius of curvature R is large compared to δ (as is usually the case), we may neglect δ and write

$$u = \frac{h}{S} \quad u' = \frac{h}{S'}, \text{ and } \phi = \frac{h}{R}$$

Moreover, since

$$u' + u = 2\phi$$

then

$$\frac{h}{S'} + \frac{h}{S} = \frac{2h}{R}$$

or

$$\boxed{\frac{1}{S'} + \frac{1}{S} = \frac{2}{R}} \qquad \textbf{(4.1)}$$

This is the general relation between the object distance S, the image distance S′, and the radius of curvature R. This equation states that all rays from O making sufficiently small angles with the axis will intersect at O′. Such rays are called *paraxial* rays. If the object O were an extremely large distance from the mirror, all of the rays from O intercepted by the mirror would be, for all intents and purposes, parallel to one another. This would

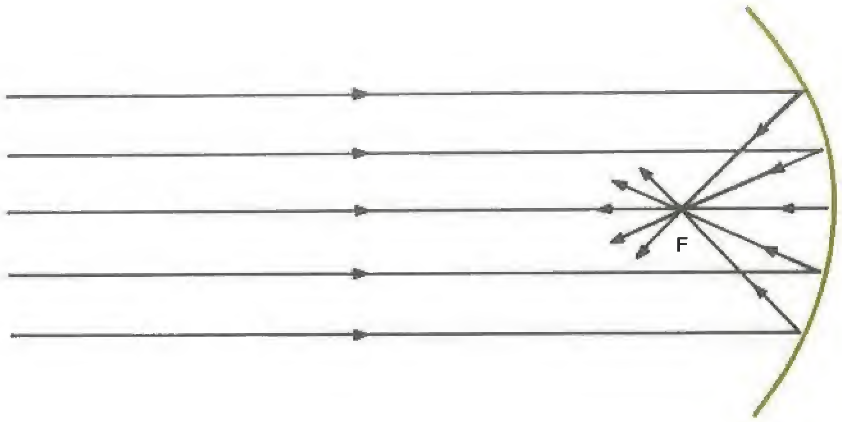

Fig. 4-2 Incident Rays Parallel to Axis Converge at $S' = \frac{R}{2} = f$

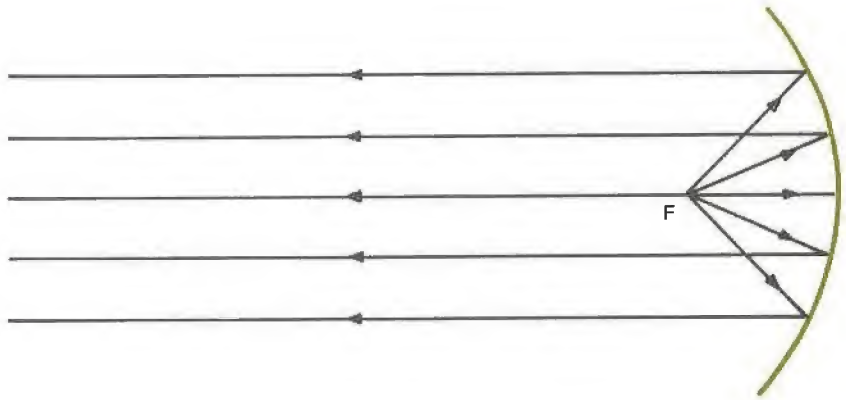

Fig. 4-3 Rays From Object Point at $S = \frac{R}{2} = f$ *are Parallel to Axis After Reflection*

correspond to $S = \infty$. For this case we would have

$$\frac{1}{\infty} + \frac{1}{S'} = \frac{2}{R}$$

or

$$S' = \frac{R}{2}$$

That is, the image distance equals one-half the radius of curvature R and has the same sign.

Conversely when $S' = \infty$, then

$$S = \frac{R}{2}$$

This is illustrated in figures 4-2 and 4-3 for a concave mirror.

The point F in figures 4-2 and 4-3 is called the *focal point* and the distance from the vertex to F is called the *focal length* and is represented as f.

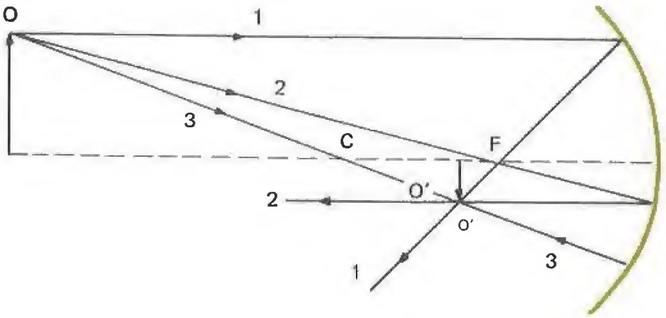

Fig. 4-4 Rays Used in Graphical Method of Locating an Image for Concave Surfaces

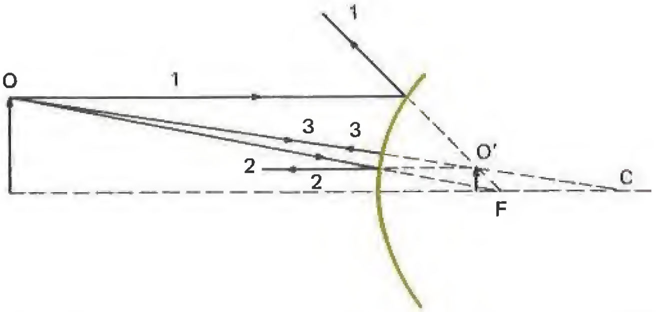

Fig. 4-5 Rays Used in Graphical Method of Locating an Image for Convex Surfaces

From equation 4.1 we have

$$\frac{1}{S'} + \frac{1}{S} = \frac{2}{R}$$

but $$f = \frac{R}{2}$$

Therefore,

$$\boxed{\frac{1}{S'} + \frac{1}{S} = \frac{1}{f}} \qquad \textbf{(4.2)}$$

We are now in a position to locate graphically the image of an object of finite size. This consists, as with the plane mirror, of finding the point of intersection, after reflection from the curved mirror, of a few rays diverging from some point on the object *not* on the mirror axis. We assume that *all* rays originating from this object point will intersect at the same point. The three rays whose paths are probably the easiest to trace are:

1. *A ray parallel to the axis.* After reflection this ray passes through F for a concave mirror and *appears* to come from F for a convex mirror.

2. *A ray from (or going toward) F.* After reflection this ray passes parallel to the axis.

3. *A ray along the radius* (through the center of curvature C). This ray intersects the surface normally and is reflected back along itself.

The graphical technique is illustrated in figures 4-4 and 4-5 for the concave and convex mirrors, respectively.

Notice that in figure 4-4, the image is *real*, *inverted*, and smaller than the object, while in figure 4-5, the image is *virtual, upright*, and smaller than the object. The image is virtual because the rays appear to come from behind the mirror after reflection.

There are two major uses for concave and convex reflecting surfaces. These are: (1) to converge to a point or to diverge a beam of light composed of parallel rays or, (2) to increase or decrease the size of an image of a finite object.

An example of the first use is shown in figure 4-2 where we may wish to concentrate the paraxial rays to a point. The second case is shown in figure 4-4.

If a curved mirror is used to increase or decrease an object's dimensions we can use

$$M = \frac{Y'}{Y} = -\frac{S'}{S} \qquad (4.3)$$

for either a concave or convex surface. Y and Y′ are heights of the object and image, S and S′ are the object and image distances, respectively. The quantity M is termed the *magnification* of the mirror.

In this experiment we are going to find the focal length of a concave mirror by locating the real and virtual images as an object is moved in relation to the mirror. Notice from equation 4.2 that if S is less than f, the sign of S′ will be minus, indicating a virtual image. Also, from equation 4.3, under the same conditions, the sign of M will be positive indicating an upright image. Locating a real image is no problem. However, we shall have to use a rather indirect means to locate a virtual image since a virtual image cannot be projected onto a screen. To locate the virtual image we shall employ the method of parallax.

When two objects nearly in line with the eye are viewed by moving the eye sideways, the apparent change in their relative positions is called parallax. While looking at two fingers this way, note the change in magnitude of the relative sideways displacement as the distance between the fingers is varied. Also note which finger moves in relatively the same direction as the eye is moved from side to side. The only way parallax is not observed is, obviously, when the fingers are against each other.

MATERIALS

1 Optical bench
1 Illuminated object box
1 Lens holder
1 Screen holder
3 Carriages
1 Image screen
1 Concave mirror
1 Wood or cardboard pointer
1 Sheet graph paper

PROCEDURE

1. Set up the optical bench as shown in figure 4-6.

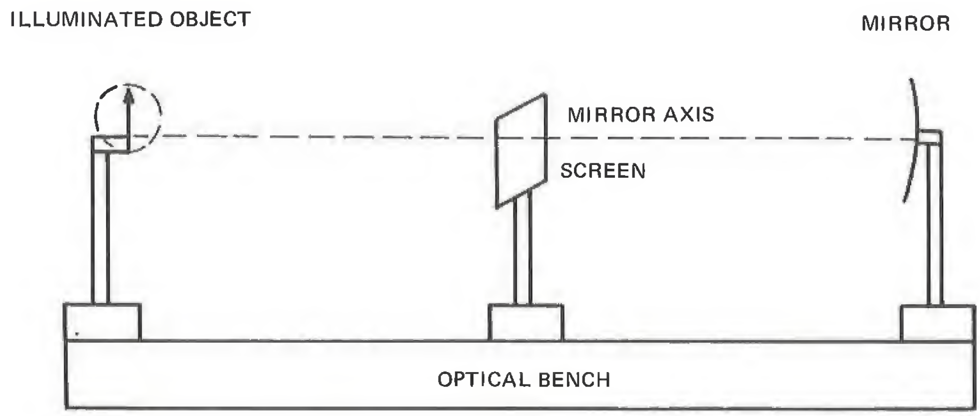

Fig. 4-6 Optical Bench Set-Up

2. With the object distance at 100 to 115 centimeters, adjust the screen position until a clear and distinct image is formed on the screen.

3. Reduce the object distance in steps of five to ten centimeters per step until a real image is no longer obtainable on the screen at any S′. Record S and S′ at each step. Take at least ten data points.

4. Set up the optical bench to measure S′ when the image is virtual. You will have to look across the object into the mirror. Find S′ by the parallax method previously discussed. Take at least five data points.

5. For each S and S′, calculate f and M. Also calculate the average f.

6. Construct a graph of S′ versus S and label each data point with its corresponding M.

ANALYSIS GUIDE. In the analysis of this experiment you should discuss the probable sources of error in the measurement of f, utilizing both the real and virtual image method. You should also describe the type of real and of virtual images observed (i.e., upright, inverted, enlarged, diminished, etc.).

PROBLEMS

1. Why can a virtual image not be projected on a screen?

2. Could the same experiment be applied to a convex mirror? Explain.

3. Is there a simpler, more direct, method for finding the focal length of the concave mirror? Explain the method and why it would work.

4. What is S/f and S′/f for a real image, amplification of unity?

REAL IMAGE

S	S′	f	M

VIRTUAL IMAGE

	AVERAGE f		

Fig. 4-7 The Data Tables

experiment 5 REFRACTION

INTRODUCTION. The phenomenon of refraction has many different uses. With it we can bend light beams more conveniently than with mirrors, or separate a beam of monochromatic light (single frequency or color) into two distinct and somewhat different beams. In this experiment we will investigate refraction under reasonably uncomplicated conditions.

DISCUSSION. To get a qualitative idea of what refraction is, let's consider the situation illustrated by figure 5-1.

Lines L_1 and L_2 are boundaries and N is the normal to L_1 and L_2. Suppose you start from O toward P at a constant velocity of 15 feet/second. At the end of one second you reach P. Distance d_1 is therefore 15 feet. Immediately after crossing L_1 you head for some point on L_2. Now, assume that you are required to reach line L_2 in *exactly* the *same* amount of time it took to go from O to P. Also assume that d'_1 equals d_1 so that if you maintained a velocity of 15 ft/sec, you would reach point O″ in one second and would satisfy the requirements.

However, upon crossing L_1 you have to slow down to, say, 10 ft/sec. You still must reach L_2 in one second but now you cannot go from P to O″ because this would take 1.5 seconds. You also cannot go along N because this would take less than one second and thus violate the requirements. You must, therefore, head to some point O′ such that d_2 is 10 feet and line L_2 is reached in exactly one second, no more or no less. In other

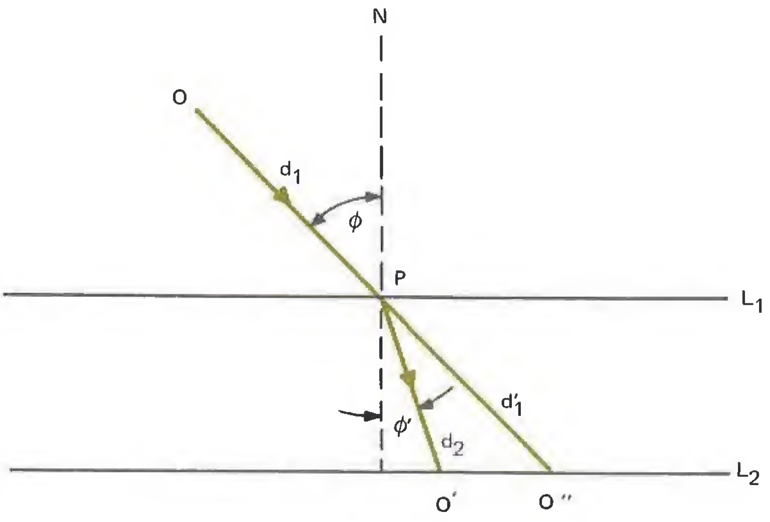

Fig. 5-1 Illustration of Refraction

words, you must deviate from d_1' toward N in order to go from L_1 to L_2 on time. Your velocity along d_1 was, say, v_1, and along d_2, v_2. Then

$$\frac{v_1}{v_2} = \frac{15 \text{ ft/sec}}{10 \text{ ft/sec}} = 1.5$$

and

$$\frac{d_1}{d_2} = \frac{15 \text{ feet}}{10 \text{ feet}} = 1.5$$

for a time t from O to P and P to O' of one second. Under these conditions it is also true that

$$\frac{\sin \phi}{\sin \phi'} = 1.5$$

where ϕ and ϕ' are the angles to the normal line N.

Light behaves in somewhat the same manner in going from one medium *into* another. Referring back to figure 5-1, above L_1 is one medium, air perhaps, and between L_1 and L_2 is another, maybe glass. The distances are such that $d_1 = d_1'$. We will assume that the velocity of light in air at room temperature is the same as it is in a vacuum.

When light traverses a transparent substance (glass, water, etc.) it interacts with the atoms of the substance. The atomic electrons are set into vibration by the incident light and these emit secondary light waves, much as an antenna does when excited by a radio wave. The combination of these secondary waves and the incident light results in a wave that travels slower in the substance than in empty space. The velocity of light in a transparent substance is given by

$$v = \frac{c}{\sqrt{\epsilon}}$$

where c is the velocity of light in a vacuum, v is the velocity of light through the transparent substance and ϵ is a constant of the substance. For most optically transparent materials

$$\sqrt{\epsilon} = \frac{c}{v} = n$$

where n is the *index of refraction* of the substance.

This means that if we have a substance such as glass whose index of refraction is n = 1.5, the velocity of light through the glass is c/1.5, or the light will travel 1.5 feet in air in the *same* amount of time it would travel one foot in glass. Thus in figure 5-1, a light ray will travel d_2 in the same amount of time it travels d_1 if the region between L_1 and L_2 is glass of n = 1.5 and $d_1 = d_1'$.

Refraction may be defined as the bending of light as it passes from a medium of one index of refraction into a medium with a different index of refraction, figure 5-2.

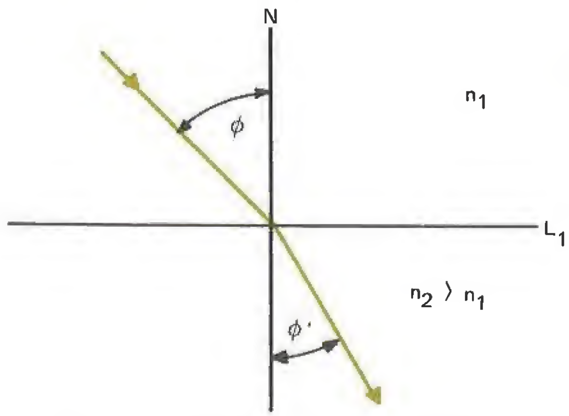

Fig. 5-2 Snell's Law of Refraction

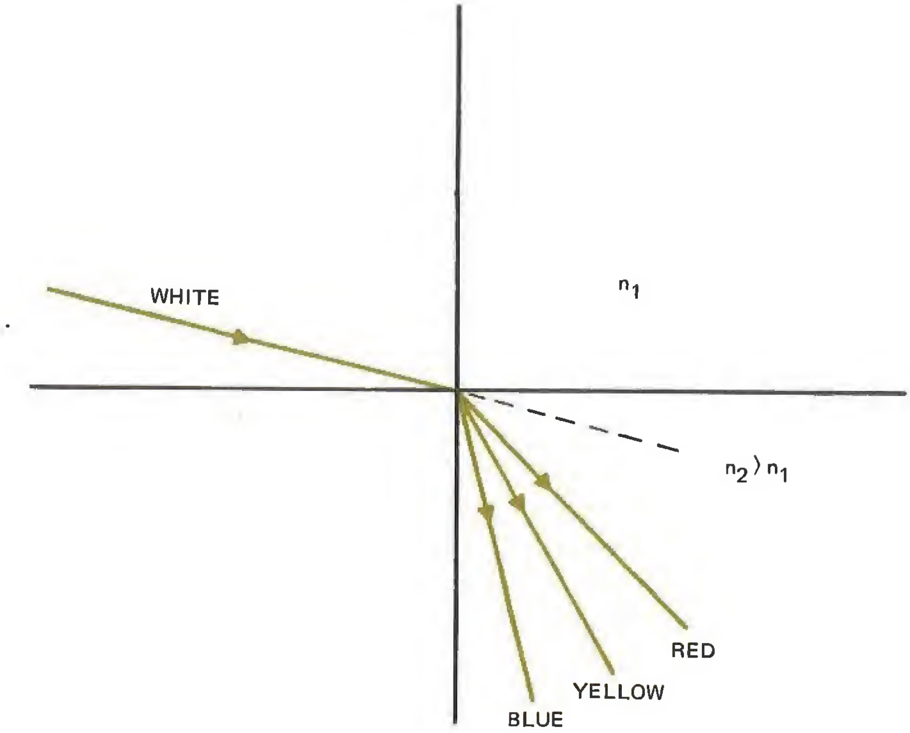

Fig. 5-3 Dispersion of White Light

Above L_1 is one medium, say water with index n_1, and below L_1 is another medium, say glass with index n_2. The general relation for this case given by *Snell's law of refraction* which is

$$n_1 \sin \phi = n_2 \sin \phi' \tag{5.1}$$

We can see that if $n_2 > n_1$, then $\phi' < \phi$, and the ray will be bent *toward* the normal. If $n_1 > n_2$ then $\phi < \phi'$ and the ray would be bent *away* from the normal.

Remember that refraction takes place because the velocity of light in a substance depends upon the atomic characteristics of the substance. There is an additional effect which takes place: the index of refraction of a given substance is *not* constant but depends upon the wavelength of the light going through it. Usually the index of refraction of a given substance *decreases* as the wavelength of the light *increases* (frequency *decreases*). This effect is called *dispersion*. Red light will not be refracted as much as blue light since the frequency of red is less than that of blue. This is shown in figure 5-3 for an incident beam of white light.

The index of refraction of most optically transparent materials is usually measured with yellow light. The factors to obtain the index for red, blue or another color will depend upon the type of material employed (crown glass, flint glass, quartz, etc.).

In most cases both reflected and refracted rays are present. In this experiment we shall determine the index of refraction for a parallel-sided glass plate, the expected beam deviation by a glass plate, and the index of refraction of a prism.

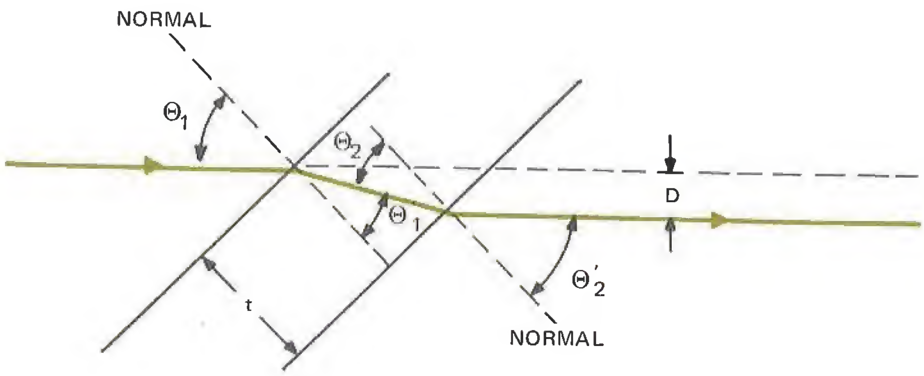

Fig. 5-4 Deviation of a Light Beam by a Glass Plate

For the glass plate illustrated in figure 5-4, we can see that an incident beam will be refracted upon passing into and out of the plate. If the sides are truly parallel, then $\Theta_1 = \Theta'_2$, and the beam leaving the plate will be parallel to, but displaced from, the incident beam. In such a case the beam displacement will be

$$D = t\,\frac{\sin(\Theta_1 - \Theta'_1)}{\cos\Theta'_1} \qquad (5.2)$$

where D is the displacement and t is the thickness of the plate.

The use of plates such as this has many applications where you wish to deviate a beam or an image from, but still retain it parallel to, some axis.

The prism is probably second only to the lens as the most useful piece of optical apparatus. We shall look at a general case for a prism as shown in figure 5-5. Here, A is the prism angle and δ is the deviation angle between the incident and exit beams. Since the sides are not parallel, we can see that the incident and exit beams will not be parallel as they were for the glass plate. The total deviation angle can be found by

$$\delta = \Theta_1 + \Theta'_2 - A \qquad (5.3)$$

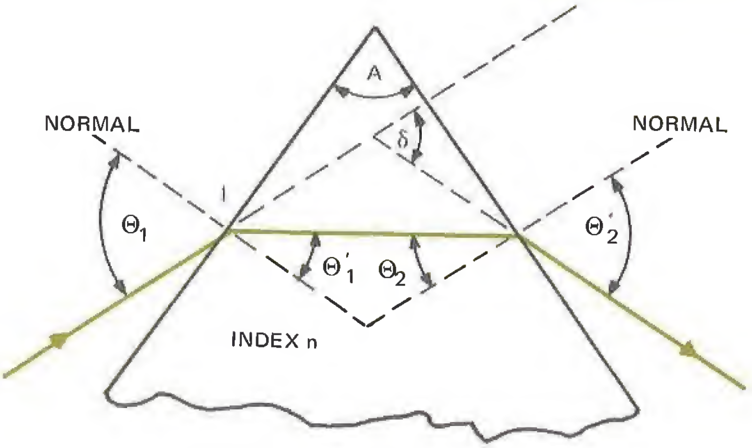

Fig. 5-5 Deviation by a Prism

If during the time a beam is being refracted by a prism, the prism is rotated about some point, say I in figure 5-5, the deviation angle δ will be observed to decrease, reach a minimum, and then increase. The smallest deviation angle δ_m, is called the angle of minimum deviation and occurs when

$$\Theta_1 = \Theta_2' \text{ and } \Theta_1' = \Theta_2$$

With a little mathematical maneuvering we could demonstrate that for a prism of index n,

$$n = \frac{\sin \frac{1}{2}(A + \delta_m)}{\sin \frac{1}{2}A} \qquad \textbf{(5.4)}$$

The index of refraction of most optically transparent substances is frequently obtained in this manner. The substance is first cut into a prism of angle A. A beam is sent through and the prism rotated until δ_m is observed. The index of refraction of the substance can be found by equation 5.4. If the measurements are precise enough, n can be found quite accurately.

Also in this experiment, we shall employ the laser. We will use the laser for two main reasons: (1) The beam emitted from the laser is extremely narrow and well collimated (very little divergence). (2) The beam is composed of monochromatic (single frequency) light. We shall, therefore, use just one frequency of light instead of the various frequencies found in a normal white light source.

For example, the visible light range runs from a wavelength of about 4000 Å to about 7000 Å. The symbol Å means an angstrom unit and 1 Å = 10^{-8}cm. The laser used here emits a very narrow beam of 6328 Å. By using the laser in an optics experiment, we can bypass some of the chromatic effects observed with white light and perform all of the measurements with one frequency. Keep in mind, however, that when the index of refraction is measured using a laser, that index is good only for that frequency, and will not equal the book value for the substance in question. The book values for n are usually stated for yellow light, not red.

A couple of other points should be mentioned. **Above all else, DO NOT under any circumstances, look directly into the laser!** Even though the laser used is a low output type, the intensity is still enough to cause retinal burns. Also, do not look at a mirrored reflection of the laser beam, that is, a reflection from a mirror or polished metal surface. You may observe the laser by watching the reflection from a piece of dull-white paper, the meter stick, or a ground glass screen.

In this experiment you will have to align the laser beam with the optical bench axis. This can be done by putting a small hole in a card or metal plate. Aim the laser down the length of the bench and move the card back and forth on the bench. Adjust the bench until the laser beam passes through the hole with the card at either end of the bench.

MATERIALS

1	He-Ne laser
1	45° prism
1	60° prism
1	Glass plate approximately 5 cm thick
1	Meter stick
1	Support platform
1	Screen holder
5	3x5 cards
2	Carriages
1	Protractor

PROCEDURE

NOTE: You may wish to use nontoxic smoke in this experiment.

1. Align the optical bench with the laser beam, both vertically and horizontally, such that the beam is parallel to the axis of the bench.

2. Lay a 3x5 card on the bench support platform and construct a line across the card parallel to the laser beam (the incident beam). Then turn the laser off.

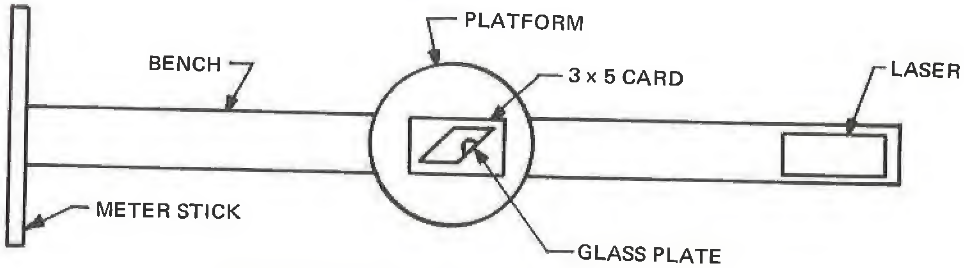

Fig. 5-6 Experiment Set-Up Top View

3. Stand a glass plate of *known* thickness on the card at some known angle (say 45°) to the incident beam line. Record the angle as Θ.

