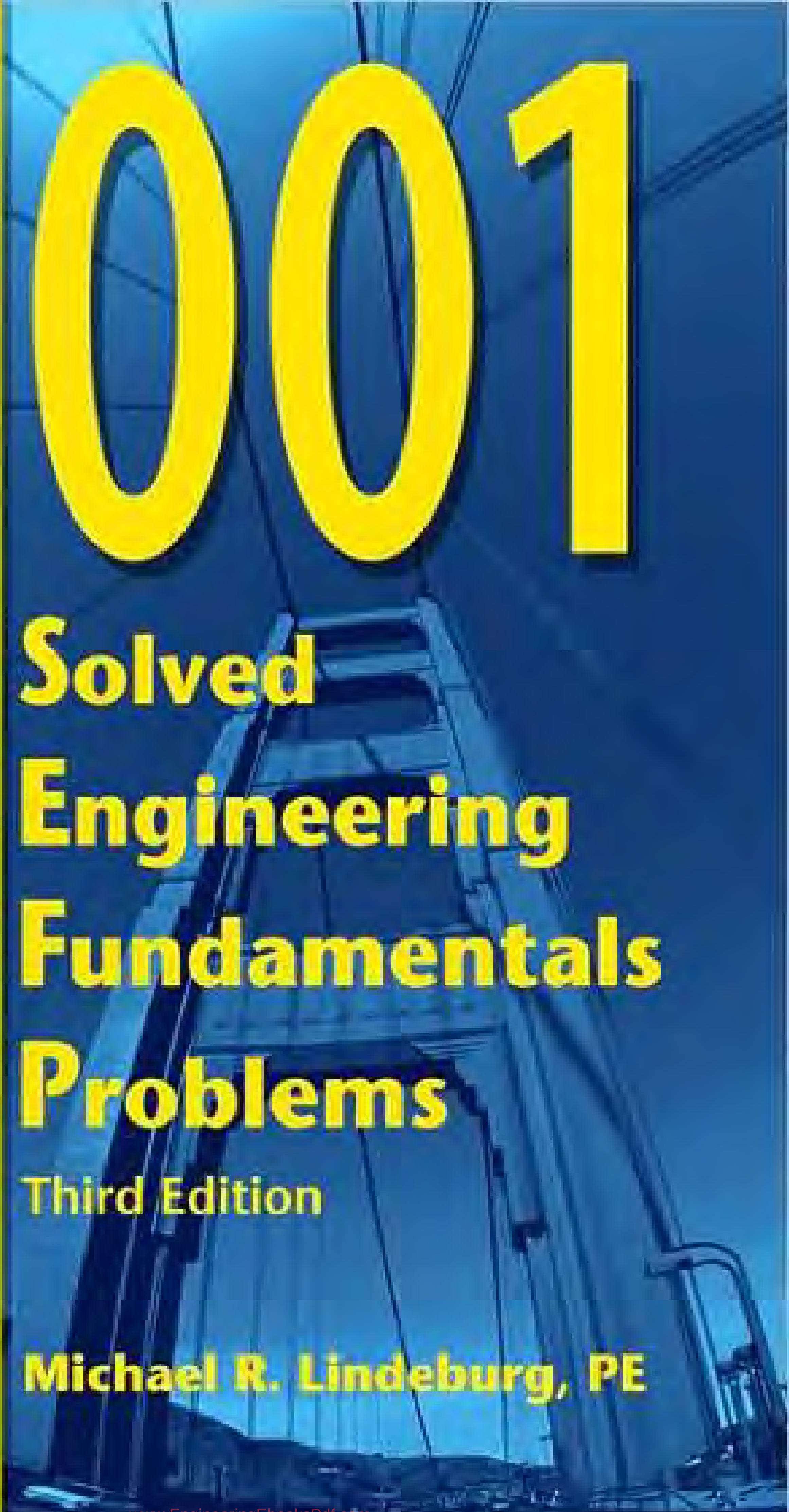

1001 Solved Engineering Fundamentals Problems

Third Edition

Michael R. Lindeburg, PE

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Third Edition

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TABLE OF CONTENTS

PREFACE TO THE THIRD EDITION

In the good old days, when examinations for engineering licensing were in their infancy, most review books were mainly compilations of problems with little supporting theory—much like this book. Such books placed the burden on the examinees to accumulate and become familiar with numerous textbooks and references.

The National Council of Examiners for Engineering and Surveying (NCEES) changed all that when it limited you to a single reference booklet that it provides. While this eliminated the "shopping cart syndrome" (wherein some examinees literally brought shopping carts of books to the exam), it also changed the nature of how most examinees conduct their review. The scope of the exam has narrowed significantly, and the scope of the examinees' review has shrunk accordingly, placing more emphasis on the ability to quickly work certain standard types of problems.

Since you cannot use your own reference books in the Fundamentals of Engineering (FE) exam, working countless practice problems has become the review method of choice for many examinees. This book contains the grist for the problem review mill.

Parallel to the actual exam, this edition features problems that all use the SI system of units. Approximately half of the problems have been revised or replaced in order to use these units. New problems have been added to cover high-probability subjects that had no representation in previous editions. The entire book has been edited to incorporate PPI's rigorous style and quality standards.

I hope you will find this book particularly useful and valuable.

Michael R. Lindeburg, PE

ACKNOWLEDGMENTS

The FE examination is a secure examination, which means problems are not made public after the exam is given. Therefore, all of the problems in this book were developed from scratch, based on our knowledge of the examination. Working from detailed outlines that I prepared, a team of more than 20 engineers and engineering students fleshed out the problems in the first edition.

This third edition shares a similar heritage with the first edition in that many people contributed to it. Timothy W. Zeigler, PE, revised, wrote, and rewrote approximately 750 problems for the Mathematics, Economics, Systems of Units, Fluid Statics and Dynamics, Thermodynamics, Chemistry, Statics Materials Science, Mechanics of Materials, Dynamics, and Physics chapters. Similarly, Yanqing Du, PhD, PE, revised, wrote, and rewrote almost 300 problems for the Power Cycles, DC Electricity, AC Electricity, Systems Modeling, Computer Science, and Atomic Theory chapters. Without their authorship, this new edition would still be sitting in the in-basket on my desk.

Christopher Lew, a master's program candidate at Stanford University and former engineering intern in the PPI editorial department, performed the first round of calculation and unit checks. You can thank him for the accuracy of the calculations.

PPI editorial and production staff who worked diligently on this book include: Sarah Hubbard, editorial director; Cathy Schrott, production director; Amy Schwertman, illustrator; Kate Hayes, typesetter; Heather Kinser, project editor; and Sean Sullivan, project editor. These people gave the book a consistency and personality of its own. The styles and conventions you see in this book are the result of their work.

Many of the thousands of readers of the previous two editions sent in their comments and suggestions. Though they are nameless, this book is better because of their help.

Michael R. Lindeburg, PE

HOW TO USE THIS BOOK

If you never read the material at the front of your books anyway, and if you are in a hurry to begin and you only want to read one paragraph, here it is.

> All chapters in this book are independent. Start with any one. Solve all the problems for which you have time. Don't peek at the answers. Work problems in your weak areas as well as in your strong areas. Keep studying until the exam. Good luck.

However, if you want a thorough review, you will probably want to know a little more about reviewing for the exam. The rest of this introduction is for you.

This book doesn't contain any supporting theory and is not really meant to be used as a stand-alone exam review. It was meant to be used in conjunction with two other references: a FE exam review book (e.g., either the *FE Review Manual* or *Engineer-In-Training Reference Manual*) and the NCEES Handbook. All three of the books listed are available from PPI.

Depending on your preference, you might decide to first review a subject and then work practice problems. Or, you might decide to jump right in and try to work the problems, reviewing only those subjects that are rusty or unfamiliar. This book can be used either way.

While working through the problems, you should try to use the NCEES Handbook as your sole reference source. Refer to the *FE Review Manual* or *Engineer-In-Training Reference Manual* when you need to refresh your memory about subjects that have become dim. However, use the NCEES Handbook when you simply need a formula or data. That way, you will become intimately familiar with the only reference that is permitted in the examination room.

1 MATHEMATICS

MATHEMATICS–1

The set A consists of elements $\{1, 3, 6\}$, and the set B consists of elements $\{1, 2, 6, 7\}$. Both sets come from the universe of $\{1, 2, 3, 4, 5, 6, 7, 8\}$. What is the intersection, $\bar{A} \cap B$?

(A) $\{2, 7\}$ (B) $\{2, 3, 7\}$ (C) $\{2, 4, 5, 7, 8\}$ (D) $\{4, 5, 8\}$

The set of "not A" consists of all universe elements not in set A: $\{2, 4, 5, 7, 8\}$.

The intersection of $\{2, 4, 5, 7, 8\}$ and $\{1, 2, 6, 7\}$ is the set of all elements appearing in both.

Thus $\bar{A} \cap B$ is $\{2, 7\}$.

The answer is (A).

MATHEMATICS–2

For a given function, $f(t) = f(-t)$. What type of symmetry does $f(t)$ have?

(A) odd symmetry
(B) even symmetry
(C) rotational symmetry
(D) quarter-wave symmetry

When $f(t) = f(-t)$, the function is "mirrored" on either side of the vertical axis. This is known as even symmetry.

The answer is (B).

MATHEMATICS–3

What is the value of each interior angle of a regular pentagon?

(A) $\frac{\pi}{5}$ (B) $\frac{2\pi}{5}$ (C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{5}$

For a regular polygon, the value of each interior angle, θ, is

$$\theta = \frac{\pi(\text{number of sides} - 2)}{\text{number of sides}}$$

For a regular pentagon,

$$\theta = \frac{\pi(5-2)}{5} = \frac{3\pi}{5}$$

The answer is (D).

MATHEMATICS–4

A cubical container that measures 2 m on a side is tightly packed with eight balls and is filled with water. All eight balls are in contact with the walls of the container and the adjacent balls. All of the balls are the same size. What is the volume of water in the container?

(A) 0.38 m^3 (B) 2.5 m^3 (C) 3.8 m^3 (D) 4.2 m^3

Since the balls are tightly packed, $r_{ball} = 0.5$ m.

$$\begin{aligned} V_{water} &= V_{box} - 8V_{ball} \\ &= (2\text{ m})^3 - (8)\left(\tfrac{4}{3}\pi(0.5\text{ m})^3\right) \\ &= 3.8\text{ m}^3 \end{aligned}$$

The answer is (C).

MATHEMATICS–5

Which number has four significant figures?

(A) 0.0014 (B) 0.01414 (C) 0.141 (D) 1.4140

The number of significant figures, or digits, for each choice is (A) 2, (B) 4, (C) 3, and (D) 5.

Only option (B) has four significant figures.

The answer is (B).

MATHEMATICS–6

What is the solution of the equation $50x^2 + 5(x-2)^2 = -1$, where x is a real-valued variable?

(A) −6.12 and −3.88 (B) −0.52 and 0.700
(C) 7.55 (D) no solution

For real-valued x, the left-hand side of the equation must always be greater than or equal to zero, since all terms containing x are squared. There is no solution to this equation for real values of x.

The answer is (D).

MATHEMATICS–7

What are the roots of the cubic equation $x^3 - 8x - 3 = 0$?

(A) $x = -7.90, -3, -0.38$
(B) $x = -3, -2, 2$
(C) $x = -3, -0.38, 2$
(D) $x = -2.62, -0.38, 3$

By inspection, +3 is a root, and $(x-3)$ is a factor. Factor out $(x-3)$.

$$\frac{x^3 - 8x - 3}{x - 3} = x^2 + 3x + 1$$

Use the quadratic equation to solve $x^2 + 3x + 1 = 0$.

$$x = 3, \frac{-3 \pm \sqrt{9-4}}{2}$$
$$= -2.62, \ -0.38, \ 3$$

The answer is (D).

MATHEMATICS–8

Naperian logarithms have a base closest to which number?

(A) 2.17 (B) 2.72 (C) 3.14 (D) 10.0

The base of Naperian logarithms is the number $e \approx 2.7183$. Of the choices given, 2.72 is the closest to e.

The answer is (B).

MATHEMATICS–9

What is the radius of the circle defined by $x^2 + y^2 - 4x + 8y = 7$?

(A) $\sqrt{3}$ (B) $2\sqrt{5}$ (C) $3\sqrt{3}$ (D) $4\sqrt{3}$

Since the general equation for a circle is $(x-a)^2 + (y-b)^2 = r^2$, rearrange the equation given to fit the general equation.

$x^2 - 4x + y^2 + 8y = 7$

Complete the binomial forms.

$x^2 - 4x + 4 + y^2 + 8y + 16 = 7 + 4 + 16$

$(x-2)^2 + (y+4)^2 = 27$

$r^2 = 27$

$r = 3\sqrt{3}$

The answer is (C).

MATHEMATICS–10

What is the natural logarithm of e^{xy}?

(A) $\frac{1}{xy}$ (B) xy (C) $2.718xy$ (D) $\frac{2.718}{xy}$

By definition, the natural logarithm of a number is

$$\ln e^G = G$$
$$\ln e^{xy} = xy$$

The answer is (B).

MATHEMATICS–11

What is the value of $(0.001)^{2/3}$?

(A) $\text{antilog}\left(\frac{3}{2}\log 0.001\right)$

(B) $\frac{2}{3}\,\text{antilog}\left(\log 0.001\right)$

(C) $\text{antilog}\left(\log \frac{0.001}{\frac{2}{3}}\right)$

(D) $\text{antilog}\left(\frac{2}{3}\log 0.001\right)$

$$\log x^a = a\log x$$
$$\log(0.001)^{2/3} = \tfrac{2}{3}\log 0.001$$
$$(0.001)^{2/3} = \text{antilog}\left(\tfrac{2}{3}\log 0.001\right)$$

The answer is (D).

MATHEMATICS–12

The salary of an employee's job has five levels, each one 5% greater than the one below it. Due to circumstances, the salary of the employee must be reduced from the top (fifth) level to the second level, which results in a reduction of \$122.00 per month. What is the employee's present salary per month?

(A) \$440/mo (B) \$570/mo (C) \$680/mo (D) \$900/mo

The salary levels represent a geometric sequence. Let S_i be the salary at level i.

$$\begin{aligned} S_3 &= 1.05S_2 \\ S_4 &= 1.05S_3 \\ S_5 &= 1.05S_4 \\ &= (1.05)^3 S_2 \\ S_5 - 122 &= S_2 \\ S_5 &= (1.05)^3 \left(S_5 - 122\ \frac{\$}{\text{mo}}\right) \\ &= \$896/\text{mo} \quad (\$900/\text{mo}) \end{aligned}$$

The answer is (D).

MATHEMATICS–13

Which of the following statements regarding matrices is FALSE?

(A) $(\mathbf{A}^{\mathrm{T}})^{\mathrm{T}} = \mathbf{A}$

(B) $\mathbf{A}(\mathbf{B}+\mathbf{C}) = \mathbf{AB} + \mathbf{AC}$

(C) $\begin{pmatrix} 2 & 5 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 12 \\ 1 \end{pmatrix}$

(D) $(\mathbf{AB})^{-1} = \mathbf{A}^{-1}\mathbf{B}^{-1}$

The inverse of a product of two matrices is the product of the inverses, in reverse order.

$$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$$

The answer is (D).

MATHEMATICS–14

What is the determinant of the following 2×2 matrix?

$$\begin{pmatrix} 5 & 9 \\ 7 & 6 \end{pmatrix}$$

(A) −33 (B) −27 (C) 27 (D) 33

The determinant, **D**, is calculated as follows.

$$\mathbf{D} = \begin{vmatrix} 5 & 9 \\ 7 & 6 \end{vmatrix}$$
$$= (5)(6) - (7)(9)$$
$$= -33$$

The answer is (A).

MATHEMATICS–15

What is the determinant of the following matrix?

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 2 & -1 \end{pmatrix}$$

(A) 0 (B) 1 (C) 5 (D) 7

To find the determinant, expand by minors across the top row.

$$\mathbf{D} = 1\begin{vmatrix} -1 & 1 \\ 2 & -1 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}$$
$$= \big((-1)(-1) - (2)(1)\big) - \big((2)(-1) - (1)(1)\big) + \big((2)(2) - (1)(-1)\big)$$
$$= 7$$

The answer is (D).

MATHEMATICS–16

What is the inverse of the matrix **A**?

$$\mathbf{A} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

(A) $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

(B) $\begin{pmatrix} -\cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

(C) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

(D) $\begin{pmatrix} \cos\theta\sin\theta & 0 \\ 0 & \sin\theta\cos\theta \end{pmatrix}$

For a 2×2 matrix, $\mathbf{X}$,

$$\mathbf{X} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

The inverse, $\mathbf{X}^{-1}$, is

$$\mathbf{X}^{-1} = \frac{1}{\mathbf{D}} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$\mathbf{D}$ is the determinant of $\mathbf{X}$. For matrix $\mathbf{A}$,

$$\begin{aligned} \mathbf{D} &= \cos^2\theta - (\sin\theta)(-\sin\theta) \\ &= \cos^2\theta + \sin^2\theta \\ &= 1 \end{aligned}$$

$$\mathbf{A}^{-1} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

The answer is (C).

MATHEMATICS–17

What is the rank of the matrix $\mathbf{A}$?

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 3 & 1 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ 2 & 0 & 1 & -2 \end{pmatrix}$$

(A) 0 (B) 1 (C) 2 (D) 3

The rank of a matrix is the number of independent vectors (rows). The rank can be found by row-reducing (diagonalizing) the matrix and counting the number of pivots in the row-reduced form of the matrix.

$$\begin{aligned}\text{Row } 2 &= (-2)(\text{Row } 2) + (3)(\text{Row } 4)\\ \text{Row } 4 &= \text{Row } 4 - \text{Row } 2\\ \text{Row } 3 &= (2)(\text{Row } 3) + \text{Row } 2\end{aligned}$$

The row-reduced form of **A** is

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & -2 & 1 & 4 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The matrix cannot be further row-reduced. There are three pivots and, therefore, three independent rows. The rank of matrix **A** is 3.

The answer is (D).

MATHEMATICS–18

Determine the values of x_1 and x_2 that satisfy the following linear system.

$$\begin{pmatrix} 3 & 7 \\ 2 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

(A) $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ (B) $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ (C) $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ (D) $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$

The linear equations represented by this system are

$$\begin{aligned} 3x_1 + 7x_2 &= 2 \\ 2x_1 + 6x_2 &= 4 \end{aligned}$$

Use Cramer's rule to solve the system of equations.

$$x_1 = \frac{\begin{vmatrix} 2 & 7 \\ 4 & 6 \end{vmatrix}}{\begin{vmatrix} 3 & 7 \\ 2 & 6 \end{vmatrix}} = \frac{-16}{4}$$

$$x_1 = -4$$

$$x_2 = \frac{\begin{vmatrix} 3 & 2 \\ 2 & 4 \end{vmatrix}}{\begin{vmatrix} 3 & 7 \\ 2 & 6 \end{vmatrix}} = \frac{8}{4}$$

$$x_2 = 2$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

The answer is (D).

MATHEMATICS–19

If $\sin\alpha = x$, what is $\sec\alpha$?

(A) $\sqrt{1-x^2}$ (B) $\dfrac{x}{\sqrt{1-x^2}}$ (C) $\dfrac{1}{\sqrt{1-x^2}}$ (D) $\dfrac{x}{\sqrt{1+x^2}}$

Since $\sin\alpha$ is the side facing angle α divided by the hypotenuse, the hypotenuse = 1. Therefore,

$$\text{side adjacent to angle } \alpha = \sqrt{1-x^2}$$

$$\sec\alpha = \frac{\text{hypotenuse}}{\text{side adjacent to angle } \alpha} = \frac{1}{\sqrt{1-x^2}}$$

The answer is (C).

MATHEMATICS–20

Experimental data show that a body's temperature declines exponentially in time according to the expression $T(t) = 50e^{-0.04t}$ (where 50 is a constant expressed in °C, 0.04 is the cooling rate in min^{-1}, and t is the cooling time expressed in minutes). How long would it take the body to reach 25°C?

(A) 12.4 min (B) 15.6 min (C) 16.5 min (D) 17.3 min

$$T(t) = 50e^{-0.04t}$$
$$25°\text{C} = (50°\text{C})e^{-0.04\ \text{min}^{-1}t}$$
$$e^{-0.04\ \text{min}^{-1}t} = \frac{25°\text{C}}{50°\text{C}} = 0.5$$

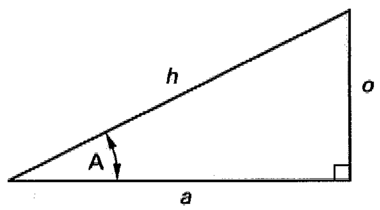
Take the natural logarithm of both sides.

$$-0.04\ \text{min}^{-1}t = \ln 0.5$$
$$t = \frac{\ln 0.5}{-0.04\ \text{min}^{-1}} = 17.3\ \text{min}$$

The answer is (D).

MATHEMATICS–21

If the sine of angle A is given as K, what is the tangent of angle A?



(A) $\frac{hK}{o}$ (B) $\frac{aK}{h}$ (C) $\frac{ha}{K}$ (D) $\frac{hK}{a}$

$$\begin{aligned}\sin A &= \frac{o}{h} \\ &= K \\ \tan A &= \frac{o}{a} \\ &= \left(\frac{h}{a}\right)\left(\frac{o}{h}\right) \\ &= \frac{hK}{a}\end{aligned}$$

The answer is (D).

MATHEMATICS–22

Which is true regarding the signs of the natural functions for angles between 90° and 180°?

