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Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, September 2022

Physics With Specialization in Nano Science/Physics With Specialization In Space Physics

PHNS 512/PHSP 512 — MATHEMATICAL PHYSICS

(2020 Admission Onwards)

Time : 3 Hours Max. Marks : 75

PART – A

(Answer any **five** questions. **Each** question carries **3** marks)

1. State the Cayley-Hamilton theorem. Illustrate it for the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

2. For u and v the real and imaginary parts of an analytic function, show that u and v are harmonic functions.

3. State the Dirichlet conditions. What is meant by convergence in the mean? Comment on 'convergence in the mean' in the context of the Fourier series of a periodic function.

4. State the axioms of probability theory.

5. Outline the Green's function method of solving inhomogeneous differential equations. Give the form of the Greens function for the Poisson equation.

6. Given the orthogonality relation $\int_{-1}^{1} \frac{T_m(x)T_n(x)}{\sqrt{1-x^2}} dx = \delta_{mn}$ prove orthogonality for the Chebyshev polynomials $T_o(x) = 1$ and $T_1(x) = x$.

P.T.O.

7. Write down the respective transformation laws for second rank covariant, contravariant and mixed tensors.

8. Distinguish between isomorphism and homomorphism between groups.

(5 × 3 = 15 Marks)

PART – B

(Answer **three** questions. **Each** question carries **15** marks)

9. (a) Starting with the general expression for the divergence of a vector function and the Laplacian of a scalar function in an orthogonal equi linear coordinates obtain explicit expression for these in spherical polar coordinates.

   (b) What are dispersion relations? Derive the dispersion relations in the case of $f(x) = u(x) + iv(x)$ where f is a complex function of the real variable x.

OR

10. (a) Obtain the Fourier expansion for $f(x)$ where $\begin{aligned} f(x) &= x \quad && 0 < x < \pi \\ &= -x \quad && -\pi < x > 0 \end{aligned}$.

    (b) Elucidate the relations between the Binomial and Poisson distribution and the normal distribution.

11. (a) Obtain the Frobenius series solution for the Bessel equation $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + (x^2 - n^2)y = 0$.

    (b) Solve the following PDE by separation of variables method after transforming using its characteristics $\dfrac{\partial z}{dx} + \dfrac{\partial z}{\partial y} + (x + y)z = 0$.

OR

12. (a) Set up the Green's function for the ODE $\dfrac{-d^2y}{dx^2} = f(x)$ with boundary conditions $y(0) = y(1) = 0$ obtain the solution for $f(x) = \sin \pi x$.

    (b) Given the generating function relation for Hermite polynomials as $e^{-t^2+2tx} = \sum_{n=0}^{\infty} H_n(x)\dfrac{t^n}{n!}$ develop recurrence relations by differentiation.

13. (a) What is the physical significance of the covariant derivative? Write down the covariant derivative of a second rank covariant tensor using Christoffel symbols.

(b) Hence obtain the Riemann – Christoffel tensor in terms of appropriate squares and derivatives of the Christoffel symbols.

OR

14. (a) State and explain the orthogonality relations and the dimensionality theorem obeyed by the irreducible representations of a group.

(b) Comment on the roles played in physics by the generators and the basis vectors for a two dimensional representation of the $su(2)$ group.

(3 × 15 = 45 Marks)

PART – C

(Answer any **three** questions. **Each** question carries **5** marks)

15. Prove that the line $z = \gamma e^{i\beta}$ (β = constant) maps into a hyperbola in the (u, v) plane under the mapping $w(z) = z + \dfrac{1}{z}$.

16. For a radioactive sample 100 decays are counted in 1,000 seconds. Estimate the probability of observing 3 decays in 10 seconds.

17. Using partial fraction expansion determine $L^{-1}\left(\dfrac{1}{(s+a)(s+b)}\right)$ and $L^{-1}\left(\dfrac{s}{(s+a)(s+b)}\right)$ where L^{-1} stands for the inverse Laplace transform.

18. Show that the spherical harmonics give the angular part of the solution to the Schrödinger equation for a particle in a central potential $V(r)$.

19. Prove that tensors that are symmetric or antisymmetric with respect to any two covariant or contravariant indices will retain the respective symmetry/skews symmetry under general coordinate transformations.

20. Prove that the set of unitary matrices, as well as the set of special unitary matrices constitute groups.

(3 × 5 = 15 Marks)
