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Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, March 2020

First Degree Programme under CBCSS

Complementary Course for Physics

MM 1431.1 : MATHEMATICS IV – COMPLEX ANALYSIS, SPECIAL FUNCTIONS AND PROBABILITY THEORY

(2018 Admission)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. Each question carries **1** mark

1. Define harmonic function.

2. Define regular point.

3. Write Cauchy's integral formula.

4. What is the residue of e^z at $z=0$?

5. Define Beta function.

6. What is the value of $\sqrt{\left(\frac{1}{2}\right)}$?

7. A single card is drawn at random from a shuffled deck. What is the probability that it is red?

P.T.O.

8. Define Poisson probability function.

9. What is the standard deviation of Binomial distribution?

10. What is the mean and variance of standard normal distribution?

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions. Each question carries **2** marks

11. Find the real and imaginary part of $\cosh z$.

12. Check whether the following function is analytic or not $f(z) = z^2$.

13. Show that $f(x, y) = e^x \sin y$ satisfies the Laplace equation.

14. Evaluate $\oint_C \frac{dz}{z-a}$ where is the curve $|z-a| = r$.

15. Expand in Laurent's series $\frac{1}{z(z-1)^2}$ at the point $z = 1$.

16. Define Gamma function of negative numbers and evaluate $\sqrt{-3}$.

17. Prove that $B(p, q) = B(q, p)$.

18. Evaluate $\int_0^\infty x^{1/4} e^{-\sqrt{x}} dx$.

19. Two dice are thrown together. What is the probability that sum is greater than 8.

20. Criticize the statement " The mean of a binomial distribution is 5 and standard deviation is 3".

21. If x is a poisson variate such that $P(x=1)=P(x=2)$, then evaluate $P(x=4)$.

22. The diameter of an electric cable is assumed to be a continuous random variate with probability density function $f(x)=6x(1-x),\ 0\le x\le 1$ find mean.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions. Each question carries **4** marks

23. Find an analytic function whose imaginary part is $3x^2y-y^3$ and which vanishes at z = 0.

24. Evaluate $\int_{-\infty}^{\infty}\frac{dx}{x^2+4x+5}$.

25. Evaluate $\int_0^{2\pi}\frac{d\theta}{13+5\sin\theta}$.

26. Prove that $\sqrt{p+1}=p\sqrt{p}$.

27. Show that $1.3.5...(2n-1)=\frac{2^n\sqrt{n+\frac{1}{2}}}{\sqrt{\pi}}$.

28. Express $\int_0^{\pi/2}\sin^p x\cos^q x\,dx$ in terms of Gamma function.

29. From group of 8 children, 5 boys and 3 girls, three children are selected at random. Calculate the probability that the selected group contains atleast one girl.

30. In a certain factory turning out razor blades, there is a chance (1/100) for any blade to be defective. The blades are in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing no, one and two defective blades respectively in a consignment of 10000 packets.

31. Find the number r such that the area under the normal distribution curve $y=f(x)$ from $\mu=r$ to $\mu+r$ is equal to 1/2.

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions. Each question carries **15** marks

32. State and prove the necessary and sufficient condition for f(z) to be analytic.

33. Prove that $B(p,q)=\frac{\sqrt{p}\sqrt{q}}{\sqrt{p+q}}$.

34. (a) Suppose it is known that 1% of the population have a certain kind of cancer. It is also known that a test for this kind of cancer is positive in 99% of the people who have it but is also positive in 2% of the people who do not have it. What is the probability that a person who tests positive has cancer of this type?

    (b) A doctor is to visit the patient and from past experience it is known that the probabilities that he will come by train, bus or scooter are respectively 3/10, 1/5 and 1/10, the probability that he will use some other means of transport being, therefore, *2/5*. If he comes by train, the probability that he will be late is 1/4 if by bus 1/3 and if by scooter 1/12, if he uses other means of transport it can be assumed that he will not be late. When he arrives he is late. What is the probability that he comes by train.

35. (a) Find the probability of exactly 52 heads in 100 tosses of a coin using the binomial distribution and using the normal approximation.

    (b) If there are 100 misprints in a magazine of 40 pages, on how many pages would you expect to find no misprints? Two misprints? Five misprints?

(2 × 15 = 30 Marks)