(A) The tangent is positive.
(B) The cotangent is positive.
(C) The cosine is negative.
(D) The sine is negative.

In the second quadrant, the natural functions and their signs are as follows.

sin	positive
cos	negative
tan	negative
cot	negative
sec	negative
csc	positive

The answer is (C).

MATHEMATICS–23

What is the inverse natural function of the cosecant?

(A) secant (B) sine (C) cosine (D) tangent

In a right triangle, the cosecant is the hypotenuse divided by the opposite side. The sine is the opposite side divided by the hypotenuse.

$$\sin\theta = \frac{1}{\csc\theta}$$

The answer is (B).

MATHEMATICS–24

What is the sum of the squares of the sine and cosine of an angle?

(A) 0 (B) 1 (C) $\sqrt{3}$ (D) 2

For any angle,

$$\cos^2 x + \sin^2 x = 1$$

The answer is (B).

MATHEMATICS–25

What is an equivalent expression for $\sin 2x$?

(A) $\frac{1}{2}\sin x\cos x$ (B) $2\sin x\cos\frac{1}{2}x$ (C) $-2\sin x\cos x$ (D) $\frac{2\sin x}{\sec x}$

The double angle formula for the sine function is

$$\sin 2x = 2\sin x\cos x = \frac{2\sin x}{\sec x}$$

The answer is (D).

MATHEMATICS–26

The Taylor series expansion for $\cos x$ contains which powers of x?

(A) 0, 2, 4, 6, 8, ...
(B) 1, 3, 5, 9, ...
(C) 1, 2, 3, 4, 5, ...
(D) 1/2, 3/2, 5/2, 7/2, ...

The Taylor series expansion for $\cos x$ is as follows.

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

Only the positive even powers of x are contained in the expansion of $\cos x$.

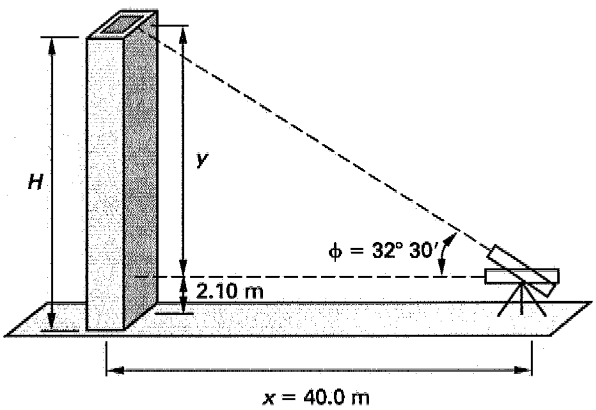
The answer is (A).

MATHEMATICS–27

A transit set up 40 m from the base of a vertical chimney reads 32°30′ with the crosshairs set on the top of the chimney. With the telescope level, the vertical rod at the base of the chimney is 2.1 m. Approximately how tall is the chimney?

(A) 15 m (B) 26 m (C) 28 m (D) 38 m

To find the height, H, of the chimney, refer to the following figure.



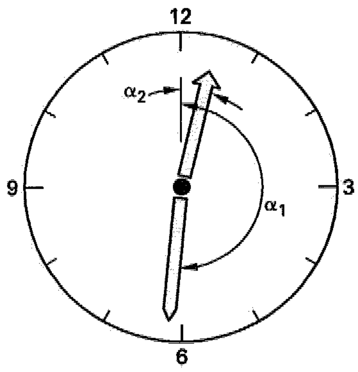
$$\tan\phi = \frac{y}{x}$$
$$y = (40.0\ \text{m})\tan 32.5^\circ$$
$$= 25.5\ \text{m}$$
$$H = 2.10\ \text{m} + y$$
$$= 2.10\ \text{m} + 25.5\ \text{m}$$
$$= 27.60\ \text{m} \quad (28\ \text{m})$$

The answer is (C).

MATHEMATICS–28

At approximately what time between the hours of 12:00 noon and 1:00 p.m. would the angle between the hour hand and the minute hand of a continuously driven clock be exactly 180°?

(A) 12:28 p.m. (B) 12:30 p.m. (C) 12:33 p.m. (D) 12:37 p.m.



The change in the angle of the minute hand between 12:00 p.m. and 1:00 p.m., α_1, is

$$\alpha_1 = \frac{360^\circ}{60\ \text{min}}t$$
$$= (6t)^\circ$$

The change in the angle of the hour hand between 12:00 noon and 1:00 p.m., α_2, is

$$\alpha_2 = \frac{360^\circ}{(12)(60\ \text{min})}t$$
$$= (0.5t)^\circ$$

In the preceding equations, t is in minutes past 12:00 noon. The angle between the two hands is $\alpha_1 - \alpha_2$.

$$\begin{aligned} \alpha_1 - \alpha_2 &= 180^\circ \\ (6t)^\circ - (0.5t)^\circ &= 180^\circ \\ (5.5t)^\circ &= 180^\circ \\ t &= 32.7 \text{ min} \end{aligned}$$

The time is approximately 12:33 p.m.

The answer is (C).

MATHEMATICS–29

In finding the distance, d, between two points, which equation is the appropriate one to use?

(A) $d = \sqrt{(x_1 - x_2)^2 - (y_2 - y_1)^2}$
(B) $d = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$
(C) $d = \sqrt{\ (x_1^2 - x_2^2) + \ (y_1^2 - y_2^2)}$
(D) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

The distance formula is defined as follows.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The answer is (D).

MATHEMATICS–30

The equation $y = a_1 + a_2 x$ is an algebraic expression for which of the following?

(A) a cosine expansion series
(B) projectile motion
(C) a circle in polar form
(D) a straight line

$y = mx + b$ is the slope-intercept form of the equation of a straight line. Thus, $y = a_1 + a_2x$ describes a straight line.

The answer is (D).

MATHEMATICS–31

Find the slope of the line defined by $y - x = 5$.

(A) $5 + x$ (B) $-1/2$ (C) $1/4$ (D) 1

The slope-intercept form of the equation of a straight line is $y = mx + b$, where m is the slope and b is the y-intercept.

$$y - x = 5$$
$$y = x + 5$$

The coefficient of x, m, is

$$m = 1$$

The answer is (D).

MATHEMATICS–32

Find the equation of a line with slope = 2 and y-intercept = -3.

(A) $y = -3x + 2$
(B) $y = 2x - 3$
(C) $y = \frac{2}{3}x + 1$
(D) $y = 2x + 3$

The slope-intercept form of the given equation is

$$y = 2x - 3$$

The answer is (B).

MATHEMATICS–33

Find the equation of the line that passes through the points $(0, 0)$ and $(2, -2)$.

(A) $y = x$ (B) $y = -2x + 2$ (C) $y = -2x$ (D) $y = -x$

Since the line passes through the origin, the y-intercept is 0. Thus, the equation simplifies to $y = mx$. Substituting for the known points,

$$\begin{aligned} y &= \left(\frac{-2-0}{2-0}\right)x \\ &= -x \end{aligned}$$

The answer is (D).

MATHEMATICS–34

What is the name for a vector that represents the sum of two vectors?

(A) scalar (B) resultant (C) tensor (D) moment

By definition, the sum of two vectors is known as the resultant.

The answer is (B).

MATHEMATICS–35

What is the resultant, $\mathbf{R}$, of the vectors $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$?

$$\begin{aligned} \mathbf{F}_1 &= 4\mathbf{i} + 7\mathbf{j} + 6\mathbf{k} \\ \mathbf{F}_2 &= 9\mathbf{i} + 2\mathbf{j} + 11\mathbf{k} \\ \mathbf{F}_3 &= 5\mathbf{i} - 3\mathbf{j} - 8\mathbf{k} \end{aligned}$$

(A) $\mathbf{R} = 18\mathbf{i} + 6\mathbf{j} + 9\mathbf{k}$
(B) $\mathbf{R} = -18\mathbf{i} - 6\mathbf{j} - 9\mathbf{k}$
(C) $\mathbf{R} = 18\mathbf{i} + 12\mathbf{j} + 25\mathbf{k}$
(D) $\mathbf{R} = 21\mathbf{i}$

The resultant of vectors given in unit-vector form is the sum of the components.

$$\begin{aligned}\mathbf{R} &= (4+9+5)\mathbf{i} + (7+2-3)\mathbf{j} + (6+11-8)\mathbf{k} \\ &= 18\mathbf{i} + 6\mathbf{j} + 9\mathbf{k}\end{aligned}$$

The answer is (A).

MATHEMATICS-36

Simplify the expression $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$, given

$$\begin{aligned}\mathbf{A} &= 3\mathbf{i} + 2\mathbf{j} \\ \mathbf{B} &= 2\mathbf{i} + 3\mathbf{j} + \mathbf{k} \\ \mathbf{C} &= 5\mathbf{i} + 2\mathbf{k}\end{aligned}$$

(A) 0 (B) 20 (C) $60\mathbf{i} + 24\mathbf{k}$ (D) $5\mathbf{i} + 2\mathbf{k}$

First find $\mathbf{A} \times \mathbf{B}$.

$$\begin{aligned}\mathbf{A} \times \mathbf{B} &= (3\mathbf{i} + 2\mathbf{j}) \times (2\mathbf{i} + 3\mathbf{j} + \mathbf{k}) \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & 0 \\ 2 & 3 & 1 \end{vmatrix} \\ &= \mathbf{i}(2-0) - \mathbf{j}(3-0) + \mathbf{k}(9-4) \\ &= 2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k} \\ (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} &= (2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}) \cdot (5\mathbf{i} + 0\mathbf{j} + 2\mathbf{k}) \\ &= (2)(5) + (-3)(0) + (5)(2) \\ &= 20\end{aligned}$$

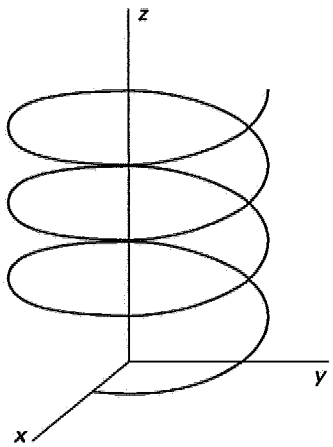
The answer is (B).

MATHEMATICS-37

What type of curve is generated by a point that moves in uniform circular motion about an axis, while travelling with a constant speed, v, parallel to the axis?

(A) a cycloid (B) an epicycloid (C) a hypocycloid (D) a helix

A curve generated by the method described is called a helix and is illustrated in the following figure.



The answer is (D).

MATHEMATICS–38

What is the term that describes a possible outcome of an experiment?

(A) a sample space (B) a random point (C) an event (D) a finite set

By definition, an event is a possible outcome of a trial or experiment.

The answer is (C).

MATHEMATICS–39

In probability theory, what is the term that describes the set of all possible outcomes of an experiment?

(A) a set of random events
(B) a fuzzy set
(C) a cumulative distribution
(D) a sample space

By definition, the sample space is the set of all possible outcomes of an experiment.

The answer is (D).

MATHEMATICS–40

How can the values of a random variable defined over a sample space be described?

(A) always continuous
(B) always numerical
(C) strictly nonzero
(D) defined only over a finite horizon

The values of a random variable can be continuous or discrete over a finite or infinite domain. The values in the sample space can be shared by other sample spaces. However, the values of a random variable must be numerical.

The answer is (B).

MATHEMATICS–41

If two random variables are independently distributed, what is their relationship?

(A) They are not identically distributed.
(B) They are uncorrelated.
(C) They are mutually exclusive.
(D) Either option (A) or option (B) is true.

By definition, two independently distributed random variables are uncorrelated. Any two random variables may or may not be identically distributed. Independent events cannot be mutually exclusive.

The answer is (B).

MATHEMATICS–42

Which of the following properties of probability is NOT valid?

(A) The probability of an event is always positive and less than or equal to one.

(B) If E_0 is an event which cannot occur in the sample space, the probability of E_0 is zero.

(C) If events E_1 and E_2 are mutually exclusive, then the probability of both events occurring is zero.

(D) If events E_1 and E_2 are events from the same sample space, then $P(E_1 + E_2) = P(E_1) + P(E_2) - P(E_1E_2)$.

The probability law given in option (D) is valid for events from two sample spaces, not events from a single sample space. The correct rule for events from a single sample space is

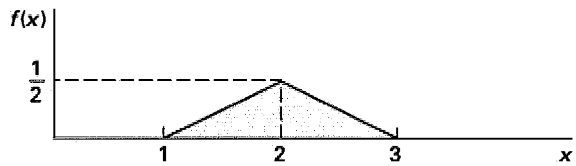
$$P(E_1 + E_2) = P(E_1) + P(E_2)$$

The answer is (D).

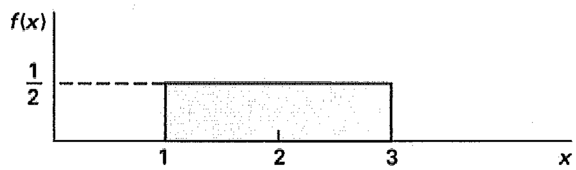
MATHEMATICS–43

Which one of the following functions cannot be a probability density function for the variable x?

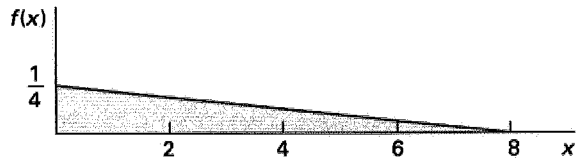
(A)



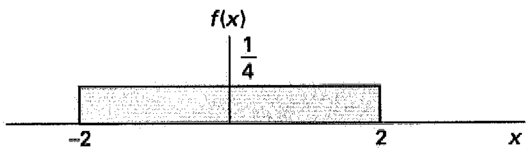
(B)



(C)



(D)



To be a probability density function, the area under the curve must equal 1. That is, the cumulative density function must sum to 1. The area under the curve for option (A) is $1/2$. Therefore, it cannot be a probability density function.

> The answer is (A).

MATHEMATICS–44

If n is the number of trials, and m is the number of successes, what is the frequency based interpretation of the probability of event E?

(A) $P(E) = \lim_{n\to\infty} \frac{n-m}{n}$

(B) $P(E) = \lim_{n\to\infty} \frac{n}{m}$

(C) $P(E) = \lim_{n\to\infty} \frac{m}{m-n}$

(D) $P(E) = \lim_{n\to\infty} \frac{m}{n}$

The probability of an event can be interpreted as the fraction of successful outcomes when the experiment is performed an infinite number of times. Thus,

$$P(E) = \lim_{n\to\infty} \frac{m}{n}$$

> The answer is (D).

MATHEMATICS–45

For a continuous random variable X with probability density function $f(x)$, what is the expected value of X?

(A) $E(X) = \int_0^\infty x f(x)\,dx$

(B) $E(X) = \int_{-\infty}^\infty x f(x)\,dx$

(C) $E(X) = \int_0^\infty f(x)\,dx$

(D) $E(X) = \int_0^\infty x dx$

The expected value or average of X can be defined mathematically as follows.

$$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$$

The answer is (B).

MATHEMATICS–46

If $P(B) \neq 1$, and A and B are not independent events, what is $P(A|B)$?

(A) $(P(A))(P(B))$ (B) $(P(B|A))\left(\frac{P(A)}{P(B)}\right)$

(C) $P(A)$ (D) $(P(A|B))\left(\frac{P(B)}{P(A)}\right)$

The probability of event A occurring, given that the dependent event B has occurred, is predicted by the conditional probability law, commonly known as Bayes theorem.

$$P(B|A) = \frac{P(AB)}{P(A)}$$

Similarly,

$$P(A|B) = \frac{P(AB)}{P(B)}$$

Therefore,

$$P(A|B) = \frac{(P(B|A))(P(A))}{P(B)}$$

The answer is (B).

MATHEMATICS–47

If the discrete random variable X has a geometric distribution parameter P and smallest mass point 0, what is the expected value of X?

(A) P (B) P^{-1} (C) P^{1-P} (D) $\frac{1-P}{P}$

The geometric distribution is a special case of the negative binomial distribution. The mean is $(1-P)/P$, and the variance is $(1-P)/P^2$. Note: Some authors define the geometric distribution with the smallest mass point being 1 (instead of 0). In that case, the mean is $1/P$ and the variance is the same as before.

The answer is (D).

MATHEMATICS–48

If the variable X has a Poisson distribution with parameter λ, what is the expected value of X?

(A) λ^2 (B) $\lambda(1-\lambda)$ (C) λ^{-1} (D) λ

For the Poisson distribution, both the mean and variance are equal to λ.

The answer is (D).

MATHEMATICS–49

If X is a binomial random variable with parameters n and p, what is the expected value of X?

(A) $n(1-p)$ (B) $np(1-p)$ (C) p^{-1} (D) np

For a binomial distribution, the mean is np, and the variance is $np(1-p)$.

The answer is (D).

MATHEMATICS–50

For a discrete random variable X with probability mass function $P(X)$, what is the expected value of X?

(A) $E(X) = \sum_{\text{all } x_i} x_i P(x_i)$

(B) $E(X) = \sum_{\text{all } x_i} x_i^2 P(x_i)$

(C) $E(X) = \sum_{\text{all } x_i} P(x_i)$

(D) $E(X) = \sum_{\text{all } x_i} (x_i - \bar{x}) P(x_i)$

The expected value of a discrete function is given by the following.

$$\begin{aligned} E(X) &= x_1 P(x_1) + x_2 P(x_2) + \cdots \\ &= \sum_{\text{all } x_i} x_i P(x_i) \end{aligned}$$

The answer is (A).

MATHEMATICS–51

An item's cost distribution is given. What is the approximate expected cost?

cost ($)	probability
1	0.07
2	0.23
3	0.46
4	0.17
5	0.04
6	0.03

(A) $2.5 (B) $2.9 (C) $3.0 (D) $3.1

The expected value is the sum of the products of the individual values and their respective probabilities.

$$\begin{aligned}E(\text{cost}) &= (\$1)(0.07) + (\$2)(0.23) + (\$3)(0.46) + (\$4)(0.17) \\ &\quad + (\$5)(0.04) + (\$6)(0.03) \\ &= \$2.97 \quad (\$3.0)\end{aligned}$$

The answer is (C).

MATHEMATICS–52

One fair die is used in a dice game. The player wins \$10 if he rolls either a 1 or a 6. He loses \$5 if he turns up any other face. What is the expected winning for one roll of the die?

(A) \$0.00 (B) \$3.33 (C) \$5.00 (D) \$6.67

For a fair die, the probability of any face turning up is $^1/_6$. Therefore, the expected value is

$$E_{\text{win}} = (\$10)\left((2)\left(\frac{1}{6}\right)\right) - (\$5)\left((4)\left(\frac{1}{6}\right)\right) = \$0.00$$

The answer is (A).

MATHEMATICS–53

An urn contains four black balls and six white balls. What is the probability of getting one black ball and one white ball in two consecutive draws from the urn without replacement?

(A) 0.040 (B) 0.24 (C) 0.27 (D) 0.53

$$\begin{aligned}P(\text{black and white}) &= P(\text{black then white}) + \text{P}(\text{white then black}) \\ &= \left(\frac{4}{10}\right)\left(\frac{6}{9}\right) + \left(\frac{6}{10}\right)\left(\frac{4}{9}\right) \\ &= 0.53\end{aligned}$$

The answer is (D).

MATHEMATICS–54

The probability that both stages of a two-stage rocket will function correctly is 0.95. The reliability of the first stage is 0.98. What is the reliability of the second stage?

(A) 0.95 (B) 0.96 (C) 0.97 (D) 0.98

In a serial system consisting of two units,

$$R_t = R_1 R_2$$

In the preceding question, R_2 is the reliability of stage 2, and R_t is the total reliability of all stages. For the second stage,

$$\begin{aligned} R_2 &= \frac{R_t}{R_1} \\ &= \frac{0.95}{0.98} \\ &= 0.97 \end{aligned}$$

The answer is (C).

MATHEMATICS–55

What is the exponential form of the complex number $3 + 4i$?

(A) $e^{i53.1°}$ (B) $5e^{i53.1°}$ (C) $5e^{i126.9°}$ (D) $7e^{i53.1°}$

Any complex number $a + bi$ can be converted to its equivalent exponential form as follows.

$$a + bi = \sqrt{a^2 + b^2}\, e^{i \arctan b/a}$$

Therefore,

$$\begin{aligned} 3 + 4i &= \sqrt{3^2 + 4^2}\, e^{i \arctan 4/3} \\ \arctan \frac{4}{3} &= 53.1° \\ 3 + 4i &= 5e^{i53.1°} \end{aligned}$$

The answer is (B).

MATHEMATICS–56

What is the product of the complex numbers $3 + 4i$ and $7 - 2i$?

(A) $10 + 2i$ (B) $13 + 22i$ (C) $13 + 34i$ (D) $29 + 22i$

$$\begin{aligned}(3 + 4i)(7 - 2i) &= 21 - 8i^2 + 28i - 6i \\ &= 21 + 8 + 28i - 6i \\ &= 29 + 22i\end{aligned}$$

The answer is (D).

MATHEMATICS–57

What is the rectangular form of the complex number $7.2e^{i^{7\pi/13}}$?

(A) $7.15 + 0.87i$ (B) $7.15 - 0.87i$ (C) $-0.87 + 7.15i$ (D) $-0.87 - 7.15i$

A complex number of the form ce^{id} can be converted to rectangular form as follows.

$$\begin{aligned}ce^{id} &= c\cos d + (c\sin d)i \\ 7.2e^{i(7\pi/13)} &= (7.2)\left(\cos\frac{7\pi}{13} + \left(\sin\frac{7\pi}{13}\right)i\right) \\ &= -0.87 + 7.15i\end{aligned}$$

The answer is (C).

