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P – 1260

Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, September 2022

First Degree Programme under CBCSS

Statistics

Complementary Course for Physics

ST 1231.2 : PROBABILITY THEORY

(2020 Admission onwards)

Time : 3 Hours | Max. Marks : 80

SECTION – A

Answer **all** questions. (**Each** question carries **1** mark)

1. What do you mean by mutually exclusive events?

2. Give the axiomatic definition of Probability.

3. Define mutual independence of three events.

4. State Addition theorem of probability for three events.

5. Define Probability mass function of a discrete random variable.

6. If X is a continuous random variable, with p.d.f

$$f(x) = c(4x - 2x^2); \quad 0 < x < 2$$
$$= 0; \quad \text{otherwise}$$

Find the value of c.

7. Define the joint p.m.f of two discrete random variables X and Y.

8. Define Mathematical expectation of a random variable.

9. Define characteristic function of a random variable.

10. Let X and Y be two discrete random variable having joint prof. $f(x, y)$. Define conditional mean of X given Y.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. (**Each** question carries **2** marks)

11. Give any two limitations of the statistical definition of probability.

12. If $P(A) = 0.4$, $P(B) = 0.7$ and $P(A \cap B) = 0.3$, find $P(A^c \cap B^c)$.

13. If from a pack of cards, a single card is drawn, what is the probability that it is either a spade or a king?

14. If $P(A) = 0.5$, $P(A \cup B) = 0.7$, find $P(B)$ if $P(A/B) = 0.5$.

15. Prove that A is independent of itself if and only if $P(A)$ is 0 or 1.

16. If A and B are mutually exclusive events and $P(A \cup B) \neq 0$ then prove that $P(A/A \cup B) = \dfrac{P(A)}{P(A) + P(B)}$.

17. Find the distribution function $F(x)$ of the random variable X with the following p.m.f

X	0	1	4
$f(x)$	1/5	2/5	2/5

18. Verify whether the following function is a p.d.f of a continuous random variable X

$$f(x) = 2x^3; \quad 0 \le x \le 1$$
$$= 0; \quad \text{otherwise}$$

19. Given the probability distribution of a discrete random variable X

X	0	1	2	5
$f(x)$	1/8	2/8	2/8	3/8

Find the probability distribution of $Y = 2X - 4$.

20. The joint distribution of X and Y is given by

$$f(x, y) = \frac{x+y}{21}; \quad x = 1, 2, 3 \quad y = 1, 2$$

Find the marginal distributions.

21. Write any two properties of joint distribution function of a bivariate random variable (X, Y).

22. The joint density function of X and Y is

$$f(x, y) = e^{-(x+y)}, \quad 0 \leq x, \; y \leq \infty$$
$$= 0, \quad \text{otherwise}$$

Are X and Y independent?

23. When an unbiased die is thrown, X denote the number that turns up. Find $E(X)$ and $E(X^2)$.

24. Show that if X_1 and X_2 are two independent random variables then $\phi_{x_1+x_2}(t) = \phi_{x_1}(t) \times \phi_{x_2}(t)$.

25. If the joint p.d.f. of (X, Y) is given by $f(x, y) = 24y(1-x)$, $0 \leq y \leq x \leq 1$, then find $E(XY)$.

26. Find the Moment Generating Function of a random variable X having pdf

$$f(x) = \frac{1}{3}, \quad -1 < x < 2$$
$$= 0, \quad \text{otherwise}$$

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. (**Each** question carries **4** marks)

27. In a random experiment of throwing two dice simultaneously, find the following probabilities.

    (a) Two faces showing the same number

    (b) Sum of the numbers is eight

    (c) Number on the first die < number on the second die

    (d) Sum of the numbers < 7

28. If A and B are two events with $P(B) \neq 1$ prove that

$$P(A/B^c) = \frac{P(A) - P(A \cap B)}{1 - P(B)}$$

    Find $P(A/B^c)$ if $P(A) = 0.4$, $P(B) = 0.6$, $P(A \cup B) = 0.8$.