4. Construct lines on either side indicating the edges of the glass plate.

5. Mount a meter stick across the bench at the far end and as nearly perpendicular to the optical bench as possible.

6. Remove the plate and turn the laser on. Note the location of the spot on the meter stick.

7. Now put the plate back into the beam. Note the lateral displacement, D, of the spot on the meter stick.

8. Turn the laser off. Construct, on the card, a line parallel to the incident beam but displaced from it by D.

9. Construct a line between the incident and exit points of the beam representing the path of the beam through the plate. The results should look somewhat like figure 5-4.

10. Record D, t, Θ_1, Θ_2, Θ'_1, Θ'_2 . If $\Theta_1 \neq \Theta'_2$ within a reasonable amount, repeat the above procedure.

11. Repeat the foregoing for three different values of Θ_1 and calculate the index of refraction of the plate in each case.

12. Calculate the expected D in each case.

13. Mount another 3x5 card on the platform and construct a line across it parallel to the laser beam. Also again note the location of the spot on the meter stick.

14. Mount the prism on the card and note the deflection of the spot along the meter stick, D.

15. Rotate the prism about point I (see figure 5-5) until the deviation of the beam along the meter stick is at its minimum.

16. At minimum δ, measure the distance from I, down the axis of the bench, to the meter stick. This should be the location of the undeviated spot. If it is not, re-align the bench and repeat steps 13 through 15.

17. From the results of 15 and 16, calculate δ_m and record it in the data table.

18. Calculate the index of refraction of the prism.

19. Repeat steps 13 through 18 for a prism of a different angle A.

Θ_1	Θ_1'	Θ_2	Θ_2'	t	D	n	CALCULATED D

AVERAGE n =

A	δ_m	n

Fig. 5-7 The Data Tables

ANALYSIS GUIDE. In the analysis of this experiment you should discuss your method of performing step one, and the importance of this step in regard to the vailidity of the results. Compare the calculated and measured values for D from the glass plate section and discuss probable causes for any differences. Also discuss the limitations in accuracy imposed on the measurement of δ_m by this method.

PROBLEMS

1. How would you modify equation 5.2 to include more than one plate?

2. If more than one plate were used to obtain a given D, would the distance between the plates affect D? Explain.

3. If a sharp beam of white light were used in the glass plate section, would there be any dispersion? Explain.

4. If a beam of white light had been used in the measurement of δ_m, you would have seen the color spectrum on the meter stick. What would have happened to the width of the spectrum as the prism was rotated?

5. What is the value of duration angle δ for a parallel-sided glass plate? Does this value change as Θ_1, is changed? Why?

6. Derive equation 5.2.

experiment 6 POLARIZATION

INTRODUCTION. Polarized light plays an important role in several types of optical instruments and in optical analysis. The purpose of this experiment is to become familiar with polarized light and its production.

DISCUSSION. In this experiment we will consider some optical phenomena which depend upon the transverse wave behavior of light. These phenomena are called polarization effects and can be observed only with transverse waves.

Let's draw an analogy to the nature of electromagnetic waves radiated by a radio antenna. Suppose we consider a vertical radiator and a portion of the wavefront in the vertical plane at some distance from the antenna. Such a situation is illustrated by figure 6-1.

The electric field intensity E is in a vertical direction at all points of this wavefront. At all points of any plane fixed in space, the electric field vector oscillates up and down in the vertical direction and the wave is said to be *linearly* polarized.* If at some instant E is at maximum in the upward direction, then in wavefronts one-half wave-length away, E will be at a maximum in the downward direction. For the sake of clarity, the magnetic field intensity H is not shown in figure 6-1. H is usually at right angles to E.

* Only true for an unlimited cross section.

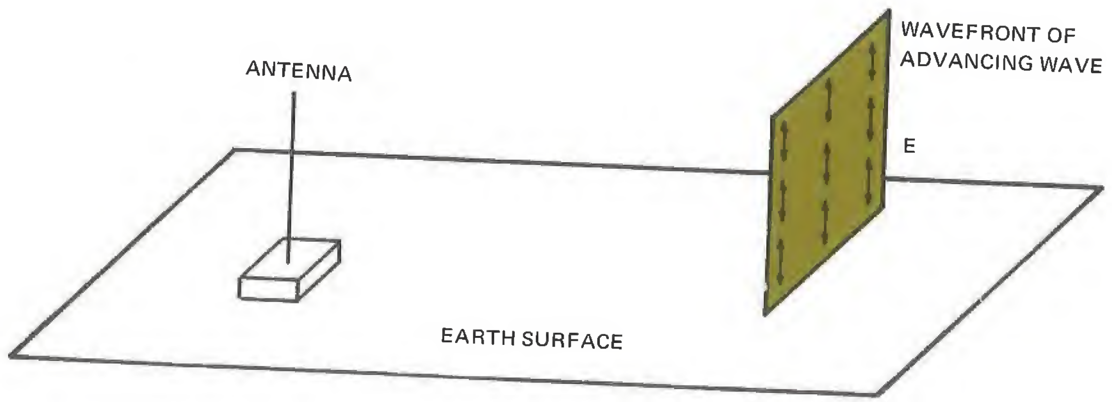

Fig. 6-1 Vertically Polarized Electromagnetic Waves Radiated by an Antenna

The "antennas" which radiate light waves are the atoms of which the light source is composed. These atoms acquire energy in some way and then radiate this acquired energy (or at least part of it) as electromagnetic waves of very short wavelength. The light waves from any *one* atom are linearly polarized similar to those from a radio antenna. However, any practical light source contains a tremendous number of these atoms which are oriented in practically all possible directions. Therefore, the emitted light is a mixture of waves linearly polarized in all possible transverse directions.

It is well to remember that for any one atom, whenever a transition from a higher to a lower energy state is made, a bundle or packet of electromagnetic energy is radiated. Since, from a practical light source, these wave packets are emitted many millions of times per second, we may assume the light to be composed of continuous electromagnetic waves. Either way you choose to look at it, the polarization considerations are the same.

In figure 6-2, assume a beam of light is coming directly out of the page. The dot in the center of the figure represents one "ray" of this beam. A beam of natural light is represented by (a) while a linearly polarized beam is represented by (b). The arrows represent the directions of oscillation of the electric field E.

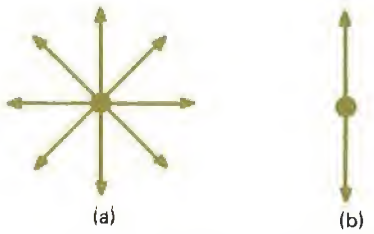

Fig. 6-2 Representation of Ordinary Light and Linearly Polarized Light

Figure 6-3 illustrates a beam of light incident upon a sheet of commercial polarizing material called a "Polaroid". There exists in a sheet of this material, a certain and definite polarizing direction illustrated by the parallel lines. The material will transmit only those waves or wavetrain components whose electric field vectors oscillate parallel to this polarizing direction and will absorb those which oscillate at right angles to this direction. That is, if a beam of normal unpolarized light is incident upon a polaroid, the emerging light will be linearly or plane-polarized. This polarizing direction is established during the manufacture of such sheets by embedding long-chain molecules in the plastic and then stretching the plastic so that the molecules are aligned parallel to each other.

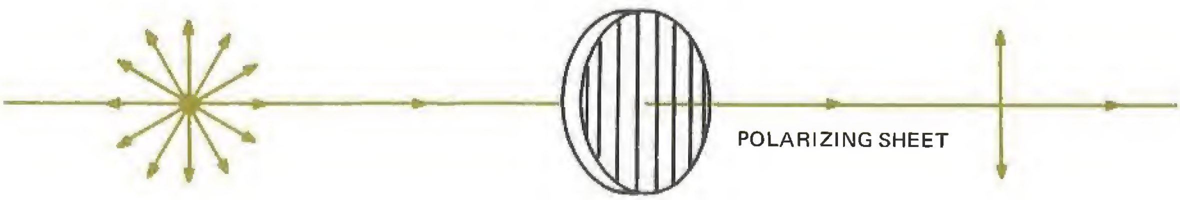

Fig. 6-3 Production, by a Polarizing Sheet, of Plane-Polarized Light from Unpolarized Light

Figure 6-4 represents a polarizing sheet or *polarizer* lying in the plane of the page. E represents the amplitude of a randomly selected wavetrain incident upon the sheet. E may be broken into two vector components of

$$E_x = E \sin\Theta \text{ and } E_y = E \cos\Theta$$

Of the two components, only E_y will be transmitted since it is parallel to the polarizing direction. E_x, being perpendicular to the polarizing direction, will be absorbed by the sheet.

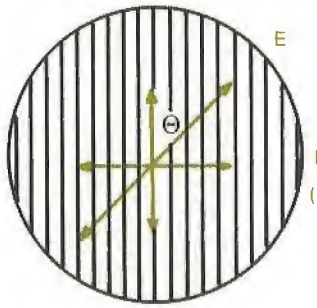

Fig 6-4 Components E_y and E_x of Random Wave E

Now let's consider the situation illustrated in figure 6-5. A second polarizing sheet has been placed crosswise to the first. This second sheet is usually called an analyzer when used as in figure 6-5. If the analyzer is rotated about the direction of light propagation as shown, it is found that there are two positions, 180° apart, at which the transmitted light intensity is almost zero. These are the positions at which the polarizing directions of the polarizer and analyzer are at right angles.

If the amplitude of the plane-polarized light falling upon the analyzer is E, the amplitude of the emergent light is $E \cos\Theta$ where Θ is the angle between the polarizing directions of the analyzer and polarizer. Since the intensity of a light beam is proportional to the square of the amplitude, we have for the transmitted intensity, I_t,

$$I_t = I \cos^2 \Theta \qquad \textbf{(6.1)}$$

where I is the intensity of the light falling upon the analyzer. The light intensity emerging from the analyzer, I_t, will be at a maximum when Θ = 0° or 180°. It will be at a minimum when Θ = 90° or 270°.

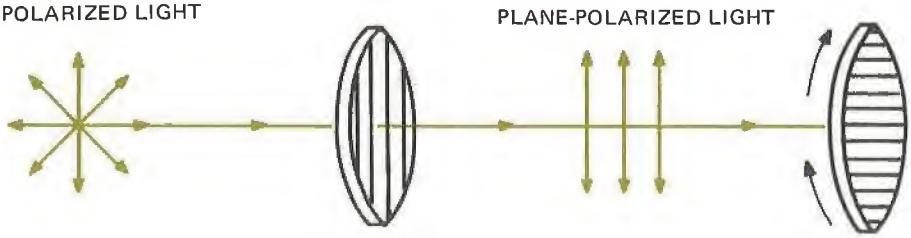

Fig. 6-5 Light is Not Transmitted by Crossed Polaroids

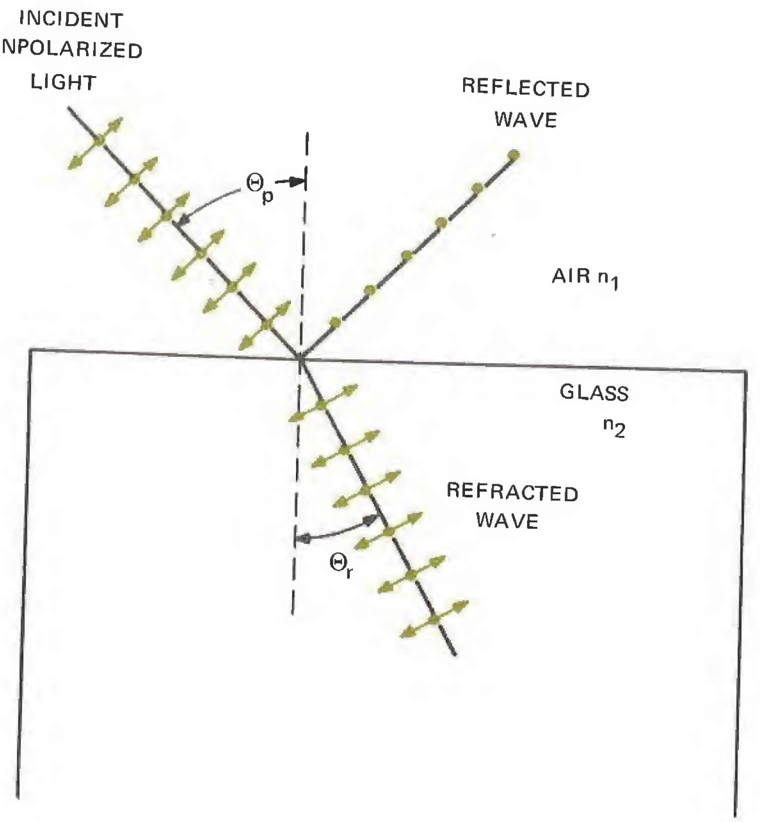

Fig. 6-6 At Θ_p, the Reflected Light is Completely Polarized

It was discovered in 1809 that light can be partially or completely polarized by reflection. If you have noticed the sun's reflection from the surface of water while wearing a pair of polarizing sun glasses, you have probably noticed the effect. If you tilt your head from side to side, you will notice the reflected light has a point of minimum intensity.

Figure 6-6 illustrates an unpolarized beam falling upon a glass surface.

The E vector for each beam or wave can be resolved into two components, one perpendicular to the plane of incidence (which is the plane of the paper) and the other parallel to, or lying in, the plane. The perpendicular component is represented by the dots while the other is represented by the arrows. On the average, for completely unpolarized light, these two components are equal.

It has been found experimentally that, for glass and other dielectric materials, there is a certain angle of incidence, called the *polarizing angle* Θ_p, at which the reflected beam is completely plane-polarized with its plane of vibration at right angles to the plane of incidence. This is shown also in figure 6-6.

Although of higher intensity, the transmitted or refracted beam is only partially polarized with a small perpendicular component and a large in-plane component.

At Θ_p, it has been found experimentally that the reflected and refracted beams are at right angles. Therefore

$$\Theta_p + \Theta_r = 90^\circ$$

From Snell's law

$$n_1 \sin \Theta_p = n_2 \sin \Theta_r$$

$$n_1 \sin \Theta_p = n_2 \sin (90^\circ - \Theta_p)$$

$$n_1 \sin \Theta_p = n_2 \cos \Theta_p$$

or

$$\tan \Theta_p = \frac{n_2}{n_1} \qquad \textbf{(6.2)}$$

If n_1 represents the index of refraction of air, then

$$\tan \Theta_p = n \qquad \textbf{(6.3)}$$

where n is the index of refraction of the glass or other material. Equation 6.3 is known as Brewster's law and Θ_p is commonly called Brewster's angle.

Let's look at an example. Suppose we wish to use a glass plate (n=1.5) as a polarizer. From equation 6.3

$$\Theta_p = \tan^{-1} 1.5$$

$$\Theta_p = 56.3^\circ$$

MATERIALS

1	He-Ne laser
1	Optical bench
3	Flat glass plates
1	Plexiglass plate
1	Lucite plate
1	Polarimeter
1	Support platform
1	White screen
1	Screen holder
2	Carriages

PROCEDURE

1. Set the screen at one end of the optical bench, and the laser at the other.

2. Turn the laser on and note the brightness of the spot.

3. Insert a polarizer set with 0° perpendicular to the bench, between the laser and the screen. Note the effect upon the spot intensity.

4. Rotate the polarizer through 360° and note the readings at which the laser intensity changes.

5. Insert in the data table, figure 6-8, either *increase* or *decrease* relative to the 0° intensity, beside the appropriate angle. If no change is noticeable, write *zero*.

6. Set up a glass plate, laser, and screen as diagrammed in figure 6-7.

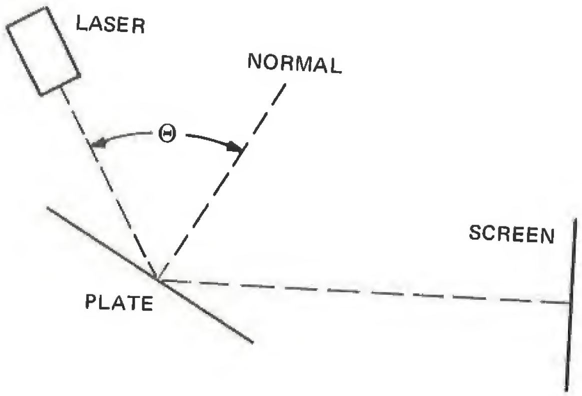

Fig. 6-7 Experimental Set-Up

7. Insert a polarizer between the plate and screen. Rotate the polarizer 360° and note the variation in intensity on the screen.

8. Adjust Θ until the reflected beam is completely plane-polarized. Record this as Θ_p for glass.

9. Repeat steps 6, 7, and 8 for the lucite and plexiglass plates.

10. Replace the glass plate as in step 6.

11. Intercept the refracted beam with the polarizer.

12. Rotate the polarizer 360°. Note the intensity changes in the refracted beam.

13. Insert another glass plate behind the first and parallel with it.

14. Repeat steps 11 and 12.

15. Insert the third glass plate behind and parallel with the first two.

16. Repeat steps 11 and 12.

ANALYSIS GUIDE. In the analysis of this experiment you should discuss the readings obtained from step five and how they show whether or not the laser beam itself is initially polarized. You should relate the measured Θ_p to the calculated Θ_p you would obtain for each material used in steps eight and nine. You should also discuss the effect upon the polarization of the refracted beam as more plates are added. Give reasons for this effect.

DEGREES	RELATIVE CHANGE IN INTENSITY
0°	
45°	
90°	
135°	
180°	
225°	
270°	
315°	

MATERIAL	Θ_p
GLASS	
LUCITE	
PLEXIGLASS	

Fig. 6-8 The Data Table

PROBLEMS

1. Why do sunglasses made of polarizing material have an advantage over those which simply depend upon absorption effects?

2. Devise a way to identify the polarizing direction of polaroid sunglasses. Draw a diagram to illustrate.

3. What is the polarizing angle for water at 20° C? n = 1.333

experiment 7 DIFFRACTION

INTRODUCTION. The phenomenon of diffraction, and its effects, is probably the most important contributor to the wave theory of light. It is the purpose of this experiment to qualitatively examine some simple instances of diffraction and the associated effects.

DISCUSSION. Diffraction is the bending of light around the edge of an opaque object. We can see the diffraction of light by looking through a crack between two fingers at a distant source of light such as a neon sign, or by looking at a street light through a cloth umbrella. Usually diffraction effects are quite small and must be very carefully looked for to be noticed. A couple of reasons for this are that most sources of light have an extended area such that diffraction effects produced by one point of the source will overlap those produced by another point; also, most sources of light are not monochromatic and the diffraction effects for various wavelengths may overlap. The size of the optical wavelengths is also a major factor.

The most notable, and often the most easily observable, case of diffraction occurs when light is incident upon a narrow slit. The pattern observed upon a screen placed at some distance from the slit will not be a geometrical outline of the slit, but a series of light and dark bands. This means that around the edges of the slit, the light is spreading into the geometrical shadow. In other words, when a wavefront is limited in cross-section, the light no longer travels in straight lines. This can be explained only by assuming a wave nature for light.

Figure 7-1 shows a general diffraction situation.

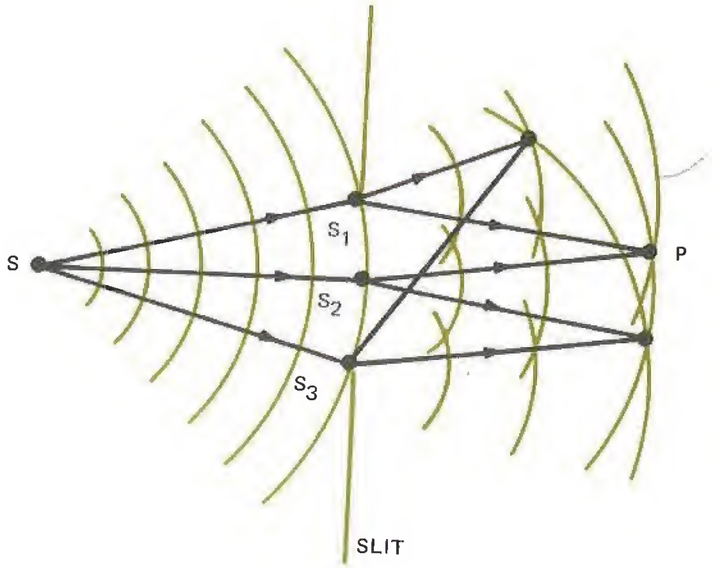

Fig. 7-1 Diffraction of Spherical Wavefronts

S is a point source illuminating the slit. The concentric rings illustrate the spherical wavefronts sent out by S. Now, it is a postulate of optics that each point on a wavefront acts as a source of secondary radiation or wavelets. Such a construction was advocated by Huygens and the secondary wavefronts are called Huygens waves. Now, on the basis of this, S_1, S_2, and S_3 represent three such Huygens sources with a few "rays" shown emanating from each. That is, S_1, S_2 and S_3 represent sources sending out their *own* spherical wavefronts. We can see from figure 12-1 that there are points to the right of the slit where wavefronts from S_1 overlap those from S_2, S_2 overlaps wavefronts from S_3 and S_3 overlaps S_1. The general technique for finding the resultant intensity of the diffraction pattern at some point P in figure 7-1 is to divide the incident wavefront into a number of Huygen's secondary radiators or sources. The resultant amplitude at P is found by adding the amplitudes of each of the secondary sources at P, and the resultant intensity is proportional to the square of this sum.

There are two general classes of diffraction. These are called Fresnel and Fraunhofer diffraction. The basic difference between the two is that Fresnel diffraction takes place relatively close to the slit while Fraunhofer diffraction is the pattern observed far from the slit. Fresnel and Fraunhofer diffraction are illustrated in figure 7-2. In this figure, we use "rays" instead of wavefronts to simplify the diagrams.

You can see from figure 7-2 that the wavefronts in (a) must be spherical in that the rays from S are diverging, while in (b) the rays are parallel so that the wavefronts are plane. Each point between the edges of the slits in figure 7-2 is a source of secondary wavefronts or secondary rays.

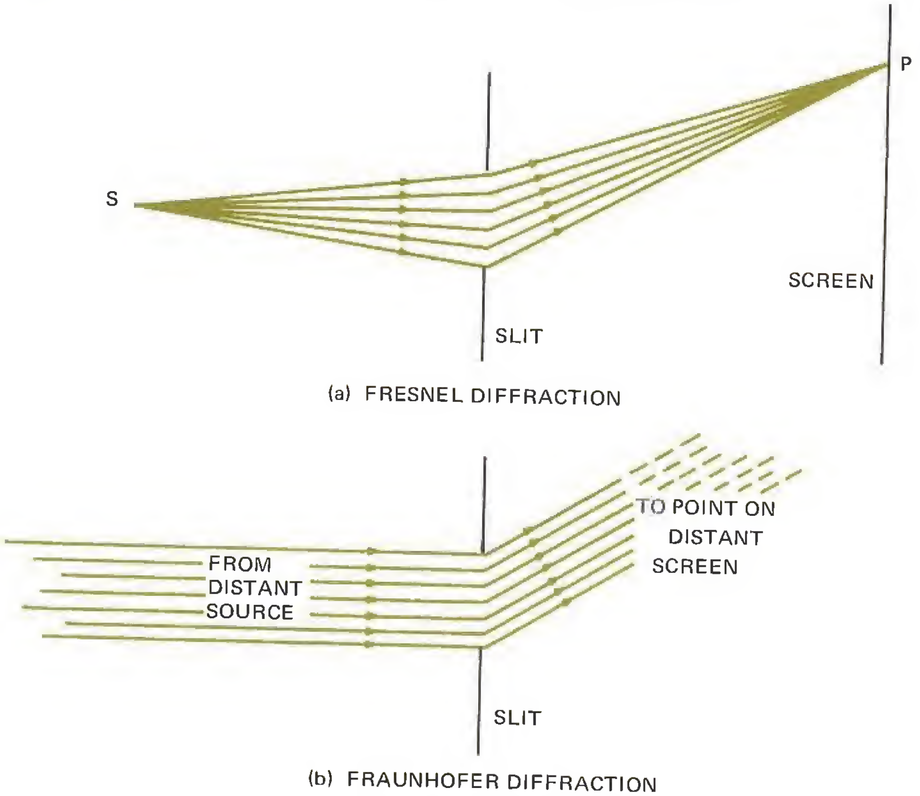

Fig. 7-2 General Classes of Diffraction

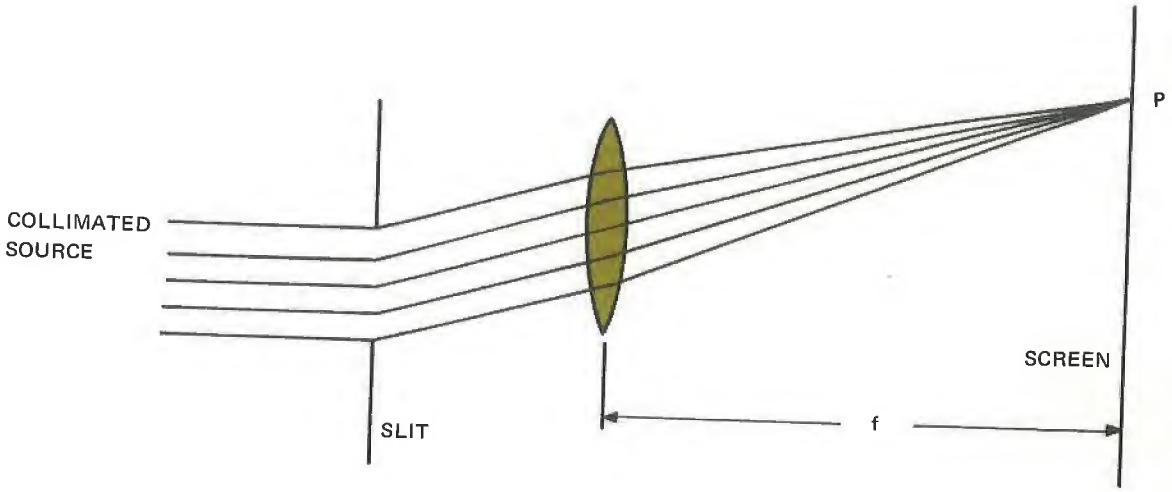

Fig. 7-3 Fraunhofer Conditions Using a Lens

Although Fraunhofer diffraction is just a special case of Fresnel diffraction, it is a very important special case and is much easier to handle mathematically. Therefore, we shall only look at Fraunhofer diffraction to any extent.

Fraunhofer diffraction may be produced by employing a converging lens as shown in figure 7-3.

Now, let's look more closely at Fraunhofer diffraction. Figure 7-4 shows a plane wave incident normally on a slit of width d. Here, we will look just at the central point, P_o, of the screen. For simplicity, only three secondary rays are shown.

