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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, May 2019

First Degree Programme under CBCSS

Complementary Course for Physics

ST 1231.2 : PROBABILITY THEORY

(2017 Admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions (Each question carries 1 mark):

1. What do you mean by “exhaustive events”? Give one example.
2. Give the empirical definition of probability.
3. Prove that the probability of the impossible event is zero.
4. If A and B are two independent events, then show that $\overline{A}$ and $\overline{B}$ are also independent.
5. Given that $P(A)=\frac{1}{3}$, $P(B)=\frac{3}{4}$ and $P(A\cup B)=\frac{11}{12}$, then find $P(B/A)$.
6. Define distribution function of a random variable X.
7. Prove that $f(x)=\alpha e^{-\alpha x}$; $x \geq 0$; $\alpha > 0$ is a probability density function.
8. Define independence of two random variables.
9. Define mathematical expectation.
10. Show that $M_{cX}(t)=M_X(ct)$, where c is a constant.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

Answer **any eight** questions (Each question carries 2 marks).

11. Write down the sample space when two dice are thrown. What is the probability that the sum of two faces greater than 10?
12. State and prove multiplication theorem of probability for independent events.
13. In a random arrangement of the letters of the word "MATHEMATICS", find the probability that all the vowels come together.
14. Find P(A/B) if (a) $A \cap B = \phi$, (b) $B \subset A$.
15. Distinguish between pairwise and mutual independence of events.
16. If $f(x) = \begin{cases} x e^{\frac{-x^2}{2}}, & x \geq 0 \\ 0, & x < 0 \end{cases}$

    Show that $f(x)$ is a pdf and also find its distribution function.
17. The distribution function of a random variable X is given by $F(x) = 1 - (1 + x) e^{-x}$; $x > 0$. Find the pdf and mean.
18. If X is a random variable, then show that $V(aX + b) = a^2 V(X)$, where a and b are constants.
19. Define moment generating function of a random variable and state its properties.
20. Show by an example that a random variable X may have no moments although its mgf exists.
21. Define conditional expectation and conditional variance.
22. If the possible values of a random variable X are 0,1,2, ..., then show that $E(X) = \sum_{n=0}^{\infty} P(X > n)$.

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions (Each question carries 4 marks):

23. A box contains tags marked $1, 2, ..., n$. Two tags are chosen at random without replacement. Find the probability that the numbers on the tag will be consecutive integers.

24. Let the random variable X has the distribution $P(X=0)=P(X=2)=p$; $P(X=1)=1-2p$ for $0 \le p \le \frac{1}{2}$. Find the variance of X and for what value of p the variance is maximum.

25. A discrete random variable X has the following pmfs:

$P(X=1)=\frac{1}{2}$, $P(X=2)=\frac{1}{4}$, $P(X=3)=\frac{1}{8}$ and $P(X=4)=\frac{1}{8}$. Find and sketch the distribution function and also find $P(1 < X \le 3)$.

26. A two-dimensional random variable (X, Y) have a bivariate distribution given by

$P(X=x, Y=y)=\frac{x^2+y}{32}$, for $x=0, 1, 2, 3$ and $y=0, 1$.

Find the marginal distributions of X and Y.

27. The joint pdf of a bivariate random variable (X, Y) is given by

$f(x, y)=\begin{cases} k, & 0 < y \le x < 1 \\ 0, & otherwise \end{cases}$, where k is a constant. Determine the value of k and find the marginal pdf's of X and Y..

28. If the moment generating function of a random variable X is $\frac{2}{2-t}$. Find the standard deviation of X.

29. If X and Y are two independent continuous random variables, then show that $E(XY)=E(X)E(Y)$.

30. State and prove Cauchy —Schwartz inequality.

31. If the joint pdf of (X, Y) is given by $f(x, y)=2-x-y$; $0 \le x \le 1$, $0 \le y \le 1$. Find $E(XY)$.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions (Each question carries 15 marks) :

32. (a) State and prove Baye's theorem.

(b) Urn I has 2 white and 3 black balls; Urn II has 4 white and 1 black balls; Urn III has 3 white and 4 black balls. An urn is selected at random and a ball drawn is found to be white. Find the probability that Urn I was selected.

33. Let X be a continuous random variable with pdf given by
$$f(x) = \begin{cases} kx, & \text{if } 0 \le x \le 1 \\ k, & \text{if } 1 \le x \le 2 \\ -kx + 3k, & \text{if } 2 \le x \le 3 \end{cases}$$ and $f(x) = 0$, otherwise.

(a) Determine the constant k

(b) Obtain the distribution function

(c) Find $P(X > 1.5)$?

34. An information source generates symbols at random from a four-letter alphabet $\{a, b, c, d\}$ with probabilities $P(a) = \frac{1}{2}$, $P(b) = \frac{1}{4}$, and $P(c) = P(d) = \frac{1}{8}$. A coding scheme encodes these symbols into binary codes as follows:

(a) 0

(b) 10

(c) 110

(d) 111

Let X be the random variable denoting the length of the code, that is, the number of binary symbols(bits).

(i) What is the range of X?

(ii) Sketch the cdf of X?

(iii) Find $P(X \le 1)$, $P(1 < X \le 2)$ and $P(X > 1)$.

(iv) Find the mathematical expectation of X.

35. The joint probability density function of two discrete random variables X and Y is given by, $f(x, y) = c(2x + y)$, where x and y can assume all integers such that, $0 \le x \le 2$ and $0 \le y \le 3$ and $f(x, y) = 0$, otherwise.

Then

(a) Find the value c.

(b) Find $P(X = 2, Y = 1)$

(c) Find the marginal distribution function of X and Y.

(d) Show that X and Y are independent.

(2 × 15 = 30 Marks)