29. Two persons A and B appear in an interview for two vacancies for the same post. The probability of A's selection is 1/7 and that of B's selection is 1/5. What is the probability that

    (a) both of them is selected

    (b) None of them will be selected

30. If A, B, C are pairwise independent and A is independent of $B \cup C$, prove that A, B and C are mutually independent.

31. A continuous random variable with density function

$$f(x) = k(1+x), \quad 2 < x < 5$$
$$= 0, \quad \text{otherwise}$$

    Find (a) k (b) $P(X < 4)$

32. If the random variable X take the value 1,2,3 and 4 such that $2P(X=1)=3P(X=2)=P(X=3)=5P(X=4)$. Find the probability distribution of X. Also find its mean.

33. The mileage (in thousands of miles) that car owner get with a certain kind of tyre is a random variable having pdf

$$f(x)=\frac{1}{20}e^{\frac{-x}{20}};\ x>0$$

Find the probability that one of these tyres will last

(a) at most 10,000 miles

(b) anywhere from 16,000 to 24,000 miles

34. If X is a continuous random variable with $p.d.f.\ f(x)=e^{-x},\ 0<x<\infty$. Find the p.d.f. of (a) $Y=2X+5$ (b) $Y=X^3$.

35. Let X be a random variable with p.m.f.

X	1	2
$f(x)$	2/3	1/3

Find its m.g.f and hence find its mean.

36. A continuous random variable X is distributed over the interval [0,1] with p.d.f ax^2+bx, where a and b are constants. If the arithmetic mean of X is 0.5, find the values of a and b?

37. A two-dimensional random variable (X, Y) has joint density function

$$f(x, y)=\frac{x+2y}{27},\ x=0, 1, 2;\ y=0, 1, 2$$

Find the conditional distribution of Y given $X=x$.

38. Find the characteristic function of a random variable X having p.m.f

$$p(x)=q^{x}p,\ x=0, 1, 2,\ldots\ldots,\ p+q=1$$

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. (Each question carries **15** marks)

39. State and prove Bayes theorem. In a bolt factory, machines A, B, C manufacture respectively 25%, 35% and 40% of the total. Of their output, 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine A?

40. Two unbiased dice are thrown. Let A denote the event that the sum of the numbers shown on the faces is odd, B denote the event that at least on dice shows the face with the number 1.

    (a) Describe the complete Sample space

    (b) Find the probabilities of the events (a) A (b) B (c) $A^c \cup B^c$ (d) $A^c \cap B^c$ (e) $A \cap B^c$.

41. If X is a discrete random variable with p.m.f.

$$p(x) = \frac{x}{15}, \quad x = 1, 2, 3, 4, 5$$
$$= 0, \quad \textit{otherwise}$$

Find :

(a) $P(X \leq 3)$

(b) $P\left[\left(\frac{1}{2} < X \leq 3\right) / (X \geq 2)\right]$

(c) $P(X^2 - 3X + 2 = 0)$

(d) Sketch $F(x)$

42. A continuous random variable X has the distribution function

$$F(x) = \begin{cases} 0 & ; \quad x < 0 \\ \dfrac{x^2}{2} & ; \quad 0 \le x < 1 \\ k(4x - x^2) - 1 & ; \quad 1 \le x < 2 \\ 1 & ; \quad x \ge 2 \end{cases}$$

Find (a) k (b) pdf of X (c) If A and B are the events $(0.5 < X < 15)$ and $(X > 1)$ respectively, verify whether A and B are independent.

43. If $f(x, y) = k$, $0 < x < 1$, $0 < y < 1$ is a joint pdf, find

(a) k

(b) $P(X > 0.4, Y > 0.6)$

(c) the joint distribution function of X and Y

(d) $P(X > 0.4)$

44. The joint pdf of a two dimensional random variable (X, Y) is given by

$$f(x, y) = xy^2 + \frac{x^2}{8}, \quad 0 \le x \le 2, \; 0 \le y \le 1$$

Find

(a) $P\left(X > 1/\left(Y < \frac{1}{2}\right)\right)$

(b) $P\left(Y < \frac{1}{2}/(X > 1)\right)$

(c) $P(X < Y)$

(2 × 15 = 30 Marks)