Since the incident wave is plane, the rays in the plane of the slit are all in phase. Therefore the rays emerging from the slit will

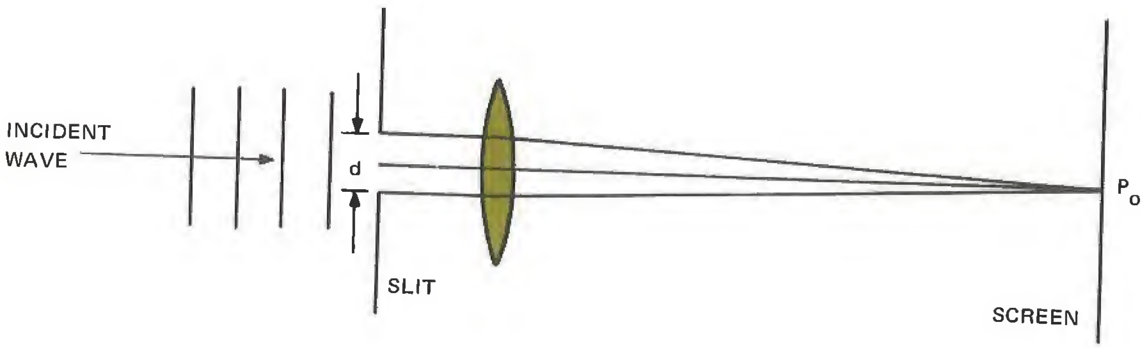

Fig. 7-4 Central Maximum of Diffraction Pattern

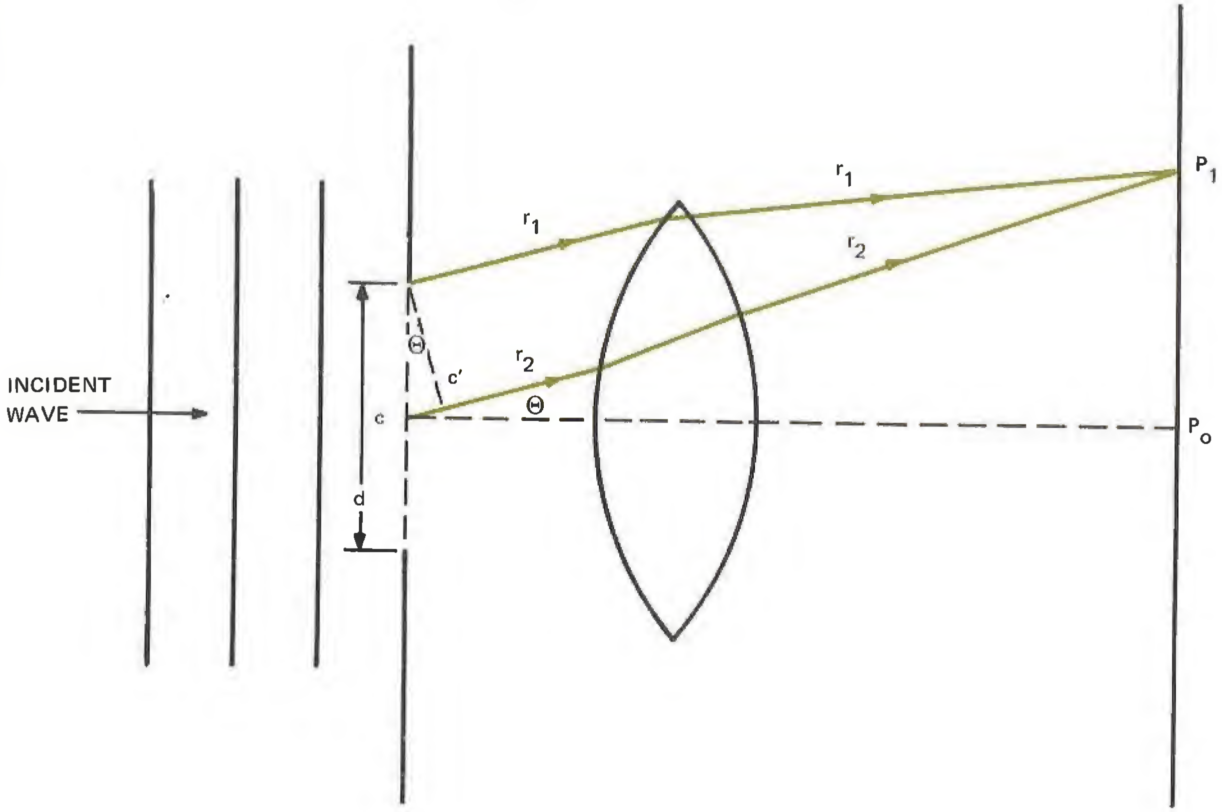

Fig. 7-5 Conditions for First Minimum of Diffraction Pattern

be in phase at P_o since all rays from the slit will have the same optical path length. Thus, the central point of the diffraction pattern, P_o, has maximum intensity.

Let's now consider another point, P_1, on the screen. Figure 7-5 illustrates this case.

Ray r_1 which originates at the top of the slit and ray r_2 which originates at the center of the slit reach P_1 after emerging from the slit at angle Θ as shown. If Θ is such that the distance cc' is one-half a wavelength, rays r_1 and r_2 will be out of phase and will destructively interfere at P_1. In other words, if r_1 and r_2 differ by 180°, there will be no effect at P_1. It can be concluded that every ray from the upper half of the slit will be cancelled by a ray originating a distance of d/2 below it. If Θ is such as to satisfy these conditions, then P_1 will be the first minimum of the pattern and will have zero intensity.

The conditions described by the foregoing are mathematically

$$\frac{d}{2}\sin\Theta = \frac{\lambda}{2}$$

or

$$d\sin\Theta = \lambda$$

for the first minimum.

Now, suppose that the slit width is as small as one wavelength ($d = \lambda$). Then

$$\sin \Theta = 1$$
$$\Theta = 90°$$

The first minimum of the diffraction pattern occurs at $\Theta = 90°$ which implies that the central maximum fills the entire forward hemisphere.

For the positions of the other minima we only need to realize that the distance cc' in figure 7-5 must be a whole number multiple of half wavelengths. In other words, for any minima

$$\frac{d}{2} \sin \Theta = m \frac{\lambda}{2}$$

or

$$\boxed{d \sin \Theta = m\lambda} \qquad (7.1)$$

where m = 1, 2, 3, . . .

For m = 1, Θ is the angular displacement of the first minimum. For m = 2, Θ is the angular displacement of the second minimum and so on for m = 3, 4, . . .

Remember, that for every minima above P_o, there is a corresponding one below it. That is, Θ is the angular displacement at the first minima (m = 1) above and below the central maximum.

Let's consider an example of Fraunhofer diffraction. Suppose we have a slit of unknown width, d. The slit is illuminated with red light of wavelength 6500 Å. It is found that the first minimum of the diffraction pattern falls at $\Theta = 30°$. What is d? Using equation 7.1 with m = 1

$$d = \frac{m\lambda}{\sin \Theta}$$

$$d = \frac{(1)(6500\text{Å})}{\sin 30°}$$

$$d = 13000 \text{ Å}$$

MATERIALS

1 He-Ne laser

1 Adjustable slit

1 White screen

1 Meter stick

1 Lens holder

1 Screen holder

2 Carriages

1 Optical bench

PROCEDURE

NOTE: You may wish to use nontoxic smoke in this experiment.

1. Very carefully align the axis of the optical bench with the laser beam.
2. Mount the screen at the far end of the bench. Mount the adjustable slit between the laser and the screen.

3. Adjust the slit such that the long side is perpendicular to the bench axis.

4. Adjust the width of the slit and the slit-to-screen distance until a good diffraction pattern with at least three minima on either side of the central maximum is observed.

5. Sketch the pattern observed in step 4.

6. Decrease the width of the slit. Note and record any changes in the diffraction pattern.

7. Sketch the pattern observed when the slit width is decreased.

8. Set up the apparatus in figure 7-6.

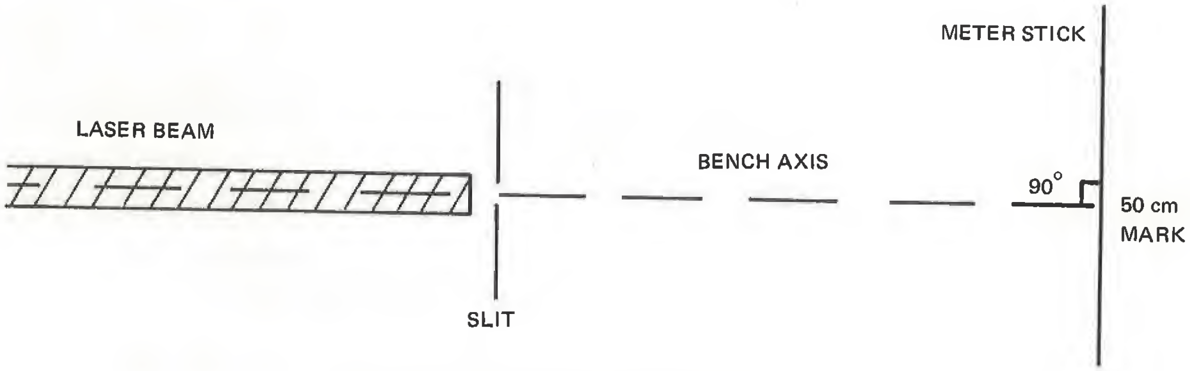

Fig. 7-6 Top View of Slit Arrangement

9. Adjust the slit width until a good diffraction pattern can be seen on the meter stick. The central maximum should fall on the 50 cm mark; if it does not, re-align the laser and bench axis.

10. Knowing the slit-to-stick distance and the distance along the stick from the central maximum to the first minimum, calculate Θ for $m = 1$.

11. Knowing λ, Θ, and m, calculate the slit width, d.

12. Measure the slit width. Record this as d′.

13. Record λ, Θ, m, d, and d′, in the data table, figure 7-7.

14. Change the slit width until a different pattern is obtained. Repeat steps 10, 11, 12, and 13.

15. Repeat step 14 for at least two more different values for d.

λ	m	Θ	d	d′

Fig. 7-7 The Data Table

ANALYSIS GUIDE. In the analysis of this experiment, you should discuss and give reasons for the changes in the observed diffraction patterns from steps 5 and 7. You should also discuss the difference between d and d′ and give the probable causes of error in the measurement of d.

PROBLEMS

1. Referring to figure 7-3, why do you think a lens was *not* needed in this experiment if we assume the laser to be a source of plane wavefronts?

2. Why is the diffraction of sound waves more evident in daily experience than that of light waves?

3. Why do radio waves diffract around buildings although light waves do not?

experiment 8 *INTERFERENCE*

INTRODUCTION. Interference patterns can arise from a single source or from multiple sources where coherent. It is the purpose of this experiment to examine the interference of light from two or more beams.

DISCUSSION. In an experiment dealing with single slit diffraction, we often assume that any point on a wavefront acts as a source of secondary or Huygen's waves. In this experiment we will consider what takes place when the slit width is negligible and we have interference from two or more distinct sources

This basic experiment was first performed by Thomas Young in 1801. His apparatus is shown in figure 8-1. The dimensions are grossly distorted for clarity.

Young allowed sunlight to fall on pinhole S_o. The emerging light spreads out by diffraction (S_o acts as a point source) and falls on pinholes S_1 and S_2. Again the waves spread out and two overlapping spherical wavefronts expand toward the screen.

In a diffraction experiment, if the width of the slit is approximately equal to a wavelength or smaller, the central maximum will fill almost the entire forward hemisphere. That is, the slit acts as a point source. Now

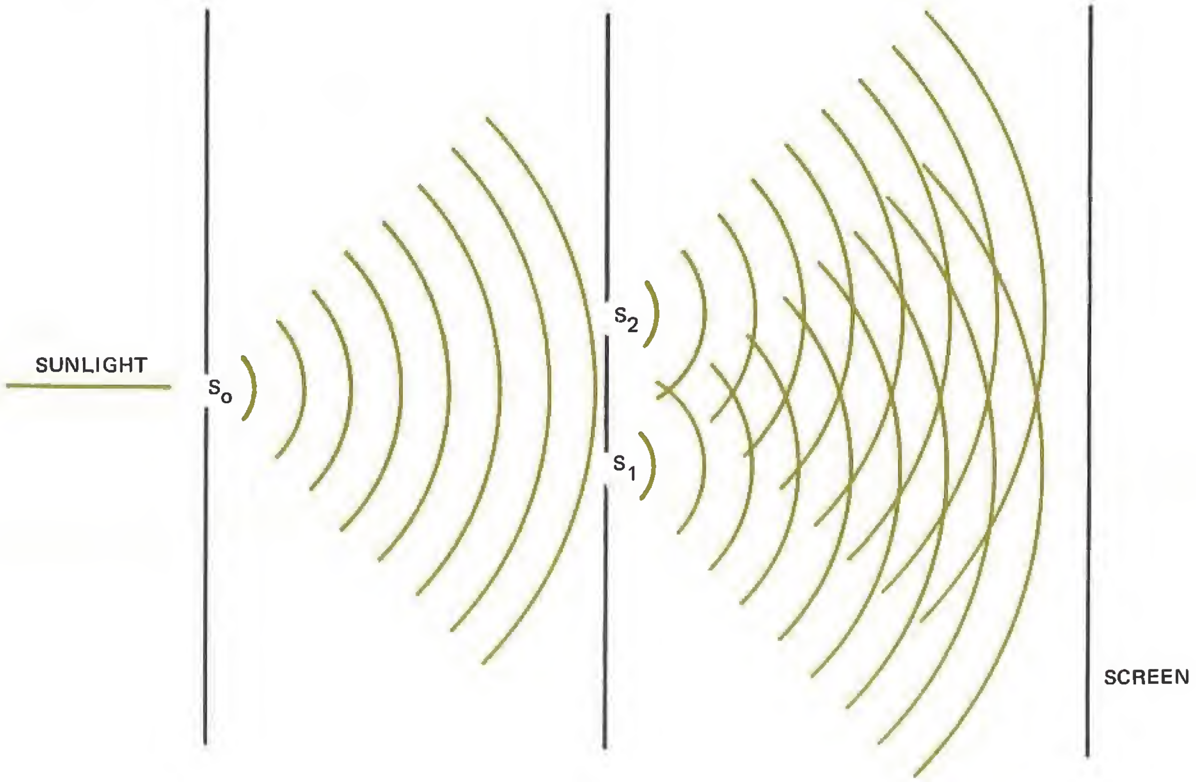

Fig. 8-1 Young's Experiment

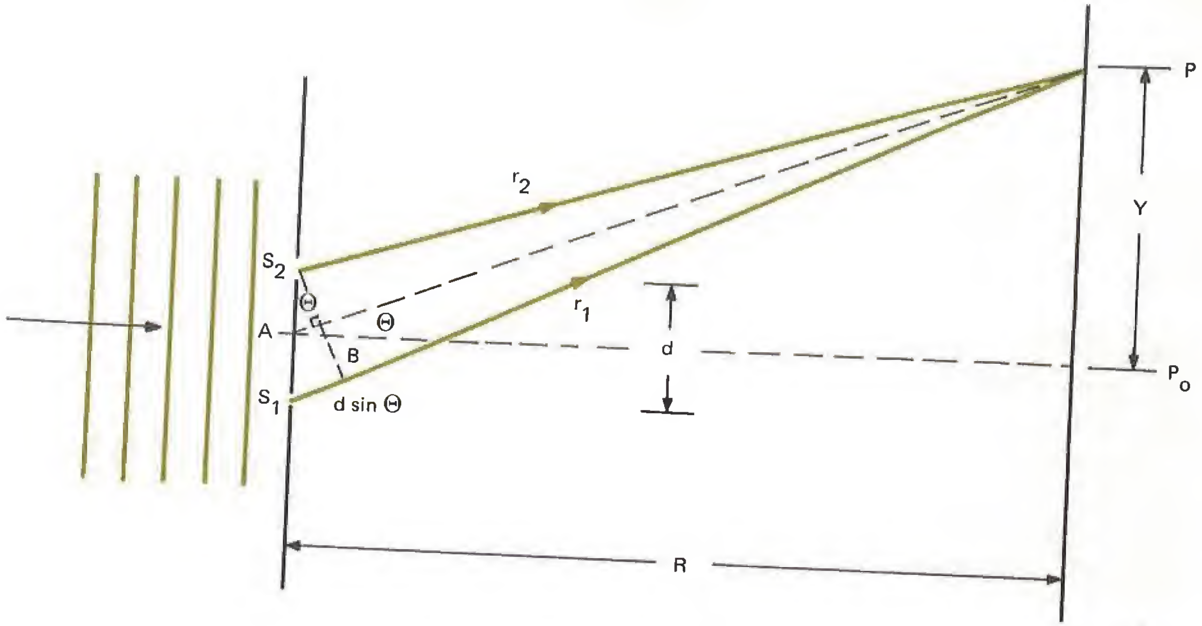

Fig. 8-2 Double Slit Light Interference

the criteria that the width of the slits, or the diameter of the pinholes, is much greater than a wavelength does not apply in this case. Therefore, S_1 and S_2 act as sources or radiators themselves and the resultant pattern of light and dark bands observed on the screen will be due to the interference of waves from two sources.

Suppose now that instead of Young's apparatus, we use a monochromatic source of plane waves and very narrow slits instead of pinholes. We now have an arrangement such as that shown in figure 8-2, again with rays from S_1 and S_2 for simplicity.

Notice that in this figure d is the slit *spacing*, not their widths, since we are considering the width of the slits to be very small.

Let's now look at Young's experiment more closely. Line S_2B is drawn such that lines $\overline{PB}$ and $\overline{PS_2}$ are equal. If d, the slit spacing, is much smaller than R, which is almost always the case, then $\overline{S_2B}$ is almost perpendicular to $\overline{PS_1}$. That is, if $d << R$, then r_1 and r_2 may be taken as parallel.

The two rays, r_1 and r_2, arriving at P are in phase at the source slits since they were both derived from the same incident wavefront. Because they have a slightly different path length, r_1 and r_2 will arrive at P with a phase difference. The number of wavelengths contained in $\overline{S_1B}$ will determine the nature of the interference at P.

To have a *maximum* at P, $\overline{S_1B}$ must contain an integral number of wavelengths. In other words, the only way r_1 and r_2 will constructively interfere is if the maximum amplitudes of the waves in r_1 and r_2 coincide at P. The only way this will happen is if their path lengths differ by either zero or by a whole multiple of a wavelength.

Therefore, for a maximum at P

$S_1B = m\lambda$ $\quad m = 0, 1, 2, \ldots$

Since

$$S_1B = d \sin \Theta$$

then

$$d \sin \Theta \doteq m\lambda \qquad m = 0, 1, 2, \ldots \qquad \textbf{(8.1)}$$

Note that each maximum is symmetrically located above and below P_o. There will be a central maximum at P_o described by m = 0. This central maximum is often called the zeroth fringe.

Keep in mind that, although the letters may be the same, equation 8.1 is *not* the same as the equation for single slit diffraction. *Make sure you know the difference between the two equations.*

If point P is the center of the mth fringe and d<<R, then Y, the distance from the zeroth to the mth fringe is

$$Y_m = R \tan \Theta_m$$

Since Θ is usually quite small,

$$Y_m = R \sin \Theta_m$$

or

$$Y_m = R \frac{m\lambda}{d}$$

and

$$\lambda = \frac{Y_m d}{mR} \qquad \textbf{(8.2)}$$

Thus, by measuring the slit spacing, the distance from the slits to the screen, and the distance from the zeroth fringe to the mth fringe on either side, we can compute the wavelength of the light producing the interference pattern.

Let's consider an example of how this may be used. With two slits spaced 0.2 mm apart, and a screen at a distance of one meter, the third bright fringe is found to be displaced 7.5 mm from the central fringe. Find the wavelength of the light used.

$$\lambda = \frac{Y_m d}{mR}$$

$$\lambda = \frac{(0.75\text{cm})(.02\text{cm})}{(3)(100\text{cm})}$$

$$\lambda = 5000 \times 10^{-10}\text{m} = 5000\ \text{Å}$$

If the apparatus of figure 8-2 is set up in the laboratory, the resulting pattern will be a series of fairly narrow bright fringes about a bright, narrow central maximum. However, if you extend beyond about m = 3, the fringes very rapidly decrease in brightness (assuming non-laser sources). This rapid decrease in intensity for the higher order fringes is due to diffraction effects which arise because of the finite width of the slits. Another disadvantage of the two-slit arrangement is that Θ is normally extremely small for reasonable fringe orders. Thus the approximations used to arrive at equation 8.2 are not conducive to high degrees of accuracy. One way to alleviate the problem, that is, to obtain a large number of fringes reasonably well displaced, is to use more than two very narrow slits; for example, five or ten thousand slits per inch.

Suppose that instead of two slits we have a very large number of parallel slits all of the same width and spaced at regular intervals. Such an arrangement is known as a *diffraction grating*. These gratings are usually constructed by ruling, with a diamond point, a large number of equidistant grooves on a metal or glass surface.

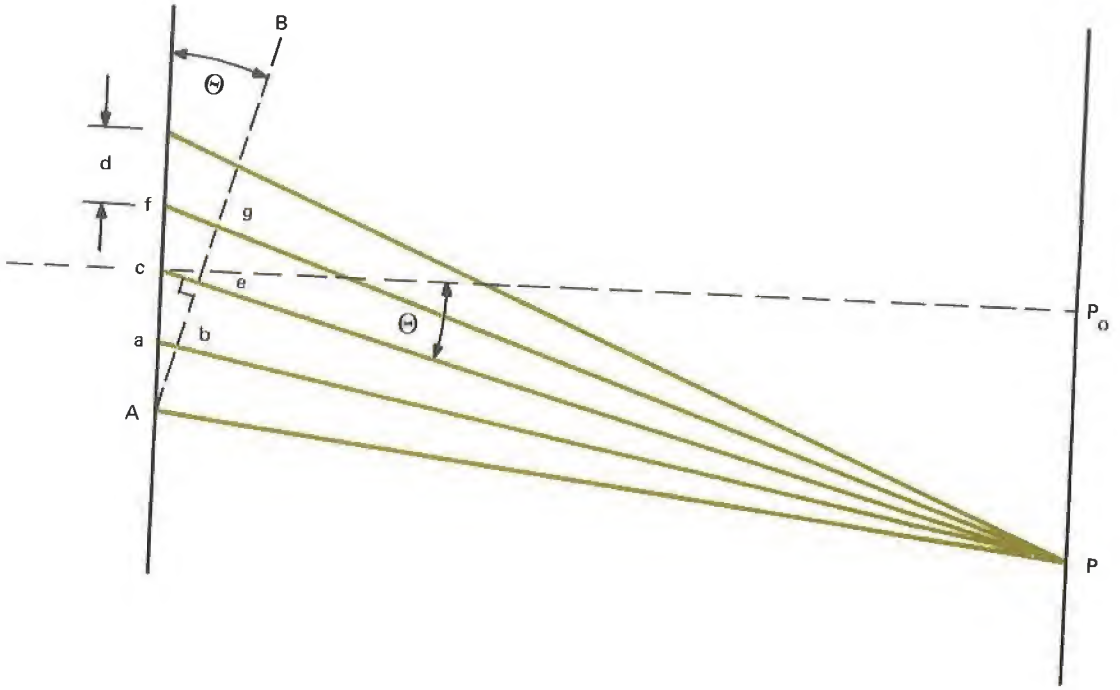

Fig. 8-3 The Plane Diffraction Grating

Figure 8-3 illustrates a five-slit grating. While only five slits are shown, an actual grating may contain upwards of ten thousand "slits" per inch.

Here again, d is the slit spacing. We also again assume that the slits are so narrow that each acts as a source.

If a train of plane waves is incident from the left, each slit will appear to radiate individually. The radiations from each of the slits will overlap the radiations of those next to it in much the same manner as with a double slit.

The rays emerging from each slit are again in phase. Now suppose that Θ is such that the distance ab = λ. Then if d is sufficiently small, ce = 2λ, fg = 3λ and so on. The waves from all these slits will arrive at line $\overline{AB}$ in phase and will thus arrive at P in phase.

If the angle Θ is increased slightly, the waves from the grating elements no longer arrive at $\overline{AB}$ in phase, and even a minutely small change in Θ will result in almost complete destructive interference of the waves, providing there is a very large number of slits in the grating. Therefore, the maximum at the angle Θ is an extremely sharp one, differing from the rather broad maxima and minima which result from the interference of diffraction with a small number of slits.

If Θ is further increased, a point will be reached where ab = 2λ. Then, under our assumptions, ce = 4λ, fg = 6λ, and so forth. The waves at $\overline{AB}$ are again in phase and another maxima results on the screen. Keep in mind the maxima will be symmetrical about P_o.

The angles of deviation for which the maxima occur are found in like manner to those of the double slit pattern. If we assume the rays leaving the slit are nearly parallel, then in a similar way to that used before, the condition for a maximum at P is

$$ab = m\lambda \qquad m = 0, 1, 2, \ldots$$

or

$$d \sin \Theta = m\lambda \qquad m = 0, 1, 2, \ldots \qquad (8.3)$$

The angle Θ is, under our assumptions, also the angle by which the rays corresponding to the maxima have been deviated from the incident light.

Since the angle Θ is quite sharply defined when the number of slits is large, diffraction gratings are used quite extensively for spectral analysis. If the grating is illuminated by a beam of white light, fringes corresponding to m = 0, 1, 2, . . . will appear. The separate fringes, instead of being white, will be broken up into finely divided bands of color due to the different wavelengths. In other words, the zeroth fringe will be broken up into different colored lines as will the first, second, third, and so on, fringes.

In this case, the m = 1 colored fringe is called the first order spectrum, the m = 2 colored fringe is called the second order spectrum, and so on.

MATERIALS

1 He-Ne laser

1 Double slit

1 Transmission type diffraction grating

1 Lens holder

1 Meter stick

1 Screen

1 Screen holder

2 Carriages

1 Optical bench

PROCEDURE

1. Align the optical bench axis with the laser beam.
2. Mount the double slit near the laser and the screen on the far end of the bench.
3. Adjust the slit until an interference pattern is observed on the screen.
4. Note any difference between this pattern and those that would be observed for the single slit.
5. Replace the screen with the meter stick.

DOUBLE SLIT

d	R	Y_m	m	λ (COMPUTED)
			AVERAGE λ	

DIFFRACTION GRATING

STATED LASER λ	m	d (COMPUTED)
	AVERAGE d	

Fig. 8-4 The Data Tables

6. Using the same techniques, measure the slit spacing, R and Y_m. You may use any m you wish.
7. Record these values in the data table, figure 8-4.
8. Knowing d, R, Y_m, and m, compute the wavelength of the laser.
9. Repeat steps 6, 7, and 8 for two more values of fringe order.
10. Replace the double slit with the diffraction grating. Note and record any differences between this multiple slit pattern and the double slit pattern.
11. Using the stated value for the laser wavelength and any m you desire, compute the slit spacing.
12. Repeat step 11 for two different values of m.

