P – 1258

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, September 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1231.1 : MATHEMATICS II – APPLICATIONS OF CALCULUS AND VECTOR DIFFERENTIATION

(2021 Admission)

Time : 3 Hours | Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Find the intervals on which $f(x) = x^2 - 4x + 3$ is increasing.

2. If $f''(a)$ exists and f has an inflection point at $x = a$, then $f''(a) =$ ————

3. Determine where the graph of the function $f(x) = x + 2\sin x$ is concave up.

4. Define critical point of a function.

5. State Mean value theorem.

6. What is the value of $\lim\limits_{x \to 0} \frac{e^x - 1}{\sin x}$?

7. Find the rectangular coordinates of the point P whose polar coordinates are given by $(r,\theta)=(6,2\pi/3)$.

8. Identify the quadric surface $z=\frac{x^2}{4}+\frac{y^2}{25}$.

9. Find the gradient of $f(x,y)=x^2+xy+z^2$.

10. Find $r'(t)$ if $r(t)=t^2i+e^tj-(2\cos\pi t)k$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions from among the questions 11 to 26. These questions carry **2** marks each.

11. Find the x-coordinates of all inflection points of $3x^4-4x^3$.

12. Find the critical points of $f(x)=3x^{5/3}-15x^{2/3}$.

13. Let $f(x)=x^2+bx+c$. Find the values of b and c such that $f(1)=5$ is an extreme value of f on [0, 2]. Is this value a maximum or minimum?

14. Find an interval [a, b] on which $f(x)=x^4+x^3-x^2+x-2$ satisfies the hypotheses of Rolle's theorem.

15. Find all values of k and l such that $\lim_{x\to 0}\frac{k+\cos lx}{x^2}=-4$.

16. Find the total area between the curve $y=1-x^2$ and the x axis over the interval [0, 2].

17. Find area of the surface that is generated by revolving the portion of the curve $y^2=x$ from origin to the point where $x=2$ about the x-axis.

18. Find the exact arc length of the curve $y=3x^{3/2}-1$ from $x=0$ to $x=1$.

19. Evaluate : $\int_2^a \int_2^b \frac{dy\, dx}{xy}$.

20. Evaluate : $\int_{-1}^{2} \int_0^2 \int_0^1 \left(x^2 + y^2 + z^2\right) dx\, dy\, dz$.

21. Find the perimeter of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

22. Find the length of the arc of the parabola $y^2 = 4ax$ from the vertex to the extremity of the latus rectum.

23. Find the area of the surface that is generated by revolving the curve $y = \sqrt{4 - x^2}$, $-1 \le x \le 1$.

24. Find the domain of $r(t)$ and the value of $r(t_0)$ if $r(t) = \sqrt{5t+1}\, i + t^2 j$; $t_0 = 1$.

25. Describe the graph of $r(t) = \langle 1 + 2t, -1 + 3t \rangle$.

26. If $f(x, y) = (1 + xy)^{3/2}$, find $D_u f(8,1)$ for the unit vector $u = \frac{1}{\sqrt{2}} i + \frac{1}{\sqrt{2}} j$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions from among the questions 27 to 38. These questions carry **4** marks each.

27. Find the relative extrema of $f(x) = 3x^5 - 5x^3$.

28. Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = x^3 - 3x^2 + 2x$ over [0, 1] is revolved about the *y*-axis.

29. Use double integral to find the area bounded by the *x*-axis, $y = 2x$ and $x + y = 1$.

30. Evaluate the triple integral $\iiint\limits_G 12xy^2 z^3\, dV$ over the rectangular box G defined b
the inequalities $-1 \le x \le 2, 0 \le y \le 3, 0 \le z \le 2$.

31. Evaluate $\lim\limits_{x \to \infty} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$.

32. Evaluate $\iint xy\, dx\, dy$ over the positive quadrant bounded by $\frac{x}{a} + \frac{y}{b} = 1$.

33. Find the length of the curve $27y^2 = 4x^3$ from (0, 0) to (3, 2).

34. Find the area of the region that is enclosed between the curves $y = x^2$ and $y = x + 6$.

35. Use polar coordinates to evaluate $\int\limits_{-1}^{1} \int\limits_{0}^{\sqrt{1-x^2}} \left(x^2 + y^2\right)^{3/2} dy\, dx$.

36. Using double integral, find the area of the region R bounded by $y = x$ and $y = x^2$ in the first quadrant.

37. If $r_1(t) = 2t\,i + 3t^2 j + t^3 k$ and $r_2(t) = t^4 k$. Find $\frac{d}{dt}(r_1 \circ r_2)$ and $\frac{d}{dt}(r_1 \times r_2)$.

38. Find the directional derivative of $f(x, y, z) = x^2 y - yz^3 + z$ at the point (1, –2, 0) in the direction of the vector $a = 2i + j - 2k$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions from among the questions 39 to 44. These question carry **15** marks.

39. (a) Show that the function $f(x) = \frac{1}{4}x^3 + 1$ satisfies the hypotheses of the Mean Value Theorem over the interval [0, 2] and find all values of c in the interval (0, 2).

(b) An open box is to be made from a 16-inch by 30-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. What size should the squares be to obtain a box with the largest volume?

40. (a) Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = x^3 - 3x^2 + 2x$ over [0, 1] is revolved about the y-axis.

(b) Find the area of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$.

41. (a) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y-axis.

(b) Find the volume of the solid that results when the region above the x-axis and below the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > 0, b > 0)$ is revolved about the x-axis.

42. (a) Use double integration to find the area of the region R enclosed between the parabola $y = \frac{1}{2}x^2$ and the line $y = 2x$.

(b) Use triple integration in cylindrical coordinates to evaluate

$$\int_0^a \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \int_0^{a^2-x^2-y^2} x^2 \, dz \, dy \, dx (a > 0).$$

43. (a) Using integration find the area of the triangular region whose sides have the equations $y = 2x + 1$, $y = 3x + 1$ and $x = 4$.

(b) Find the volume of the solid that results when the region enclosed by the curves $f(x) \sin x$ and $g(x) = \cos x$ is revolved about the x-axis from $x = 0$ to $x = \pi/4$.

44. If the electrical potential at a point (*x*, *y*) in the *xy*-plane is $V(x,y)$, then the electrical intensive vector at the point (*x*, *y*) is $E = -\nabla V(x,y)$. Suppose that $V(x,y) = e^{-2x}\cos 2y$.

(a) Find the electric intensity vector at $(\pi/4, 0)$.

(b) If $f(x,y,z) = 3x^2y - y^3z^2$, find the maximum value of direction derivative of *f* at the point (1, –2, –1).

(2 × 15 = 30 Marks)