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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2021

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1331.1 : MATHEMATICS III — CALCULUS AND LINEAR ALGEBRA

(2019 Admission Regular)

Time : 3 Hours Max. Marks : 80

SECTION — I

All the **ten** questions are compulsory. They carry **1** mark each.

1. Write the differential equation of all straight lines in a plane.

2. What is the general form of Clairaut's equation?

3. Solve $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + 1 = 0$.

4. What is the electrostatic potential energy gained by moving a charge q along a path C in an electric field $\overline{E}$?

5. Define a conservative vector field.

P.T.O.

6. $f(t)=\sin 2\pi t$ is periodic with period ________

7. If $f(x)=\sum_{-\infty}^{\infty} c_n e^{inx}$, then what is the average of $|f(x)|^2$ over a period?

8. Define rank of a matrix.

9. If A is an invertible matrix of order n and b is an $n\times 1$ matrix, then the number of solutions of $Ax=b$ is ________.

10. Wronskian of $\{\sin x, \cos x\}$ is ________.

SECTION — II

Answer **any eight** questions. Each question carries **2** marks.

11. Solve $\frac{dy}{dx}=\frac{y}{x}+\tan\left(\frac{y}{x}\right)$.

12. Solve $\frac{dy}{dx}=(x+y+1)^2$.

13. Solve $\left(1-x^2\right)y^1+2xy=\left(1-x^2\right)^{3/2}$.

14. Solve $(y-px)(p-1)=p$.

15. Solve $\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+3\frac{dy}{dx}-y=0$.

16. Compute the work done by the force

$\overline{F}=xy^2\hat{i}+2\hat{j}+x\hat{k}$ over a path C parameterized by $x=ct, y=c/t, z=d; 1\le t\le 2$.

17. If $\bar{r} = x\hat{i} + y\hat{j} + 3\hat{k}$, show that (a) $\nabla \cdot \bar{r} = 3$ (b) $\nabla \times \bar{r} = \bar{0}$.

18. Prove that for any scalar valued function $\phi(x, y, z)$ whose second order partial derivatives are continuous, $Curl(grad\phi) = 0$.

19. Find the vector area of the surface of the Hemisphere $x^2 + y^2 = a^2$, $z \geq 0$, by evaluating the line integral $S = \frac{1}{2}\oint_C \bar{r} \times d\bar{r}$ around its perimeter.

20. State Dirichlet's conditions for a Fourier series.

21. Find the coefficients in the Fourier series of the function $f(x) = x, -\pi \leq x \leq \pi$ where $f(x + 2\pi) = f(x)$.

22. If A and B are symmetric matrices, prove that $AB - BA$ is an antisymmetric matrix.

23. Determine the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$.

24. Find a unit vector perpendicular to both $\hat{i} + \hat{j}$ and $i - 2\hat{k}$.

25. Show that the vectors $(1, 1, 1), (0, 1, 1)$ and $(0, 0, 1)$ are linearly independent.

26. Find the equation of the line through $(3, 4, -1)$ and parallel to $2\hat{i} - 3\hat{j} + 6\hat{k}$.

SECTION — III

Answer **any six** questions. Each question carries **4** marks.

27. Solve $xdy - y\,dx = \sqrt{x^2+y^2}\,dx$.

28. Solve $x\frac{dy}{dx} + y - \frac{y^2}{x^{3/2}} = 0$, subject to $y(1)=1$.

29. Solve $\frac{dy^2}{dx^2} - 2\frac{dy}{dx} + y = e^x$.

30. Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^{-x}$, subject to the boundary conditions $y(0)=2, y'(0)=1$.

31. Show that the area of a region R enclosed by a simple closed curve C is given by $A = \frac{1}{2}\oint_C x\,dy - y\,dx$. Hence prove that the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab.

32. Find the volume enclosed between a sphere of radius a centred on the origin and a circular cone of half angle α with its vertex at the origin.

33. Show that the vector field $\overline{Q} = (x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$ is conservative and find ϕ such that $\overline{Q} = \nabla\phi$.

34. A periodic function $f(t)$ with period 2π is defined within the period $-\pi < t < \pi$ by

$$f(t) = \begin{cases} -1, & -\pi < t < 0 \\ 1, & 0 < t < \pi \end{cases}$$

Find its Fourier series expansion.

35. Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$.

36. Find the distance from $P(1, 2, -1)$ to the line joining $P_1(0, 0, 0)$ and $P_2(-1, 0, 2)$.

37. Solve the equations $x+y+z=6, x-y+2z=5, 3x+y+z=8$.

38. Find what transformation corresponds to the matrix

$$A=\frac{1}{2}\begin{bmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{bmatrix}.$$

SECTION — IV

Answer **any two** questions. Each question carries **15** marks.

39. (a) Solve $(1+y^2)dx=(\tan^{-1}y-x)dy$

(b) Solve $(1-x^2)\frac{d^2y}{dx^2}-3x\frac{dy}{dx}-y=1$.

40. (a) Use Green's functions to solve $\frac{d^2y}{dx^2}+y=\operatorname{cosec} x$, subject to the boundary conditions $y(0)=y(\pi/2)=0$.

(b) Solve $4x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+(x^2-1)y=0$.

41. (a) Evaluate the surface integral $\int_S \bar{a}.d\bar{s}$, where $\bar{a}=(y-x)\hat{i}+x^2z\hat{j}+(z+x^2)\hat{k}$ and S is the open surface of the hemisphere $x^2+y^2+z^2=a^2, z\geq 0$.

(b) Verify Stoke's theorem for the vector field $\bar{a}=(2x-y)\hat{i}-yz^2\,\hat{j}-y^2z\,\hat{k}$ over the upper half surface of the unit sphere $x^2+y^2+z^2=1, z\geq 0$.

42. A periodic function $f(t)$ of period 2π is defined within the period $0<t<2\pi$ by

$$f(t)=\begin{cases} t, & 0\leq t\leq \frac{\pi}{2} \\ \frac{\pi}{2}, & \frac{\pi}{2}\leq t\leq \pi \\ \pi-\frac{t}{2}, & \pi\leq t\leq 2\pi \end{cases}$$

find a Fourier series expansion of it.

43. Represent $f(x)=\begin{cases} 1, & 0<x<1/2 \\ 0, & 1/2<x<1 \end{cases}$ by

(a) A Fourier sine series

(b) A Fourier cosine series

(c) A Fourier series.

44. (a) Diagonalize the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

(b) Find the equation relative to the principal axes of the quadric surface

$$x^2+5y^2+z^2+2xy+2yz+6xz=48.$$