ANALYSIS GUIDE. In the analysis of this experiment you should discuss any noticeable characteristics between the double slit interference pattern and the diffraction grating pattern. You should also compare the calculated wavelengths from steps 8 and 9 with the stated value for the laser. List and justify the most probable sources of error in the experimental results.

PROBLEMS

1. Using the average value obtained for the slit spacing of the diffraction grating, calculate the number of slits per inch.

2. In what way are interference and diffraction similar? In what way are they different?

3. Two slits are spaced 0.3 mm apart and are placed 50 cm from a screen. What is the distance between the second and third maxima when the illuminating light has a wavelength of 6000 Å?

experiment 9 TOTAL INTERNAL REFLECTION

INTRODUCTION. Total internal reflection has a number of interesting applications. Among the devices which employ this effect are light guides and fiber optic applications. The purpose of this experiment is to investigate total internal reflection and its effects.

DISCUSSION. Suppose we have some light rays originating in an optically dense medium of index n_1 which fall on the interface of this medium and another *less* dense medium of index n_2. This situation, for a few rays, is shown in figure 9-1.

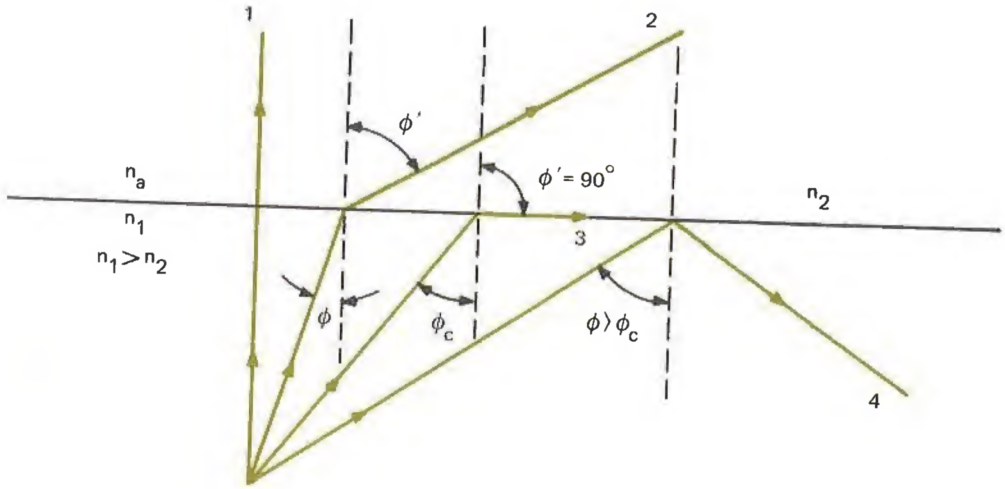

Fig. 9-1 Total Internal Reflection
ϕ_c = critical angle

We see that for rays 1, 2 and 3, 4 passing from n_1 to n_2, the angle of incidence ϕ is increasing. Since $n_1 > n_2$, then the angle of refraction $\phi' > \phi$. As the angle of incidence is increased, a point will be reached when the angle of refraction, ϕ', will equal 90°. The angle of incidence at the point when ϕ' equals 90° is called the critical angle and is designated by ϕ_c. For any $\phi > \phi_c$, *no refracted ray exists*, giving rise to *total internal reflection*. Such a reflected ray is shown as number 4.

Recalling Snell's law,

$$n_1 \sin \phi = n_2 \sin \phi'$$

The critical angle is found by setting $\phi' = 90°$.

$$n_1 \sin \phi_c = n_2 \sin 90°$$

$$\boxed{\sin \phi_c = \frac{n_2}{n_1}} \qquad (9.1)$$

For glass and air surfaces where the light is going from the glass into the air

$$n_{air} \simeq 1.0 = n_2$$

$$n_{glass} = 1.5 = n_1$$

Therefore

$$\sin \phi_c = \frac{1.0}{1.5} = 0.67$$

and

$$\phi_c = 41.8°$$

Using this effect, light can be piped from one point to another with very little loss by allowing it to enter one end of a transparent plastic rod. The light will undergo total internal reflection at the boundary of the rod and will follow its contour, emerging at its far end. This situation is illustrated in figure 9-2, for one incoming ray.

Images may be transferred from one point to another using a bundle of such plastic rods, each rod or fiber transmitting a small fraction of the image. Such bundles can be made in which a flexible seven-foot length delivers approximately one-half the intensity entering at one end. In a fiber of such a bundle, a typical ray may undergo about 48,000 reflections. Most of the loss in intensity is due to absorption within the rods, the reflections being almost truly total. Measurable amounts of light intensity may be transmitted through 'fiber optics' up to 150 feet long. Fiber optics makes it possible to transmit light or images into "hard-to-get-at" places without the use of an elaborate mirror arrangement.

Recall equation 9.1. The critical angle for a glass-air surface, assuming $n_1 = 1.50$ for glass, is

$$\sin \phi_c = \frac{1.0}{1.5} = 0.67$$

$$\phi_c = 41.8°$$

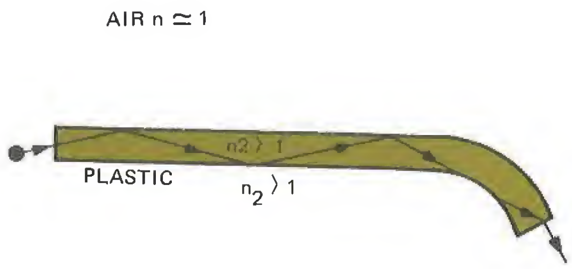

Fig. 9-2 'Light Guide' Using Total Internal Reflection

This angle, being less than 45°, makes it possible for a 45°-90°-45° prism to be used as a totally reflecting surface as shown in figure 9-3.

The totally reflecting prism has two primary advantages over metallic surfaces. No metallic surface reflects 100% of the light incident upon it. And the reflecting properties of this device are permanent and are not affected by tarnish. Figure 9-3 shows the simpler type. Light incident, normally on one of the shorter faces, strikes the longer face at 45°. This is greater than the critical angle, so the light is totally reflected and leaves the second of the shorter faces at a deviation of 90°.

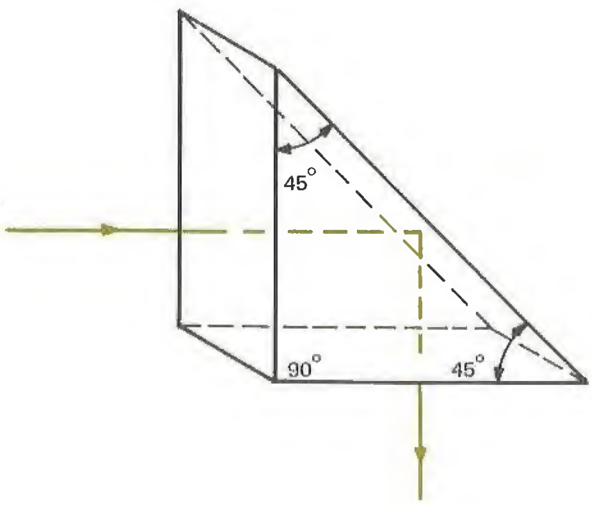

Fig. 9-3 A Totally Reflecting Prism

A 45°-90°-45° prism may also be used as shown in figure 9-4.

Here, the light enters and leaves the longer face normally and is reflected at each of the shorter faces. The total deviation is 180°. When a prism is used in this way, it is called a Porro prism.

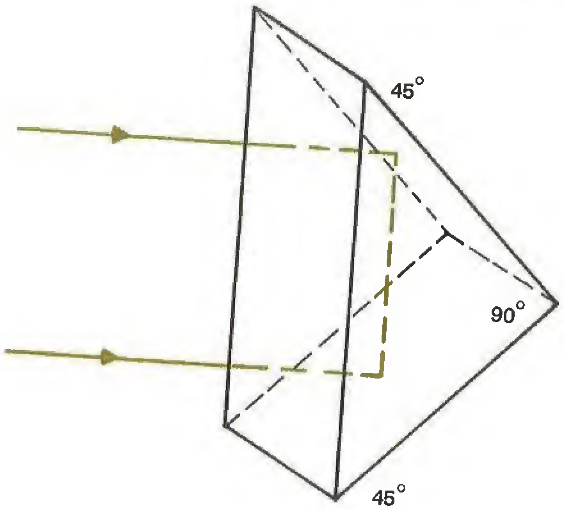

Fig. 9-4 A Porro Prism

MATERIALS

1	Optical bench	1	Support platform
1	Carriage	1	45°-90°-45° prism
1	He-Ne laser	4	Lucite rods (angles 45°, 90°, 135°, 180°)

PROCEDURE

NOTE: You may wish to use nontoxic smoke in this experiment.

1. Align, both vertically and horizontally, the optical bench with the laser beam.
2. Insert a lucite rod into the laser beam such that the beam enters the face normally. Observe the brightness at the exit face.
3. Repeat step 2 with the other three rods, observing the relative brightness at the exit face in each case.

4. Repeat steps 2 and 3, only this time let the beam enter the rod at some angle (see figure 9-2). Observe the path the laser beam takes down the rods and again the relative brightness at the exit ends. You should adjust the entrance angle so that you can observe at least three internal reflections of the laser beam.

5. Sketch, to scale if possible, each lucite rod and the beam path from step 4.

6. Insert the 45°-90°-45° prism into the beam such that the beam enters one of the shorter faces normally.

7. Sketch the prism and beam path from the observations of step 6.

8. Insert the prism again, but let the beam enter the longer face normally and to one side of the 90° apex. Slide the prism from side to side (perpendicular to the entrance beam) and observe the change in linear distance between the entrance and exit beams.

9. Sketch the results of step 8.

ANALYSIS GUIDE. Along with the sketches, you should discuss which rod appeared to transmit the most light and which appeared to transmit the least, You should also give reasons why the rods transmitted different amounts of light when the entering light stayed constant. You should discuss the results and implications of step eight.

PROBLEMS

1. A light guide normally consists of a bundle of glass or plastic fibers instead of a solid piece. Aside from a structural standpoint, is there any optical reason for this? If so, what?

2. Is there an electrical analogy to the question of problem 1? What is it?

3. Aside from structural considerations, is there any optical limit on the radius of bend of a light guide, either fibrous or solid rod? Why?

4. The Porro prism is sometimes called a "gain-of-one image inverter." Why?

5. If, in figure 9-1, the lower medium were water of $n = 1.33$, $\phi_c = ?$

6. Could a 45°-90°-45° prism be made of ice and function as a Porro prism (ice has $n = 1.309$)? Why?

Sketch of First "Rod" Results

Sketch of Second "Rod" Results

Sketch of Third "Rod" Results

Sketch of Fourth "Rod" Results

Fig. 9-5 Sketches of the Results

Sketch of First "Prism" Results

Sketch of Second "Prism" Results

Fig. 9-5 Sketches of the Results (Con't) experiment **10** SIMPLE LENSES

INTRODUCTION. The lens is probably the most widely used component in optics. It is used to amplify or intensify images while maintaining the same relative direction of a light beam. The purpose of this experiment is to investigate some of the properties of simple lenses.

DISCUSSION. Before considering lenses, let's briefly discuss refraction at a spherical surface. A typical case is illustrated in figure 10-1.

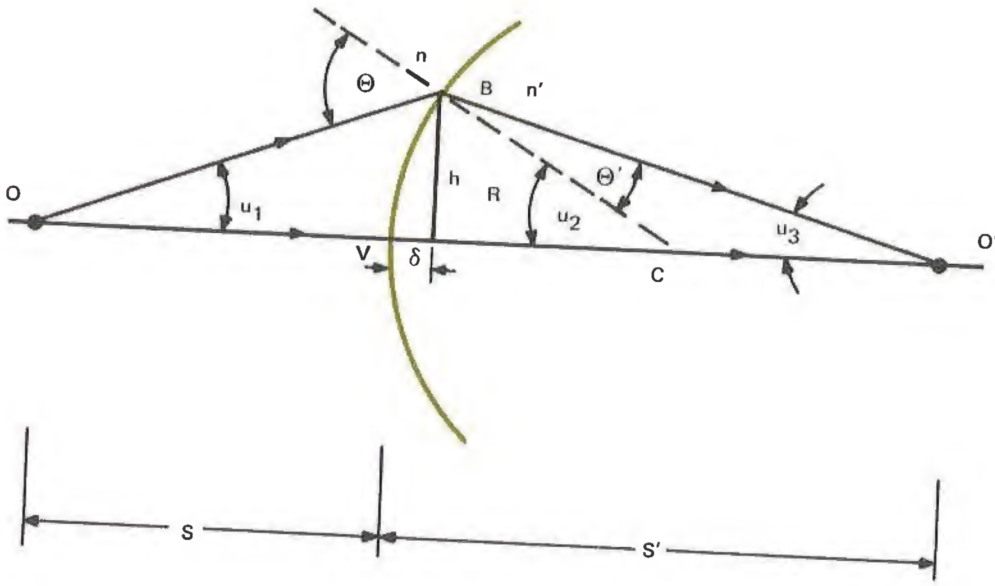

Fig. 10-1 Refraction at a Spherical Surface

O is an object point a distance S from the spherical surface in a medium of index n. O' is the image of O a distance S' from the spherical surface in a different medium of index n'. Ray OB is incident at angle Θ' and is refracted at angle Θ', while ray OV is incident normally and is thus not refracted and passes into the second medium undeviated. We see that

$$\Theta = u_1 + u_2 \text{ and } u_2 = u_3 + \Theta'$$

From Snell's law

$$n \sin \Theta = n' \sin \Theta'$$

The tangents of u_1, u_2 and u_3 are

$$\tan u_1 = \frac{h}{S+\delta}, \quad \tan u_2 = \frac{h}{R-\delta}$$

$$\text{and } \tan u_3 = \frac{h}{S'-\delta}$$

If the angles thus involved are small, we may approximate both the sine and tangent of the angles by the angles themselves. Also, if the angles are sufficiently small, we may neglect δ.

Therefore, for paraxial rays, Snell's law is approximately

$$n\Theta = n'\,\Theta'$$

Substituting into the first of the first two equations,

$$\Theta' = \frac{n}{n'}(u_1 + u_2)$$

and this value of Θ' in the second equation yields

$$nu_1 + n'\,u_3 = (n' - n)u_2$$

Using the small angle approximations for u_1, u_2 and u_3

$$n\frac{h}{S} + n'\frac{h}{S'} = (n' - n)\frac{h}{R}$$

or

$$\boxed{\frac{n}{S} + \frac{n'}{S'} = \frac{n' - n}{R}} \qquad \textbf{(10.1)}$$

A lens, however, is an optical system bounded by two refracting surfaces such as shown in figure 10-2.

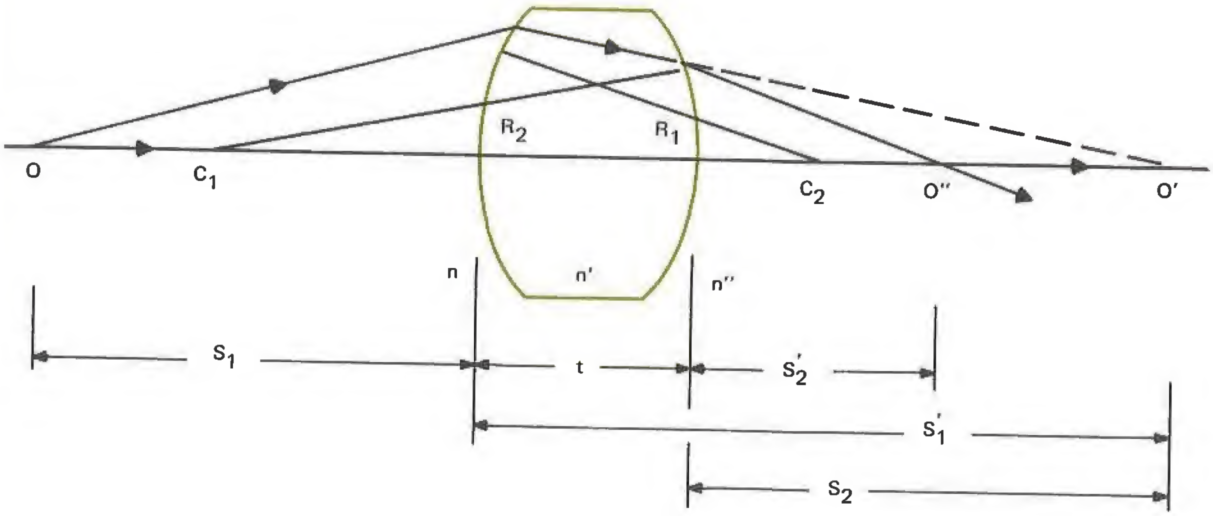

Fig. 10-2 The Thick Lens

Here O′ is the image of O if the lefthand surface of the lens is treated separately by equation 10.1, that is, if the righthand surface were not there and medium n′ extended infinitely to the right. Now O′ can be considered as the object for the righthand surface with the left surface missing. The image thus formed with O′ as the object is O″. A technique similar to that used before can be employed for finding O″.

For the first surface we have equation 10.1,

$$\frac{n}{S_1} + \frac{n'}{S'_1} = \frac{n' - n}{R_1}$$

And for the second surface by similar manipulation, we have

$$\frac{n'}{S_2} + \frac{n''}{S'_2} = \frac{n'' - n'}{R_2}$$

where

$$S_2 = S'_1 - t$$

These then, are the equations one would use for finding the object of a *thick* lens; in other words, a lens for which t is not negligible compared with the other dimensions.

Suppose now, that both media n and n'' are air of index one. We may then call the index of refraction of the thick lens just n, and we have, for a thick lens in air

$$\frac{1}{S_1} + \frac{n}{S'_1} = \frac{n-1}{R_2}$$

for the first surface.

If the lens is so thin that its thickness t is negligible compared to S_1, S'_1, S_2 and S'_2, we have very nearly

$$S_2 = -S'_1$$

If this substitution is made into the two previous equations and then they are added, we have for a *thin* lens:

$$\frac{1}{S_1} + \frac{1}{S'_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Since S_1 is the object distance for the thin lens and S'_2 the final image distance, we may drop the subscripts and obtain

$$\boxed{\frac{1}{S} + \frac{1}{S'} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} \quad \textbf{(10.2)}$$

The usual sign conventions apply to this equation (that is, the sign conventions employed for curved reflecting surfaces). In fact, the substitution $S_2 = -S'_1$ was made because O′ is a virtual object for surface two.

The *focal length f* of a thin lens may be defined as the object distance of a point on the lens axis whose image is at infinity, or the image distance of a point object at an infinite distance from the lens on the lens axis. When we set either S or S′ equal to infinity, we have for the focal length

$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} \quad \textbf{(10.3)}$$

Equation 10.3 is known as the *lensmaker's equation* for thin lenses.

Our final equation is, substituting from 10.3 into 10.2,

$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f} \quad \textbf{(10.4)}$$

Notice that this equation has exactly the same form as that for a spherical mirror.

The image magnification for a finite object can be found to be

$$M = -\frac{S'}{S} \quad \textbf{(10.5)}$$

in the same manner as that for a spherical mirror.

Remember, that the sign conventions employed for spherical mirrors are the same ones used here.

In regard to the focal point, the object point for which the image is at infinity is termed the first focal point of the lens. The image point for an object at infinity is called the second focal point. The focal points of a thin lens lie on opposite sides of the lens at a distance from it equal to its focal length. The first and second focal points, labeled F and F′, are shown for a converging lens in figure 10-3.

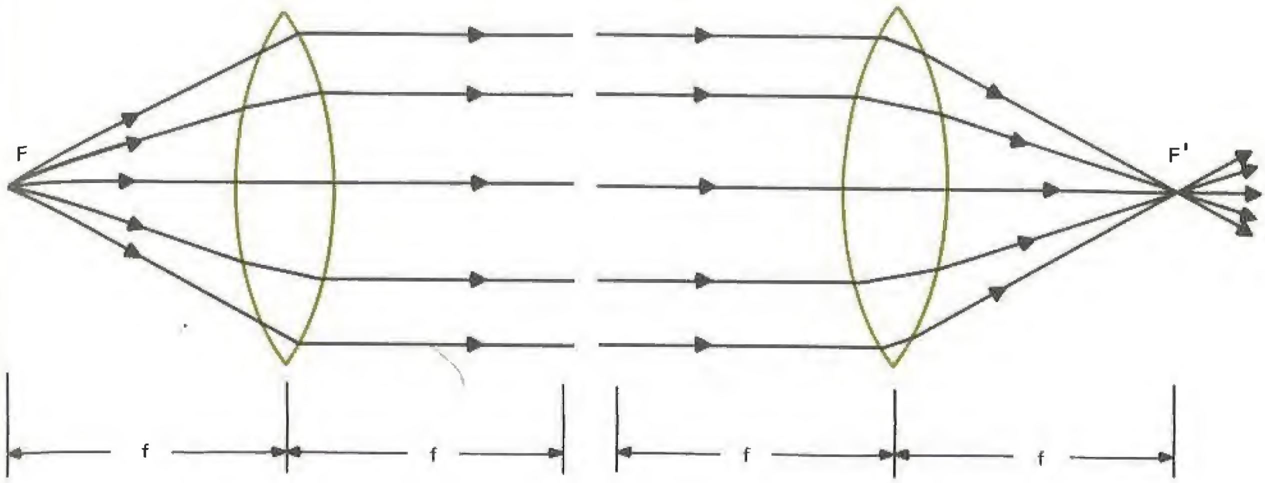

Fig. 10-3 First and Second Focal Points of a Thin Lens

Planes, perpendicular to the lens axis through F and F′, are called the *first and second focal planes*. In other words, an object of finite height lying in the first focal plane will have its image at infinity. Likewise, the image of an infinitely distant object of finite size will lie in the second focal plane. In this experiment, we will deal only with converging thin lenses.

Physically, a converging lens is always thicker in the center than at the edge. This can be seen in figure 10-4 for three basic types of converging lenses.

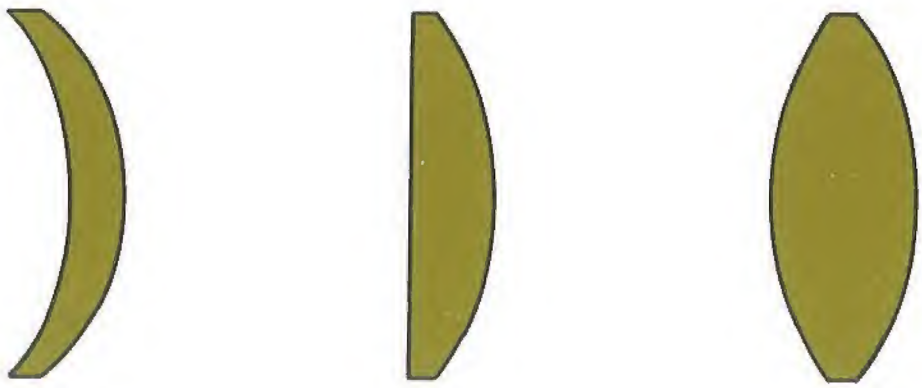

Fig. 10-4 Meniscus, Plano-Convex, and Double Convex Converging Lenses

A converging lens always has a *positive* focal length. This can be seen from equation 10.3 and figure 10-2. Following our sign convention, R_1 is positive and R_2 is negative, which will always yield a positive focal length.

The position and size of a converging thin lens image may be found by one of two ways. One is graphical and the other is mathematical. The graphical method consists of finding the point of intersection, after passage through the lens, of a few rays diverging from some point on the object. Then, ideally, all rays from this same object point will intersect at the same point after passing through the lens. In using this method, the total deviation of a ray is assumed to take place at a plane passing through the center of the lens. Three rays whose paths may be readily traced are:

1. *A ray parallel to the axis.* After refraction, this ray passes through the second focal point of the lens.
2. *A ray through the center of the lens.* If the lens is thin, a ray through the center will not be appreciably deviated.
3. *A ray through the first focal point.* This ray emerges parallel to the lens axis.

These methods are illustrated in figure 10-5.

Notice that in figure 10-5, the image is real, inverted and diminished. The image produced by a converging lens when $S>f$ is always real and inverted.

Of course, the other way of finding an image is by use of equation 10.4, keeping in mind the sign conventions.

To find the image magnification, one may either measure the object and image or use equation 10.5.

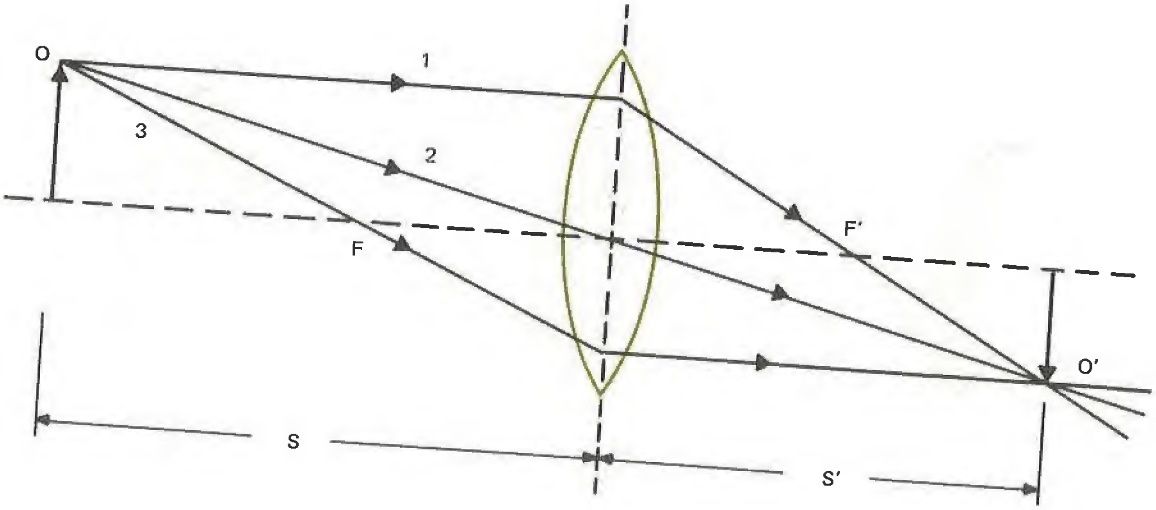

Fig. 10-5 Graphical Method of Locating an Image

A converging lens may also form a virtual image. From equation 10.4, if $S<f$, then the sign of S' will be negative indicating a virtual image. If S' is negative, M, from equation 10.5, will be positive, indicating that the image is upright.

We have discussed magnification and determined that by using equation 10.5

$$M = -\frac{S'}{S}$$

It appears that we can magnify an image to any degree desired. One important consideration to deal with here is the *resolving power* of a system of lenses. There is an upper limit to the ability of a lens system to magnify an object clearly. If two points of an object P_1 and P_2 are two separate distinct points of an object, their interference pattern must be such that the maximum of the diffraction pattern of, say P_1 just coincides with the first minimum of P_2. This is Rayleigh's criterion. If the separation of P_1 and P_2 is not large enough, then the magnified pattern of P_1 and P_2 will appear as a blur. See figure 10-6. The aperture of the magnifying system must be large enough to produce an interference pattern that will meet Rayleigh's criterion. If the aperture is too small, the magnified image will appear as that shown in figure 10-6.