MATHEMATICS–58

What is the product of the complex numbers $2 - 2i$ and $\sqrt{32}e^{i\pi/4}$?

(A) 16 (B) $16i$ (C) $16e^{i\pi/4}$ (D) $16(1 - i)$

$$\begin{aligned}2 - 2i &= \sqrt{2^2 + 2^2}e^{i\ \text{arctan}\ -2/2} \\ &= \sqrt{8}e^{-i\pi/4}\end{aligned}$$

Therefore,

$$\begin{aligned}(2-2i)\sqrt{32}e^{i\pi/4} &= \sqrt{8}e^{-i\pi/4}\sqrt{32}e^{i\pi/4}\\ &= \sqrt{8}\sqrt{32}e^{i(\pi/4-\pi/4)}\\ &= 16\end{aligned}$$

The answer is (A).

MATHEMATICS–59

What is the rationalized value of the following complex quotient?

$$\frac{6+2.5i}{3+4i}$$

(A) $-0.32+0.66i$ (B) $0.32-0.66i$ (C) $1.1-0.66i$ (D) $-1.7+1.1$

In order to rationalize a complex number, multiply the numerator and denominator by the complex conjugate of the denominator.

$$\begin{aligned}\frac{6+2.5i}{3+4i} &= \frac{(6+2.5i)(3-4i)}{(3+4i)(3-4i)}\\ &= \frac{28-16.5i}{25}\\ &= 1.1-0.66i\end{aligned}$$

The answer is (C).

MATHEMATICS–60

What is the first derivative with respect to x of the function $g(x)=4\sqrt{9}$?

(A) 0 (B) 4/9 (C) 4 (D) 12

The derivative of a constant is zero. Therefore $g'(x)=0$.

The answer is (A).

MATHEMATICS–61

If a is a simple constant, what is the derivative of $y = x^a$?

(A) ax (B) x^{a-1} (C) ax^{a-1} (D) $(a-1)x$

$$\begin{aligned} y &= x^a \\ y' &= ax^{a-1} \end{aligned}$$

The answer is (C).

MATHEMATICS–62

What is the derivative of $f(x) = \left(x^3 - (x-1)^3\right)^3$?

(A) $3x^2 - 3(x-1)^2$

(B) $3\left(x^3 - (x-1)^3\right)^2$

(C) $9\left(x^3 - (x-1)^3\right)\left(x^2 - (x-1)^2\right)$

(D) $9\left(x^3 - (x-1)^3\right)^2\left(x^2 - (x-1)^2\right)$

$$\begin{aligned} f(x) &= \left(x^3 - (x-1)^3\right)^3 \\ f'(x) &= 3\left(x^3 - (x-1)^3\right)^2 \frac{d}{dx}\left(x^3 - (x-1)^3\right) \\ &= 3\left(x^3 - (x-1)^3\right)^2\left(3x^2 - 3(x-1)^2(1)\right) \\ &= 9\left(x^3 - (x-1)^3\right)^2\left(x^2 - (x-1)^2\right) \end{aligned}$$

The answer is (D).

MATHEMATICS–63

Differentiate $f(x) = \sqrt{2x^2 + 4x + 1}$.

(A) $2x + 2$

(B) $\frac{1}{2}\sqrt{2x^2 + 4x + 1}$

(C) $\frac{2x + 2}{\sqrt{2x^2 + 4x + 1}}$

(D) $\frac{4x + 4}{\sqrt{2x^2 + 4x + 1}}$

$$\begin{aligned} f(x) &= \sqrt{2x^2 + 4x + 1} \\ &= \left(2x^2 + 4x + 1\right)^{1/2} \\ f'(x) &= \frac{1}{2}\left(2x^2 + 4x + 1\right)^{-1/2} \frac{d}{dx}\left(2x^2 + 4x + 1\right) \\ &= \frac{1}{2}\left(2x^2 + 4x + 1\right)^{-1/2}(4x + 4) \\ &= \frac{2x + 2}{\sqrt{2x^2 + 4x + 1}} \end{aligned}$$

The answer is (C).

MATHEMATICS–64

Find the second derivative of $y = \sqrt{x^2} + x^{-2}$.

(A) $1 - 2x^{-3}$ (B) $1 - 6x^{-4}$ (C) 3 (D) $\frac{6}{x^4}$

$$\begin{aligned} y &= \sqrt{x^2} + x^{-2} \\ y' &= \frac{x}{\sqrt{x^2}} - 2x^{-3} \\ &= \pm 1 - 2x^{-3} \\ y'' &= 6x^{-4} \end{aligned}$$

Note: $x/\sqrt{x^2} = \pm 1$ because by definition, $\sqrt{x^2} = |x|$.

The answer is (D).

MATHEMATICS–65

Find dy/dt given the following two simultaneous differential equations.

$$2\frac{dx}{dt} - 3\frac{dy}{dt} + x - y = k$$
$$3\frac{dx}{dt} + 2\frac{dy}{dt} - x = \cos t$$

(A) $\frac{2}{13}\left(\cos t + \frac{5}{2}x - \frac{3}{2}y - \frac{3}{2}k\right)$

(B) $\frac{1}{3}\left(\sin t + \frac{1}{9}x - y^3 - \frac{3}{2}k\right)$

(C) $-\frac{1}{6}\left(\sin t + \frac{1}{9}x + y^2 - \frac{3}{2}k\right)$

(D) $\frac{2}{9}\left(\cos t + \frac{3}{2}x - \frac{5}{2}y - \frac{3}{2}k\right)$

Solve both equations for dx/dt.

$$\frac{dx}{dt} = \frac{1}{2}\left(k + y - x + 3\frac{dy}{dt}\right)$$
$$\frac{dx}{dt} = \frac{1}{3}\left(\cos t + x - 2\frac{dy}{dt}\right)$$

Combine and solve for dy/dt.

$$\frac{1}{2}\left(k + y - x + 3\frac{dy}{dt}\right) = \frac{1}{3}\left(\cos t + x - 2\frac{dy}{dt}\right)$$
$$9\frac{dy}{dt} + 4\frac{dy}{dt} = -3k - 3y + 3x + 2\cos t + 2x$$
$$13\frac{dy}{dt} = 2\cos t + 5x - 3y - 3k$$
$$\frac{dy}{dt} = \frac{2}{13}\left(\cos t + \frac{5}{2}x - \frac{3}{2}y - \frac{3}{2}k\right)$$

The answer is (A).

MATHEMATICS–66

If $y = \cos x$, what is dy/dx?

(A) $\sec x$ (B) $-\sec x$ (C) $\csc x$ (D) $-\sin x$

$$\frac{d}{dx}\cos x = -\sin x$$

The answer is (D).

MATHEMATICS–67

If the second derivative of the equation of a curve is proportional to the negative of the equation of the same curve, what is that curve?

(A) a hyperbola (B) a square wave (C) a sinusoid (D) a cycloid

The only type of function that fits the description is a sinusoidal one.

$$\begin{aligned}\frac{d^2}{dx^2}\sin x &= \frac{d}{dx}\cos x \\ &= -\sin x\end{aligned}$$

The answer is (C).

MATHEMATICS–68

Given $P = 2R^2S^3T^{1/2} + R^{1/3}S\sin 2T$, what is $\partial P/\partial T$?

(A) $R^2S^3T^{3/2} + 2R^{1/3}\cos 2T$

(B) $6RS^2T^{-1/2} + \frac{2}{3}R^{-2/3}\cos 2T$

(C) $2R^2S^3T^{1/2} + R^{1/3}S\cos 2T$

(D) $R^2S^3T^{-1/2} + 2R^{1/3}S\cos 2T$

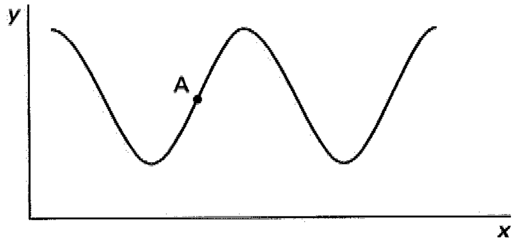
All variables other than T are treated as constants.

$$\begin{aligned}\frac{\partial P}{\partial T} &= 2R^2S^3\left(\frac{1}{2}T^{-1/2}\right) + R^{1/3}S(\cos 2T)(2) \\ &= R^2S^3T^{-1/2} + 2R^{1/3}S\cos 2T\end{aligned}$$

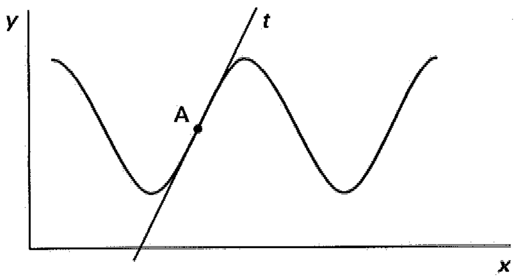
The answer is (D).

MATHEMATICS–69

Which of the following describes the first derivative at point A of the function shown in the figure?



(A) positive only
(B) negative only
(C) zero
(D) positive or negative

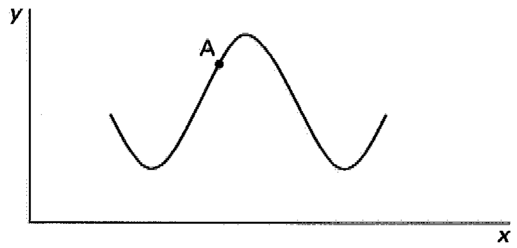


The first derivative corresponds to the slope of a tangent line at the point. The slope of this tangent line is positive. Therefore, the first derivative of the function at point A is also positive.

The answer is (A).

MATHEMATICS–70

Which of the following describes the second derivative at point A of the function shown?



(A) positive only
(B) negative only
(C) zero
(D) positive or negative

The second derivative corresponds to the concavity of the function. Since the curvature at this point is concave down, the second derivative is negative. The second derivative also indicates what is happening to the first derivative, the slope. Since the slope is decreasing at point A, the second derivative must be negative.

The answer is (B).

MATHEMATICS–71

What is the slope of the graph $y = -x^2$ at $x = 2$?

(A) -4 (B) -2 (C) 1 (D) 3

The slope of a curve is given by the first derivative.

$$y(x) = -x^2$$
$$y'(x) = -2x$$

At $x = 2$,

$$y'(2) = (-2)(2) = -4$$

The answer is (A).

MATHEMATICS–72

Given the function $f(x) = x^3 - 5x + 2$, find $f'(2)$, the value of the first derivative at $x = 2$.

(A) 2 (B) $3x^2 - 5$ (C) 7 (D) 8

$$\begin{aligned} f(x) &= x^3 - 5x + 2 \\ f'(x) &= 3x^2 - 5 \\ f'(2) &= (3)(2)^2 - 5 \\ &= 7 \end{aligned}$$

The answer is (C).

MATHEMATICS–73

Find the slope of the tangent to a parabola, $y = x^2$, at a point on the curve where $x = ^1/_2$.

(A) −1/2 (B) 0 (C) 1/4 (D) 1

$$\begin{aligned} y &= x^2 \\ y' &= 2x \\ y'\left(\frac{1}{2}\right) &= (2)\left(\frac{1}{2}\right) \\ &= 1 \end{aligned}$$

The answer is (D).

MATHEMATICS–74

What is the slope of the curve $y = x^2 - 4x$ at the origin?

(A) −4 (B) −3 (C) 0 (D) 4

$$y = x^2 - 4x$$
$$\frac{dy}{dx} = 2x - 4$$
$$\left.\frac{dy}{dx}\right|_{x=0} = (2)(0) - 4$$
$$= -4$$

The answer is (A).

MATHEMATICS–75

Find the slope of the line tangent to the curve $y = x^3 - 2x + 1$ at $x = 1$.

(A) 1/4 (B) 1/3 (C) 1/2 (D) 1

$$y = x^3 - 2x + 1$$
$$y' = 3x^2 - 2$$
$$y'(1) = 3 - 2$$
$$= 1$$

The answer is (D).

MATHEMATICS–76

Determine the equation of the line tangent to the graph $y = 2x^2 + 1$ at the point (1,3).

(A) $y = 2x + 1$
(B) $y = 4x - 1$
(C) $y = 2x - 1$
(D) $y = 4x + 1$

First, determine the slope of the graph at $x = 1$.

$$y = 2x^2 + 1$$
$$y' = 4x$$
$$y'(1) = 4$$

Since the tangent line intersects the graph at (1,3), the equation of the tangent line is

$$\begin{aligned} y &= 4x + b \\ 3 &= (4)(1) + b \\ b &= -1 \\ y &= 4x - 1 \end{aligned}$$

The answer is (B).

MATHEMATICS–77

Given $y_1 = 4x + 3$ and $y_2 = x^2 + C$, find C such that y_2 is tangent to y_1.

(A) 2 (B) 4 (C) 5 (D) 7

The slope of $y_1 = 4x + 3$ is 4 everywhere. Therefore, y_2 has a slope of 4 at the tangent point.

$$\begin{aligned} y_2' &= 2x \\ 4 &= 2x \\ x &= 2 \end{aligned}$$

$x = 2$ at the tangent point. Find $y_1 = y_2$ at the tangent point and substitute in to find C.

$$\begin{aligned} y_1 &= (4)(2) + 3 \\ &= 11 \\ y_2 &= 11 \\ 11 &= (2)^2 + C \\ C &= 7 \end{aligned}$$

The answer is (D).

MATHEMATICS–78

Given

$$\frac{dy_1}{dx} = \tfrac{2}{13}\left(1 + \tfrac{5}{2}x - \tfrac{3}{2} - \tfrac{3}{4}k\right)$$

What is the value of k such that y_1 is perpendicular to the curve $y_2 = 2x$ at $x = 1$?

(A) 2 (B) 3 (C) 6 (D) 7

For two lines to be perpendicular, $m_1 m_2 = -1$, where m_n is the slope of line n.

$$\frac{dy_2}{dx} = 2$$

Therefore, at $(1, 1)$,

$$\begin{aligned}
\frac{dy_1}{dx} &= -\frac{1}{2} \\
&= \frac{2}{13}\left(1 + \tfrac{5}{2}x - \frac{3}{2} - \tfrac{3}{4}k\right) \\
-\frac{1}{2} &= \left(\frac{2}{13}\right)\left(1 + \left(\frac{5}{2}\right)(1) - \frac{3}{2} - \tfrac{3}{4}k\right) \\
-\frac{13}{4} &= 2 - \tfrac{3}{4}k \\
\tfrac{3}{4}k &= \frac{21}{4} \\
k &= 7
\end{aligned}$$

The answer is (D).

MATHEMATICS–79

The location of a body as a function of time is $x(t) = 18t + 9t^2$. Find the body's velocity at $t = 2$.

(A) 20 (B) 24 (C) 36 (D) 54

Velocity is the first time derivative of the position function.

$$\begin{aligned} x(t) &= 18t + 9t^2 \\ \text{v}(t) &= x'(t) \\ &= 18 + 18t \\ \text{v}(2) &= 18 + (18)(2) \\ &= 54 \end{aligned}$$

The answer is (D).

MATHEMATICS–80

A particle moves according to the following functions of time.

$$\begin{aligned} x(t) &= 3\sin t \\ y(t) &= 4\cos t \end{aligned}$$

What is the resultant velocity at $t = \pi$?

(A) 0 (B) 3 (C) 4 (D) 9

$$\text{v} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$\frac{dx}{dt} = 3\cos t$$

$$\frac{dy}{dt} = -4\sin t$$

$$\text{v}(t) = \sqrt{9\cos^2 t + 16\sin^2 t}$$

$$\begin{aligned} \text{v}(\pi) &= \sqrt{(9)(-1)^2 + (16)(0)^2} \\ &= 3 \end{aligned}$$

The answer is (B).

MATHEMATICS–81

Water is pouring at a varying rate into a swimming pool that is initially empty. After t hours, there are $t + \sqrt{t}$ liters in the pool. At what rate is the water pouring into the pool when $t = 9$ h?

(A) 1/6 L/h (B) 1/2 L/h (C) 1 L/h (D) 7/6 L/h

Let V = volume of water in the tank in liters and Q = flow rate in liters per hour.

$$V = t + \sqrt{t}$$

$$Q = \frac{dV}{dt} = 1 + \frac{1}{2\sqrt{t}}$$

At $t = 9$ h,

$$Q = 1 + \frac{1}{2\sqrt{9}} = 7/6 \text{ L/h}$$

The answer is (D).

MATHEMATICS–82

If x increases uniformly at the rate of 0.001 per unit time, at what rate is the expression $(1+x)^3$ increasing when x becomes 9?

(A) 0.001 (B) 0.003 (C) 0.3 (D) 1

$$\frac{dx}{dt} = 0.001$$

$$f(x) = (1+x)^3$$

$$\frac{df}{dx} = 3(1+x)^2$$

$$\frac{df}{dt} = \frac{df}{dx}\frac{dx}{dt} = 0.003(1+x)^2$$

$$\left.\frac{df}{dt}\right|_{x=9} = (0.003)(1+9)^2 = 0.3$$

The answer is (C).

MATHEMATICS–83

A spherical balloon is filled with air at a rate of 1 m^3/s. Compute the time rate of change of the surface area of the balloon at the instant the balloon's volume is 113.1 m^3.

(A) 0.67 m^2/s (B) 1.7 m^2/s (C) 3.1 m^2/s (D) 3.7 m^2/s

$$V = \tfrac{4}{3}\pi r^3$$

$$A = 4\pi r^2$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \left(\frac{1}{4\pi r^2}\right)\frac{dV}{dt}$$

$$\begin{aligned}\frac{dA}{dt} &= \frac{dA}{dr}\frac{dr}{dt} \\ &= 8\pi r\frac{dr}{dt} \\ &= \left(\frac{8\pi r}{4\pi r^2}\right)\frac{dV}{dt} \\ &= \left(\frac{2}{r}\right)\frac{dV}{dt}\end{aligned}$$

Solve for the radius of the balloon when the volume is 113.1 m^3, and substitute into the equation for the rate of change of the surface area.

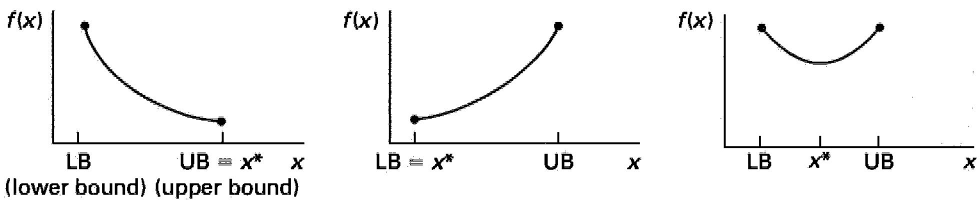
$$\begin{aligned}r &= \left(\frac{3V}{4\pi}\right)^{1/3} \\ &= \left(\frac{(3)(113.1\ \text{m}^3)}{4\pi}\right)^{1/3} \\ r &= 3\ \text{m} \\ \frac{dA}{dt} &= \left(\frac{2}{r}\right)\frac{dV}{dt} \\ &= \left(\frac{2}{3\ \text{m}}\right)\left(1\ \frac{\text{m}^3}{\text{s}}\right) \\ &= 0.67\ \text{m}^2/\text{s}\end{aligned}$$

The answer is (A).

MATHEMATICS–84

Consider a strictly concave up function of one variable, x, with lower and upper bounds on x. At what value(s) of x will the function be minimized?

(A) at the upper bound of x
(B) at the lower bound of x
(C) strictly between the upper and lower bounds of x
(D) at any of the above



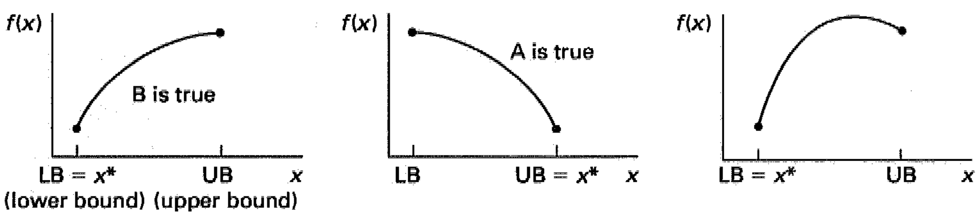
The examples given demonstrate that, for a concave up function, the minimum could occur at the lower bound, the upper bound, or somewhere between. Option (A), (B), or (C) could be correct.

> The answer is (D).

MATHEMATICS–85

Consider a strictly concave down function in one variable, x, with lower and upper bounds on x. At what value(s) of x will the function be minimized?

(A) at the upper bound of x
(B) at the lower bound of x
(C) strictly between the upper and lower bounds of x
(D) at either the upper or lower bound of x



The illustrations demonstrate that for a concave down curve, the minimum could occur at either the lower or the upper bound. Therefore, option (D) is correct.

The answer is (D).

MATHEMATICS–86

What is the maximum of the function $y = -x^3 + 3x$, for $x \geq -1$?

(A) -2 (B) -1 (C) 0 (D) 2

The maximum occurs where $y' = 0$ and $y'' < 0$ or at an endpoint.

$$\begin{aligned} y &= -x^3 + 3x \\ y' &= -3x^2 + 3 \\ y'' &= -6x \\ y' &= 0 \\ 0 &= -3x^2 + 3 \\ x^2 &= 1 \\ x &= \pm 1 \quad [-1 \text{ is also an endpoint}] \\ y(-1) &= -(-1)^3 + (3)(-1) \\ &= -2 \\ y(1) &= -(1)^3 + (3)(1) \\ &= 2 \end{aligned}$$

Therefore,

$$y_{\max} = 2$$

The answer is (D).

