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Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, November 2021

Physics with specialization in Nano Science/ Physics with specialization in Space Physics

PHNS 512/PHSP 512: MATHEMATICAL PHYSICS

(2020 Admission)

Time : 3 Hours Max. Marks : 75

SECTION – A

Answer any **five** questions. Each question carries 3 marks:

I. (a) Prove that the matrix $\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary.

(b) Express the gradient of a function f in cylindrical coordinates.

(c) Using the Cauchy -Riemann equations show that $f(z) = z^3$ is analytic in the entire z-plane.

(d) Write down the complex form of Fourier integral.

(e) Find the inverse Laplace transform of $\frac{1}{s^2 - 5s + 6}$.

(f) Prove the recurrence relation $nP_n = (2n-1)xP_{n-1} - (n-1)P_{n-2}$.

(g) What is meant by contraction of a tensor?

(h) What are cyclic groups?

(5 × 3 = 15 Marks)

P.T.O.

SECTION – B

Answer **all** questions. Each question caries **15** marks.

II. (A) (a) Find the Fourier transform of the Gaussian distribution function $f(x) = Ne^{-\propto x^2}$ where N and $\propto$ are constants.

(b) Prove that the Fourier transform of a product of two functions is given by a convolution integral.

OR

(B) (a) Find the Eigen values and normalised eigen vectors of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

(b) Find the characteristic equation of the following matrix and verify the Cayley — Hamilton theorem. $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1 \end{bmatrix}$.

III. (A) (a) Apply calculus of residues to evaluate $\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta}$ where a >b>0.

(b) Evaluate $\int_{-\infty}^{+\infty} \frac{dx}{(x^2 - 1)^3}$ using complex analysis.

OR

(B) (a) Prove the orthogonality of Legendre polynomial.

(b) Prove the recurrence relation for the Bessel function.

$$xJ'(x) = nJ_n(x) - xJ_{n+1}(x)$$

IV. (A) (a) Discuss the three theoretical probability distributions.

(b) Define mean, median, and mode of a distribution.

OR

(B) (a) Find the Laplace transform of (i) t **sin** at (ii) $t^n e^{at}$ (iii) $\frac{\sin at}{t}$

(b) Solve $\frac{dx}{dy} + xy = x^3 y^3$

(3 × 15 = 45 Marks)

SECTION – C

Answer any **three** of the following questions. Each question carries **5** marks:

V. (a) Find the Green's function for the boundary value problem $\frac{d^2y}{dx^2} - k^2 y = f(x)$ with boundary conditions, $y(\pm\infty) = 0$.

(b) Determine metric tensor in (i) spherical coordinates and (ii) cylindrical

(c) Diagonalise the matrix $A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(d) Using the Rodrigue's formula for Hermite polynomial, find $H_0(x), H_1(x), H_2(x)$ and $H_3(x)$.

(e) Show that $V(x,y,z) = 2xyzi + (x^2 z + 2y)j + x^2 yk$ is irrotational and find a scalar function $u(x,y,z)$ such that $V = grad(u)$

(f) Find the inverse Laplace transform of $\frac{1}{(s+1)(s^2+1)}$.

(3 × 5 = 15 Marks)

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