Fig. 10-6 Resolving Power of a Lens System

MATERIALS

1 Double-convex lens

1 Plano-convex lens

1 Positive meniscus lens

1 Optical bench

1 Illuminated object box

1 Lens holder

1 Screen

1 Screen holder

1 Cardboard pointer

PROCEDURE

NOTE: You may wish to use nontoxic smoke in this experiment.

1. Set up the optical bench as diagrammed in figure 10-7.

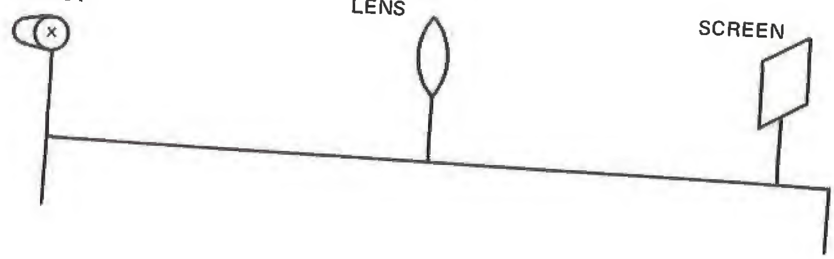

Fig. 10-7 Optical Bench Set-Up

2. Start with the largest possible S to obtain a real image on the screen.
3. Record S and S′.
4. Decrease S in steps of 5 cm, recording S′ in each step. Continue until a real image is no longer obtained.
5. Insert the pointer into the screen holder and place the assembly behind the object box.
6. Using the pointer assembly, locate the virtual image by the parallax method.
7. Record at least five well-spaced data points for the virtual image (S and S′).
8. Repeat steps 1 through 7 for the other two lenses.
9. For each lens, compute f and M at each S and S′. Find the average f.
10. Construct a plot of S/f versus S′/f for each lens. Label each data point on the plot with the M at that point.

S	S′	f	M

AVERAGE f	

Fig. 10-8 The Data Table for Lens I

S	S′	f	M
	AVERAGE f		

Fig. 10-8 (Con't) The Data Table for Lens II

S	S′	f	M
	AVERAGE f		

Fig. 10-8 (Con't) The Data Table for Lens III

ANALYSIS GUIDE. In the analysis of this experiment you should discuss the condition of the observed images (upright, inverted, enlarged, etc.). You should also discuss the relative reliability of the real image method versus the virtual image method. Also, discuss any observed defects in the images (fuzzy, distorted, etc.).

PROBLEMS

1. The curves drawn in step ten could be called "universal lens curves" for a converging thin lens. Why?

2. What would have to be taken into account if the lenses used here were not considered thin?

3. Why does a lens have two focal points, and a curved mirror only one?

4. An unsymmetrical thin lens (plano-convex) forms a real image on a screen. Is the image location changed if the lens rotates 180°?

5. Equation 10-4 applies only to what type of rays?

experiment 11 BASIC LENS SYSTEMS

INTRODUCTION. Most optical systems, such as telescopes, microscopes or collimators, employ two or more lenses. In each case, the image formed by one lens serves as the object for another. In this experiment we will examine a simple lens system and some of its characteristics.

DISCUSSION. Before discussing lens systems, let's begin with diverging thin lenses.

A bundle of parallel rays incident on the left lens of figure 11-1 becomes divergent after refraction, and the lens is thus called a diverging lens.

The focal length calculated by the equation

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

is a negative quantity, since by our sign convention, R_1 is negative and R_2 is positive. Therefore the lens is called a negative lens. Since the focal length of the diverging lens is negative, the focal points are reversed relative to those of a positive lens.

The second focal point, F′, in figure 11-1, is the point from which rays, originally parallel to the axis, *appear* to diverge after refraction by the lens. Rays converging to the first focal point in figure 11-1 will emerge parallel to the axis after refraction. Therefore, F′ is to the left of the lens while F is to the right, just opposite from a converging lens.

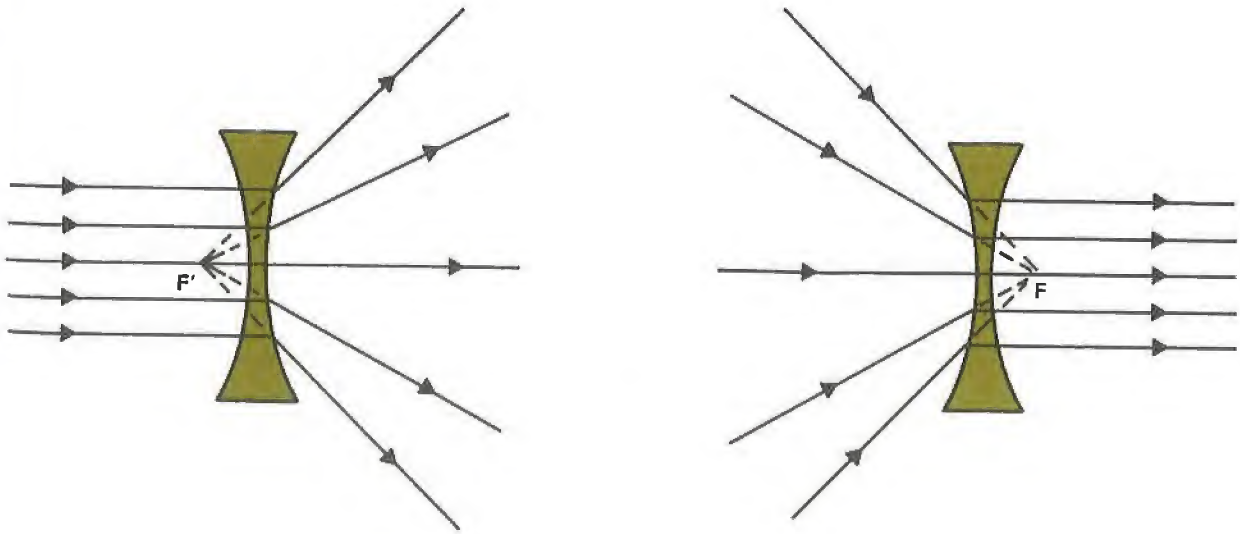

Fig. 11-1 Focal Points of a Diverging Lens

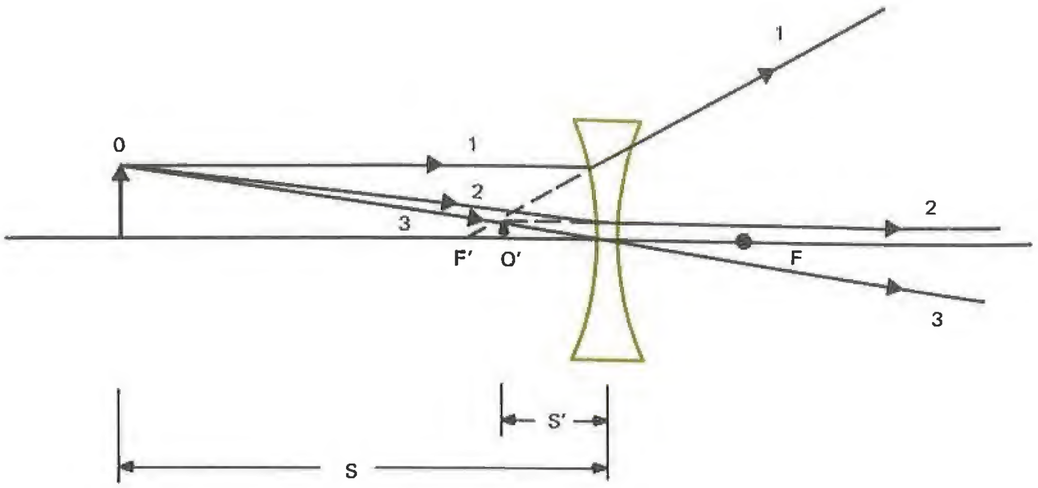

Fig. 11-2 Graphical Method of Locating an Image

Equation 11.1 applies equally well to a diverging lens and a converging lens, but one must keep in mind that the focal length is negative for the diverging lens.

$$\frac{1}{S} + \frac{1}{S'} = \frac{1}{f} \qquad (11.1)$$

The graphical method of locating an image formed by a negative lens is illustrated in figure 11-2.

The three rays most easily traced for a diverging lens are:

1. *A ray parallel to the axis.* After refraction by the lens, this ray appears to come from the second focal point.

2. *A ray proceeding toward the first focal point.* This ray emerges parallel to the axis.

3. *A ray through the center of the lens.* If the lens is considered thin, this ray is not appreciably deviated.

Notice that the image is virtual, upright, and diminished. For real objects, this type of lens will *always* produce this type of image.

There are several types of diverging lenses. Physically, however, a diverging lens is always thicker at the edge than at the center. The three basic types are shown in figure 11-3.

Fig. 11-3 Meniscus, Plano-Concave, and Double-Concave Diverging Lenses

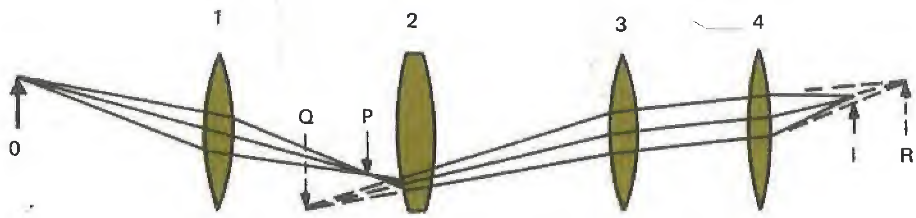

Fig. 11-4 The Object for Each Lens, After the First, Is the Image Formed by the Preceeding Lens

Within a lens the image formed by one refracting surface acts as the object for the other. In the majority of optical systems employing lenses, more than one lens is used, and the image formed by any one lens acts as the object for the next one. Figure 11-4 shows some of the various possibilities.

Lens 1 forms a real image at P of real object O. Lens 2 forms a virtual image at Q of the image at P. The virtual image at Q is a real object for lens 3. Lens 3 forms an image at R of Q and this image (R) acts as a virtual object for lens 4. Lens 4 forms a real image of virtual object R at I.

All optical instruments employing more than one lens may be analyzed in the preceeding manner. The final image produced by the system can be found either graphically, or by using equation 11.1 for each lens in turn. Some simplification can be made in the case of telescopes where the object distance of the first (actual) object is close to infinity. The image formed by the first lens (objective) is thus located at the focal plane of the lens. The second lens (eyepiece) acts upon this image and forms either a real or a virtual image of the apparent object at the focal plane of the first lens.

Many optical instruments, such as telescopes, microscopes or even simple magnifiers, will form virtual images as the final image. The location of the image formed by, say a microscope, is usually at least 25 centimeters from the eye. The reason for this is that the eye can accommodate this distance quite comfortably. Most telescopes will form final enlarged virtual images anywhere from 25 centimeters to infinity from the eye. Remember that the eye itself is a lens system that will process the enlarged virtual image formed by a telescope or microscope into a real image on the retina.

In this experiment we will use real images to find the focal length of a diverging lens. This is not contradictory to what was said before about the images formed by such lenses. We shall use the virtual image formed by the diverging lens as a real object for a converging lens of known focal length.

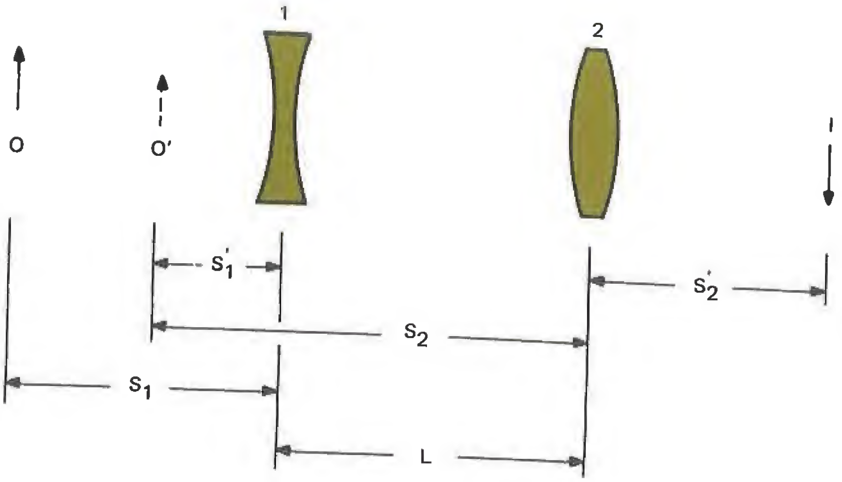

Fig. 11-5 Focal Length Measurement of Divergent Lens

The basic setup for this type of system is illustrated in figure 11-5, where L is the distance between the lenses.

If the focal length of lens 2 is known, then S_2 may be found. Knowing S_2 and the lens separation, the image distance for lens 1 may be found. Thus having S_1' and S_1, the focal length of lens 1 may be computed.

This method may seem rather indirect since f_1 could be measured by the parallax method. However, generally, the use of real images yields more consistent and accurate results.

MATERIALS

1 Optical bench
1 Illuminated object
1 Screen
1 Screen holder
2 Lens holders
4 Carriages
1 Converging lens
1 Double-concave lens
1 Plano-concave lens
1 Meniscus negative lens

PROCEDURE

NOTE: You may wish to use nontoxic smoke in this experiment.

1. If the focal length of the converging lens is not known, measure it.

2. Set up the optical bench to measure the focal length of a negative lens using real images. Refer to the discussion and figure 11-5.

3. Adjust the system until a real image is obtained on the screen.

4. Record L, S'_2 and S_1.

5. Change S_1 by approximately five centimeters. Adjust S'_2 until a new image is formed on the screen. Record S_1 S'_2 and L.

6. Record at least five data points in Data Table, figure 11-6.

7. Replace the negative lens used before by one of the other two and repeat steps three through six.

8. Perform steps three through six for the third negative lens.

9. Check to see if rotating the axis of lens 1 by 180° yields a different S'_2.

10. Compute S_2, S'_1 and f_1 for each data run.

11. Compute the average f for each negative lens used.

ANALYSIS GUIDE. In the analysis of this experiment you should discuss any apparent difficulties in obtaining the final real image. You should discuss the limiting factors of this method, such as, under what conditions a real image would not be obtained. You should also discuss the type of real images observed and any apparent defects in the images.

PROBLEMS

1. Did S'_2 change in step nine? Why or why not?

2. From the type of image observed on the screen, can you tell what type of image lens 1 produced? How?

3. Should S'_1 change as L is varied?

4. As L is varied, what quantities will change?

5. Construct a diagram to illustrate the method you would employ to measure the focal length of a convex mirror using the location of a real image.

FIRST LENS

L	S'_2	S_1	S_2	S'_1	f_1
				AVERAGE f_1	

SECOND LENS

				AVERAGE f_1	

THIRD LENS

				AVERAGE f_1	

Fig. 11-6 The Data Tables

experiment 12 LENS ABERRATIONS

INTRODUCTION. In optics we frequently make some basic assumptions regarding curved mirrors and thin lenses. When these assumptions no longer hold, aberrations are present. It is the purpose of this experiment to observe some of these aberrations.

DISCUSSION. The first type of aberration we shall consider is *chromatic aberration.* In most discussions of lenses, no account is taken of the change of refractive index with color. The assumption that n is constant applies only to the behavior of the lens for monochromatic light. Because the refractive index of transparent substances normally does vary with color, a single lens not only forms one image of an object, but a series of images, one for each color present in the beam. Such a series of colored images of an infinitely distant object point on the lens axis is illustrated in figure 12-1.

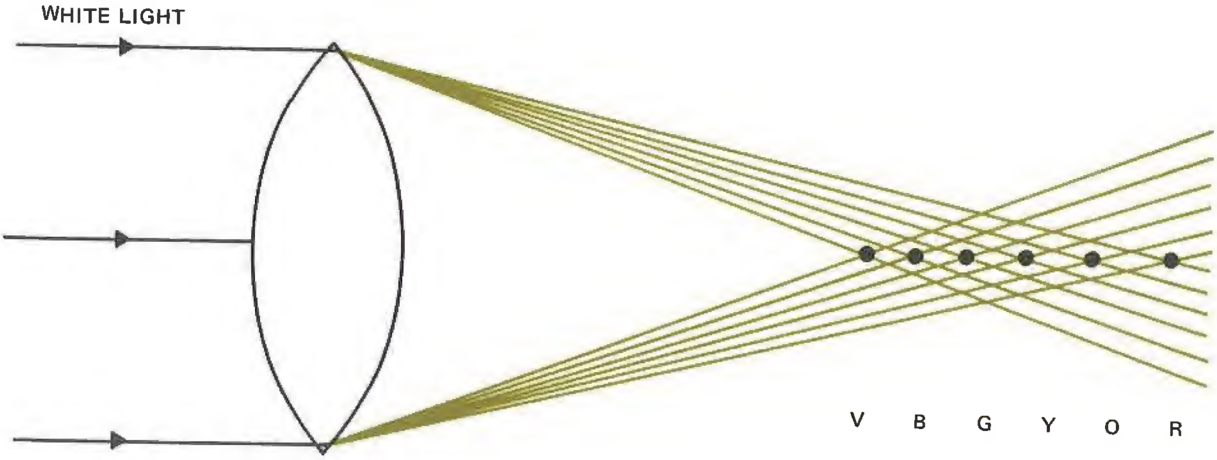

Fig. 12-1 Chromatic Aberration of a Single Lens

The prismatic action of the lens, which increases toward its edge, causes dispersion and brings violet light to a focus nearest to the lens.

Since the focal length of a lens varies with color, the lateral image magnification must vary as well. This is illustrated in figure 12-2 which shows the off-axis image points of an object O for red and violet.

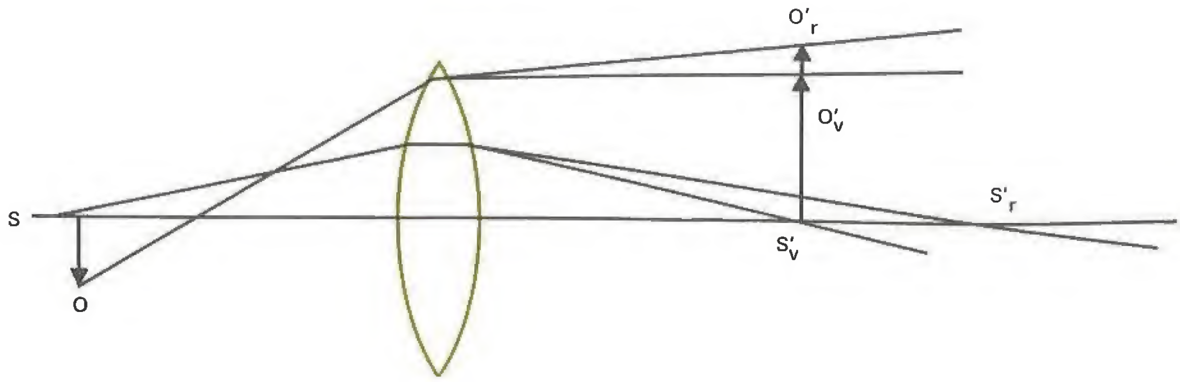

Fig. 12-2 Longitudinal and Lateral Chromatic Aberration

The horizontal distance between the axial images is called longitudinal chromatic aberration. The vertical difference in height is called the lateral chromatic aberration.

There are several methods for correcting chromatic aberration. The most common is the method of employing two thin lenses in contact, one made of crown glass and the other of flint glass. This combination, illustrated in figure 12-3, is usually called an achromatic doublet or simply an achromat.

If a lens is corrected for longitudinal chromatic aberration, it will be corrected for lateral chromatic aberration also.

In figure 12-3 the crown glass lens (C) has a small positive focal length while the flint glass lens (F) has a large negative focal length. The combined focal length is positive since, for the above combination,

$$\frac{1}{f} = \frac{1}{f_C} + \frac{1}{f_F} \qquad (12.1)$$

where the subscripts C and F refer to a crown and flint, respectively.

The crown glass lens has approximately the same dispersion as the flint glass lens. Since f_C is positive and f_F is negative, the dispersion tends to be neutralized, bringing all colors to approximately the same focus.

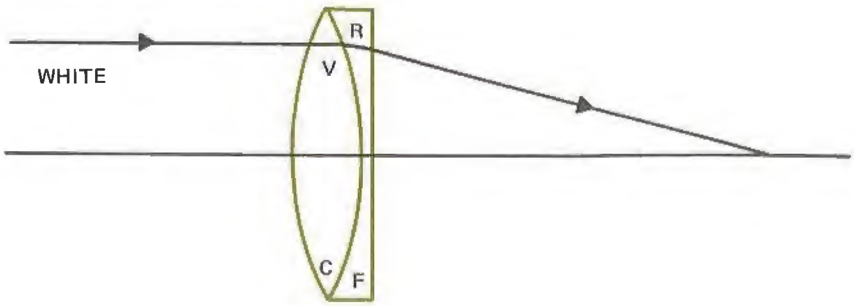

Fig. 12-3 Cemented Doublet Correcting for Chromatic Aberration

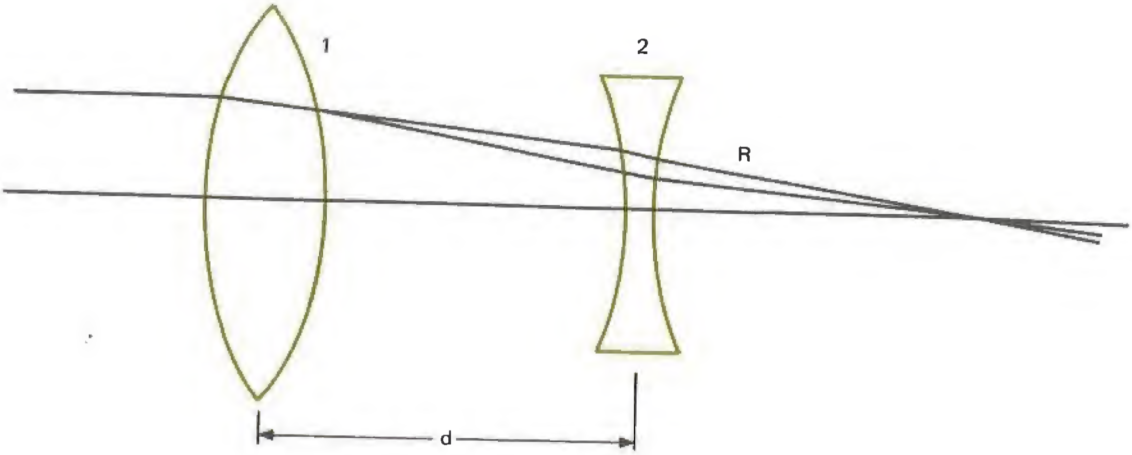

Fig. 12-4 Separated Doublet for Correcting Longitudinal Chromatic Aberration

Another method, which is sometimes used to obtain an achromatic system, is to use two thin lenses made of the same type of glass and separated by a distance equal to half the sum of their focal lengths. This is illustrated in figure 12-4.

This configuration is termed a spaced achromatic doublet and is normally used with lenses of the same type glass. The operation of this system is almost the same as the cemented type with the net positive focal length. The distance between the lenses is

$$d = \frac{f_1 + f_2}{2} \qquad (12.2)$$

The focal length of the negative lens is always such that d is positive.

In this experiment we will simulate a lens using two prisms to emphasize the chromatic aberration effect.

The relatively simple equations that are frequently used to determine object and image distances, focal length, radii of curvature, etc., are usually based upon the approximation of paraxial rays. However, a lens may be used for image points which lie either on or off its axis. Also, because of the finite size of the lens, the cone of rays which form the image of any point is of finite size. Non-paraxial rays proceeding from a given object point do not, in general, all intersect at the same point after refraction by the lens. Therefore, the image formed by these rays is not a sharp one. These considerations apply to monochromatic as well as polychromatic light. However, here, we shall assume monochromatic light so as to eliminate any chromatic aberration effects.

To get an overview of this, let's consider an expansion of a sine function:

$$\sin \Theta = \Theta - \frac{\Theta^3}{3!} + \frac{\Theta^5}{5!} + \frac{\Theta^7}{7!} - \frac{\Theta^9}{9!} \cdot \cdot \cdot$$

For small angles this is a rapidly converging series. It shows that for paraxial rays where the slope angles are small, we may neglect all terms beyond the first and write

$$\sin \Theta \simeq \Theta$$

For a single refracting surface, this yields a familiar expression

$$\frac{n}{S} + \frac{n'}{S'} = \frac{n' - n}{R} \qquad (12.3)$$

This equation and those derived from it form the basis of what is usually called first-order theory.

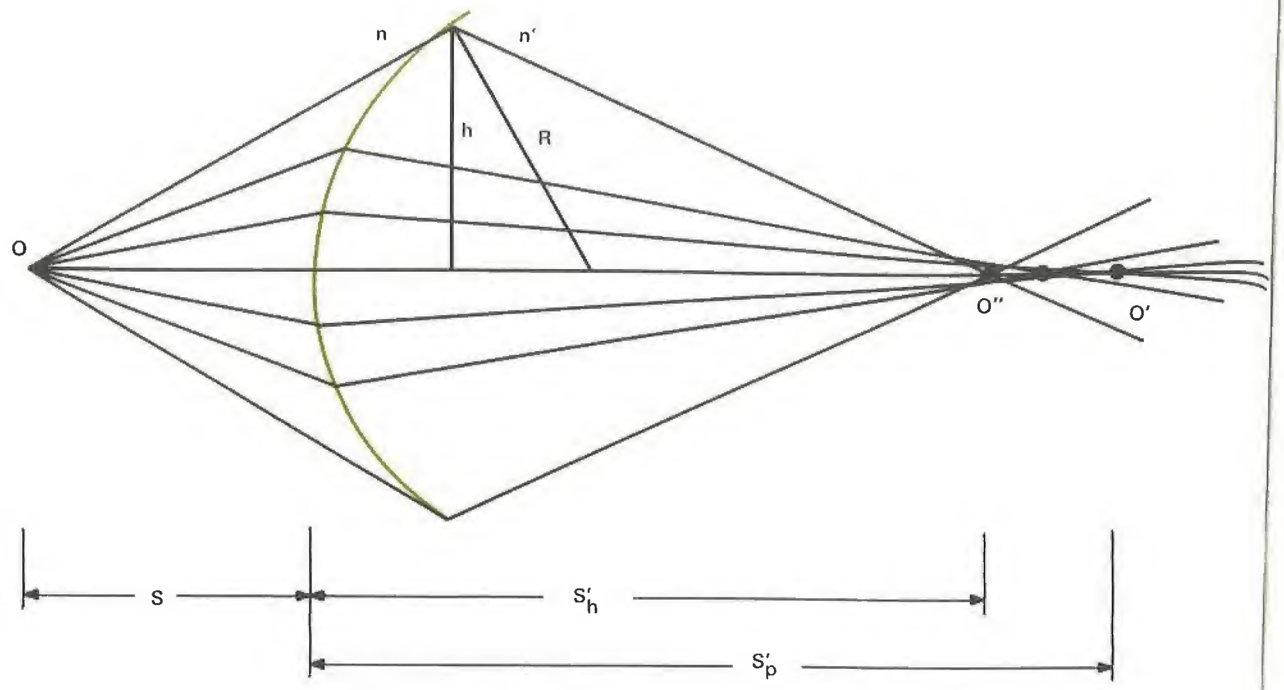

Fig. 12-5 Spherical Aberration in the Image of an Axial Object Point as Formed by a Single Spherical Refracting Surface

If all the sines of angles in the basic lens formulas are replaced by

$$\sin \Theta \simeq \Theta - \frac{\Theta^3}{3!}$$

the resultant equations represent what we may call third-order theory. Third-order theory applies just as well to curved mirrors as it does to lenses.