MATHEMATICS–87

The cost, C, of an item is a function of the quantity, x, of the item: $C(x) = x^2 - 4000x + 50$. Find the quantity for which the cost is minimum.

(A) 1000 (B) 1500 (C) 2000 (D) 3000

$$\begin{aligned} C &= x^2 - 4000x + 50 \\ C' &= 2x - 4000 \\ C'' &= 2 \\ C' &= 0 \\ 2x - 4000 &= 0 \\ x &= 2000 \\ C'' &> 0 \end{aligned}$$

Thus, cost is a minimum when $x = 2000$.

The answer is (C).

MATHEMATICS–88

Compute the following limit.

$$\lim_{x\to\infty} \frac{x+2}{x-2}$$

(A) 0 (B) 1 (C) 2 (D) ∞

Divide both the numerator and denominator by x, and allow x to approach infinity.

$$\begin{aligned} \lim_{x\to\infty} \frac{x+2}{x-2} &= \lim_{x\to\infty} \frac{1+\frac{2}{x}}{1-\frac{2}{x}} \\ &= \frac{1+0}{1-0} \\ &= 1 \end{aligned}$$

The answer is (B).

MATHEMATICS–89

Simplify the following expression.

$$\lim_{x\to 4} \frac{x^2-16}{x-4}$$

(A) 0 (B) 8 (C) 12 (D) 16

Factor the numerator, and simplify the fraction before taking the limit.

$$\begin{aligned}\lim_{x\to 4} \frac{x^2-16}{x-4} &= \lim_{x\to 4} \frac{(x-4)(x+4)}{x-4} \\ &= \lim_{x\to 4}(x+4) \\ &= 8\end{aligned}$$

The answer is (B).

MATHEMATICS–90

Compute the following limit.

$$\lim_{x\to 0} \frac{1-\cos x}{x^2}$$

(A) 0 (B) 1/4 (C) 1/2 (D) 1

Since both the numerator and denominator approach zero, use L'Hôpital's rule. L'Hôpital's rule states that the derivative of the numerator divided by the derivative of the denominator has the same limit as the original fraction, provided that both the numerator and denominator of the original fraction approach zero.

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \lim_{x\to 0} \frac{\sin x}{2x}$$

Since the numerator and denominator both approach zero, apply L'Hôpital's rule again.

$$\begin{aligned}\lim_{x\to 0} \frac{1-\cos x}{x^2} &= \lim_{x\to 0} \frac{\cos x}{2} \\ &= \frac{\cos 0}{2} \\ &= 1/2\end{aligned}$$

The answer is (C).

MATHEMATICS–91

The existence of the two equations, $y' = f(x)$ and $y = \phi(x)$, implies that which of the following equations is true?

(A) $\phi(x) = \int f(x)dx + C$
(B) $\phi(x) = f(x)$
(C) $\phi'(x) = \int f(x)dx + C$
(D) $\phi'(x) = y$

$$y = \phi(x)$$
$$\frac{d\phi}{dx} = y'$$
$$\phi(x) = \int y'dx + C$$

Since

$$y' = f(x)$$
$$\phi(x) = \int f(x)dx + C$$

The answer is (A).

MATHEMATICS–92

Fill in the blank in the following statement.

The integral of a function between certain limits divided by the difference in abscissas between those limits gives the ________ of the function.

(A) average (B) middle (C) intercept (D) asymptote

$$\frac{1}{b-a}\int_a^b f(x) = \text{the average value of the function}$$

The answer is (A).

MATHEMATICS–93

Find the area under the curve $y = 1/x$ between the limits $y = 2$ and $y = 10$.

(A) 1.61 (B) 2.39 (C) 3.71 (D) 3.97

The area under the curve $f(x)$ between x_1 and x_2, A, is given by

$$A = \int_{x_1}^{x_2} f(x)dx$$

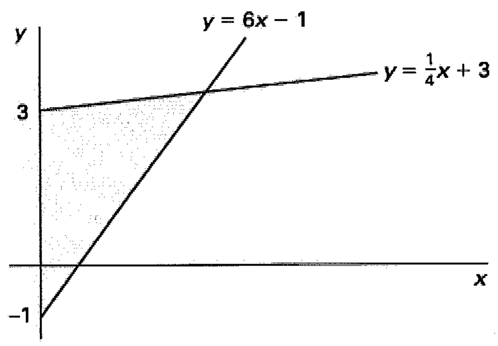
The x limits corresponding to the y limits are $x = 1/2$ and $x = 1/10$.

$$\begin{aligned} A &= \int_{1/10}^{1/2} \frac{1}{x} dx \\ &= \ln x \Big|_{1/10}^{1/2} \\ &= 1.61 \end{aligned}$$

The answer is (A).

MATHEMATICS–94

Find the area of the shaded region between $y = 6x - 1$ and $y = {}^{1}\!/_{4}x + 3$, bounded by $x = 0$ and the intersection point.



(A) 32/529 (B) 16/23 (C) 32/23 (D) 1440/529

The area between curve 1 and curve 2 is equal to the area under curve 1 minus the area under curve 2. The intersection point of the two curves is found by equating both functions.

$$\begin{aligned} 6x - 1 &= \tfrac{1}{4}x + 3 \\ \tfrac{23}{4}x &= 4 \\ x &= 16/23 \end{aligned}$$

The area, A, is

$$\begin{aligned} A &= \int_0^{16/23} \left(\tfrac{1}{4}x + 3 - 6x + 1\right) dx \\ &= \int_0^{16/23} \left(-\tfrac{23}{4}x + 4\right) dx \\ &= \left| \left(-\tfrac{23}{8}x^2 + 4x\right) \right|_0^{16/23} \\ &= \left(-\frac{23}{8}\right)\left(\frac{16}{23}\right)^2 + (4)\left(\frac{16}{23}\right) \\ &= -\frac{32}{23} + \frac{64}{23} \\ &= 32/23 \end{aligned}$$

The answer is (C).

MATHEMATICS–95

If it is known that $y = 1$ when $x = 1$, what is the constant of integration for the following integral?

$$y(x) = \int \left(e^{2x} - 2x\right) dx$$

(A) $C = 2 - e^2$

(B) $C = 3 - e^2$

(C) $C = 4 - e^2$

(D) $C = \frac{1}{2}\left(4 - e^2\right)$

$$\begin{aligned} y(x) &= \int e^{2x}dx - 2\int xdx \\ &= \tfrac{1}{2}e^{2x} - x^2 + C \\ &= \frac{1}{2}\left(e^{2x} - 2x^2\right) + C \end{aligned}$$

However, $y(1) = 1$.

$$\begin{aligned} 1 &= \frac{1}{2}\left(e^{2(1)} - (2)(1)^2\right) + C \\ &= \frac{1}{2}\left(e^2 - 2\right) + C \\ C &= 1 + 1 - \tfrac{1}{2}e^2 \\ &= \frac{1}{2}\left(4 - e^2\right) \end{aligned}$$

The answer is (D).

MATHEMATICS–96

It is known that $y(x)$ passes through the points (0,2) and (1,4). Solve for $y(x)$ if the second derivative is

$$\frac{d^2y}{dx^2} = 1$$

(A) $y = \left(x^2 + 3x\right) + 2$

(B) $y = \frac{1}{2}\left(x^2 + 3x\right) + 2$

(C) $y = \frac{1}{2}\left(x^2 - 3x\right) - 2$

(D) $y = \frac{1}{2}\left(x^2 + 3x\right) - 2$

Integrate twice to get the general form of the equation.

$$\begin{aligned} \frac{d^2y}{dx^2} &= 1 \\ \frac{dy}{dx} &= \int 1dx \\ &= x + C_1 \\ y &= \int (x + C_1) \\ &= \tfrac{1}{2}x^2 + C_1x + C_2 \end{aligned}$$

Now solve for C_1 and C_2 using the given conditions.

$$\begin{aligned}
2 &= \frac{1}{2}(0) + C_1(0) + C_2 \\
C_2 &= 2 \\
4 &= \frac{1}{2}(1)^2 + C_1(1) + 2 \\
C_1 &= 3/2 \\
y &= \tfrac{1}{2}x^2 + \tfrac{3}{2}x + 2
\end{aligned}$$

The answer is (B).

MATHEMATICS–97

What is a solution of the first-order difference equation $y(k+1) = y(k) + 5$?

(A) $y(k) = 4 - \dfrac{5}{k}$

(B) $y(k) = C - k$, where C is a constant

(C) $y(k) = 5^k + \dfrac{1-5^k}{-4}$

(D) $y(k) = 20 + 5k$

Assume the solution has the form

$$y(k) = 20 + 5k$$

Substitute the assumed solution into the difference equation.

$$\begin{aligned}
y(k+1) &= 20 + 5(k+1) \\
&= 20 + 5k + 5 \\
&= y(k) + 5
\end{aligned}$$

The answer is (D).

MATHEMATICS–98

What is the solution of the linear difference equation $y(k+1) = 15y(k)$?

(A) $y(k) = \dfrac{15}{1+15k}$

(B) $y(k) = \dfrac{15k}{16}$

(C) $y(k) = C + 15^k$, where C is a constant

(D) $y(k) = 15^k$

Assume the solution has the form

$$y(k) = 15^k$$

Substitute into the difference equation.

$$\begin{aligned} y(k+1) &= 15^{k+1} \\ &= (15)\left(15^k\right) \\ &= 15y(k) \end{aligned}$$

Note: If $y(k) = C + 15^k$, then $y(k+1) = C + 15^{k+1} \neq 15y(k)$.

The answer is (D).

MATHEMATICS–99

What is the solution of the linear difference equation $(k+1)y(k+1) - ky(k) = 1$?

(A) $y(k) = 12 - \dfrac{1}{k}$

(B) $y(k) = 1 - \dfrac{12}{k}$

(C) $y(k) = 12 + 3k$

(D) $y(k) = 3 + \dfrac{1}{k}$

Assume the solution has the form

$$y(k) = 1 - \frac{12}{k}$$

Substitute the solution into the difference equation.

$$\begin{aligned}(k+1)\big(y(k+1)\big) - k\big(y(k)\big) &= 1\\ (k+1)\left(1-\frac{12}{k+1}\right) - k\left(1-\frac{12}{k}\right) &= 1\\ (k+1)\left(\frac{k+1-12}{k+1}\right) - k\left(\frac{k-12}{k}\right) &= 1\\ k+1-12-k+12 &= 1\\ 1 &= 1\end{aligned}$$

Thus, $y(k) = 1 - 12/k$ solves the difference equation.

The answer is (B).

MATHEMATICS–100

Which of the following is a differential equation of the first order?

(A) $(y'')^3 + 2y' = -3$

(B) $\dfrac{\partial Q}{\partial x} - \dfrac{\partial Q}{\partial y} = 0$

(C) $\dfrac{dy}{dx} + \dfrac{9-x}{x} = y^3$

(D) $\left(\dfrac{dy}{dx}\right)^2 = -y + x$

A first-order differential equation contains only first derivatives and does not have partial derivatives. The only choice that fulfills this requirement is option (C).

The answer is (C).

MATHEMATICS–101

How can the following differential equation best be described?

$$a\frac{d^2x}{dt^2} + B(t)\frac{dx}{dt} + C = D(t)$$

(A) linear, homogeneous, and first order
(B) homogeneous and first order
(C) linear, second order, and nonhomogeneous
(D) linear, homogeneous, and second order

The differential equation has a second derivative, so it is of second order. The forcing function is nonzero, so the equation is nonhomogeneous. All of the terms on the left-hand side only have coefficients that are either constant or a function of the independent variable. Therefore the equation is also linear.

The answer is (C).

MATHEMATICS–102

The differential equation given is correctly described by which of the following choices?

$$a\frac{d^2y}{dx^2} + bxy\frac{dy}{dx} = f(x)$$

(A) linear, second order, homogeneous
(B) nonlinear, second order, homogeneous
(C) linear, second order, nonhomogeneous
(D) nonlinear, second order, nonhomogeneous

Since there is a second derivative, the differential equation is of second order. Since the coefficient of one of the terms contains the dependent variable, y, the equation is nonlinear. Since the forcing function, $f(x)$, is implied to be nonzero, the differential equation is also nonhomogeneous.

The answer is (D).

MATHEMATICS–103

Determine the solution of the following differential equation.

$$y' + 5y = 0$$

(A) $y = 5x + C$ (B) $y = Ce^{-5x}$ (C) $y = Ce^{5x}$ (D) (A) or (B)

This is a first-order linear equation with characteristic equation $r + 5 = 0$. Therefore, the form of the solution is

$$y = Ce^{-5x}$$

In the preceding equation, the constant, C, could be determined from additional information.

The answer is (B).

MATHEMATICS–104

What is the general solution of the following differential equation?

$$\frac{d^2y}{dx^2} + 4y = 0$$

(A) $y = \sin x + 2\tan x + C$
(B) $y = e^x - 2e^{-x} + C$
(C) $y = 2x^2 - x + C$
(D) $y = \sin 2x + \cos 2x + C$

Examination of the differential equation shows that a multiple of the function and its second derivative must sum to zero. Sines and cosines have the property that their second derivatives are the negatives of the original natural function.

If $y = \sin 2x + \cos 2x$, then

$$\begin{aligned} y' &= 2\cos 2x - 2\sin 2x \\ y'' &= -4\sin 2x - 4\cos 2x \\ y'' + 4y &= -4\sin 2x - 4\cos 2x + 4(\sin 2x + \cos 2x) \\ &= 0 \end{aligned}$$

The function in option (D) solves the differential equation.

The answer is (D).

MATHEMATICS–105

In the following differential equation with the initial condition $x(0) = 12$, what is the value of $x(2)$?

$$\frac{dx}{dt} + 4x = 0$$

(A) 3.35×10^{-4} (B) 4.03×10^{-3} (C) 3.35 (D) 6.04

This is a first-order, linear, homogeneous differential equation with characteristic equation $r + 4 = 0$.

$$\begin{aligned} x' + 4x &= 0 \\ x &= x_0 e^{-4t} \\ x(0) &= x_0 e^{(-4)(0)} \\ &= 12 \\ x_0 &= 12 \\ x &= 12e^{-4t} \\ x(2) &= 12e^{(-4)(2)} \\ &= 12e^{-8} \\ &= 4.03 \times 10^{-3} \end{aligned}$$

The answer is (B).

MATHEMATICS–106

A curve passes through the point (1,1). Determine the absolute value of the slope of the curve at $x = 25$ if the differential equation of the curve is the exact equation $y^2dx + 2xydy = 0$.

(A) 1/250 (B) 1/125 (C) $1/50\sqrt{5}$ (D) $1/\sqrt{125}$

$$\begin{aligned} y^2dx + 2xydy &= 0 \\ 2xydy &= -y^2dx \\ 2\frac{dy}{y} &= \frac{-dx}{x} \end{aligned}$$

Integrating both sides,

$$2\ln y = -\ln x + \ln C$$
$$\ln y^2 + \ln x = \ln C$$
$$\ln xy^2 = \ln C$$
$$xy^2 = C$$

Use the fact that the curve passes through the point (1,1) to solve for C, then determine the slope at $x = 25$.

$$(1)(1)^2 = C$$
$$C = 1$$
$$xy^2 = 1$$
$$y = \pm\sqrt{\frac{1}{x}}$$
$$y(25) = \pm\sqrt{\frac{1}{25}}$$
$$= \pm 1/5$$
$$y^2dx + 2xydy = 0$$
$$\frac{dy}{dx} = -\frac{y}{2x}$$
$$\left.\frac{dy}{dx}\right|_{x=25} = -\frac{\pm\frac{1}{5}}{(2)(25)}$$
$$= \pm 1/250$$

The answer is (A).

MATHEMATICS–107

Determine the constant of integration for the separable differential equation $xdx + 6y^5dy = 0$. It is known that $x = 0$ when $y = 2$.

(A) 12 (B) 16 (C) 24 (D) 64

Since this differential equation is already separated, integrate to find the solution.

$$\int xdx + \int 6y^5dy = \int 0$$
$$\tfrac{1}{2}x^2 + y^6 = C$$

Use the initial conditions to solve for C.

$$\left(\frac{1}{2}\right)(0)^2 + (2)^6 = C$$
$$C = 64$$

The answer is (D).

MATHEMATICS–108

What is the Laplace transform of e^{-6t}?

(A) $\frac{1}{s+6}$ (B) $\frac{1}{s-6}$ (C) e^{-6+s} (D) e^{6+s}

The Laplace transform of a function, $\mathcal{L}(f)$, can be calculated for the definition of a transform. However, it is easier to refer to a table of transforms.

$$\begin{aligned}\mathcal{L}(e^{-6t}) &= \int_0^\infty e^{-(s+6)t}dt \\ &= -\frac{e^{-(s+6)t}}{s+6}\Bigg|_0^\infty \\ &= \frac{1}{s+6}\end{aligned}$$

The answer is (A).

2 ECONOMICS

ECONOMICS–1

How is the capital recovery factor $(A/P,\ i,\ n)$ related to the uniform series sinking fund factor $(A/F,\ i,\ n)$? i is the effective annual rate of return, and n is the number of periods.

(A) $(A/P, i, n) = (A/F, i, n) + i$

(B) $(A/P, i, n) = (A/F, i, n) \ - i$

(C) $(A/P, i, n) = \dfrac{(A/F, i, n)}{i}$

(D) $(A/P, i, n) = \dfrac{(A/F, i, n) + i}{n}$

By definition,

$$(A/P, i, n) = (A/F, i, n) + i$$

The answer is (A).

ECONOMICS–2

What is an annuity?

(A) the future worth of a present amount
(B) an annual repayment of a loan
(C) a series of uniform amounts over a period of time
(D) a lump sum at the end of the year

The answer is (C).

ECONOMICS–3

Which of the following expressions is INCORRECT?

(A) The future worth of a present amount, $(F/P, i, n), = \dfrac{1}{(P/F, i, n)}$

(B) The future worth of an annuity, $(F/A, i, n), = \dfrac{1}{(A/F, i, n)}$

(C) The present worth of an annuity, $(P/A, i, n), = \dfrac{1}{(A/P, i, n)}$

(D) $(A/F, i, n) - i = (A/P, i, n)$

$$(A/F, i, n) + i = (A/P, i, n)$$

Therefore, option (D) is false.

The answer is (D).

ECONOMICS–4

When using net present worth calculations to compare two projects, which of the following could invalidate the calculation?

(A) differences in the magnitudes of the projects
(B) evaluating over different time periods
(C) mutually exclusive projects
(D) nonconventional cash flows

Options (A), (C), and (D) are all problems with internal rate of return calculations that net present worth handles nicely. However, the net present worth of two projects must be calculated for the same time period.

The answer is (B).

ECONOMICS–5

What is most nearly the present worth of a \$100 annuity over a 10 yr period if the interest rate is 8%?

(A) \$450 (B) \$530 (C) \$670 (D) \$700

$$
\begin{aligned}
P &= A(P/A,i,n) \\
&= (\$100)(P/A,8\%,10) \\
&= (\$100)(6.71) \\
&= \$671 \quad (\$670)
\end{aligned}
$$

The answer is (C).

ECONOMICS–6

With a 12% interest rate, approximately how much money must be invested today in order to withdraw $1000 per year at the end of each year for 10 yr?

(A) $4800 (B) $5650 (C) $5800 (D) $6150

$$
\begin{aligned}
P &= A(P/A,i,n) \\
&= (\$1000)(P/A,12\%,10) \\
&= (\$1000)(5.650) \\
&= \$5650
\end{aligned}
$$

The answer is (B).

ECONOMICS–7

A machine is under consideration for purchase. The cost of the machine is $25,000. Each year it operates, the machine will generate a savings of $15,000. Given an effective annual interest rate of 18%, what is the discounted payback period on the purchase in the machine?

(A) 1.67 yr (B) 1.75 yr (C) 2.15 yr (D) 3.17 yr

$$
\begin{aligned}
P &= A(P/A,i,n) \\
&= A\left(\frac{(1+i)^n-1}{i(1+i)^n}\right)
\end{aligned}
$$

Substituting,

$$\begin{aligned}\$25{,}000 &= (\$15{,}000)\left(\frac{(1+0.18)^n-1}{(0.18)(1+0.18)^n}\right)\\(0.3)(1.18)^n &= 1.18^n-1\\(0.7)(1.18)^n &= 1\\n &= \frac{\ln\frac{1}{0.7}}{\ln 1.18}\\&= 2.15\ \text{yr}\end{aligned}$$

The answer is (C).

ECONOMICS–8

What is the present worth of two \$100 payments at the end of the third and fourth years if the annual interest rate is 8%?

(A) \$122 (B) \$153 (C) \$160 (D) \$162

$$\begin{aligned}P &= A(P/A,i,n)\\&= (\$100)\big((P/A,8\%,4)-(P/A,8\%,2)\big)\\&= (\$100)(3.31-1.78)\\&= \$153\end{aligned}$$

The answer is (B).

ECONOMICS–9

Consider a project that involves the investment of \$100,000 now and \$100,000 at the end of year 1. Revenues of \$150,000 will be generated at the end of years 1 and 2. What is most nearly the net present value of this project if the effective annual interest rate is 10%?

(A) \$43,300 (B) \$50,900 (C) \$69,500 (D) \$78,500

$$\begin{aligned} P &= -\$100{,}000 + (\$150{,}000 - \$100{,}000)(P/F, 10\%, 1) + (\$150{,}000)(P/F, 10\%, 2) \\ &= -\$100{,}000 + (\$50{,}000)(0.9091) + (\$150{,}000)(0.8264) \\ &= \$69{,}415 \quad (\$69{,}500) \end{aligned}$$

The answer is (C).

