

Reg. No. :

Name :

Fourth Semester B.Sc. Degree Examination, August 2022

First Degree Programme Under CBCSS

Mathematics

Complementary Course for Physics

MM 1431.1 : MATHEMATICS IV – COMPLEX ANALYSIS, SPECIAL
FUNCTIONS AND PROBABILITY THEORY

(2019 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

(Answer all questions. Each carries 1 mark)

1. Find the real and imaginary part of e^z .
2. Define an Analytic function.
3. Evaluate $\int_i^{i+1} z dz$ along straight line parallel to x-axis.
4. Find the residue of $\frac{\cos z}{z}$ at $z = 0$.
5. Find $\Gamma(10)/\Gamma(8)$.
6. Define $\beta(p, q)$.

7. In three tosses of a coin, find the probability that all three are heads?
8. What is probability of getting a white ball from a box containing 3 white balls and 2 red balls?
9. Out of 50 members of a club in how many ways can a president, vice president and a secretary can be chosen.
10. A number is selected from numbers 1 to 11 at random. What is the probability of choosing an odd number?

(10 × 1 = 10 Marks)

SECTION – II

(Answer **any eight** questions. **Each** carries **2** marks).

11. Check whether the function $\frac{y - ix}{x^2 + y^2}$ is analytic or not.
12. Using polar co-ordinate find out whether $f(z) = \sqrt{z}$ satisfy Cauchy-Riemann equation.
13. Evaluate $\int_C \frac{\sin z dz}{2z - \pi}$ where C is the circle $|z| = 2$.
14. Calculate the residue of $\frac{Z+1}{Z^2 - 2Z}$ at its poles.
15. Evaluate $\int_C \frac{e^{2z} dz}{(z-2)^2}$ where C is the circle $|z| = 3$.
16. Define (a) Pole (b) Essential singularity.
17. Evaluate $\int_C \frac{z+2}{z-2} dz$ where C is the circle $|z-1| = 2$.

18. Find the residue of $f(z) = \frac{z}{(2z+1)(5-z)}$ at $Z = 5$.

19. Find $\Gamma\left(\frac{1}{4}\right) + \Gamma\left(\frac{9}{4}\right)$.

20. Express $\int_0^{\infty} x^{\frac{2}{3}} e^{-x} dx$ as Γ function.

21. A letter of English alphabet is chosen at random. What is the probability that it is one of the letter in the word 'PROBABILITY'.

22. Find the probability of drawing an ace or a spade from a pack of cards.

23. One bag contains 5 red and 3 white balls. A second contains 4 red and 7 black balls. If one ball is drawn at random from each bag, what is the probability that both are of the same colour?

24. If a die is rolled three times, what is the probability of getting 5 at least once?

25. Distinguish between discrete and continuous probability functions.

26. When two dice are thrown find the probability that the product of the numbers on the top of the dice is 12.

(8 × 2 = 16 Marks)

SECTION – III

(Answer **any six** questions. **Each** question **4** marks).

27. Derive Cauchy-Reimann equation.

28. Show that the function $x^2 - y^2$ is harmonic. Find the function $f(z)$ for which the given function is the real part.

29. Expand $\frac{1}{z^2(1+z)^2}$ as Laurent series in the region

(a) $0 < |z| < 1$

(b) $|z| > 1$.

30. Find the function $f(z) = u + iv$ such that $f(z)$ is analytic given that $u = e^x \cos y$.

31. Evaluate $\int_C \frac{e^{-3\pi z} dz}{2z+i}$ where C is the boundary of the Square whose sides lie along the lines $x = \pm 1, y = \pm 1$.

32. Solve $\int_0^{\infty} x^5 e^{-x^2} dx$.

33. Evaluate $\Gamma\left(\frac{1}{2}\right)$.

34. Express $\int_0^1 \frac{x^4 dx}{\sqrt{1-x^2}}$ as beta function and solve.

35. A committee of 5 persons is to be selected randomly from a group of 5 men and 10 women. Find the probability that the committee consists of 2 men and 3 women.

36. A player is to toss three coins. He wins Rs.10 if three heads appear, Rs. 5 if two heads appear, Rs. 1 if one head appears. He will lose Rs. 12 if no head appears. What is the expected amount?

37. If 3% of electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs exactly five bulbs are defective.

38. Find the mean and variance of a random variable x which takes values 0,1,2,3 with respective probabilities $\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}$.

(6 × 4 = 24 Marks)

SECTION – IV

(Answer **any two** questions. **Each** question carries **15** marks).

39. (a) Evaluate using Residue theorem $\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$.

(b) Find the residue of $f(z) = \frac{z \sin z}{(z-\pi)^3}$ at $z = \pi$.

40. (a) Prove that $\beta(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$.

(b) Find $\int_0^{\infty} \frac{y^2 dy}{(1+y)^6}$.

41. Evaluate $\int_0^{\infty} \frac{\cos x}{1+x^2} dx$.

42. Expand $\frac{-1}{(z-1)(z-2)}$ as a power series in z in the regions.

(a) $|z| < 1$

(b) $1 < |z| < 2$

(c) $|z| > 2$.

43. (a) Find the probability of exactly 52 heads in 100 tosses of a coin using the binomial distribution and using normal approximation.
- (b) Find the probability $P(45, 55)$ of between 45 and 55 heads in 100 tosses of a coin. That is $45 \leq x \leq 55$.
44. Eight unbiased coins were tossed simultaneously. Find the probability of getting
- (a) exactly 4 heads
- (b) no heads at all
- (c) 6 or more heads
- (d) at most two heads
- (e) number of heads ranging from 3 to 5.

(2 × 15 = 30 Marks)