Aberration due to non-paraxial rays can often be evaluated by third-order theory and is usually called *spherical aberration.* Spherical aberration of a single refracting surface is illustrated in figure 12-5.

O is an object point on the axis of the surface and O′ is the paraxial image point. Oblique rays incident on the surface in a zone of radius h are brought to a focus closer to, and at a distance S'_h from, the vertex of the surface.

The distance, $S'_p - S'_h$, is a measure of the *longitudinal spherical aberration,* and its magnitude may be found from the third-order relationship

$$\frac{n}{S} + \frac{n'}{S'_h} = \frac{n' - n}{R} + \left[\left(\frac{h^2 n^2 R}{2f' n'}\right) \left(\frac{1}{S} + \frac{1}{R}\right)^2 \left(\frac{1}{R} + \frac{n' - n}{nS}\right) \right] \qquad \textbf{(12.4)}$$

Since, from equation 12.3, we have for paraxial rays,

$$\frac{n}{S} + \frac{n'}{S'_p} = \frac{n' - n}{R}$$

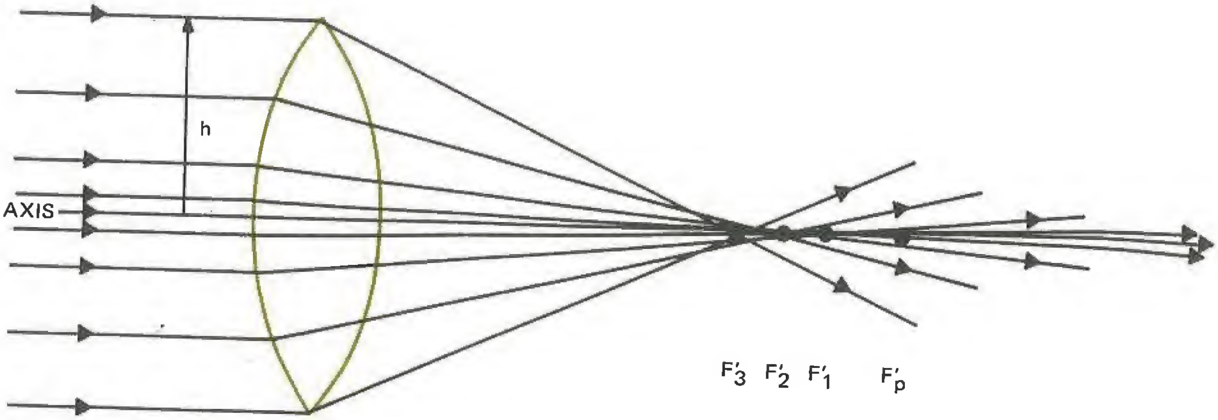

Fig. 12-6 Spherical Aberration of a Thin Lens

the right hand bracket in equation 12.4 is a measure of the deviations from first-order theory. Its magnitude varies with S, and for any fixed point is approximately proportional to h^2.

If $S = \infty$, so that the incident rays are parallel to the axis, equation 12.4 reduces to

$$\frac{n'}{S'_h} = \frac{n'}{f'} + \frac{h^2 n^2}{2f'R^2 n'} \qquad \textbf{(12.5)}$$

Again, the magnitude of the spherical aberration is proportional to h^2, the square of the height of the ray above the axis.

The existence of spherical aberration for a single spherical surface implies that it may also occur in thin lenses. Since many of the thin lenses in optical instruments are used to focus parallel ray light, spherical aberration in thin lenses is usually determined for parallel ray light. Figure 12-6 shows this case and shows the paraxial focal point F'_p as well as F'_1, F'_2 and F'_3 which are the focal points for for zones of increasing diameter.

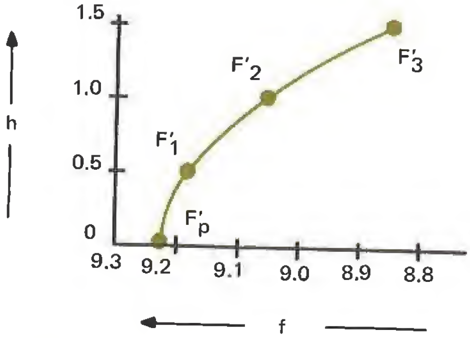

Fig. 12-7 Plot of Variation of Focal Length with Ray Height h

As an example of the actual magnitudes involved, figure 12-7 is a plot of f versus h for a typical lens whose paraxial focal length is 9.22 cm.

For this lens, the focal length for a zone of radius 1.5 cm was 8.88 cm. The spherical aberration is, therefore,

$$SA = F'_p - F'_3$$

$$SA = (9.22 - 8.88)\ \text{cm}$$

$$SA = .34\ \text{cm}$$

A similar curve for a concave lens would bend to the right, indicating negative spherical aberration.

You may have wondered why lenses come in different shapes, such as double-convex, plano-convex, meniscus, etc. This is known as lens bending, that is, finding the shape for which spherical aberration is at a minimum. We will, in this experiment, investigate a few different-shaped lenses to see which has the least amount of spherical aberration.

To perform this experiment, we will employ a very useful optical device, the beam splitter. There are two basic forms of beam splitters, one of which is the prism type shown in figure 12-8.

This type consists of two 45°-90°-45° prisms cemented together at their hypotenuses to form a cube. Sometimes one hypotenuse is silvered to improve the reflectance of one beam. Of course, the main purpose of the beam splitter is to divide an incoming light beam into two parts, one of which is deviated by 90°, the other by 0°. Typical values for this type of beam splitter are

Transmission	30%
Reflectivity	30%
Absorption	40%

In other words, this type ol beam splitter will absorb about 40% of the incoming beam while transmitting the outputs of about 30% each.

The other type of beam splitter is the so-called "mirror type". It is not a mirror in the sense that it does not actually have a silvered surface. Instead, one face of a flat glass plate is coated with a dielectric film so as to transmit part of the light impinging upon it while reflecting the rest. A beam splitter of this type is illustrated in figure 12-9.

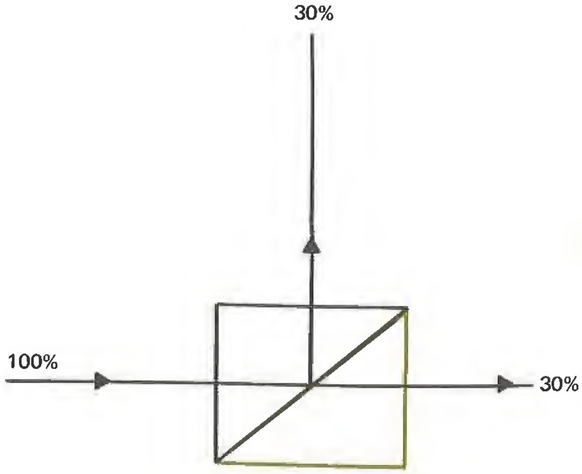

Fig. 12-8 Prism Beam Splitter

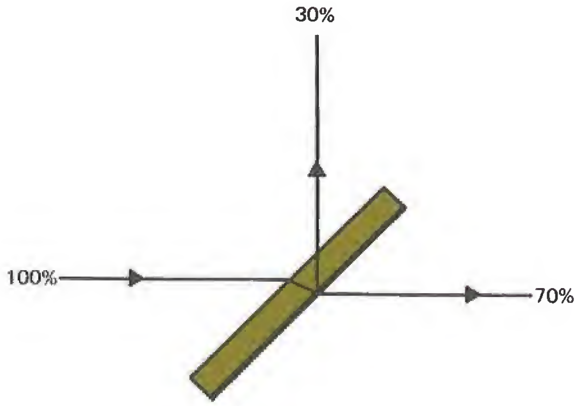

Fig. 12-9 Mirror Type Beam Splitter

As shown, this type of beam splitter reflects about one-third of the incoming light and transmits almost all of the remainder. There is virtually no light loss due to absorption; however, there may be some interference generated in the reflecting film which is not present in a prism type of beam splitter.

MATERIALS

2 60° prisms

1 Positive lens of known focal length

1 Negative lens of known focal length

1 Achromatic telescope objective

1 Collimated white light source

1 Mechanical breadboard

1 Beam splitter

3 Positive lenses, large diameter (about 4 cm), identical focal lengths but different shapes (meniscus, plano-convex, double-convex)

1 He-Ne laser

1 45°-90°-45° prism

1 White screen

1 Screen holder

2 Lens holders

4 Carriages

1 Optical bench

PROCEDURE

NOTE: You may wish to use nontoxic smoke in this experiment.

Part I Chromatic Aberration

1. Set up the optical bench according to figure 12-10.

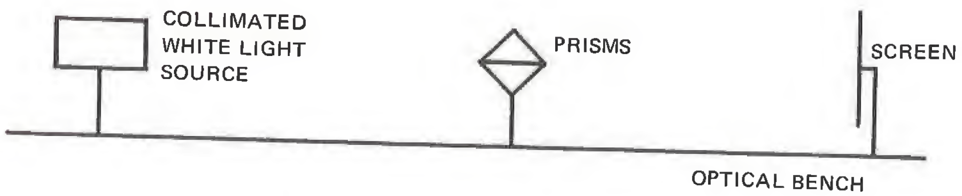

Fig. 12-10 Experimental Set-up / Top View of Simulated Lens Arrangement

2. Turn on the white light source and note the pattern on the screen.

3. Move the screen toward the "lens" (prism pair) and measure the focal length for red, green and violet. Tabulate these values in the data table, figure 12-11.

	f_R	f_G	f_V
PRISMS			
POSITIVE LENS			
ACHROMAT			
	f_1	f_2	d

Fig. 12-11 Data Table I

4. Replace the simulated lens by a large diameter positive lens.

5. Repeat step 3 for this lens.

6. Replace the lens in step 4 by the achromatic telescope objective.

7. Repeat, if possible, step 3 for the achromat. If no aberration is observed, then f_R, f_G and f_V will be the same.

8. Replace the achromat with the lens used in step 4.

9. Place the negative lens on the screen side of the positive lens and adjust the spacing until minimum chromatic aberration is observed on the screen.

10. Measure the distance between the positive and negative lenses and compare this to equation 12.2.

Part II Spherical Aberration

11. Very carefully align the laser with the optical bench axis.

12. On the mechanical breadboard, construct the set-up illustrated in figure 12-12.

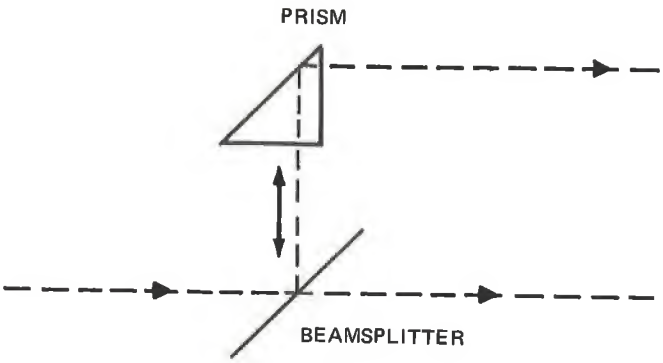

Fig. 12-12 Experimental Set-up II Prism and Beam Splitter Set-Up

NOTE: The prism should be movable in the direction shown and the beam splitter should be rigidly mounted. Although a mirror type is shown, a prism beam splitter will work equally well.

13. Mount the assembly on the optical bench as illustrated in figure 12-13. Insert the plano- convex lens.

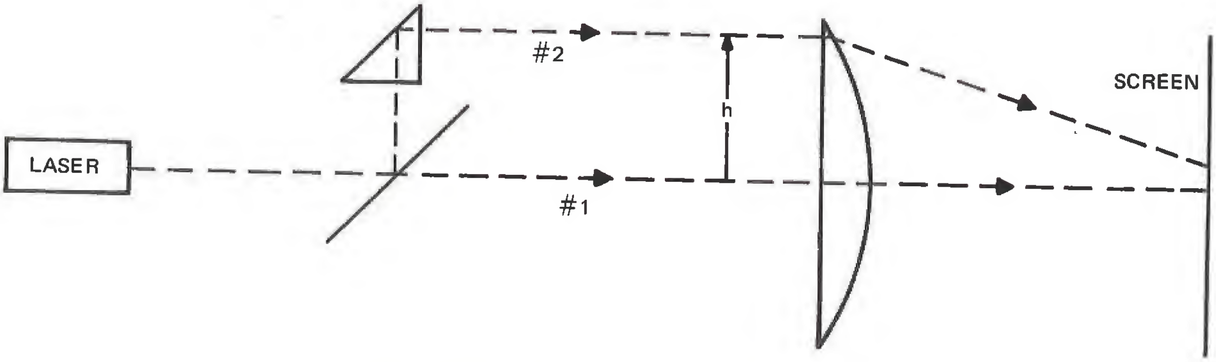

Fig. 12-13 Experimental Set-Up III

14. Beam #1 should be aligned with the axis of the bench and lens. Beam #2 must be parallel to #1 in the distance between the splitter assembly and the lens. To check this, insert a card or the screen between the lens and the splitter assembly. Move the screen back and forth to see if h changes. Adjust the prism until h is constant.

15. Set h to some convenient value by using a ruled card or screen as outlined in step 14.

LENS	h	f
		
		

Fig. 12-14 Data Table II

16. Move the screen toward or away from the rear of the lens until beams #1 and #2 coincide.

17. Record the value for h from step 15 and the corresponding f from step 16 in the data table, figure 12-14.

18. Repeat steps 15, 16, and 17 for at least three different values of h, one of which should almost equal the diameter of the lens.

19. Now, rotate the lens by 180° and repeat steps 15, 16, 17, and 18.

20. Repeat steps 15, 16, 17, 18, and 19 for the other two lenses.

21. In the data table, figure 12-14, sketch each lens with the light entrance side to the left and in the position it was in when the h and f measurements were made. The first case is shown for clarity.

ANALYSIS GUIDE. In the analysis of these data you should discuss the validity of using the two prisms in Part I to simulate a lens and why they emphasize chromatic aberration. You should also relate d to f_1 and f_2 and discuss any conditions on f_2 in the spaced achromat. For Part II you should discuss any unusual observations seen during the course of this experiment and any construction difficulties encountered.

PROBLEMS

1. Why is chromatic aberration a prime consideration in designing a telescope objective?

2. Would a concave mirror produce chromatic aberration? Why?

3. How would you measure f_R and f_V for a negative lens?

4. Of the first two lens configurations used in Part II, which would you prefer for a telescope objective? Why?

5. Could the spherical aberration of a converging lens be corrected by a diverging lens in a system? Explain.

6. Would a concave mirror exhibit spherical aberration? Why?

experiment 13 CAMERA PARAMETERS

INTRODUCTION. Practical lens systems are found in a considerable variety of instruments. Telescopes, binoculars, cameras, projectors, and surveying transits are only a few of the ones you have probably encountered. In this experiment we will examine one of these systems.

DISCUSSION. Cameras are perhaps the most frequently encountered optical system. Even so, few people (including most camera fans) have a good understanding of how they work.

A camera lens is composed of an objective only. Its function is to provide a sharp image of the appropriate size at the plane of the film.

This apparently simple function is often difficult to provide because:

1. The light intensity available to the lens will vary widely from time-to-time.
2. The field of view may be anything from less than 10° to over 120°. Most cameras have a field of view of around 35° which is relatively wide.
3. The field must be very flat over its entire area.
4. Distortion must not be noticeable in the final photo.

Because of these requirements, camera lenses are often quite complex.

Normally, camera lenses have four somewhat-interrelated parameters that can be used to describe the system. These parameters are:

1. *The focal length.*
2. *The aperture size.*
3. *The shutter speed.*
4. *The range of focus.*

Let's look at each of these parameters one at a time.

The focal length of a camera lens system is actually the focal length of an equivalent thin lens which would produce the same image of an object at infinity.

Sometimes we refer to this distance as the *effective* focal length of the lens system. Actually any combination of lenses will have a definite focal length. For a two-lens system the result will be

$$\frac{1}{f_T} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1 f_2} \quad \textbf{(13.1)}$$

where

f_T = focal length of the system

f_1 = focal length of the first lens

f_2 = focal length of the second lens

L = spacing between lenses

In a system employing more than two lenses, we can use this relationship with successive pairs until we get a final effective focal length.

The effective focal length is one of the parameters that we use to identify a camera lens. That is, we might say that we have a 35mm lens, a 50mm lens or perhaps a 135mm lens. In each case the distance named is the effective focal length.

We also occasionally refer to lenses as being *wide angle* or *telephoto.* In general, wide angle lenses have short effective focal lengths (28 or 35mm, for example) while telephoto lenses normally have longer focal lengths (135mm or 200mm, for example). A so-called *standard* lens would have an intermediate focal length (40 - 50mm, perhaps).

The relative aperture of a camera lens is the ratio of the effective focal length to the effective entrance diameter of the lens. That is

$$a = \frac{f}{D} \tag{13.2}$$

For example, a 135mm lens with an entrance diameter of 38.57mm would have an aperture of

$$a = \frac{135}{38.57} = 3.5$$

We would say that was an f/3.5 lens (pronounced f 3.5).

To describe the lens more fully we would call it a 135mm f/3.5 lens.

Many camera lenses have adjustable apertures with stops at f/2, f/2.8, f/4, f/5.6, f/8, f/11, f/16, and f/22. The various settings are achieved by adjusting a diaphragm-type iris located in the lens assembly. The particular f/stop numbers are selected to allow the light admitted to the camera to change in steps of two. That is, f/8 admits twice as much light as f/11 and half as much as f/5.6.

Some cameras have f/stops between the values given above (f/3.5 and f/4.8 are common).

In describing a lens we always use the largest aperture (smallest f/stop). So if a lens had stops at f/3.5, f/4, f/8, f/11 and f/16, we would refer to it as an f/3.5 lens.

Because the lower f/stop numbers let in more light (bigger aperture), the f/stop is sometimes referred to as the *speed* of the lens. An f/1.5 lens would be called a *fast* lens while an f/8 lens would be a *slow* one.

Shutter speed is not actually a lens property; however, it is closely connected to the f/stop setting and light intensity of the lens. Consequently we will discuss shutter speed briefly.

The appropriate shutter speed to use with a camera depends primarily on the nature of the object to be photographed. If the object is still, then shutter speed can be of any desired value. However, the use of shutter speed below about 1/25 sec (yes, we talk about shutter speeds in seconds) usually requires a stable camera mount of some kind to prevent camera movement from spoiling the photograph. For slowly moving objects, about 1/125 sec is usually appropriate, while 1/250 sec may be required for fast moving objects (say, up to about 30 mph). For extremely fast-moving objects shutter speeds of 1/500 or 1/1000 sec or even higher may be required.

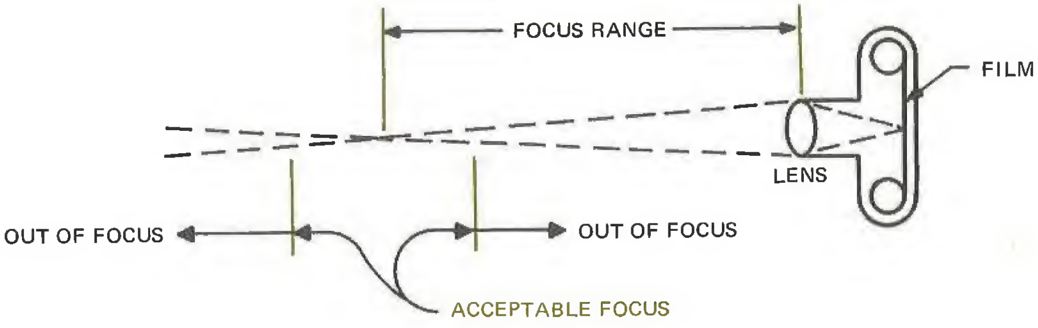

Fig. 13-1 Depth of Field

It is best to have the object move across the field of view rather than along it because this will normally allow somewhat lower shutter speeds to be used.

With the shutter speed selected, the f/stop can then be set to a value suitable for use with the available light intensity, the type of film being used and the selected shutter speed.

Typical shutter speeds available on many cameras are 1, 1/2, 1/4, 1/8, 1/15, 1/30, 1/60, 1/125, 1/250, 1/500 sec. A typical camera setting using a popular color film outside on a bright day might be 1/125 sec and f/8. In fact, this is approximately how inexpensive nonadjustable cameras are set.

Most cameras have some provision for adjusting the range at which objects are in focus. For any selected range there will be some distance in front of and beyond the object in which focus is reasonably sharp. This condition is illustrated in figure 13-1.

The range of acceptable focus is called the depth-of-field of the lens at that particular range and aperture.

Because the depth of field of a lens is a more-or-less subjective parameter, it will vary from one camera to another. Each designer decides how much unfocusing will be accepted and this establishes the depth-of-field for that camera. Figure 13-2 shows curves taken from the depth-of-field scale of a popular brand camera employing a 28mm, 50mm, and 200mm lens.

To use these curves you start at the f/stop setting on the upper scale. Project across to the appropriate focus curve then down to the max range. Similarly, you use the lower f/stop to find the min range.

For example, suppose that we have a range setting of 10 feet and f/8. The 28mm lens has a depth-of-field of from five feet to infinity, while for the 50mm lens it is from 7.5 feet to 14 feet.

Notice from the curves that the depth of field depends on the focal length of the lens, the focus range and the f/stop setting. Also notice that for each lens there is a range beyond which the lens sees everything at infinity. For most lenses (except telephoto) this range is about 50 feet.

In addition to the main lens assembly, most cameras have two other optical systems. These two systems are the view finder and the light meter. In this experiment we will not examine these systems further. However, you may wish to investigate them on your own.