ECONOMICS–10

At an annual rate of return of 8%, what is the future worth of $100 at the end of year 4?

(A) $130 (B) $132 (C) $135 (D) $136

$$\begin{aligned} F &= A(F/A, i, n) \\ &= (\$100)(F/P, 8\%, 4) \\ &= (\$100)(1.3605) \\ &= 136.05 \quad (\$136) \end{aligned}$$

The answer is (D).

ECONOMICS–11

A person invests $450 to be collected in 8 yr. Given that the interest rate on the investment is 14.5%/yr, compounded annually, most nearly what sum will be collected 8 yr from now?

(A) $1050 (B) $1130 (C) $1240 (D) $1330

$$\begin{aligned} F &= A(F/P, i, n) \\ &= (\$450)(F/P, 14.5\%, 8) \\ &= (\$450)(2.954) \\ &= \$1329 \quad (\$1330) \end{aligned}$$

The answer is (D).

ECONOMICS–12

An investment of x dollars is made at the end of each year for 3 yr, at an interest rate of 9% per year compounded annually. What will the dollar value of the total investment be, most nearly, upon the deposit of the third payment?

(A) $\$0.77x$ (B) $\$1.3x$ (C) $\$2.3x$ (D) $\$3.3x$

$$\begin{aligned} F &= A(F/A,i,n) \\ &= A\left(\frac{(1+i)^n-1}{i}\right) \\ &= x\left(\frac{(1+0.09)^3-1}{0.09}\right) \\ &= \$3.278x \quad (\$3.3x) \end{aligned}$$

The answer is (D).

ECONOMICS–13

If \$500 is invested at the end of each year for 6 yr at an effective annual interest rate of 7%, what is most nearly the total dollar amount available upon the deposit of the sixth payment?

(A) \$3000 (B) \$3210 (C) \$3580 (D) \$4260

$$\begin{aligned} F &= A(F/A,i,n) \\ &= (\$500)(F/A,7\%,6) \\ &= (\$500)(7.153) \\ &= \$3577 \quad (\$3580) \end{aligned}$$

The answer is (C).

ECONOMICS–14

Assuming i = annual rate of return, n = number of years, F = future worth, and P = present worth, what is the future worth of a present amount P?

(A) $P(1+i)^n$ (B) $P(1+i)^{n-1}$ (C) $P(1+i)^{-n}$ (D) $P(1+n)^i$

This situation corresponds to a single payment compound amount. Therefore,

$$F = P(1+i)^n$$

The answer is (A).

ECONOMICS–15

$1000 is deposited into a 9% account today. At the end of 2 yr, another $3000 will be deposited. In 5 yr, a $4000 purchase will be made. Approximately how much will be left in the account 1 yr after the purchase?

(A) $1230 (B) $1420 (C) $1540 (D) $1690

year	cash flow ($)
0	1000
1	0
2	3000
3	0
4	0
5	−4000
6	0

$$\begin{aligned} F &= A(F/A,i,n) \\ &= (\$1000)(F/P,9\%,6) + (\$3000)(F/P,9\%,4) - (\$4000)(F/P,9\%,1) \\ &= (\$1000)(1.6671) + (\$3000)(1.4116) - (\$4000)(1.0900) \\ &= \$1542 \quad (\$1540) \end{aligned}$$

The answer is (C).

ECONOMICS–16

A student needs $4000/yr for 4 yr to attend college. Her father invested $5000 in a 7% account for her education when she was born. If the student withdraws $4000 at the end of her 17th, 18th, 19th, and 20th years, how much money will be left in the account at the end of her 21st year?

(A) $1700 (B) $2500 (C) $3400 (D) $4000

$$
\begin{aligned}
F &= A(F/A, i, n) \\
&= (\$5000)(F/P, 7\%, 21) \\
&\quad \times (-\$4000)(F/P, 7\%, 4) - (\$4000)(F/P, 7\%, 3) \\
&\quad \times (-\$4000)(F/P, 7\%, 2) - (\$4000)(F/P, 7\%, 1) \\
&= (\$5000)(4.1406) - (\$4000)(1.3108) \\
&\quad \times (-\$4000)(1.2250) - (\$4000)(1.1449) \\
&\quad \times (-\$4000)(1.0700) \\
&= \$1700
\end{aligned}
$$

The answer is (A).

ECONOMICS–17

The following schedule of funds is available to form a sinking fund.

$t = 0$ yr	\$5000
$t = 1$ yr	\$4000
$t = 2$ yr	\$3000
$t = 3$ yr	\$2000

At the end of the fourth year, equipment costing \$25,000 will have to be purchased as a replacement for old equipment. Money is valued at 20% by the company. At the time of purchase, how much money will be needed?

(A) \$820 (B) \$1000 (C) \$2000 (D) \$8200

First, find the future worth of the available funds.

$$
\begin{aligned}
F &= P(F/P, i, n) \\
&= (\$5000)(F/P, 20\%, 4) \\
&\quad + (\$4000)(F/P, 20\%, 3) + (\$3000)(F/P, 20\%, 2) \\
&\quad + (\$2000)(F/P, 20\%, 1) \\
&= (\$5000)(2.074) + (\$4000)(1.728) + (\$3000)(1.44) \\
&\quad + (\$2000)(1.20) \\
&= \$24{,}000
\end{aligned}
$$

The additional funds needed are

$$\text{equipment cost} - F = \$25{,}000 - \$24{,}000 = \$1000$$

The answer is (B).

ECONOMICS–18

$10,000 is invested at the beginning of a year in a 15% security and held for 5 yr. During that time, the average annual inflation is 6%. Approximately how much, in terms of year zero dollars, will be in the account at maturity ?

(A) $11,700 (B) $13,400 (C) $15,000 (D) $15,400

First, find the future worth, F, without accounting for inflation.

$$\begin{aligned} F &= P(F/P, i, n) \\ &= (\$10{,}000)(F/P, 15\%, 5) \\ &= (\$10{,}000)(2.0114) \\ &= \$20{,}114 \end{aligned}$$

Next, figure in inflation and express F in terms of real dollars.

$$\begin{aligned} F_{\text{real}} &= (\$20{,}114)(P/F, 6\%, 5) \\ &= (\$20{,}114)(0.7473) \\ &= \$15{,}030 \quad (\$15{,}000) \end{aligned}$$

The answer is (C).

ECONOMICS–19

A firm borrows $2000 for 6 yr at 8%, to be repaid in a lump sum at the end of 6 yr. At the end of 6 yr, the firm renews the loan for the amount due plus $2000 more for 2 yr at 8%. What is most nearly the amount of the loan renewal?

(A) $5280 (B) $5750 (C) $5510 (D) $6140

$$\begin{aligned} F &= (\$2000)(F/P, 8\%, 6) + (\$2000)(F/P, 8\%, 2) \\ &= (\$2000)(1.587) + (\$2000)(1.166) \\ &= \$5506 \quad (\$5510) \end{aligned}$$

The answer is (C).

ECONOMICS–20

A company invests $10,000 today to be repaid in 5 yr in one lump sum at 12% compounded annually. If the rate of inflation is 3% compounded annually, approximately how much profit, in present day dollars, is realized over the 5 yr?

(A) $3200 (B) $5200 (C) $5630 (D) $7620

First, find the future worth of the investment without accounting for inflation.

$$\begin{aligned} F &= P(F/P, i, n) \\ &= (\$10{,}000)(F/P, 12\%, 5) \\ &= (\$10{,}000)(1.7623) \\ &= \$17{,}623 \end{aligned}$$

Next, find the present worth accounting for inflation.

$$\begin{aligned} F_{\text{real}} &= (\$17{,}623)(P/F, 3\%, 5) \\ &= (\$17{,}623)(0.8626) \\ &= \$15{,}202 \\ \text{profit} &= \$15{,}202 - \$10{,}000 \\ &= \$5202 \quad (\$5200) \end{aligned}$$

The answer is (B).

ECONOMICS–21

What must two investments with the same present worth and unequal lives have?

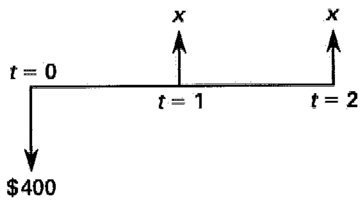
(A) identical salvage values
(B) different salvage values
(C) identical equivalent uniform annual cash flows
(D) different equivalent uniform annual cash flows

Two investments with the same present worth and unequal lives must have different equivalent uniform annual cash flows.

The answer is (D).

ECONOMICS–22

The following cash-flow diagram represents an investment of \$400 and a revenue of x at the end of years one and two. Given a discount rate of 15% compounded annually, what must x approximately be for this set of cash flows to have a net present worth of zero?



(A) \$246 (B) \$255 (C) \$257 (D) \$281

$$\begin{aligned} P = \$0 &= A(P/A, i, n) \\ &= -\$400 + x(P/A, 15\%, 2) \\ &= -400 + x(1.6257) \end{aligned}$$

Rearranging to solve for x,

$$400 = 1.6257x$$
$$x = \$246$$

The answer is (A).

ECONOMICS–23

A replacement electric motor is being considered for purchase. It is capable of providing 200 hp. The pertinent data are as follows.

cost	$3200
electrical efficiency	0.85
maintenance cost per year	$50
life expectancy	14 yr
salvage value after 14 yr	$0

The motor is used for 400 h/yr and the cost of electricity is $0.04/kW·h. (1 hp = 0.746 kW.) What is most nearly the effective annual cost using an interest rate of 10%?

(A) $2920 (B) $3250 (C) $3290 (D) $3610

The capital recovery factor for 10% and 14 yr is 0.1357.

$$A = P(A/P, i, n) + \text{annual maintenance cost} + \text{annual operating cost}$$

$$\begin{aligned}\text{annual cost} &= (\$3200)(0.1357) + \$50 \\ &\quad + \frac{\left(400\ \frac{\text{h}}{\text{yr}}\right)\left(0.04\ \frac{\$}{\text{kW}\cdot\text{h}}\right)(200\ \text{hp})\left(0.746\ \frac{\text{kW}}{\text{hp}}\right)}{0.85} \\ &= \$3293 \quad (\$3290)\end{aligned}$$

The answer is (C).

ECONOMICS–24

What annuity over a 10 yr period at 8% interest is most nearly equivalent to a present worth of $100?

(A) $12.50 (B) $13.80 (C) $14.10 (D) $14.90

$$\begin{aligned}A &= P(A/P, i, n) \\ &= (\$100)(A/P, 8\%, 10) \\ &= (\$100)(0.149) \\ &= \$14.90\end{aligned}$$

The answer is (D).

ECONOMICS–25

Mr. Richardson borrowed \$15,000 two years ago. The repayment terms of the loan are 10% interest for 10 yr and uniform annual payments. He just made his second payment. How much principal, most nearly, does he still owe?

(A) \$10,100 (B) \$11,700 (C) \$12,000 (D) \$13,000

The annual payments, A, are

$$\begin{aligned} A &= P(A/P, i, n) \\ &= (\$15{,}000)(A/P, 10\%, 10) \\ &= (\$15{,}000)(0.1627) \\ &= \$2441 \end{aligned}$$

year	amount owed (\$)		interest owed (\$)		payment (\$)		balance (\$)
1	15,000	+	1500	−	2441	=	\$14,059
2	14,059	+	1406	−	2441	=	\$13,024

Thus, Mr. Richardson still owes \$13,024 (\$13,000) on the principal.

The answer is (D).

ECONOMICS–26

Given that the discount rate is 15%, what is the equivalent uniform annual cash flow for the following stream of cash flows?

year 0	−\$100,000
year 1	−\$200,000
year 2	−\$50,000
year 3	−\$75,000

(A) −\$158,100 (B) −\$124,200 (C) −\$106,250 (D) −\$90,260

$$\begin{aligned}
\text{EUAC} &= (A/P,i,n)P \\
&= (A/P,i,n)\left(\sum F(P/F,i,n)\right) \\
&= (A/P,15\%,3)\big(-\$100{,}000 - \$200{,}000)(P/F,15\%,1) \\
&\quad - (\$50{,}000(P/F,15\%,2) - (\$75{,}000)(P/F,15\%,3)\big) \\
&= 0.4380\big(-\$100{,}000 - (\$200{,}000)(0.8696) \\
&\quad - (\$50{,}000)(0.7561) - (\$75{,}000)(0.6575)\big) \\
&= (0.4380)(-\$361{,}000) \\
&= -\$158{,}100
\end{aligned}$$

The answer is (A).

ECONOMICS–27

A company must relocate one of its factories in 3 yr. Equipment for the loading dock is being considered for purchase. The original cost is \$20,000, and the salvage value after 3 yr is \$8000. The company's rate of return on money invested is 10%. The capital recovery rate per year is most nearly

(A) \$4810/yr (B) \$4950/yr (C) \$5120/yr (D) \$5630/yr

$$\begin{aligned}
\text{CR} &= P(A/P,i,n) - F(A/F,i,n) \\
&= (\$20{,}000)(A/P,10\%,3) - (\$8000)(A/F,10\%,3) \\
&= (\$20{,}000)(0.4021) - (\$8000)(0.3021) \\
&= \$5625/\text{yr}
\end{aligned}$$

The answer is (D).

ECONOMICS–28

In 5 yr, \$18,000 will be needed to pay for a building renovation. In order to generate this sum, a sinking fund consisting of three annual payments is established now. No further payments will be made after the third year. What payments are most nearly necessary if money is worth 15% per year?

(A) \$2670 (B) \$2870 (C) \$3920 (D) \$5100

The present worth of $18,000 at the end of the third year is

$$\begin{aligned} P_3 &= F_5(P/F, i, n) \\ &= (\$18{,}000)(P/F, 15\%, 2) \\ &= (\$18{,}000)(0.7561) \\ &= \$13{,}610 \end{aligned}$$

The sinking fund must generate $13,610 in 3 yr. The payments that are necessary are

$$\begin{aligned} A &= P(A/P, i, n) \\ &= (\$13{,}610)(A/F, 15\%, 3) \\ &= (\$13{,}610)(0.2880) \\ &= \$3920 \end{aligned}$$

The answer is (C).

ECONOMICS–29

Mr. Johnson borrows $100,000 at 10% effective annual interest. He must pay back the loan over 30 yr with uniform monthly payments due on the first day of each month. Approximately what amount does Mr. Johnson pay each month?

(A) $840 (B) $850 (C) $870 (D) $880

An effective annual interest rate of 10% is equivalent to an effective monthly rate of

$$(1+i)^{12} - 1 = 0.1$$

$$i = (1.1)^{1/12} - 1 = 0.007974 \quad (0.7974\%/\text{mo})$$

The number of months that Mr. Johnson has to pay off his loan is

$$\begin{aligned} n &= (30 \text{ yr})\left(12 \, \frac{\text{mo}}{\text{yr}}\right) \\ &= 360 \text{ mo} \end{aligned}$$

$$\begin{aligned} \text{end of month payment} &= P(A/P, i, n) \\ &= (\$100{,}000)(A/P, 0.7974\%, 360) \\ &= (\$100{,}000)\left(\frac{(0.007974)(1+0.007974)^{360}}{(1+0.007974)^{360} - 1}\right) \\ &= \$846 \end{aligned}$$

$$\begin{aligned}\text{beginning of month payment} &= F(P/F,i,n) = (\$846)(P/F, 0.7974\%, 1)\\ &= (\$846)(1+0.007974)^{-1}\\ &= \$839 \quad (\$840)\end{aligned}$$

The answer is (A).

ECONOMICS–30

What is the formula for a straight-line depreciation rate?

(A) $\dfrac{100\% - \%\text{ net salvage value}}{\text{estimated service life}}$

(B) $\dfrac{\%\text{ net salvage value}}{\text{estimated service life}}$

(C) $\dfrac{100\%\text{ net salvage value}}{\text{estimated service life}}$

(D) $\dfrac{\text{average net salvage value}}{\text{estimated service life}}$

$$\text{straight-line depreciation rate} = \frac{100\% - \%\text{ net salvage value}}{\text{estimated service life}}$$

The answer is (A).

ECONOMICS–31

What is the book value of equipment purchased 3 yr ago for \$15,000 if it is depreciated using the sum of years' digits (SOYD) method? The expected life is 5 yr.

(A) \$3000 (B) \$4000 (C) \$6000 (D) \$9000

In the SOYD method, the digits corresponding to n, the number of years of estimated life, are added. The total sum of years' digits, t, is

$$\begin{aligned}t &= \frac{n(n+1)}{2}\\ &= \frac{(5\text{ yr})(5\text{ yr}+1)}{2}\\ &= 15\text{ yr}\end{aligned}$$

The depreciation charge for the first year is

$$\begin{aligned} D_1 &= \left(\frac{n}{t}\right)P \\ &= \left(\frac{5 \text{ yr}}{15 \text{ yr}}\right)(\$15{,}000) \\ &= \$5000 \end{aligned}$$

The depreciation charge for year two is

$$\begin{aligned} D_2 &= \left(\frac{n-1}{t}\right)P \\ &= \left(\frac{4 \text{ yr}}{15 \text{ yr}}\right)(\$15{,}000) \\ &= \$4000 \end{aligned}$$

For year 3, the depreciation charge is

$$\begin{aligned} D_3 &= \left(\frac{n-2}{t}\right)P \\ &= \left(\frac{3 \text{ yr}}{15 \text{ yr}}\right)(\$15{,}000) \\ &= \$3000 \end{aligned}$$

The total depreciation is

$$\begin{aligned} D_{\text{total}} &= D_1 + D_2 + D_3 \\ &= \$5000 + \$4000 + \$3000 \\ &= \$12{,}000 \end{aligned}$$

The book value is

$$\begin{aligned} \text{BV} &= P - D_{\text{total}} \\ &= \$15{,}000 - \$12{,}000 \\ &= \$3000 \end{aligned}$$

The answer is (A).

ECONOMICS–32

The purchase of a motor for \$6000 and a generator for \$4000 will allow a company to produce its own energy. The configuration can be assembled for \$500. The service will operate for 1600 h/yr for 10 yr. The maintenance cost is \$300/yr, and the cost to operate is \$0.85/h for fuel and related costs. There is \$400 in salvage value for the system at the end of 10 yr. Using straight-line depreciation, what is the annual cost for the operation?

(A) \$2480/yr (B) \$2630/yr (C) \$2670/yr (D) \$2710/yr

$$\begin{aligned}\text{total initial cost} &= \$6000 + \$4000 + \$500 \\ &= \$10{,}500 \\ \text{straight-line depreciation} &= \frac{\text{total initial cost} - \text{salvage value}}{t} \\ &= \frac{\$10{,}500 - \$400}{10\ \text{yr}} \\ &= \$1010/\text{yr} \\ \text{operation cost} &= \left(0.85\ \frac{\$}{\text{h}}\right)\left(1600\ \frac{\text{h}}{\text{yr}}\right) \\ &= \$1360/\text{yr} \\ \text{annual cost} &= \text{straight-line depreciation} + \text{maintenance cost} \\ &\quad + \text{operation cost} \\ &= \$1010 + \$300 + \$1360 \\ &= \$2670/\text{yr}\end{aligned}$$

The answer is (C).

ECONOMICS–33

Company A purchases \$200,000 of equipment in year 0. It decides to use straight-line depreciation over the expected 20 yr life of the equipment. The interest rate is 14%. If its overall tax rate is 40%, what is the present worth of the after-tax depreciation recovery?

(A) \$23,500 (B) \$24,000 (C) \$26,500 (D) \$39,700

$$\begin{aligned}
\text{straight-line depreciation} &= \frac{\text{equipment cost}}{\text{equipment life expectancy}} \\
&= \frac{\$200{,}000}{20 \text{ yr}} \\
&= \$10{,}000/\text{yr write off} \\
P &= A(P/A, i, n)(\text{tax rate}) \\
&= (\$10{,}000)(P/A, 14\%, 20)(\text{tax rate}) \\
&= (\$10{,}000)(6.623)(0.40) \\
&= \$26{,}500
\end{aligned}$$

The answer is (C).

ECONOMICS–34

Which of the following is true regarding the minimum attractive rate of return used in judging proposed investments?

(A) It is the same for every organization.

(B) It is larger than the interest rate used to discount expected cash flow from investments.

(C) It is frequently a policy decision made by an organization's management.

(D) It is not relevant in engineering economy studies.

The answer is (C).

ECONOMICS–35

What is most nearly the effective annual interest rate on a loan if the nominal interest rate is 12%/yr compounded quarterly?

(A) 11.8% (B) 12.0% (C) 12.3% (D) 12.6%

The effective interest rate, i, is

$$\begin{aligned}
i &= \left(1 + \frac{r}{m}\right)^m - 1 \\
&= \left(1 + \frac{0.12}{4}\right)^4 - 1 \\
&= 0.1255 \quad (12.55\%)
\end{aligned}$$

In the preceding equation, r is the nominal interest rate and m is the number of times the nominal interest rate is compounded per year.

The answer is (D).

ECONOMICS–36

A person pays interest on a loan semiannually at a nominal annual interest rate of 16%. What is most nearly the effective annual interest rate?

(A) 15.5% (B) 15.7% (C) 16.4% (D) 16.6%

The effective interest rate, i, is

$$\begin{aligned} i &= \left(1+\frac{r}{m}\right)^m - 1 \\ &= \left(1+\frac{0.16}{2}\right)^2 - 1 \\ &= 0.1664 \quad (16.64\%) \end{aligned}$$

In the preceding equation, r is the nominal interest rate and m is the number of times the nominal interest rate is compounded per year.

The answer is (D).

