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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, December 2021

First Degree Programme Under CBCSS

Statistics

Complementary Course for Physics

ST 1231.2 : PROBABILITY THEORY

(2020 Admission Regular)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. (Each question carries **1** mark)

1. Define Sample Space.

2. Give the statistical definition of probability.

3. State the multiplication theorem of probability for three events.

4. What are the properties of p.d.f. of a continuous random variable X?

5. Define a random variable.

6. A random variable X has the following probability mass function

X	0	1	2	3	4
$P(x)$	a	$3a$	$5a$	$7a$	$9a$

   Find the value of a.

7. Define the joint p.d.f for two continuous random variables X and Y.

P.T.O.

8. Define Moment Generating Function of a random variable.

9. When an unbiased die is thrown, let X denote the number that turns up. Find $E(X)$.

10. Give any two properties of the characteristic function of a random variable.

($10 \times 1 = 10$ Marks)

SECTION – B

Answer **any eight** questions (Each question carries **2** marks)

11. If $P(A) = 0.3$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$, find $P(A \cap B^C)$ and $P(B \cap A^C)$.

12. A problem in mathematics is given to 3 students A, B, C whose chances of solving it are $\frac{1}{4}, \frac{1}{2}$ and $\frac{3}{4}$ respectively. What is the probability that the problem is solved?

13. Define random experiment. Give an example.

14. A card is chosen at random from a well shuffled pack of cards. Find the probability of getting a card of (a) A King (b) A red Queen

15. Define pairwise independence and mutual independence of three events.

16. If $P(A) = 1/3$, $P(B) = 3/4$ and $P(A \cup B) = 11/12$, find $P(A/B)$ and $P(B/A)$.

17. Consider a die throwing experiment. Let A be the event of getting a face marked 2 and let B be the event that the number is ≤ 4. Check whether A and B are independent.

18. Consider the random experiment of tossing two coins. Write down the p.m.f of Y, the number of tails.

19. Let X be a continuous random variable with pdf

$$f(x) = 1, \quad 0 < x < 1$$
$$= 0, \quad otherwise$$

Find the cumulative distribution function of X.

20. If X is a random variable with p.d.f, $f(x) = 2x;\ 0 < x < 1$, Find the p.d.f of $Y = 3X + 1$.

21. Find the marginal density functions of X and Y if

$f(x,y)=\frac{2}{5}(2x+3y);\ 0\le x\le 1,\ 0\le y\le 1$

22. Let X and Y have the following joint p.m.f.

Y/X	2	4
1	0.1	0.15
3	0.2	0.3
5	0.1	0.15

Check whether X and Y are independent.

23. If the joint p.d.f of X and is

$f(x,y)=kx(y-x);\ 0<x<4, 4<y<8$

Find the value of k.

24. For a continuous random variable X having p.d.f

$$f(x)=\frac{1}{18}(3+2x);\quad 2\le x\le 4$$
$$=0;\quad otherwise$$

Find $E(X)$.

25. Define raw moments and central moments.

26. Find the m.g.f. of a random variable X whose probability function is

$p(x)=\frac{1}{2^x};\ x=1,2,3....$

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions. (**Each** question carries **4** marks).

27. What are the advantages and disadvantages of classical definition of probability?

28. If $A_1, A_2, ..., A_n$ are n independent events with probabilities $p_1, p_2, ..., p_n$, then show that $P(A_1 \cup A_2 \cup .... \cup A_n) = 1-(1-p_1)(1-p_2)....(1-p_n)$.

29. State and prove addition theorem of probability for two events.

30. If A and B are two independent events, then show that

(a) A^C and B

(b) A^C and B^C are also independent

31. If A and B are two possible outcomes of an experiment such that $P(A) = 0.4$, $P(B) = p$ and $P(A\ B) = 0.7$. Find the value of p when (a) A and B are mutually exclusive (b) A and B are independent.

32. A random variable X has the following Probability Function

X	0	1	2	3	4	5	6	7	8
$P(X)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

(a) Find a.

(b) Find $P(X < 3)$, $P(X \geq 3)$, $P(0 < X < 5)$

33. For the following probability distribution

(a) Find the distribution function of X

(b) What is the smallest value of x for which $P(X \leq x) > 0.5$.

X	0	1	2
P(X)	¼	2/4	1/4

34. The two-dimensional random variable (X, Y) has the joint density function

$$f(x,y) = \frac{x+2y}{27}, x = 0,1,2,\ y = 0,1,2$$

Find the conditional distribution of X given $Y = 1$.

35. The joint p.d.f of a two-dimensional variable is

$$f(x,y) = x + y; \quad 0 < x < 1,\ 0 < y < 1$$
$$= 0; \qquad \textit{otherwise}$$

Check whether X and Y are independent.

36. Let X be a random variable with $E(X) = 10$, $V(X) = 25$. Find the positive values of a and b such that $Y = ax - b$ has expectation zero and variance one.

37. Find the Moment Generating Function of a random variable X having p.d.f

$$f(x) = 2/3, \quad x = 1$$
$$= 1/3, \quad x = 2$$
$$= 0, \quad otherwise$$

Hence find it's mean.

38. Two random variables X and Y have joint p.d.f

$$f(x,y) = 2 - x - y; \quad 0 \le x \le 1, 0 \le y \le 1$$
$$= 0; \quad otherwise$$

Find (a) $f_1(x)$ (b) $f_2(y)$ (c) $E(X/Y = y)$.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions. (Each question carries **15** marks)

39. State and prove Bayes Theorem. An insurance company insured 3000 scooter drivers, 2000 car drivers and 5000 truck drivers. The probabilities of accident by the drivers of these types of vehicles are 0.04,0.02 and 0.03 respectively. One of the insured people meets an accident. What is the probability that he is a truck driver?

40. From a bag containing 4 white and 6 black balls, two balls are drawn at random. If the balls are drawn one after the other without replacement, find the probability that

(a) both balls are white

(b) both balls are black

(c) the first ball is white and the second ball is black

(d) the first ball is black and the second ball is white

(e) one ball is white and the other is black

41. A random variable X has the following probability distribution

X	–2	–1	0	1	2	3
P(X)	0.1	k	0.2	2k	0.3	3k

Find

(a) The value of k

(b) Evaluate $P(X<2)$ and $P(-2<X<2)$

(c) Find the cumulative distribution of x

(d) Find the mean of X

42. For the following density function, $f(x) = ae^{-|x|}, -\infty < x < \infty$

Find the value of (a) a (b) mean of X (c) Variance of X

43. Two random variables X and Y have joint p.d.f

$$f(x,y) = \frac{xy}{96}, \quad 0 < x < 4, 1 < y < 5$$
$$= 0, \quad \text{otherwise}$$

(a) $E(X)$

(b) $E(Y)$

(c) $E(XY)$

(d) $V(X)$

(e) $V(Y)$

44. A random variable X has the density function given by

$$f(x) = \frac{1}{k}, \quad 0 < x < k$$
$$= 0, \quad \text{otherwise}$$

Find

(a) m.g.f

(b) r^{th} raw moment about zero

(c) mean

(d) variance

(2 × 15 = 30 Marks)