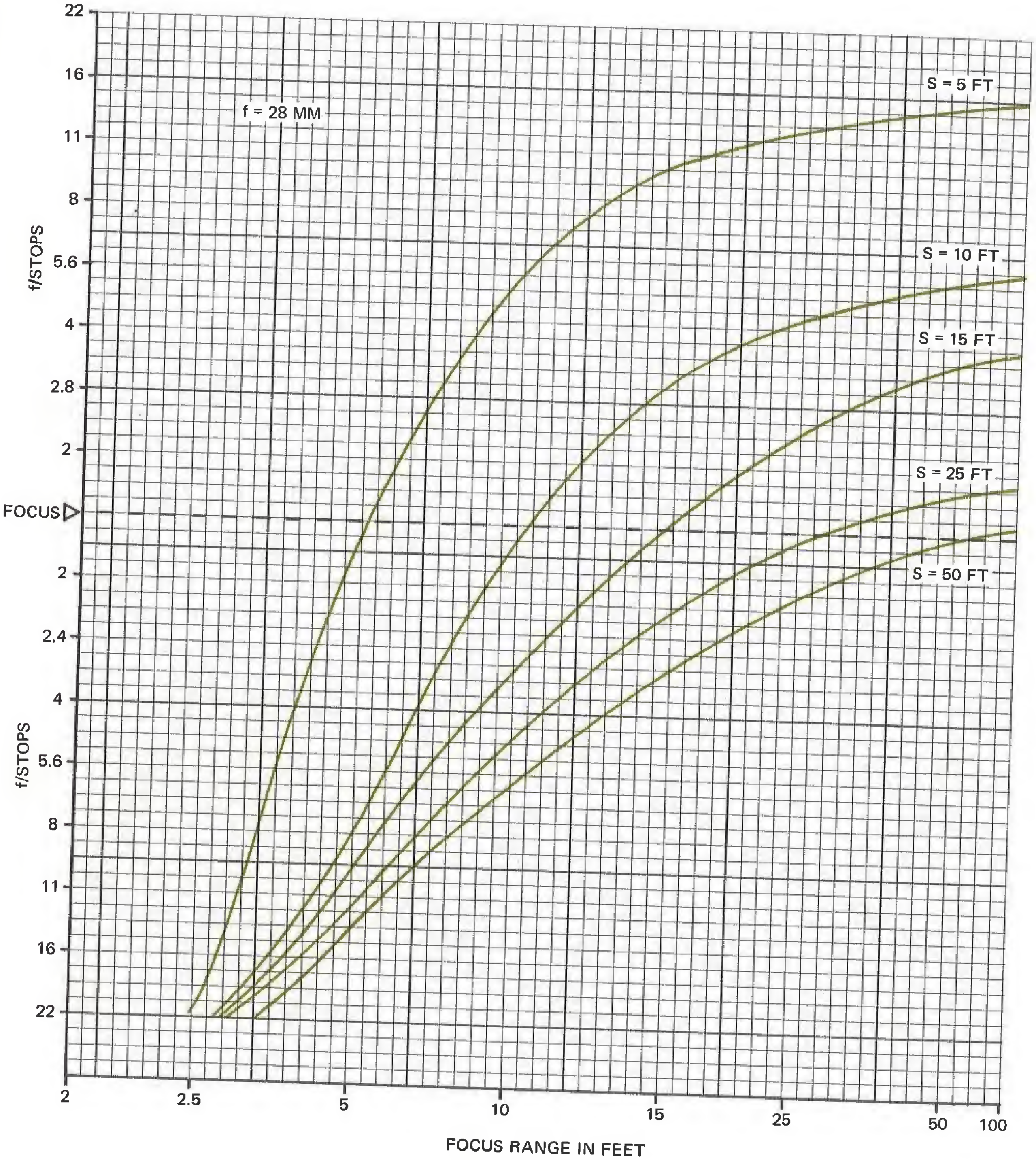

Fig. 13-2A Focus Curves for a 28 mm Lens

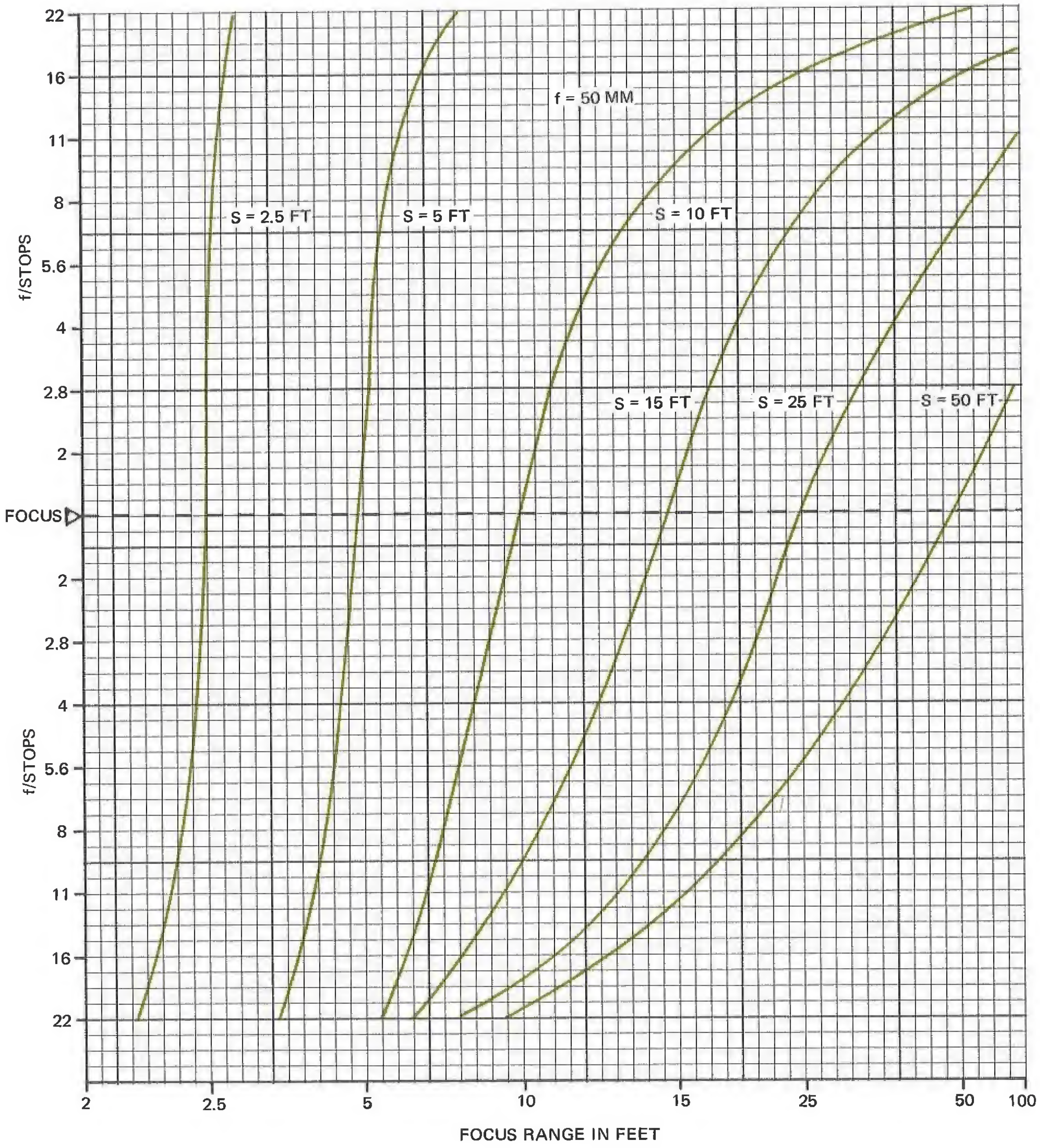

Fig.. 13-2B Focus Curves for a 50 mm Lens

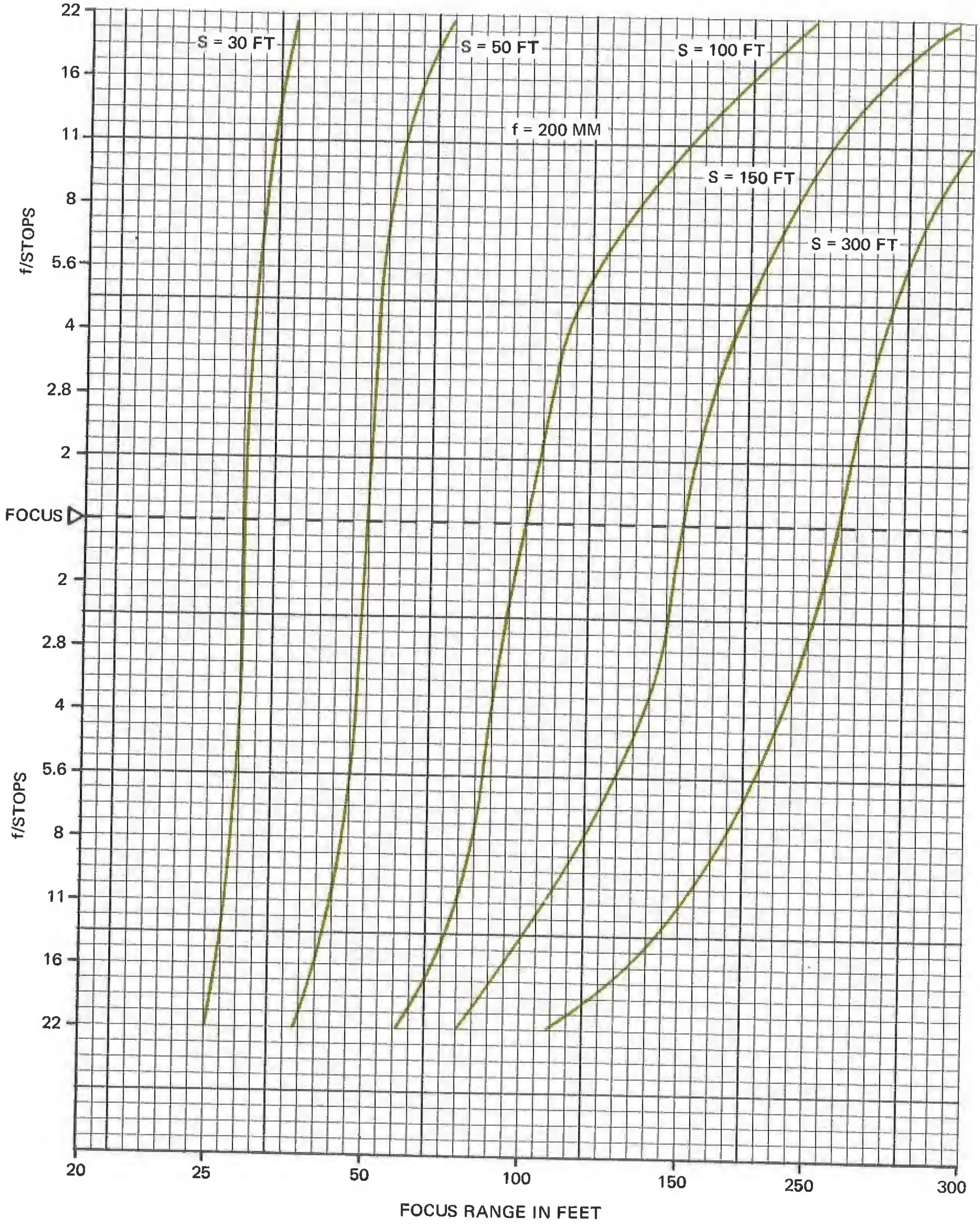

Fig. 13-2C Focus Curves for a 200 mm Lens

MATERIALS

1 Photographic camera (with adjustable focus, aperture, and shutter speed. It should also have a depth-of-field scale, a lens of known focal length and known maximum aperture)

1 Photographic light meter (with f/stop and shutter speed readout)

1 Steel rule approximately 15 cm long

PROCEDURE

1. Examine the camera carefully. Determine that it is not loaded with film before you proceed. Record the focal length of the lens and the maximum aperture.

2. Identify each of the following controls.
   (a) The shutter speed control.
   (b) The aperture adjusting control.
   (c) The focus control.

3. Record the numbers shown on the shutter speed, f/stop, and focus scales.

4. Being very careful not to touch the lens, lay the scale across the lens opening and record its approximate diameter in millimeters.

5. Compute the approximate maximum aperture using equation 14.2. Round your results off to the next larger standard f/stop (2, 2.8, 4, 5.6, 8, etc.).

6. Record both your computed f/stop and the rounded-up value.

7. Using the light meter, determine the shutter speed required to take a photograph at each f/stop on your camera. (You may not be able to get them all).

8. Similarly, determine the aperture setting required for each shutter speed of your camera. (You probably won't be able to get them all due to low light levels.)

9. Examine the depth-of-field scale on your camera. Observe how the depth-of-field scale changes with focus range and aperture setting.

10. For a shutter speed of 1/125 sec (or the nearest value on your camera) and a focus range of 10 feet (3 meters), record the minimum and maximum range from the depth-of-field scale for each f/stop on your camera.

11. Plot a curve similar to figure 13-2 for your camera.

LENS DATA

FOCAL LENGTH	MAX. APERTURE
mm	f/

LENS MEASUREMENT

APPROX. LENS DIAMETER	COMPUTED MAX. APERTURE	ROUNDED UP APERTURE
mm	f/	f/

CAMERA DATA

SHUTTER SPEEDS	
f/STOPS	
RANGE MARKS	

DEPTH-OF-FIELD DATA

f/STOP	MINIMUM RANGE	MAXIMUM RANGE

Fig. 13-3 The Data Tables

LIGHT METER DATA

CAMERA f/STOP	SHUTTER SPEED (LIGHT METER)

LIGHT METER DATA

CAMERA SHUTTER SPEED	f/STOP (LIGHT METER)

Fig. 13-3 (Con't) The Data Tables

ANALYSIS GUIDE. In the analysis of these data you should consider several points:

(a) How well did your values of the maximum aperture agree with the one specified on the camera? Which one of your values was nearest the specified value?

(b) Did your f/stop and shutter speed data tend to confirm that you must halve the shutter speed when you go to the next higher f/stop number?

(c) Did your depth-of-field data tend to agree with the curves given in the discussion? How was yours similar to those given? How was it different?

PROBLEMS

1. A certain camera has a 90mm f/3.5 lens. Approximately what is the diameter of the lens?

2. The 28mm lens in figure 13-2A is set to f/4 and focused at 15 feet. What is the depth-of-field?

3. Compute the depth-of-field for each of the three lenses in figure 13-2 using f/8 and S = 25 feet. *(Assume that the 200mm lens has approximately the same depth-of-field at 25 and 30 feet.)* Which one has the greatest depth-of-field? What makes you think so?

4. In problem three, which lens had the shortest minimum range?

5. A camera is set for a shutter speed of 1/60 sec, aperture of f/11, and a range of 10 feet. Two objects are located three feet and 15 feet from the lens. Which of the lenses in figure 13-2 would be best for photographing both objects at the same time?

6. Suppose that you take the photograph in problem five and both objects are slightly out of focus but one at 10 feet is in focus. How could you correct the problem? (Give new shutter speed and aperture settings.)

experiment 14 PHOTOELECTRIC DEVICES

INTRODUCTION. With the coming of the Space Age and the need for energy generation in outer space, the use of photoelectricity became very important. In this experiment we will examine the characteristics and operation of some *photoelectric devices.*

DISCUSSION. Electron emission from metals due to thermal agitation is known as thermionic emission. Electrons may also acquire enough energy to escape from a metal, even at low temperatures, if the material is illuminated by light of sufficiently short wavelength. This phenomenon is known as the photoelectric effect. It was first discovered by Heinrich Hertz in 1887 and later investigated by Hallwachs, to whom the discovery is usually credited. The solar sensor was soon discovered and shortly after that came the selenium light meter which is used extensively today by photographers.

Industrial applications make use of many photoemissive devices. Photoelectric devices are classified by the way in which the electric output is furnished to the circuit. Devices which emit electrons are *photoemissive.* Devices which change their resistance as a function of light intensity are *photoconductive* or *photoresistive.* Devices which produce potential differences are *photovoltaic.*

It has been found that with a given photosensitive material, the wavelength of light must be shorter than a critical value, which is different for each material. This critical wavelength, or the corresponding frequency, is called the threshold wavelength or frequency of the particular material. The threshold wavelength for most metals is in the *ultraviolet* spectrum (2000 - 3000 *angstroms* where one angstrom equals 10^{-10} meters), but for *potassium* and *cesium oxide* it lies in the visible spectrum (4000 - 7000 angstroms).

Seen most often in "electric eyes", photoemission is a process whereby electron flow is generated when light strikes a light-sensitive surface. Figure 14-1 illustrates this phenomena.

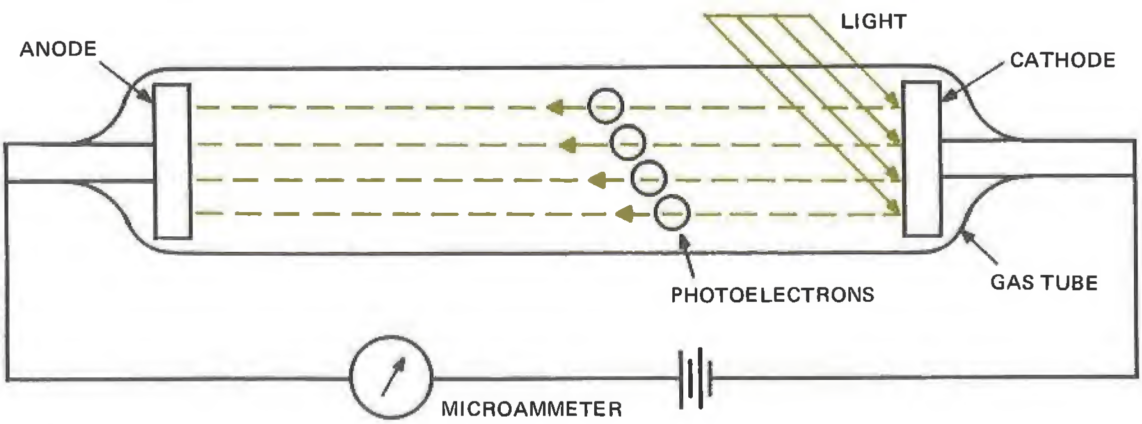

Fig. 14-1 Photoelectron Emission

The device in figure 14-1 consists of a gas tube (sometimes vacuum) with two metal plates. The cathode is light-sensitive and it will give up electrons to the anode. A power supply must be connected so as to give the anode a positive charge and the cathode a negative charge. The tube will not conduct until a light beam strikes the cathode plate surface. As the light strikes the surface, the *molecular* action within the material causes electrons to be freed from the cathode surface. These electrons are attracted to the anode. The electrons that are given off are referred to as *"photoelectrons."* The internal resistance of a photoelectric cell (tube) is in the order of 10^{11} to 10^{12} ohms in darkness, which, for practical purposes, is an open circuit. Figure 14-2 shows another arrangement of a photoelectric tube.

The photoconductive cell has the property such that its internal resistance is a function of the light intensity falling on its photosensitive material. When operating at low light intensities, the resistance is low. Photoconductive cells cover the entire spectrum from ultraviolet to *infra-red*. As pointed out before, each specific material to be used will possess a different threshold frequency.

For this device, the corresponding long-wavelength limit is most often in the infra-red range between 10,000 and 8000 angstroms. Since the long-wavelength limit is longer than the visible spectrum wavelengths, the photoconductive devices can be used in the visible as well as the untraviolet regions. The sensitivity to infra-red radiation is of great practical importance. Photoemissive cells normally cannot compete in this region because there is not enough energy present. Therefore, photoconductive cells are most often used as infra-red detectors.

Before examining applications of the various photocells, some fundamentals of photoelectric emission will be discussed.

Light, an electromagnetic radiation, is a form of energy. This energy can be considered to be made up of discrete packages. The energy per package is related to the frequency of the light by

$$\text{Energy} = hf \text{ (quantum energy or joules)} \tag{14.1}$$

where h is Planck's constant (6.624×10^{-34} joules per second) and f is the frequency of radiation in Hertz.

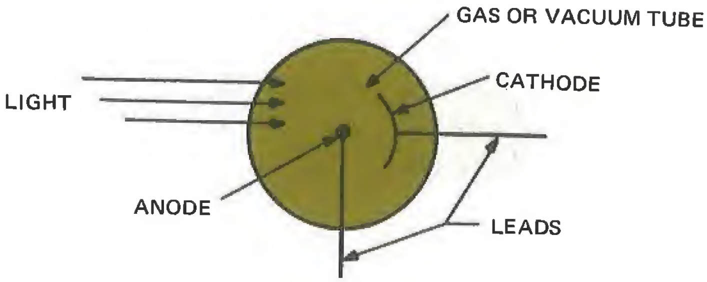

Fig. 14-2 Illustration of Photoelectric Tube

Each package of energy is called a photon and the amount of energy is called a quantum. If the energy in a photon striking a metal surface is equal to or greater than the work function of the metal, an electron will be emitted.

Einstein developed this mathematically as

$$hf = eE_W + \frac{mv^2}{2} \text{ (joules)} \tag{14.2}$$

where

E_W = energy of the work function

m = mass of the electron, kg

v = velocity of the electron, meters/sec

e = 1.6×10^{-19} coulombs

For any given photosensitive material, the work function is a constant. Therefore, there must exist a frequency of radiation (threshold frequency) below which emission will not occur. At the threshold frequency, the emitted electrons have no kinetic energy so equation 14.2 becomes

$$f_o = \frac{eE_W}{h} \tag{14.3}$$

where f_o = threshold frequency.

To illustrate the use of these equations, let's determine the work function of a metal which gives off an electron with an energy of 0.68 electron volts when it is exposed to light having a frequency of 8.2×10^{14} Hertz. Using equation 14.2 we have

$$hf = eE_W + \frac{mv^2}{2}$$

Solving for E_W gives

$$E_W = \frac{hf}{e} - 0.68$$

$$E_W = \frac{6.624 \times 10^{-34} \times 8.2 \times 10^{14}}{1.602 \times 10^{-19}} - 0.68$$

$$E_W = 3.33 - 0.68$$

$$E_W = 2.65 \text{ electron volts}$$

Semiconductor photocells are perhaps the ones most often encountered. They have nonlinear and temperature-sensitive calibration curves, and the choice of a suitable type depends on the particular use. Figure 14-3 shows a simple photoconductor circuit.

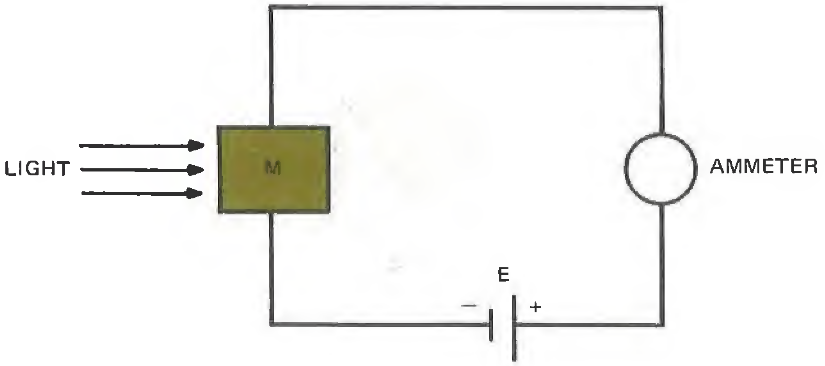

Fig. 14-3 Basic Photoconductor Circuit

While the material M is in darkness, its resistance is high and very little current is flowing from the battery. When the light strikes the surface of the material, the resistance will decrease and, consequently, the current will increase.

The photoconductive process is initiated by the absorption of an *incident photon* which causes excitation of an electron. The electron is then free to move. As more electrons are free to move, the *conductivity* of the device increases. As the conductivity increases, the resistance of the device decreases, allowing more current to flow.

The internal resistance of the photoconductor is in the order of 1×10^6 to 30×10^6 ohms in darkness. Figure 14-4 shows a symbol that is frequently used for a photoconductor (photoresistor).

A third type of photoelectric cell is one in which a voltage is produced directly by the application of light upon the photosensitive material. The most important of this class is the dry type, or the barrier-layer type, in which the light acts upon the boundary layers between a metal and a semiconductor.

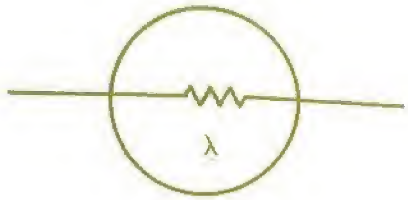

Fig. 14-4 Symbol for Photoresistor

Figure 14-5 shows a cut-a-way view of a selenium-on-iron cell which is often used in commercial photographic exposure meters and in *foot-candle meters.* In this cell the selenium is placed on an iron disk and then a thin translucent layer of metal, such as gold or silver, is placed on the selenium to act as a front electrode.

As light strikes the light-sensitive material, the electrons near the front electrode tend to be displaced from the selenium to the metal electrode. The electrons cannot return easily through the boundary, so they return through the external circuit. If the circuit is open, the electron flow across the barrier will increase the difference of potential between the terminals until an equilibrium is reached and the net electron flow is zero.

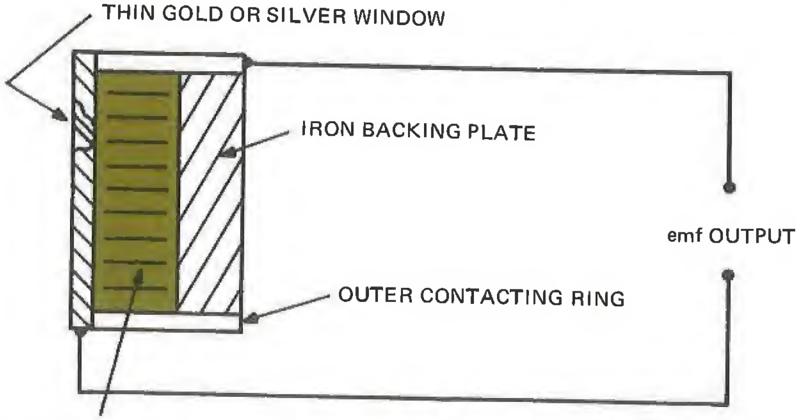

Fig. 14-5 Basic Photovoltaic Cell

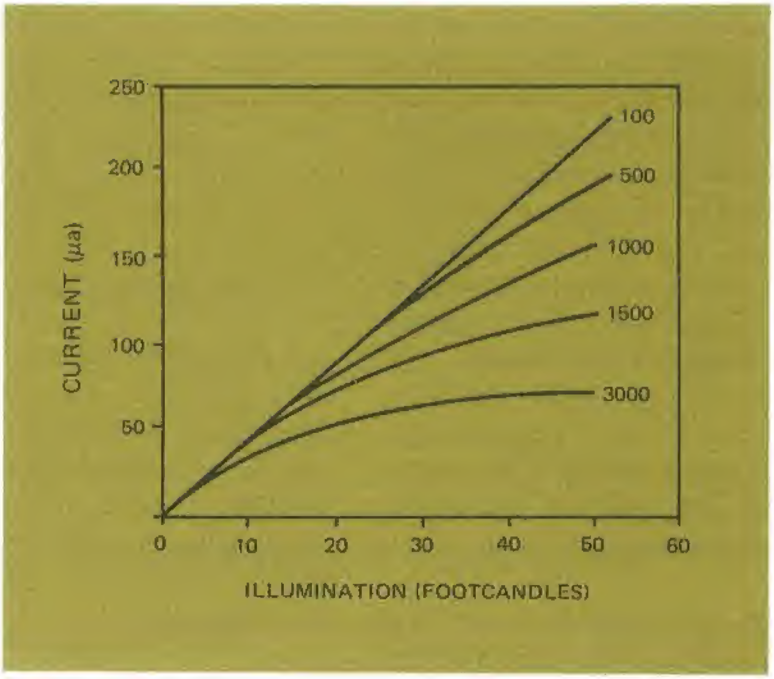

Fig. 14-6 Output Current for a Typical Photovoltaic Cell

The potential produced is relatively small and is not a linear function of the illumination. However, if the cell is connected to a low resistance microammeter, the circuit current is nearly a straight-line function of the illumination. The behavior of the cell is shown in figure 14-6 with the current as a function of the illumination, and the external resistance as an additional variable.

The photovoltaic cell can generate enough power to actuate a relay. The relay must be very sensitive and its resistance must be chosen so that the cell delivers approximately maximum power output. These relays are usually slow in action and are normally used where high speed is not essential.

The photovoltaic cell can be used as a source to produce electrical energy. In the space industry they are called *solar cells*. Through these cells, scientists have been able to put man into space and recharge the batteries on board his space craft everytime the craft is sunlit.

Because small voltages and currents are produced from fairly large sized cells, about 0.6 volts per cell in full daylight, many cells are required to produce appreciable power.

The internal resistance of this device is in the range of 300 to 6000 ohms, and its surface temperature should not exceed 122° F.

Photoelectric cells of one type or another are being used in many places around the home and community. Some examples would be in the automatic eye which controls outside lights around the home, automatic opening and closing of doors at the supermarket, burglar alarms in various establishments, flame indicators for fire alarms, in heat control, and also in fluid level controllers.

MATERIALS

1 VOM or FEM

2 Photovoltaic cells (type 10 4LC or equivalent)

1 Photoconductive cell (type CL5M3 or equivalent)

1 20 Ω resistor 2W

1 White light source

1 Optical bench

1 Lens holder

3 Sheets of white paper

1 Sheet of black paper

PROCEDURE

1. Set up the apparatus shown in figure 14-7 with one of the photovoltaic cells.

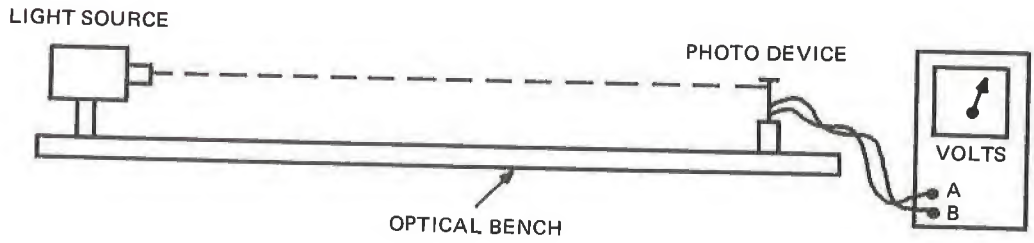

Fig. 14-7 Photovoltaic Setup

2. With the light source off, place a sheet of black paper over the photovoltaic cell. Measure the voltage and record in the data table, figure 14-9.
3. Remove the paper and record the voltage for normal room light.
4. With the light source on, record the voltage for distances of 80, 60, 40, 20, and 10 cm between the light source and the cell.
5. Replace the cell with the other photovoltaic cell.
6. Repeat steps two, three and four.
7. Place a 20-ohm resistor between points A and B in figure 14-7.
8. Repeat the experiment recording the voltage in the data table.