ECONOMICS–37

Which of the following statements is NOT correct?

(A) A nominal rate of 12% per annum compounded quarterly is the same as $12\%/4 = 3\%/\text{quarter}$.

(B) \$1 compounded quarterly at 3% for n yr has a future value of $(1.03)^{4n}$.

(C) Compounding quarterly at a nominal rate of 12%/yr is equivalent to compounding annually at a rate of 12.55%.

(D) Effective rate of return in options (A), (B), and (C) is the difference between 12.55% and 12%.

The statements given in options (A), (B), and (C) are correct. Only option (D) is false.

The answer is (D).

ECONOMICS–38

A bank is advertising 9.5% accounts that yield 9.84% annually. How often is the interest compounded?

(A) daily (B) monthly (C) bimonthly (D) quarterly

The formula for effective interest rate, i, is

$$i = \left(1 + \frac{r}{m}\right)^m - 1$$

Substituting for i,

$$0.0984 = \left(1 + \frac{0.095}{m}\right)^m - 1$$

In the preceding equation, r is the nominal interest rate and m is the number of times interest is compounded per year.

Solve for m algebraically, or by trial and error using the five choices. The solution is $m = 4$.

The answer is (D).

ECONOMICS–39

A firm is considering renting a trailer at $300/mo. The unit is needed for 5 yr. The leasing company offers a lump sum payment of $24,000 at the end of 5 yr as an alternative payment plan, but is willing to discount this figure. The firm places a value of 10% (effective annual rate) on invested capital. How large should the discount be in order to be acceptable as an equivalent?

(A) $750 (B) $820 (C) $980 (D) $1030

For a 10% effective rate per year, the effective monthly rate, i, is

$$\begin{aligned} 0.1 &= (1+i)^{12} - 1 \\ i &= 0.007974/\text{mo} \end{aligned}$$

At $300/mo for 5 yr,

$$\begin{aligned} F &= A(F/A, i, n) \\ &= (\$300)(F/A, 0.7974\%, 60) \\ &= (\$300)(76.561) \\ &= \$22{,}970 \end{aligned}$$

$$\begin{aligned}\text{discount} &= \$24{,}000 - F \\ &= \$24{,}000 - \$22{,}970 \\ &= \$1030\end{aligned}$$

The answer is (D).

ECONOMICS–40

Consider a deposit of \$1000, to be paid back in 1 yr by \$975. What are the conditions on the rate of interest, i, in %/yr compounded annually, such that the net present worth of the investment is positive? Assume $i \geq 0\%$.

(A) $0\% \leq i < 50\%$
(B) $0\% \leq i < 90\%$
(C) $12.5\% \leq i < 100\%$
(D) There are no conditions on i that will make this possible.

$$\begin{aligned}\text{Set}: P = 0 &= F(P/F, i, n) \\ &= -\$1000 + (\$975)(P/F, i, 1) \\ &= -\$1000 + (\$975)\left(\frac{1}{1+i}\right) \\ i &= -0.0256\end{aligned}$$

Therefore, there is no solution such that $i \geq 0\%$, and $P \geq 0$.

The answer is (D).

ECONOMICS–41

Consider a deposit of \$600, to be paid back in 1 yr by \$700. What are the conditions on the rate of interest, i, in %/yr compounded annually, such that the net present worth of the investment is positive? Assume $i \geq 0\%$.

(A) $12.5\% \leq i < 14.3\%$
(B) $0\% \leq i < 14.3\%$
(C) $0\% \leq i < 16.7\%$
(D) $16.7\% \leq i \leq 100\%$

$$\begin{aligned}
\text{Set}: P > 0 &> F(P/F, i, n) \\
&> (-\$600 + \$700)(P/F, i, 1) \\
&> (-\$600 + \$700)\left(\frac{1}{1+i}\right) \\
\frac{7}{6} &> 1 + i \\
i &< 0.167 \quad (16.7\%)
\end{aligned}$$

Thus, for P > \$0, $0\% \leq i < 16.7\%$.

The answer is (C).

ECONOMICS–42

A company has \$100,000 to spend on the various projects listed. Using these projects only, what should this company consider its minimum attractive rate of return to be?

project	investment required (\$)	expected return (%)
A	10,000	14
B	25,000	10
C	50,000	12
D	40,000	16
E	25,000	11
F	30,000	10
G	20,000	12

(A) 10% (B) 11% (C) 12% (D) 14%

The highest-return projects should be chosen until all of the company's money is spent.

project	investment (\$)	return (%)	cumulative investment (\$)
D	40,000	16	40,000
A	10,000	14	50,000
C	50,000	12	100,000

The minimum attractive rate of return is 12%, the return on project C.

The answer is (C).

ECONOMICS–43

What is most nearly the internal rate of return on the following cash flow?

$t = 0$ yr	spend \$100,000
$t = 1$ yr	spend \$50,000
$t = 2$ yr	receive \$100,000
$t = 3$ yr	receive \$103,000

(A) 15.0% (B) 17.5% (C) 18.2% (D) 20.0%

The IRR is the interest rate which makes the present worth of the cash flow zero. It must be found by trial and error using the answer choices provided.

$$\begin{aligned}\text{IRR} = 0 = F(P/F,i,n) &= (-\$100{,}000 - \$50{,}000)(P/F,i,1)\\ &\quad + (\$100{,}000)(P/F,i,2) + (\$103{,}000)(P/F,i,3)\end{aligned}$$

For an interest rate of 15%, the equation for the present worth of the cash flow is

$$\begin{aligned}P &= -\$100{,}000 - (\$50{,}000)(0.8696)\\ &\quad + (\$100{,}000)(0.7561) + (\$103{,}000)(0.6575)\\ &= -\$100{,}000 - \$43{,}480 + \$75{,}610 + \$67{,}772\\ &= -\$98 \quad (\approx 0)\end{aligned}$$

The IRR is approximately 15%.

The answer is (A).

ECONOMICS–44

Which of the following situations has a conventional cash flow so that an internal rate of return can be safely calculated and used?

(A) You purchase a house and pay the bank in monthly installments.

(B) You lease a car and pay by the month.

(C) Your company undertakes a mining project in which the land must be reclaimed at the end of the project.

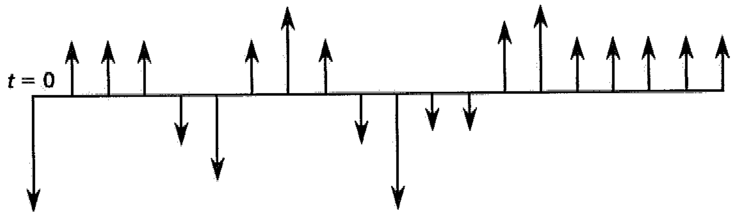
(D) You invest in a safe dividend stock and receive dividends each year.

The situation in option (D) has a negative cash flow, one sign change, then a positive cash flow. Thus, it is the only situation that has a conventional cash flow so that an IRR can be safely calculated and used.

The answer is (D).

ECONOMICS–45

A project has the cash flow shown. Theoretically, how many internal rates of return can be calculated for it?



(A) 2 (B) 3 (C) 4 (D) 5

There are five places in the cash flow where there are sign changes. Thus, theoretically, five internal rates of return could be calculated for it.

The answer is (D).

ECONOMICS–46

An investment of \$350,000 is made, to be followed by revenue of \$200,000 each year for three years. What is most nearly the annual rate of return on investment for this project?

(A) 15% (B) 33% (C) 42% (D) 57%

The formula for annual payment, A, is

$$\begin{aligned} A &= P(A/P, i, n) \\ \$200{,}000 &= \$350{,}000(A/P, i, 3) \\ (A/P, i, 3) &= \frac{\$200{,}000}{\$350{,}000} \\ &= 0.57143 \end{aligned}$$

Interpolating from the tables,

i (%)	$(A/P, i, 3)$
30	0.55063
32.66	0.57143
35	0.58966

Thus, the IRR is approximately 33%.

The answer is (B).

ECONOMICS–47

A steel drum manufacturer incurs a yearly fixed operating cost of \$200,000. Each drum manufactured costs \$160 to produce and sells for \$200. What is the manufacturer's break-even sales volume in drums per year?

(A) 1000 (B) 1250 (C) 2500 (D) 5000

Given that x is the number of drums sold per year, the cash flow per year is

operating cost	−\$200,000
manufacturing cost	$-\$160x$
sales	$\$200x$

In order to break even, total cash flow must be zero.

$$\begin{aligned} \text{operating cost} + \text{manufacturing cost} + \text{sales} &= 0 \\ -\$200{,}000 - \$160x + \$200x &= 0 \\ \$40x &= \$200{,}000 \\ x &= 5000 \end{aligned}$$

The answer is (D).

ECONOMICS–48

XYZ Corporation manufactures bookcases that it sells for \$65 each. It costs XYZ \$35,000/yr to operate its plant. This sum includes rent, depreciation charges on equipment, and salary payments. If the cost to produce one bookcase is \$50, how many bookcases must be sold each year for XYZ to avoid taking a loss?

(A) 539 bookcases/yr (B) 750 bookcases/yr (C) 2333 bookcases/yr (D) 2334 bookcases/yr

Determine the quantity of bookcases sold, Q.

$$Q = \frac{f}{p - a} = \frac{\frac{\$35{,}000}{\text{yr}}}{\frac{\$65}{\text{bookcase}} - \frac{\$50}{\text{bookcase}}} = 2333.3 \text{ bookcases/yr}$$

In the preceding equation, f is the fixed cost, p is the fixed revenue, and a is the incremental cost.

Therefore, XYZ must sell 2334 bookcases/yr to avoid taking a loss.

The answer is (D).

ECONOMICS–49

A manufacturing firm maintains one assembly line to produce signal generators. Weekly demand for the generators is 35 units, and the line operates for 7 h/d, 5 d/wk. What is the maximum production time per unit required of the line in order to meet demand?

(A) 0.750 h/unit (B) 1.00 h/unit (C) 2.25 h/unit (D) 5.00 h/unit

The maximum production time per unit, t, can be derived from

$$\text{weekly demand} = t(\text{operating hours per day})(\text{operating days per week})$$

$$\frac{1 \text{ wk}}{35 \text{ units}} = t\left(\frac{1 \text{ d}}{7 \text{ h}}\right)\left(\frac{1 \text{ wk}}{5 \text{ d}}\right)$$

Rearranging to solve for t,

$$t = \left(\frac{1 \text{ d}}{7 \text{ h}}\right)\left(\frac{1 \text{ wk}}{5 \text{ d}}\right)\left(35 \, \frac{\text{units}}{\text{wk}}\right)$$
$$= 1.00 \text{ h/unit}$$

The answer is (B).

ECONOMICS–50

In determining the cost involved in fabricating subassembly B within XYZ Corporation, the following data have been gathered.

XYZ's costs of manufacturing subassembly B

item	cost
direct material	\$0.30/unit
direct labor	\$0.50/unit
tooling setup	\$300/setup

It is decided to subcontract the manufacturing of subassembly B to an outside company. For an order of 100 units, which one of the following unit price bids from outside companies is unacceptable to XYZ Corporation?

(A) \$3.50/unit (B) \$3.65/unit (C) \$3.75/unit (D) \$4.10/unit

For 100 units,

$$\text{cost per unit} = \frac{\text{total setup cost}}{\text{no. of units}} + \text{direct material cost} + \text{direct labor cost}$$
$$= \frac{\frac{\$300}{\text{setup}}}{100 \text{ units}} + \frac{\$0.30}{\text{unit}} + \frac{\$0.50}{\text{unit}}$$
$$= \$3.80/\text{unit}$$

Thus, a bid of \$4.10 is unacceptable.

The answer is (D).

ECONOMICS–51

The economic order quantity (EOQ) is defined as the order quantity that minimizes the inventory cost per unit time. Which of the following is NOT an assumption of the basic EOQ model with no shortages?

(A) The demand rate is uniform and constant.
(B) There is a positive cost on each unit inventoried.
(C) The entire reorder quantity is delivered instantaneously.
(D) There is an upper bound on the quantity ordered.

$$\text{EOQ} = \sqrt{\frac{2aK}{h}}$$

In the preceding equation, a is the constant depletion rate (items per unit time), K is the fixed cost per order in dollars, and h is the inventory storage cost (dollars per item per unit time).

Thus, there is no upper bound on the quantity ordered.

The answer is (D).

ECONOMICS–52

Which of the following events will cause the optimal lot size, given by the classic EOQ model with no shortages, to increase?

(A) a decrease in inventory carrying cost
(B) a decrease in demand
(C) an increase in demand
(D) either option (A) or (C)

$$\text{EOQ} = \sqrt{\frac{2aK}{h}}$$

In the preceding equation, a is the constant depletion rate (items per unit time), K is the fixed cost per order in dollars, and h is the inventory storage cost (dollars per item per unit time).

Thus, a decrease in inventory carrying cost, h, or an increase in demand, a, will cause the optimal lot size to increase.

The answer is (D).

ECONOMICS–53

A manufacturer of sports equipment produces tennis rackets for which there is a demand of 200/mo. The production setup cost for each batch of rackets is \$300. In addition, the inventory carrying cost for each racket is \$24/yr. Using the EOQ model, which is most nearly the best production batch size for the rackets?

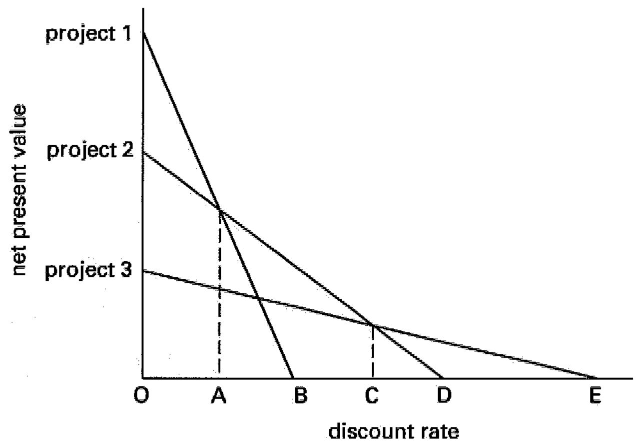
(A) 120 units (B) 170 units (C) 250 units (D) 350 units

$$\text{EOQ} = \sqrt{\frac{2aK}{h}}$$
$$= \sqrt{\frac{(2)\left(200\ \frac{\text{rackets}}{\text{mo}}\right)(\$300)}{\left(\frac{\$24}{\text{yr}}\right)\left(\frac{1\ \text{yr}}{12\ \text{mo}}\right)}}$$
$$= 244.9\ \text{units}\quad (250\ \text{units})$$

The answer is (C).

ECONOMICS–54

For what range of discount rates is project 2 the most attractive project?



(A) OA (B) OD (C) AC (D) AD

Project 2 has the highest net present worth over the range AC.

The answer is (C).

ECONOMICS–55

The internal rate of return of a project that involves an initial investment with subsequent positive cash flows is 18%. Five companies are considering the project. Given the following respective minimum attractive rates of return (MARR), which company will be most likely to accept the project?

company	MARR (%)
A	12
B	15
C	18
D	19

(A) company A (B) company B (C) company C (D) company D

The project is good for companies A, B, and C. However, it is the best for company A.

The answer is (A).

ECONOMICS–56

Two mutually exclusive projects are being considered. Project A requires an investment of \$1,000,000 at year zero. Project A will pay \$200,000/yr forever. Project B also requires an investment of \$1,000,000 at year zero. However, it pays \$1,500,000 the next year, and nothing after that. The internal rate of return (IRR) on project A is 20%. The IRR for project B is 50%. Which is the better project? The borrowing rate is 5%.

(A) Project A, because it has a lower IRR.
(B) Project B, because it has a higher IRR.
(C) The two projects are equivalent.
(D) Project A, because its net present worth is higher.

The use of IRR breaks down for mutually exclusive projects. Therefore, consider the net present worth, P, of each project. The net present worth of project A is

$$\begin{aligned}
P_\text{A} &= A(P/A,i,n) \\
&= -\$1{,}000{,}000 + (\$200{,}000)(P/A, 5\%, n \text{ approaches } \infty) \\
&= -\$1{,}000{,}000 + (\$200{,}000)\lim_{n\to\infty}\left(\frac{1.05^n - 1}{(0.05)(1.05)^n}\right) \\
&= -\$1{,}000{,}000 + (\$200{,}000)(20) \\
&= \$3{,}000{,}000
\end{aligned}$$

The net present worth of project B is

$$\begin{aligned} P_B &= -\$1{,}000{,}000 + (\$1{,}500{,}000)(P/F, 5\%, 1) \\ &= -\$1{,}000{,}000 + (\$1{,}500{,}000)(0.9524) \\ &= \$428{,}600 \end{aligned}$$

Since the net present worth of project A is higher, the company should choose project A.

The answer is (D).

ECONOMICS–57

Which plan is the least expensive way to purchase plant maintenance equipment? The discount rate is 11%.

plan A: $50,000 down, equal payments of $25,115.12 for 20 yr

plan B: nothing down, equal payments of $31,393.91 for 20 yr

plan C: $100,00 down, equal payments of $21,975.74 for 20 yr

(A) plan A (B) plan B (C) plan C (D) plan A or B

plan A

$$\begin{aligned} P &= A(P/A, i, n) = \$50{,}000 + (\$25{,}115.12)(P/A, 11\%, 20) \\ &= \$250{,}000 \end{aligned}$$

plan B

$$\begin{aligned} P &= (\$31{,}393.91)(P/A, 11\%, 20) \\ &= \$250{,}000 \end{aligned}$$

plan C

$$\begin{aligned} P &= \$100{,}000 + (\$21{,}975.74)(P/A, 11\%, 20) \\ &= \$275{,}000 \end{aligned}$$

Thus, plan C is the most expensive and plans A and B are equivalent.

The answer is (D).

ECONOMICS–58

The volatility, β, of a stock is found to be 1.5 times the stock market average. If the risk premium for buying stocks averages 8.3% and the present treasury bill rate (assumed to be risk free) is 7%, what is most nearly the expected return (ER) on the stock?

(A) 12.5% (B) 15.3% (C) 18.9% (D) 19.5%

$$\begin{aligned} \text{ER} &= \text{risk-free rate} + \beta(\text{market premium}) \\ &= 7\% + (1.5)(8.3\%) \\ &= 19.45\% \quad (19.5\%) \end{aligned}$$

The answer is (D).

ECONOMICS–59

What is a borrower of a particular loan almost always required to do during repayment?

(A) pay exactly the same amount of interest each payment
(B) repay the loan over an agreed-upon amount of time
(C) pay exactly the same amount of principal each payment
(D) both options (A) and (C)

The answer is (B).

ECONOMICS–60

What is "work in process" classified as?

(A) an asset (B) a liability (C) an expense (D) a revenue

Work in process is included in the working fund investments. The working fund investments is an asset not subject to depreciation.

The answer is (A).

ECONOMICS–61

What is the indirect product cost (IPC) spending variance?

(A) the difference between actual IPC and IPC absorbed

(B) the difference between actual IPC and IPC volume-adjusted budget

(C) the IPC volume-adjusted budget (fixed + (volume)(variable IPC rate))

(D) the IPC volume-adjusted budget minus the total IPC absorbed

The IPC spending variance is the difference between actual IPC and IPC volume-adjusted budget.

The answer is (B).

ECONOMICS–62

Firm A uses full absorption costing while firm B uses variable product costing. How will the financial statements of these companies differ?

(A) Firm A has a higher cost of goods sold and, therefore, a smaller profit.

(B) Firm A has a higher cost of goods sold, higher inventory value, and higher retained earnings.

(C) Firm A has a smaller cost of goods sold and a larger profit.

(D) Firm A has a smaller cost of goods sold, no change in inventory value, and no change in retained earnings.

Full absorption costing includes all direct and indirect, fixed and variable production costs. Variable product costing leaves fixed costs for the expense accounts. Therefore, the cost of goods sold is less under variable costing. Inventory value (an asset) is higher under full absorption. Since assets equal liabilities plus owner's equity, the owner's equity (retained earnings) must increase as the assets have.

The answer is (B).

ECONOMICS–63

How is the material purchase price variance defined?

(A) (quantity purchased)(actual price)
  – (quantity purchased)(standard price)

(B) (quantity issued – standard quantity)(standard price)

(C) (actual price – standard price)(quantity used)

(D) (quantity purchased – quantity used)(actual price)

The definition of material purchase price variance is given in option (A).

Note: Option (B) is the material usage variance.

The answer is (A).

ECONOMICS–64

Which of the following does NOT affect owner's equity?

(A) dividends paid
(B) license to start operation
(C) invested capital
(D) expense to get license to start operation

The license to start business is a company asset, not a part of owner's equity.

The answer is (B).

ECONOMICS–65

Higrow Company is planning to grow 30% during the next fiscal year. What has to increase if Higrow is to achieve their goal?

(A) the ratio of sales to total assets
(B) the ratio of total assets to equity
(C) equity
(D) any combination of (A), (B), and (C)

$$\text{sales} = \left(\frac{\text{sales}}{\text{total assets}}\right)\left(\frac{\text{total assets}}{\text{equity}}\right)(\text{equity})$$

Sales can grow only if at least one of the three terms on the right hand side of the equation grows.

The answer is (D).

ECONOMICS–66

Tops Corporation's gross margin is 45% of sales. Operating expenses such as sales and administration are 15% of sales. Tops is in a 40% tax bracket. What percent of sales is their profit after taxes?