9. Set up the circuit with the photoconductive cell as shown in figure 14-8.

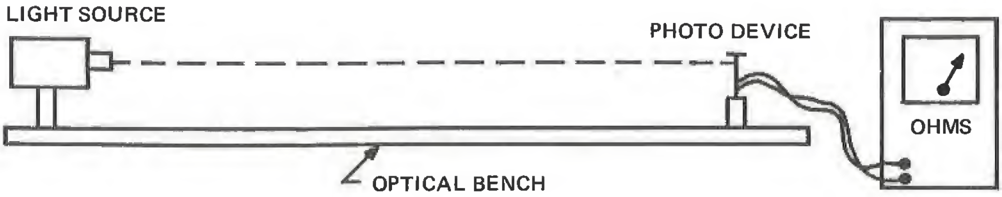

Fig. 14-8 Photoconductive Circuit

10. With the light source 20 cm away, place one sheet of white paper between the light source and the cell.

11. Record the resistance in the data table.

12. Repeat for two and three sheets of paper.

	BLACK PAPER	ROOM LIGHT	80 cm	60 cm	40 cm	20 cm	10 cm
PHOTOVOLTAIC ONE							
PHOTOVOLTAIC TWO							
WITH 20 Ω R VOLTAGE							
PHOTO-CONDUCTOR							

VOLTAGE OUTPUT VERSUS ILLUMINATION

NO. OF SHEETS	1	2	3
RESISTANCE			

CELL RESISTANCE VERSUS OBSTRUCTION

Fig. 14-9 The Data Tables

ANALYSIS GUIDE. Plot graphs of the voltage output versus illumination distance. Also plot a graph of voltage output versus the obstructions for the photoconductive material. It should be apparent from the graphs that the voltage and illumination are related.

PROBLEMS

1. How many cells hooked in series like the first photovoltaic cell would be needed to produce five volts at a distance of two and a half feet from the light source used?

2. Give two applications of a photo cell used in the community.

3. Draw a circuit for each application given.

experiment 15 *PHOTOELASTICITY*

INTRODUCTION. In this experiment we shall examine the phenomenon of double refraction as a means of producing polarized light. We will also observe how this phenomenon is useful in optical stress analysis.

DISCUSSION. Most materials encountered as optical media are approximately homogenous and isotropic. That is, they are of uniform content throughout and the speed of light through the media does not depend upon the propagation direction. There are, however, many transparent crystalline substances which, while being homogeneous, are anisotropic. In other words, the velocity of light waves in them is not the same in all directions. Crystals possessing this property are said to be *double refracting* or *birefringent.* Some of the crystals which possess this property are calcite, quartz, tourmaline, and, to a small extent, ice.

Figure 15-1 shows a beam of unpolarized light falling upon one of the faces of a polished calcite crystal.

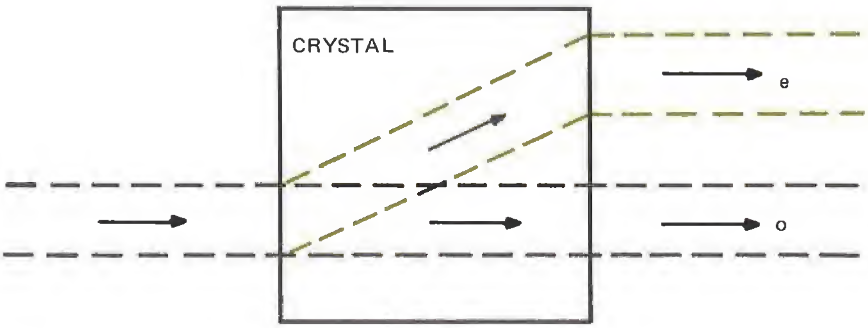

Fig. 15-1 A Beam of Unpolarized Light Can be Split into Two Beams by a Doubly Refracting Crystal

If the two emerging beams in figure 15-1 are analyzed with a polaroid, they are found to be plane-polarized with their planes of vibration at right angles to each other. Also, if experiments are carried out at various angles of incidence, it is found that one of the beams will completely obey Snell's law of refraction while the other will not. For example, the angle of incidence in figure 15-1 is 0° so that if the medium were truly isotropic, the angle of refraction would also be 0°. We see that for one beam, the angle of refraction is indeed 0°

while for the other, it is not. The ray or beam which obeys Snell's law of refraction is called the *ordinary* or *o-ray*, while the ray or beam which violates Snell's law is termed the *extraordinary* or *e-ray*.

The basic differences between the e and o rays in calcite may be summarized as follows:

1. The o-ray travels with the **same** speed in the crystal in all directions. That is, for the waves in this ray, the crystal has a single and definite index of refraction, n_o.

2. The e-ray travels in the crystal with a speed which **varies** with direction. The velocity variation is from v_o (the velocity of the o-ray) in one direction to some larger value, v_e, in another direction. The index of refraction, therefore, varies from n_o to a smaller value, n_e.

Now, from the foregoing, there is one direction through the crystal in which the o-and e-rays will have the same velocity and, hence, the same indices of refraction. That is, there is one particular direction through the crystal in which both the e- and o-rays obey Snell's law of refraction. This direction is called the *optic axis* of the crystal. The characteristics of the emerging e- and o-rays will depend upon the angle of incidence with the optic axis of the crystal.

Three different cases for a calcite crystal are illustrated in figure 15-2. Note that the incident beam is resolved into its vertical and horizontal polarization components.

We can see from figure 15-2 that the angle of incidence with the crystal face is 0°. However, the angle of incidence with the

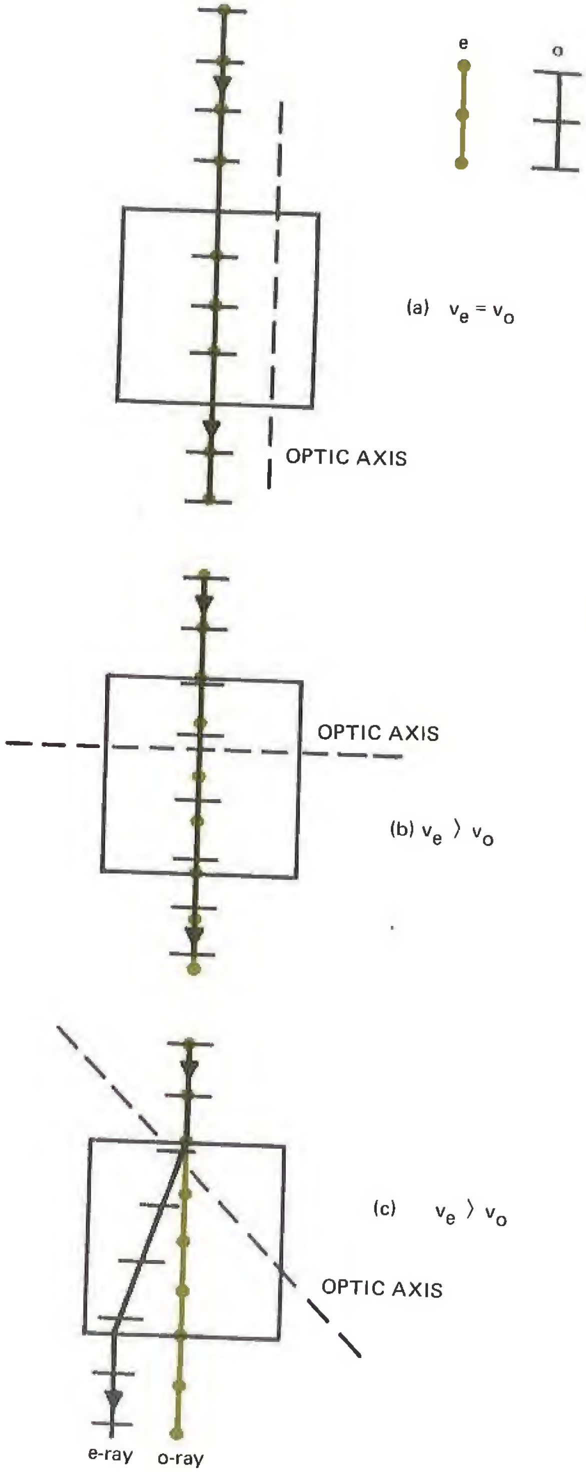

Fig. 15-2 Action of e-ray and o-ray for Various Angles of Incidence with the Optic Axis

optic axis varies and thus the characteristics of the emerging e and o-rays will vary. We may summarize figure 15-2 as follows:

1. Angle of incidence with optic axis = 0°, $v_e = v_o$, no double refraction.

2. Angle of incidence with optic axis = 90°, $v_e > v_o$, no double refraction.

3. Angle of incidence with optic axis = Θ such that, $0° < \Theta < 90°$, $v_e > v_o$, double refraction occurs.

In other words, in (1) there will be no detectable difference between the incident and emerging rays. In (2) the speed of the horizontal polarization component exceeds that of the vertical polarization component inside the crystal. Therefore, in the emerging beam, the horizontal polarization component will be displaced *ahead* of the vertical component. Both will still lie along the same line however. In (3) the horizontal component is displaced *ahead* of and *laterally* from the vertical component. Only in (3) will two distinct beams be observable.

Let's digress for a moment and see what happens when we have two simple harmonic motions vibrating at right angles to each other. This can be visually demonstrated using an oscillator and an oscilloscope. If the horizontal sweep is disconnected and a sine wave signal inserted at the vertical terminals, a vertical line will be displayed. If the signal is removed and inserted at the horizontal terminals, a horizontal line will be displayed. Now, if sine wave signals of exactly the same frequency are applied to both the vertical and horizontal terminals, the resultant pattern will be a combination of a simple harmonic vibration in the vertical and horizontal directions. Suppose that we can adjust the phase between the vertical and horizontal signals. If this is done, the pattern will be changed. These figures are called Lissajous patterns and are well known in electronic circuit analysis.

Figure 15-3 shows the results obtained by combining a vertical and horizontal simple harmonic motion of the same amplitude and frequency for nine different phase relationships.

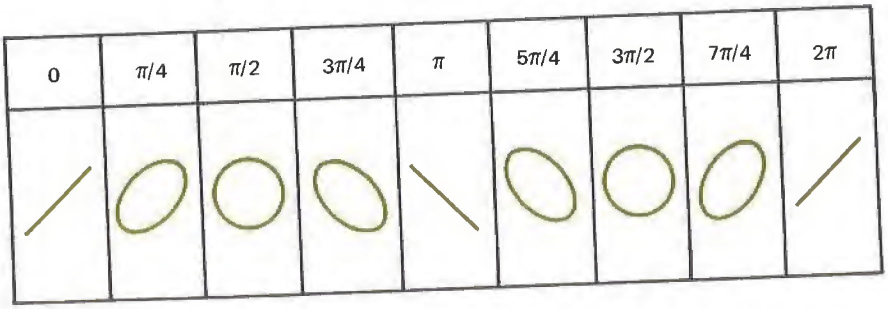

Fig. 15-3 Lissajous Patterns

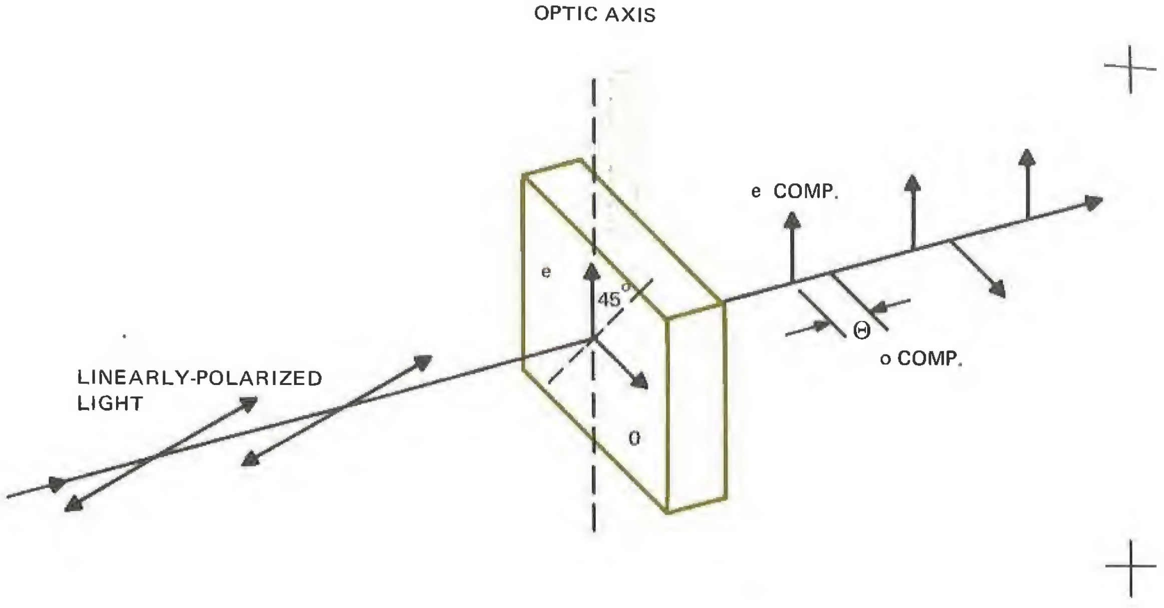

Fig. 15-4 Plane-Polarized Light Incident upon a Doubly-Refracting Crystal

Now, suppose we aim linearly-polarized, single-frequency light incident upon a polished doubly-refracting crystal plate as shown in figure 15-4.

The crystal plate is cut such that the optic axis is parallel to the incident face. Therefore, the incident beam is perpendicular to the optic axis. The polarization direction of the incident beam is at a 45° angle to the optic axis. The linearly-polarized incident beam may, therefore, be broken up into two components; one of which will lie in the direction of the optic axis, the other perpendicular to it. Since the angle is 45°, the e and o components of the incident beam will be equal. For calcite, the o waves or the o component of the incident beam will have a higher speed through the crystal than the e component. Therefore, the emerging beam will be composed of an e component and an o component which are displaced by some phase angle Θ as shown. The combination of the e and o component vibrations will produce a resultant vibration which may be circular elliptical, or a straight line depending on the value of Θ. For example, if $\Theta = \pi/4$, the emerging beam will have a resultant ellipsoidal vibration as shown in figure 15-3. A beam such as this we say is elliptically polarized. If $\Theta = \pi/2$, the emerging beam has a circular vibration about the propagation axis and we say the beam is circularly polarized. And finally, if $\Theta = \pi$, the emerging beam will be plane-polarized but will have its polarization direction at right angles to that of the incident beam.

Suppose we now have the arrangement diagrammed in figure 15-5.

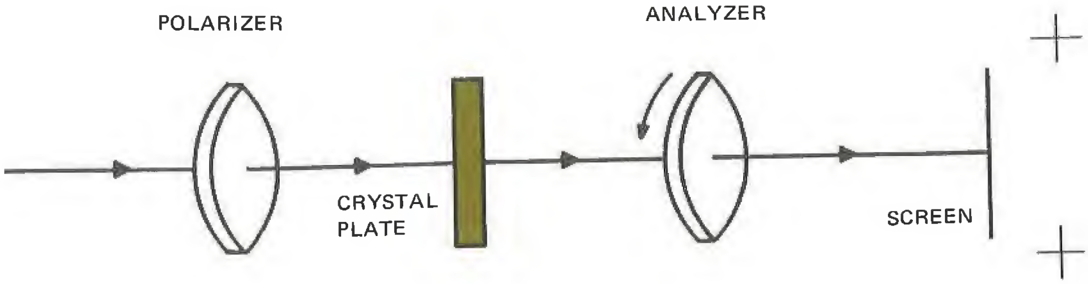

Fig. 15-5 Crystal Plate Between Two Polaroids

The polarizer does nothing but produce plane-polarized light in some direction which is incident upon the crystal plate. The plate is cut as in figure 15-4 and the e and o components traverse the crystal with different speeds. The emerging beam will, therefore, have its e and o components displaced in phase by some angle Θ and may have either circular, elliptical or linear resultant polarization. The beam then passes through the analyzer and hits the screen.

Now, the phase difference between the e and o rays after traversing a crystal plate as in figure 15-4 is given by

$$\Theta = \frac{2\pi}{\lambda} t\,(n_o - n_e) \qquad \textbf{(15.1)}$$

where λ is the wavelength in air and t is the thickness of the crystal. Therefore, if the light is of single frequency, then Θ will be determined by t. If t is such that $\Theta = \pi/2$, the beam emerging from the crystal will be circularly polarized and there will be no intensity change on the screen if the analyzer is rotated. That is, if an analyzer alone is used to analyze circularly polarized light, it will give the same result as when used to analyze unpolarized light. If the beam leaving the crystal is linearly polarized, $\Theta = \pi$ or 2π, then rotation of the analyzer will produce a screen intensity which varies from zero to maximum.

If, for the emerging beam from the crystal, $\Theta = \pi/2$, the light is circularly polarized and we say that the crystal is a *quarter-wave plate* at that frequency. For example, if $\Theta = \pi/2$ for red light and thus is a quarter-wave plate for red, the crystal will not act as a quarter-wave plate for blue or any other color. This can be seen from equation 15.1 in that Θ also depends upon λ.

Consider that in figure 15-5 we have a crystal which is a *half-wave plate* ($\Theta = \pi$) for red. If the analyzer is crossed with the polarizer, the red light will be transmitted to the screen.

What happens when the incident beam is composed of white light? Only the red component of the white light will emerge from the half-wave plate as plane-polarized light. All of the other wavelengths will emerge as either circularly or elliptically polarized light. If the analyzer is in a position to transmit this red light completely, it will block out a portion of the other wavelengths. The light transmitted by the analyzer will therefore be mostly red and will have a pinkish hue. When the analyzer is rotated 90° so as to block out the red, the other wavelengths will be partially transmitted and the resulting hue will be the complement of pink, or a blue-green.

Now, if in figure 15-5, our crystal is not of uniform thickness, the light on the screen will be composed of patches of different colors. A small piece of the non-uniform crystal may act as a half-wave plate for red, another for yellow, another for blue, etc. Therefore, the patches of color on the screen will correspond to regions of different thickness of the crystal. These colors will change to complementary values when the analyzer is rotated through 90°.

All this leads to a very important tool in industry known as optical stress analysis. We know that if a polarizer and an analyzer are mounted with their transmission directions at right angles to each other, no light will be transmitted through the combination. However, if a doubly refracting crystal is inserted between the combination, we also know that some light will be transmitted by the analyzer. Some of the wavelengths will be circularly or elliptically polarized and thus portions of these will pass through the analyzer.

Some substances, such as celluloid, glass, and certain plastics, while not normally so, become doubly refracting under mechanical stress. From a study of the object between crossed polaroids, much information about the mechanical stresses can be obtained.

The double refraction produced by mechanical stress forms the basis of the science of photoelasticity. The stress in opaque engineering materials such as beams, girders, gear teeth, etc., can be analyzed by constructing a plastic model of the object and examining it between crossed polaroids. Extremely complicated stress patterns such as those around a hole or gear tooth may be quite easily studied by optical methods.

MATERIALS

1 He-Ne laser	1 Support platform
1 Polished calcite crystal	2 Carriages
1 White light source	1 Plexiglass, U or horseshoe shape
1 Polarimeter	1 Plexiglass, rectangular bar
1 Optical bench	1 Wooden wedge
1 Lens holder	2 Pieces cellophane

PROCEDURE

1. Mount the calcite crystal in a lens holder on the optical bench.
2. Turn on the laser and observe the emergent face of the crystal.

3. Adjust the calcite crystal until two emergent beams are clearly visible.

4. Sketch the emergent face of the calcite crystal and show the location of the emerging e and o beams.

5. Insert a polaroid between the laser and the calcite crystal.

6. Rotate the polaroid 360° and note the degree readings at which e and o beams disappear.

7. In the data table, figure 15-7, record the degree readings that corresponds to the complete extinction of the respective beam.

8. Set up the white light source, polarizer and analyzer as depicted in figure 15-5. Instead of a screen, you may wish to look into the analyzer directly.

9. "Cross" the analyzer and the polarizer. That is, adjust the analyzer until no light is transmitted through the combination.

10. Crumple a piece of cellophane and place it between the polarizer and analyzer. Observe the color pattern through the analyzer.

11. Pick out a certain portion of the cellophane and note the color change as the analyzer is rotated 360°.

12. Place the plexiglass U between the polarizer and analyzer. The two polaroids should still be crossed.

13. Apply a small force to the open ends of the U and observe the optical pattern through the analyzer. Sketch this pattern. Indicate the colors.

14. Increase the force and note how the pattern changes. Sketch the stress pattern observed with a stronger force on the ends.

15. With the polaroids still crossed, insert the set-up of figure 15-6 between them.

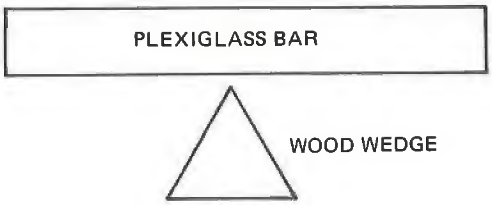

Fig. 15-6 Bar and Wedge

16. While viewing through the analyzer, apply a downward force to each end of the plastic bar. Sketch the resultant stress pattern and indicate the colors.

17. Increase the force and repeat step 16.

EXTINCTION	DEGREES	
e		
o		

Fig. 15-7 The Data Table

ANALYSIS GUIDE. In the analysis of this experiment you should discuss how the readings in the data table show how the e-vibration and o-vibration planes are oriented with respect to each other. You should also discuss the various colors observed in step 11. You should discuss the stress patterns observed with the two models and try to correlate color or color changes with lines of maximum stress.

PROBLEMS

1. What method, other than that used in step six, could you use to show how the e and o polarization states are related?

2. Show that the minimum thickness of a quarter-wave plate is given by

$$t = \lambda/4\,(n_o - n_e)$$

3. What is the minimum thickness for a quarter-wave calcite plate for a light wavelength of 6328 Å? (n_o = 1.65836, n_e = 1.48641)

EXPERIMENT 1 ______________ Name ______________________________

Date: ______________________ Class __________ Instructor ______________

d (mm)	A (cm^2)	r (cm)	r^2 (cm^2)

Fig. 1-5 The Data Table

EXPERIMENT 2 ______________ Name ______________________________

Date: ______________________ Class __________ Instructor ______________

THE FIRST COLLIMATOR

THE SECOND COLLIMATOR

THE THIRD COLLIMATOR

Fig. 2-5 Sketches of the Results

EXPERIMENT 3

Date: ______________________

Name ______________________

Class ______________ Instructor ______________

Θ_i	Θ_r	$\overline{OT}$	$\overline{O_1'T}$	Θ_i'	Θ_r'	ϕ

Fig. 3-6 The Data Table

EXPERIMENT 4

Date: ____________ Name ____________

Class ________ Instructor ____________

REAL IMAGE

S	S′	f	M

VIRTUAL IMAGE

	AVERAGE f		

Fig. 4-7 The Data Tables

EXPERIMENT 5

Date: ______________________

Name ______________________

Class __________ Instructor ______________

Θ_1	Θ_1'	Θ_2	Θ_2'	t	D	n	CALCULATED D

AVERAGE n =

A	δ_m	n

Fig. 5-7 The Data Tables

EXPERIMENT 6

Name ______________________

Date: ______________________ Class ____________ Instructor ______________

DEGREES	RELATIVE CHANGE IN INTENSITY
0°	
45°	
90°	
135°	
180°	
225°	
270°	
315°	

MATERIAL	Θ_p
GLASS	
LUCITE	
PLEXIGLASS	

Fig. 6-8 The Data Table

EXPERIMENT 7 Name ______________________

Date: ______________ Class __________ Instructor __________

λ	m	Θ	d	d′

Fig. 7-7 The Data Table

EXPERIMENT 8

Date: ____________________ Name ____________________

Class ____________ Instructor ____________________

DOUBLE SLIT

d	R	Y_m	m	λ (COMPUTED)
			AVERAGE λ	

DIFFRACTION GRATING

STATED LASER λ	m	d (COMPUTED)
	AVERAGE d	

Fig. 8-4 The Data Tables

EXPERIMENT 9 Name ______________________

Date: ______________ Class __________ Instructor ______________

Sketch of First "Rod" Results	Sketch of Second "Rod" Results
Sketch of Third "Rod" Results	Sketch of Fourth "Rod" Results

Fig. 9-5 Sketches of the Results

Sketch of First "Prism" Results

Sketch of Second "Prism" Results

Fig. 9-5 Sketches of the Results (Con't)

EXPERIMENT 10 ____________ Name ______________________

Date: ______________________ Class ________ Instructor ____________

S	S′	f	M
	AVERAGE f		

Fig. 10-8 The Data Table for Lens I

S	S′	f	M
	AVERAGE f		

Fig. 10-8 (Con't) The Data Table for Lens II

EXPERIMENT 10 ____________ Name ______________________

Date: ______________________ Class ________ Instructor ______________

S	S′	f	M
	AVERAGE f		

Fig. 10-8 (Con't) The Data Table for Lens III

EXPERIMENT 11

Name ______________________

Date: ______________________ Class ____________ Instructor ______________________

FIRST LENS

L	S'_2	S_1	S_2	S'_1	f_1
				AVERAGE f_1	

SECOND LENS

				AVERAGE f_1	

THIRD LENS

				AVERAGE f_1	

Fig. 11-6 The Data Tables

EXPERIMENT 12

Date: ______________________

Name ______________________

Class __________ Instructor ______________

	f_R	f_G	f_V
PRISMS			
POSITIVE LENS			
ACHROMAT			
	f_1	f_2	d

Fig. 12-11 Data Table I

LENS	h	f
		
		

Fig. 12-14 Data Table II

EXPERIMENT 13

Date: ____________________

Name ____________________

Class __________ Instructor __________

LENS DATA

FOCAL LENGTH	MAX. APERTURE
mm	f/

LENS MEASUREMENT

APPROX. LENS DIAMETER	COMPUTED MAX. APERTURE	ROUNDED UP APERTURE
mm	f/	f/

CAMERA DATA

SHUTTER SPEEDS	
f/STOPS	
RANGE MARKS	

DEPTH-OF-FIELD DATA

f/STOP	MINIMUM RANGE	MAXIMUM RANGE

Fig. 13-3 The Data Tables

LIGHT METER DATA

CAMERA f/STOP	SHUTTER SPEED (LIGHT METER)

LIGHT METER DATA

CAMERA SHUTTER SPEED	f/STOP (LIGHT METER)

Fig. 13-3 (Con't) The Data Tables

EXPERIMENT 14

Date: ______________ Name ______________

Class __________ Instructor ______________

	BLACK PAPER	ROOM LIGHT	80 cm	60 cm	40 cm	20 cm	10 cm
PHOTOVOLTAIC ONE							
PHOTOVOLTAIC TWO							
WITH 20 Ω R VOLTAGE							
PHOTO-CONDUCTOR							

VOLTAGE OUTPUT VERSUS ILLUMINATION

NO. OF SHEETS	1	2	3
RESISTANCE			

CELL RESISTANCE VERSUS OBSTRUCTION

Fig. 14-9 The Data Tables

EXPERIMENT 15 Name ____________________

Date: ____________________ Class ____________ Instructor ____________________

EXTINCTION	DEGREES	
e		
o		

Fig. 15-7 The Data Table