(A) 0.0% (B) 5.0% (C) 18% (D) 24%

$$\begin{aligned}\text{before tax profit} &= \text{gross margin} - \text{sales and administrative costs}\\ &= 45\% - 15\%\\ &= 30\%\\ \text{after tax profit} &= (1 - \text{tax bracket})(\text{before tax profit})\\ &= (1 - 0.40)(30\%)\\ &= 18\%\end{aligned}$$

The answer is (C).

ECONOMICS–67

Z Corporation is applying for a short-term loan. In reviewing Z Corporation's financial records, the banker finds a current ratio of 2.0, an acid test ratio of 0.5, and an accounts receivable period of 70 d. What should the banker do?

(A) be concerned that Z Corporation will be unable to meet the payments
(B) suggest that Z Corporation lower its inventories
(C) suggest that Z Corporation be more aggressive in collecting on its invoices
(D) both options (B) and (C)

Z Corporation has invested heavily in inventory and accounts receivable. If it could change its accounts receivable collection period to between 30 and 60 days and invest less in inventory, the company would probably not need the loan.

The answer is (D).

ECONOMICS-68

Companies A and B are identical except for their inventory accounting systems. Company A uses the last-in, first-out convention while company B uses the first-in, first-out convention. How will their financial statements differ in an inflationary environment?

(A) Company A's profits will be higher and the book value of their inventory will be higher than for company B.

(B) Company A's profits will be higher and the book value of their inventory will be lower than for company B.

(C) Company B's profits and inventory book value will be higher than for company A.

(D) Company B's profits will be higher than A's, but inventory book value will be lower.

Last-in, first-out (LIFO) puts a higher value on the inventory that went into the cost of goods sold. Thus, the gross margin is lowered and profits are lowered. The remaining inventory is still valued at old prices, so its value is also low.

The answer is (C).

ECONOMICS-69

What is the acid test ratio?

(A) the ratio of owner's equity to total current liabilities
(B) the ratio of all assets to total liabilities
(C) the ratio of current assets (exclusive of inventory) to total current liabilities
(D) the ratio of gross margin to operating, sales, and administrative expenses

The answer is (C).

ECONOMICS–70

The balance sheet of Allied Company is as follows.

assets		liabilities	
cash	\$10,000	payables	\$17,000
receivables	\$12,000	notes due	\$6,000
inventory	\$7,000	long term debt	\$3,000
capital equipment	\$20,000	owner's equity	\$23,000
total	\$49,000	total	\$49,000

What is most nearly its acid test ratio?

(A) 0.39 (B) 0.59 (C) 0.85 (D) 1.1

$$\begin{aligned}\text{acid test ratio} &= \frac{\text{cash} + \text{accounts receivable}}{\text{total liabilities}}\\ &= \frac{\$10{,}000 + \$12{,}000}{\$17{,}000 + \$6000 + \$3000}\\ &= 0.846 \quad (0.85)\end{aligned}$$

The answer is (C).

3 SYSTEMS OF UNITS

UNITS–1

An iron block weighs 5 N and has a volume of 200 cm^3. What is most nearly the density of the block?

(A) 875 kg/m^3 (B) 988 kg/m^3 (C) 1250 kg/m^3 (D) 2550 kg/m^3

$$\rho = \frac{m}{V}$$

$$W = mg$$

$$m = \frac{W}{g} = \frac{5\ \text{N}}{9.81\ \frac{\text{m}}{\text{s}^2}} = 0.51\ \text{kg}$$

$$\rho = \frac{0.51\ \text{kg}}{200\ \text{cm}^3} = 0.00255\ \text{kg/cm}^3 \quad (2550\ \text{kg/m}^3)$$

The answer is (D).

UNITS–2

If the density of a gas is 0.003 $slugs/ft^3$, what is most nearly the specific weight of the gas?

(A) 9.0 N/m^3 (B) 15 N/m^3 (C) 76 N/m^3 (D) 98 N/m^3

The specific weight, γ, is defined as follows.

$$\begin{aligned}\gamma &= \rho g \\ &= \left(0.003\ \frac{\text{slug}}{\text{ft}^3}\right)\left(32.2\ \frac{\text{ft}}{\text{sec}^2}\right)\left(14.59\ \frac{\text{kg}}{\text{slug}}\right)\left(\frac{1\ \text{ft}}{0.3048\ \text{m}}\right)^2 \\ &= 15.2\ \text{kg/s}^2\text{·m}^2 \quad (15\ \text{N/m}^3)\end{aligned}$$

The answer is (B).

UNITS–3

The specific gravity of mercury relative to water is 13.55. What is most nearly the specific weight of mercury? (The specific weight of water is 62.4 lbf/ft^3.)

(A) 82.3 kN/m^3 (B) 102 kN/m^3 (C) 133 kN/m^3 (D) 151 kN/m^3

The specific weight, γ, is

$$\gamma = \rho g$$

The specific gravity, SG, of mercury is the ratio of the density (or unit weight) of mercury to the density (or unit weight) of water.

$$\text{SG} = \frac{\rho_{\text{mercury}}}{\rho_{\text{water}}} = \frac{\gamma_{\text{mercury}}}{\gamma_{\text{water}}} = 13.55$$

Rearranging,

$$\begin{aligned}\gamma_{\text{mercury}} &= 13.55\gamma_{\text{water}} \\ &= (13.55)\left(62.4\ \frac{\text{lbf}}{\text{ft}^3}\right)\left(4.449\ \frac{\text{N}}{\text{lbf}}\right)\left(\frac{1\ \text{ft}}{0.3048\ \text{m}}\right)^3 \\ &= 132.9\ \text{kN/m}^3 \quad (133\ \text{kN/m}^3)\end{aligned}$$

The answer is (C).

UNITS–4

If the specific weight of a liquid is 58.5 lbf/ft^3, what is most nearly the specific volume of the liquid?

(A) 0.532 cm^3/g (B) 0.675 cm^3/g (C) 0.950 cm^3/g (D) 1.07 cm^3/g

The specific weight, γ, is

$$\gamma = \left(58.5\ \frac{\text{lbf}}{\text{ft}^3}\right)\left(4.449\ \frac{\text{N}}{\text{lbf}}\right)\left(\frac{1\ \text{ft}}{0.3048\ \text{m}}\right)^3$$
$$= 9191.2\ \text{N/m}^3$$
$$\rho = \frac{\gamma}{g} = \frac{9191.2\ \frac{\text{N}}{\text{m}^3}}{9.81\ \frac{\text{m}}{\text{s}^2}}$$
$$= 936.9\ \text{kg/m}^3$$

The specific volume, υ, is

$$\upsilon = \frac{1}{\rho}$$
$$= \left(\frac{1}{936.9\ \frac{\text{kg}}{\text{m}^3}}\right)\left(100\ \frac{\text{cm}}{\text{m}}\right)^3\left(\frac{1\ \text{kg}}{1000\ \text{g}}\right)$$
$$= 1.0673\ \text{cm}^3/\text{g} \quad (1.07\ \text{cm}^3/\text{g})$$

The answer is (D).

UNITS–5

Which of the following are NOT units of pressure?

(A) kPa (B) N/cm^2 (C) bars (D) kg/m^2

All of the above are units of pressure (force over area) except for option (D), which has units of mass over area.

The answer is (D).

UNITS–6

A cylinder weighs 150 lbf. Its cross-sectional area is 40 in^2. When the cylinder stands vertically on one end, approximately what pressure does the cylinder exert on the floor?

(A) 14 kPa (B) 26 kPa (C) 63 kPa (D) 90 kPa

The pressure, p, is

$$\begin{aligned} p &= \frac{F}{A} \\ &= \left(\frac{150 \text{ lbf}}{40 \text{ in}^2}\right)\left(4.448 \ \frac{\text{N}}{\text{lbf}}\right)\left(\frac{1 \text{ in}}{2.54 \text{ cm}}\right)^2\left(100 \ \frac{\text{cm}}{\text{m}}\right)^2\left(\frac{1 \text{ kPa}}{1000 \text{ Pa}}\right) \\ &= 25.9 \text{ kPa} \quad (26 \text{ kPa}) \end{aligned}$$

The answer is (B).

UNITS–7

A column of water 100 cm high is most nearly equivalent to what pressure?

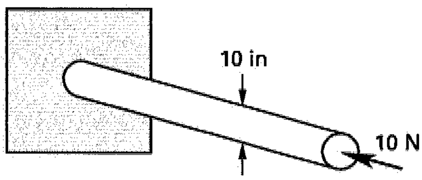
(A) 9800 dyne/cm^2 (B) 9800 Pa (C) 0.10 bar (D) 0.10 atm

$$\begin{aligned} p &= \gamma h \\ \gamma &= \rho g \\ p &= \rho g h \\ &= \left(1000 \ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(100 \text{ cm})\left(\frac{1 \text{ m}}{100 \text{ cm}}\right) \\ &= 9810 \text{ kg/m·s}^2 \quad (9800 \text{ N/m}^2 \text{ or } 9800 \text{ Pa}) \end{aligned}$$

The answer is (B).

UNITS–8

What is most nearly the direct compressive stress on a circular rod 10 in in diameter when a force of 10 N acts on one end?



(A) 0.050 kPa (B) 0.10 kPa (C) 0.15 kPa (D) 0.20 kPa

$$
\begin{aligned}
A &= \pi r^2 \\
&= \pi \left((5 \text{ in}) \left(2.54 \ \frac{\text{cm}}{\text{in}} \right) \left(\frac{1 \text{ m}}{100 \text{ cm}} \right) \right)^2 \\
&= 0.0507 \text{ m}^2
\end{aligned}
$$

Determine the stress, σ.

$$
\begin{aligned}
\sigma &= \frac{F}{A} \\
&= \frac{10 \text{ N}}{0.0507 \text{ m}^2} \\
&= 197 \text{ N/m}^2 \quad (0.20 \text{ kPa})
\end{aligned}
$$

The answer is (D).

UNITS–9

Water is flowing at a velocity of 5 m/s in a pipe with a radius of 10 in. At the temperature in the pipe, the density and viscosity of the water are as follows.

$$
\begin{aligned}
\rho &= 997.9 \text{ kg/m}^3 \\
\mu &= 1.131 \text{ Pa·s}
\end{aligned}
$$

What is most nearly the Reynolds number for this situation?

(A) 441 (B) 882 (C) 1140 (D) 2240

$$
\begin{aligned}
D &= 2r = (2)(10 \text{ in}) \left(0.0254 \ \frac{\text{m}}{\text{in}} \right) \\
&= 0.508 \text{ m} \\
\text{v} &= 5 \text{ m/s} \\
\mu &= 1.131 \text{ Pa·s} \quad (1.131 \text{ kg/m·s}) \\
\text{Re} &= \frac{\rho \text{v} D}{\mu} \\
&= \frac{\left(998 \ \frac{\text{kg}}{\text{m}^3} \right) \left(5 \ \frac{\text{m}}{\text{s}} \right) (0.508 \text{ m})}{1.131 \ \frac{\text{kg}}{\text{m·s}}} \\
&= 2241 \quad (2240)
\end{aligned}
$$

The answer is (D).

UNITS–10

What is most nearly the volumetric flow rate through a pipe 4 in in diameter carrying water at a velocity of 11 ft/sec?

(A) 595 cm^3/s (B) 726 cm^3/s (C) 993 cm^3/s (D) 27 200 cm^3/s

$$\begin{aligned} A &= \pi r^2 \\ &= \pi\left(\frac{4\text{ in}}{2}\right)^2\left(\frac{1\text{ ft}}{12\text{ in}}\right)^2 \\ &= 0.0872\text{ ft}^2 \\ Q &= \text{v}A \\ &= \left(11\ \frac{\text{ft}}{\text{sec}}\right)(0.0872\text{ ft}^2)\left(12\ \frac{\text{in}}{\text{ft}}\right)^3\left(2.54\ \frac{\text{cm}}{\text{in}}\right)^3 \\ &= 27\,155.9\text{ cm}^3/\text{s} \quad (27\,200\text{ cm}^3/\text{s}) \end{aligned}$$

The answer is (D).

UNITS–11

How long must a current of 5.0 A pass through a 10 Ω resistor until a charge of 1200 C passes through the resistor?

(A) 1 min (B) 2 min (C) 3 min (D) 4 min

$$\begin{aligned} I &= \frac{\text{charge}}{t} \\ t &= \left(\frac{1200\text{ C}}{5.0\text{ A}}\right)\left(\frac{1\text{ min}}{60\text{ sec}}\right) \\ &= 4\text{ min} \end{aligned}$$

The answer is (D).

UNITS–12

A car moving at 70 km/h has a mass of 1700 kg. The force necessary to decelerate it at a rate of 40 cm/s^2 is most nearly

(A) 0.680 N (B) 42.5 N (C) 680 N (D) 4250 N

Use Newton's second law.

$$F = ma$$

$$a = \left(40\ \frac{\text{cm}}{\text{s}^2}\right)\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)$$

$$= 0.4\ \text{m/s}^2$$

$$F = (1700\ \text{kg})\left(0.4\ \frac{\text{m}}{\text{s}^2}\right)$$

$$= 680\ \text{kg·m/s}^2 \quad (680\ \text{N})$$

The answer is (C).

UNITS–13

100 mL of water in a plastic bag of negligible mass is to be catapulted upward with an initial acceleration of 20.0 m/s^2. What initial force is necessary to do this?

(A) 2.0 N (B) 3.0 N (C) 15 N (D) 2.0 kN

Use Newton's second law. 100 mL is the same volume as 100 cm^3.

$$F - mg = ma$$

$$F = m(g + a)$$

$$m = \rho V$$

$$= \left(1\ \frac{\text{g}}{\text{cm}^3}\right)(100\ \text{cm}^3)\left(\frac{1\ \text{kg}}{1000\ \text{g}}\right)$$

$$= 0.10\ \text{kg}$$

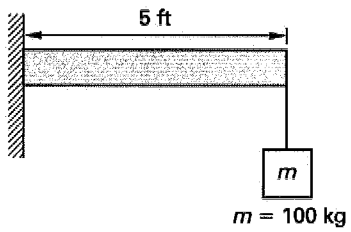
$$F = (0.1\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2} + 20\ \frac{\text{m}}{\text{s}^2}\right)$$

$$= 2.98\ \text{kg·m/s}^2 \quad (3.0\ \text{N})$$

The answer is (B).

UNITS–14

For the cantilever beam and applied force shown, what is the approximate resisting moment at the wall?



(A) 1500 N·m (B) 15 000 N·m (C) 49 000 N·m (D) 170 000 N·m

For the cantilever to be in static equilibrium, the sum of the moments taken at the wall must be zero.

$$\sum \text{moments} = 0$$

$$M_{\text{resist}} - M_{\text{applied}} = 0$$

$$M_{\text{resist}} = M_{\text{applied}}$$

$$= LF_{\text{applied}}$$

$$L = (5 \text{ ft})\left(0.3048 \ \frac{\text{m}}{\text{ft}}\right)$$

$$= 1.524 \text{ m}$$

$$F = ma$$

$$= (100 \text{ kg})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)$$

$$= 981 \text{ kg·m/s}^2 \quad (981 \text{ N})$$

$$M_{\text{resist}} = (1.524 \text{ m})(981 \text{ N})$$

$$= 1495 \text{ N·m} \quad (1500 \text{ N·m})$$

The answer is (A).

UNITS–15

Which of the following is NOT a unit of work?

(A) N·m (B) erg (C) kg·m^2/s^2 (D) dyne

The units of work are force times distance or power multiplied by time. Therefore, all of the choices are units of work except for the dyne. A dyne is a unit of force.

The answer is (D).

UNITS–16

Which of the following is the definition of a joule?

(A) a unit of power
(B) a N·m
(C) a kg·m/s^2
(D) a rate of change of energy

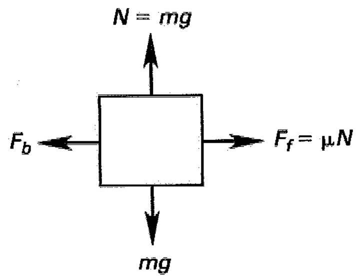
A joule is a unit of energy and is defined as a newton·meter (N·m). None of the other choices are units of energy.

The answer is (B).

UNITS–17

A boy pulls a sled with a mass of 20 kg horizontally over a surface with a coefficient of friction of 0.20. It takes him 10 min to pull the sled 100 yd. What is his average power output over these 10 min?

(A) 4 W (B) 6 W (C) 8 W (D) 10 W



N is the normal force, and μ is the coefficient of friction.

The force that the boy must pull with, F_b, must be large enough to overcome the frictional force.

$$\begin{aligned} F_b &= F_f = \mu N \\ &= (20\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.20) \\ &= 39.24\ \text{kg·m/s}^2 \quad (39.24\ \text{N}) \end{aligned}$$

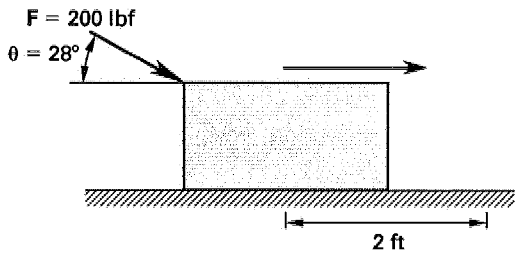
$$\begin{aligned} W &= F{\cdot}x = (39.24\ \text{N})(100\ \text{yd})\left(3\ \frac{\text{ft}}{\text{yd}}\right)\left(0.3048\ \frac{\text{m}}{\text{ft}}\right) \\ &= 3588\ \text{J} \end{aligned}$$

$$\begin{aligned} \overline{P} &= \frac{W}{t} = \left(\frac{3588\ \text{J}}{10\ \text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) \\ &= 5.98\ \text{W} \quad (6\ \text{W}) \end{aligned}$$

The answer is (B).

UNITS–18

A force of 200 lbf acts on a block at an angle of 28° with respect to horizontal. The block is pushed 2 ft horizontally. The work done by this force is most nearly



(A) 210 J (B) 320 J (C) 480 J (D) 540 J

The work done by the force is

$$\begin{aligned} W &= Fx\cos\theta \\ &= (200\ \text{lbf})(2\ \text{ft})\cos 28°\left(4.45\ \frac{\text{N}}{\text{lbf}}\right)\left(0.3048\ \frac{\text{m}}{\text{ft}}\right) \\ &= 479.04\ \text{J} \quad (480\ \text{J}) \end{aligned}$$

The answer is (C).

UNITS–19

Two particles moving in the same direction collide, stick together, and continue their motion together. Each particle has a mass of 10 g, and their respective velocities before the collision were 10 m/s and 100 m/s. The energy of the system after the collision is most nearly

(A) 22 J (B) 30 J (C) 48 J (D) 78 J

$$E_f = \tfrac{1}{2} m_{\text{total}} \text{v}_f^2$$

$$m_{\text{total}} = m_1 + m_2$$

v_f is the velocity of the two masses after the collision. Use the principle of conservation of momentum.

$$m_1 \text{v}_1 + m_2 \text{v}_2 = m_{\text{total}} \text{v}_f$$

$$(10 \text{ g}) \left(10 \ \frac{\text{m}}{\text{s}} + 100 \ \frac{\text{m}}{\text{s}}\right) = (20 \text{ g}) \text{v}_f$$

$$\text{v}_f = 55 \text{ m/s}$$

$$E_f = \left(\frac{1}{2}\right) (0.02 \text{ kg}) \left(55 \ \frac{\text{m}}{\text{s}}\right)^2$$

$$= 30.25 \text{ kg·m}^2/\text{s}^2 \quad (30 \text{ J})$$

The answer is (B).

UNITS–20

Two protons, each of charge 1.6×10^{-19} C, are 3.4 μm apart. What is most nearly the change in the potential energy of the protons if they are brought 63 nm closer together?

(A) 6.4×10^{-29} J (B) 7.2×10^{-29} J (C) 1.3×10^{-24} J (D) 3.6×10^{-21} J

The potential energy of a system of two charges is given by the following.

$$U = k \frac{q_1 q_2}{r}$$

$$k = 8.99 \times 10^9 \text{ N·m}^2/\text{C}^2$$

q_1 and q_2 are the charges, and r is the distance between the charges.

$$\begin{aligned}\Delta U &= U_f - U_i \\ &= k\frac{q_1q_2}{r_f} - k\frac{q_1q_2}{r_i} \\ &= kq_1q_2\left(\frac{1}{r_f} - \frac{1}{r_i}\right) \\ &= \left(8.99 \times 10^9 \ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.60 \times 10^{-19} \ \text{C})^2 \\ &\quad \times \left(\frac{1}{3.4 \times 10^{-6} \ \text{m}} - \frac{1}{63 \times 10^{-9} \ \text{m}}\right) \\ &= -3.6 \times 10^{-21} \ \text{J}\end{aligned}$$

The answer is (D).

UNITS–21

According to the Bohr model of the atom, the energy of the atom when the electron is at the first Bohr radius is given by

$$E_0 = \frac{mk^2e^4}{2\hbar^2}$$

m	mass of the electron	9.1095×10^{-31} kg
k	Coulomb's constant	8.99×10^9 N·m²/C²
e	electron charge	1.602×10^{-19} C
$\hbar$	$\frac{h}{2\pi}$	
h	Planck's constant	6.626×10^{-34} J·s

Using these values in metric units, calculate the value of E_0.

(A) 2.18×10^{-36} W (B) 2.18×10^{-36} J (C) 2.18×10^{-18} J (D) 2.18×10^{-18} W

$$E_0 = \frac{\left(9.1095 \times 10^{-31}\ \text{kg}\right)\left(8.99 \times 10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)^2 \left(1.602 \times 10^{-19}\ \text{C}\right)^4}{(2)\left(\frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{2\pi}\right)^2}$$

$$= 2.18 \times 10^{-18}\ \text{J}$$

The answer is (C).

UNITS–22

A copper bar is 90 cm long at 86°F. What is most nearly the increase in its length when the bar is heated to 95°F? The linear expansion coefficient for copper is 1.7×10^{-5} 1/°C.

(A) 2.1×10^{-5} m
(B) 3.2×10^{-5} m
(C) 5.3×10^{-5} m
(D) 7.7×10^{-5} m

The change in length of the bar is given by the following.

$$\Delta L = \alpha L_0 \Delta T$$

Convert the temperatures from °F to °C.

$$°\text{C} = \left(\frac{5}{9}\right)(°\text{F} - 32°)$$

$$T_1 = \left(\frac{5}{9}\right)(95°\text{F} - 32°) = 35°\text{C}$$

$$T_2 = \left(\frac{5}{9}\right)(86°\text{F} - 32°) = 30°\text{C}$$

$$\begin{aligned}\Delta T &= T_2 - T_1 \\ &= 35°\text{C} - 30°\text{C} \\ &= 5°\text{C}\end{aligned}$$

$$\begin{aligned}\Delta L &= \left(1.7 \times 10^{-5}\ \frac{1}{°\text{C}}\right)(0.9\ \text{m})(5°\text{C}) \\ &= 7.7 \times 10^{-5}\ \text{m}\end{aligned}$$

The answer is (D).

UNITS–23

A slab of iron with initial temperature $T_{i1} = 48°\text{C}$ is used to heat a flat glass plate that has an initial temperature of $T_{g1} = 18°\text{C}$. No heat is lost to the environment. The masses are $m_i = 0.49$ kg for the slab and $m_g = 310$ g for the plate. What is the amount of heat transferred when the two have reached the equilibrium temperatures? The specific heats are $c_i = 0.11$ kcal/kg·°C for iron, and $c_g = 0.20$ kcal/kg·°C for glass.

(A) 860 cal (B) 32 kcal (C) 53 kcal (D) 320 kcal

The heat transferred by an object is given by the following equation.

$$Q = mc\Delta T$$

Q is the heat transferred, m is the mass of the object, ΔT is the change in temperature of the object, and c is the specific heat capacity of the object.

Since no heat is lost, $Q_i = Q_g$.

$$m_i c_i \Delta T_i = m_g c_g \Delta T_g$$

The final temperature of each object is the same.

$$-m_i c_i (T_2 - T_{i1}) = m_g c_g (T_2 - T_{g1})$$

Solve the equation for T_2, then substitute to find Q.

$$\begin{aligned} T_2 &= \frac{m_i c_i T_{1i} + m_g c_g T_{1g}}{m_i c_i + m_g c_g} \\ &= \frac{(0.49 \text{ kg})\left(110 \, \frac{\text{cal}}{\text{kg}\cdot°\text{C}}\right)(48°\text{C}) + (0.310 \text{ kg})\left(200 \, \frac{\text{cal}}{\text{kg}\cdot°\text{C}}\right)(18°\text{C})}{(0.49 \text{ kg})\left(110 \, \frac{\text{cal}}{\text{kg}\cdot°\text{C}}\right) + (0.310 \text{ kg})\left(200 \, \frac{\text{cal}}{\text{kg}\cdot°\text{C}}\right)} \\ &= 32.0°\text{C} \end{aligned}$$

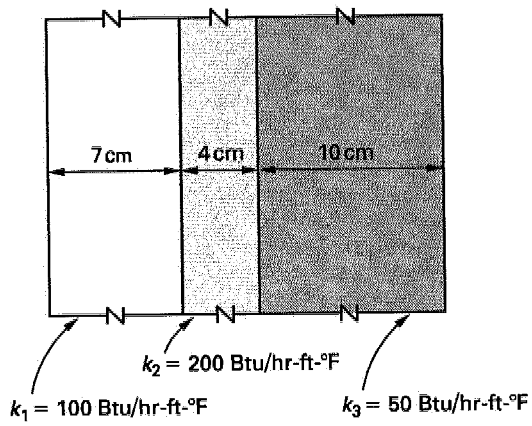
$$\begin{aligned} Q_i &= m_i c_i (T_2 - T_{i1}) \\ &= (0.490 \text{ kg})\left(0.11 \times 10^3 \, \frac{\text{cal}}{\text{kg}\cdot°\text{C}}\right)(32°\text{C} - 48°\text{C}) \\ &= -860 \text{ cal} \end{aligned}$$

Thus, 860 calories were transferred from the iron slab to the glass plate.

The answer is (A).

UNITS–24

What is the average thermal conductivity for the composite material shown?



(A) 42 W/m·K (B) 75.5 W/m·K (C) 115 W/m·K (D) 155 W/m·K

$$\frac{L_{\text{total}}}{\bar{k}A} = \frac{1}{A}\left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}\right)$$

$$\frac{L_{\text{total}}}{\bar{k}} = \left(\frac{0.07\ \text{m}}{100\ \frac{\text{Btu}}{\text{hr-ft-°F}}} + \frac{0.04\ \text{m}}{200\ \frac{\text{Btu}}{\text{hr-ft-°F}}} + \frac{0.10\ \text{m}}{50\ \frac{\text{Btu}}{\text{hr-ft-°F}}}\right)\left(1.7305\ \frac{\frac{\text{Btu}}{\text{hr-ft-°F}}}{\frac{\text{W}}{\text{m·K}}}\right)$$

$$= 0.0050\ \text{m}^2\text{·K/W}$$

$$L_{\text{total}} = 0.21\ \text{m}$$

$$\bar{k} = \frac{0.21\ \text{m}}{0.0050\ \frac{\text{m}^2\text{·K}}{\text{W}}}$$

$$= 42\ \text{W/m·K}$$

The answer is (A).

UNITS–25

Approximate the energy transfer rate across a 6 in wall of firebrick with a temperature difference across the wall of 50°C. The average thermal conductivity of firebrick is 0.65 Btu/hr-ft-°F.

(A) 110 W/m² (B) 290 W/m² (C) 370 W/m² (D) 430 W/m²

$$\Delta T = \left(\frac{9}{5}\right)(50°\text{C}) = 90°\text{F}$$

$$\begin{aligned}\dot{Q} &= -k\,\frac{dT}{dx} \\ &= -k\,\frac{\Delta T}{\Delta x} \quad \text{[for linear profile]} \\ &= \frac{\left(0.65\,\frac{\text{Btu}}{\text{hr-ft-°F}}\right)(90°\text{F})}{(6\text{ in})\left(\frac{1\text{ ft}}{12\text{ in}}\right)} \\ &= 117\text{ Btu/hr-ft}^2\end{aligned}$$

$$\begin{aligned}Q &= \left(117\,\frac{\text{Btu}}{\text{hr-ft}^2}\right)\left(0.2931\,\frac{\text{W-hr}}{\text{Btu}}\right)\left(3.281\,\frac{\text{ft}}{\text{m}}\right)^2 \\ &= 369\text{ W/m}^2 \quad (370\text{ W/m}^2)\end{aligned}$$

The answer is (C).

UNITS–26

A house has brick walls 15 mm thick. On a cold winter day, the temperatures of the inner and outer surfaces of the walls are found to be 20°C and −12°C, respectively. There is 120 m² of exterior wall space. The thermal conductivity of brick is 0.711 J/m·s·°C. Most nearly how much heat is lost through the walls per hour?

(A) 180 J (B) 13 kJ (C) 660 kJ (D) 660 MJ

Calculate the temperature difference, ΔT.

$$\begin{aligned}\Delta T &= T_{\text{inner}} - T_{\text{outer}} \\ &= 20°\text{C} - (-12°\text{C}) \\ &= 32°\text{C}\end{aligned}$$

The thickness of the wall, x, is 0.015 m.

$$\begin{aligned} Q &= \frac{kA\Delta T}{x} \\ &= \frac{\left(0.711\ \frac{\text{J}}{\text{m}\cdot\text{s}\cdot{}^\circ\text{C}}\right)(120\ \text{m}^2)\left(3600\ \frac{\text{s}}{\text{h}}\right)(32^\circ\text{C})}{0.015\ \text{m}} \\ &= 655\times 10^6\ \text{J/h} \quad (660\ \text{MJ/h}) \end{aligned}$$

Thus, the heat transferred per hour is most nearly 660 MJ.

The answer is (D).

UNITS–27

Air has a specific heat, c_p, of 1 kJ/kg·K. If 2 Btu of energy are added to 100 g of air, what is most nearly the change in air temperature?

(A) 10°C (B) 21°C (C) 44°C (D) 88°C

$$Q = mc_p\Delta T$$

$$\Delta T = \frac{Q}{mc_p}$$

$$\begin{aligned} Q &= (2\ \text{Btu})\left(1.055\ \frac{\text{kJ}}{\text{Btu}}\right) \\ &= 2.11\ \text{kJ} \end{aligned}$$

$$\begin{aligned} m &= (100\ \text{g})\left(\frac{1\ \text{kg}}{1000\ \text{g}}\right) \\ &= 0.10\ \text{kg} \end{aligned}$$

$$c_p = 1\ \text{kJ/kg}\cdot\text{K}$$

$$\begin{aligned} \Delta T &= \frac{2.11\ \text{kJ}}{(0.10\ \text{kg})\left(1\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)} \\ &= 21.1\text{K} \quad (21^\circ\text{C}) \end{aligned}$$

Note: The temperature differences of 1K and 1°C are equivalent.

The answer is (B).

UNITS–28

Air has a specific heat, c_p, of 1 kJ/kg·K. If 100 g of air are heated with a 1500 W heater, which of the following occurs?

I. The air heats up at a rate of 15K/s.
II. The air reaches a final temperature of 1500K.
III. The air undergoes a nonisentropic process.

(A) I only (B) I and II (C) I, II, and III (D) I and III

$$Q = mc_p\Delta T$$

$$\dot{Q} = mc_p\Delta\dot{T}$$

$$\Delta\dot{T} = \frac{\dot{Q}}{mc_p}$$

$$= \frac{1500\ \frac{\text{J}}{\text{s}}}{(0.1\ \text{kg})\left(1000\ \frac{\text{J}}{\text{kg·K}}\right)}$$

$$= 15\ \text{K/s}$$

Thus, I is correct.

It is not possible to predict the final temperature of the air without knowing the length of time it is being heated. Therefore, II is false.

The addition of heat is a nonisentropic process (the entropy of the air changes). Therefore, III is correct.

The answer is (D).

UNITS–29

The change in enthalpy of an incompressible liquid with constant specific heat is given by

$$h_2 - h_1 = c(T_2 - T_1) + v(p_2 - p_1)$$

T_n is the temperature at a state n, p_n is the pressure at state n, and v is the specific volume of the liquid.

Water, with c_p = 4.18 kJ/kg·K and $v = 1.00 \times 10^{-3}$ m^3/kg, has the following final states.

state 1: $T_1 = 19°\text{C}$ $p_1 = 0.1013$ MPa
state 2: $T_2 = 30°\text{C}$ $p_2 = 0.113$ MPa

What is the change in enthalpy from state 1 to state 2?

(A) 46 kJ/kg (B) 46 kN/kg (C) 46 kPa/kg (D) 56 kJ/kg

$$p_1 = 101\,300 \text{ Pa}$$
$$p_2 = 111\,300 \text{ Pa}$$

Temperature differences need not be in K, since a temperature difference of 1°C equals a temperature difference of 1K. The specific volume, v, and specific heat, c_p, are already in consistent units.

$$h_2 - h_1 = \left(4180 \frac{\text{J}}{\text{kg·K}}\right)(30°\text{C} - 19°\text{C}) + (111\,300 \text{ Pa} - 101\,300 \text{ Pa})\left(0.001 \frac{\text{m}^3}{\text{kg}}\right)$$
$$= 46\,000 \text{ J/kg} \quad (46.0 \text{ kJ/kg})$$

The answer is (A).

UNITS–30

In a constant-temperature, closed-system process, 100 Btu of heat are transferred to the working fluid at 100°F. What is the change in entropy of the working fluid?

(A) 0.18 kJ/K (B) 0.25 kJ/K (C) 0.34 kJ/K (D) 0.57 kJ/K

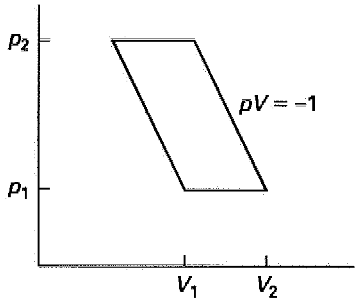
For a closed system at a constant temperature,

$$S_2 - S_1 = \frac{dQ}{T}$$
$$= \left(\frac{100 \text{ Btu}}{100°\text{F} + 460°}\right)\left(1.06 \frac{\text{kJ}}{\text{Btu}}\right)\left(1.8 \frac{°\text{R}}{\text{K}}\right)$$
$$= 0.34 \text{ kJ/K}$$

The answer is (C).

UNITS–31

In the illustration, pressure is plotted using a scale of 10 000 Pa per unit, volume is plotted using a scale of 1 L per unit, and the area enclosed by the cycle is 3 units^2. What is the work done during the cycle?



(A) 3×10^{-2} W·s (B) 3×10^{7} ergs (C) 30 J (D) 30 000 N·m

The area enclosed by the cycle, W, is

$$\begin{aligned} W &= F \cdot x \\ &= pV \\ &= (3)\left(10\,000\ \frac{\text{N}}{\text{m}^2}\right)(1\ \text{L})\left(0.001\ \frac{\text{m}^3}{\text{L}}\right) \\ &= 30\ \text{N·m} \quad (30\ \text{J}) \end{aligned}$$

The answer is (C).

UNITS–32

If a $^1/_3$ hp pump runs for 20 min, what is the energy used?

(A) 0.060 ergs (B) 0.25 kW (C) 0.30 MJ (D) 0.11 kW·h

$$\begin{aligned} W = Pt &= \left(\frac{1}{3}\ \text{hp}\right)\left(0.7457\ \frac{\text{kW}}{\text{hp}}\right)(20\ \text{min})\left(\frac{1\ \text{h}}{60\ \text{min}}\right)\left(3600\ \frac{\text{s}}{\text{h}}\right) \\ &= 0.298\ \text{MJ} \quad (0.30\ \text{MJ}) \end{aligned}$$

The answer is (C).

UNITS–33

A machine repeatedly accelerates a 1 kg mass at 1 m/s^2 for 1 min. The machine runs at 60 rpm. What is the power output of the machine?

(A) 1 erg (B) 1 cal (C) 1 J (D) 1 W

$$P = \frac{W}{T}$$
$$W = Fx$$
$$F = ma$$

T is the period of revolution.

$$\begin{aligned} f &= \left(60\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) \\ &= 1\ \text{Hz} \\ T &= \frac{1}{f} = \frac{1}{\frac{1}{1\ \text{s}}} \\ &= 1\ \text{s} \\ P &= \frac{max}{T} \\ &= \frac{(1\ \text{kg})\left(1\ \frac{\text{m}}{\text{s}^2}\right)(1\ \text{m})}{1\ \text{s}} \\ &= 1\ \text{W} \end{aligned}$$

The answer is (D).

UNITS–34

A power of 6 kW is supplied to the motor of a crane. The motor has an efficiency of 90%. With most nearly what constant speed does the crane lift an 800 lbf weight?

(A) 0.091 m/s (B) 0.32 m/s (C) 0.98 m/s (D) 1.5 m/s

$$\begin{aligned} P &= \frac{dW}{dt} \\ &= \frac{Fx}{t} \\ &= F\text{v} \\ \text{v} &= \frac{P}{F} \end{aligned}$$

P_{in} is the input power, and η is the efficiency.

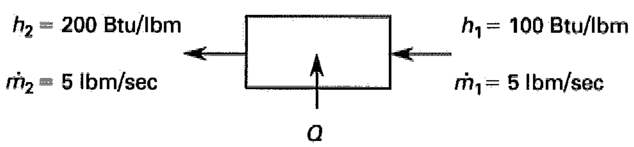
The useful power, P_r, is

$$\begin{aligned} P_r &= \eta P_{in} \\ &= (0.90)(6 \text{ kW})\left(1000 \frac{\text{W}}{\text{kW}}\right) \\ &= 5400 \text{ W} \\ \text{v} &= \frac{P_r}{F} \\ &= \frac{P_r}{mg} \\ &= \left(\frac{5400 \text{ W}}{800 \text{ lbf}}\right)\left(\frac{1 \text{ lbf}}{4.45 \text{ N}}\right) \\ &= 1.52 \text{ W/N} \quad (1.5 \text{ m/s}) \end{aligned}$$

The answer is (D).

UNITS–35

Given a heat exchanger with specified inlet and outlet enthalpies, what is most nearly the energy requirement for the heating coil?



(A) 500 kW (B) 530 kW (C) 560 kW (D) 600 kW

$$\begin{aligned} Q &= \dot{m}(h_2 - h_1) \\ &= \left(5 \frac{\text{lbm}}{\text{s}}\right)\left(200 \frac{\text{Btu}}{\text{lbm}} - 100 \frac{\text{Btu}}{\text{lbm}}\right)\left(1.055 \frac{\text{kJ}}{\text{Btu}}\right) \\ &= 527.5 \text{ kJ/s} \quad (530 \text{ kW}) \end{aligned}$$

The answer is (B).

UNITS–36

An engine has an efficiency of 26%. The engine uses 2 gal/hr of gasoline. Gasoline has a heating value of 20 500 Btu/lbm and a specific gravity of 0.8. What is most nearly the power output of the engine?

(A) 0.33 kW (B) 21 kW (C) 26 kW (D) 42 kW

First, find P_{in}, the input power of the engine.

$$\begin{aligned} P_{\text{in}} &= \dot{m}c_p = \dot{V}\rho c_p = \dot{V}(\text{SG})\rho_{\text{water}}c_p \\ &= \left(2\ \frac{\text{gal}}{\text{hr}}\right)\left(\frac{1\ \text{ft}^3}{7.48\ \text{gal}}\right)\left(0.8\ \frac{\text{lbm}}{\text{ft}^3}\right)\left(62.4\ \frac{\text{lbm}}{\text{ft}^3}\right) \\ &\quad \times \left(20{,}500\ \frac{\text{Btu}}{\text{lbm}}\right)\left(1.054\ \frac{\text{kJ}}{\text{Btu}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) \\ &= 80\ \text{kW} \end{aligned}$$

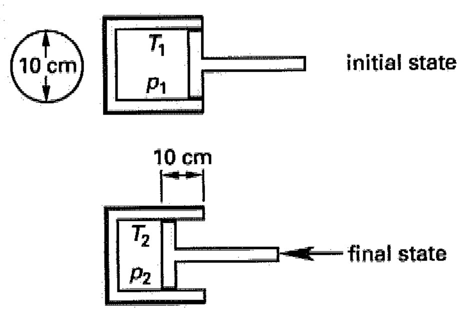
Next, find P_{out}, the power output of the engine.

$$\begin{aligned} P_{\text{out}} &= \eta P_{\text{in}} \\ &= (0.26)(80\ \text{kW}) \\ &= 20.8\ \text{kW} \quad (21\ \text{kW}) \end{aligned}$$

The answer is (B).

UNITS–37

2 L of an ideal gas, at a temperature of $T_1 = 25°\text{C}$ and a pressure of $p_1 = 0.101$ MPa, are in a 10 cm diameter cylinder with a piston at one end. The piston is depressed, so that the cylinder is shortened by 10 cm. The temperature increases by 2°C. What is most nearly the change in pressure?



(A) 0.16 MPa (B) 0.17 MPa (C) 0.25 MPa (D) 0.33 MPa

Apply the ideal gas law to the gas in the cylinder.

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

$$p_2 = \frac{p_1V_1T_2}{T_1V_2}$$

$$T_1 = 25°\text{C} + 273° = 298\text{K}$$

$$T_2 = 27°\text{C} + 273° = 300\text{K}$$

$$\Delta V = \Delta LA = \Delta L(\pi r^2) = (-10\text{ cm})\pi(5\text{ cm})^2 = -785\text{ cm}^3$$

Note that $\Delta V < 0$ because the piston is depressed.

$$V_2 = V_1 + \Delta V = 2000\text{ cm}^3 - 785\text{ cm}^3 = 1215\text{ cm}^3$$

$$p_2 = \frac{(0.101\text{ MPa})(2000\text{ cm}^3)(300\text{K})}{(298\text{K})(1215\text{ cm}^3)} = 0.167\text{ MPa} \quad (0.17\text{ MPa})$$

The answer is (B).

UNITS–38

The average power output of a cylinder in a combustion engine is given by

$$\overline{P} = pLAN$$

p is the average pressure on the piston during the piston stroke, L is the length of the piston stroke, A is the area of the piston head, and N is the number of strokes per second.

An eight-cylinder engine has the following specifications at optimum speed.

p	283 kPa
L	14 cm
d	diameter of piston head, 12 cm
N	1500 strokes/min

What is the average power output of this engine?

(A) 89.5 N/s (B) 89.5 kW (C) 89.5×10^3 J·m/s (D) 89.5 kJ

$$\overline{P} = pLAN \quad \text{[for one cylinder]}$$

$$\overline{P}_{\text{total}} = 8pLAN \quad \text{[for eight cylinders]}$$

$$\begin{aligned} A &= \pi r^2 = \pi \left(\frac{d}{2}\right)^2 \\ &= \pi \left(\frac{12\text{ cm}}{2}\right)^2 \\ &= 0.0113\text{ m}^2 \end{aligned}$$

$$\begin{aligned} \overline{P}_{\text{total}} &= (8)(283\,000\text{ Pa})(0.14\text{ m})(0.0113\text{ m}^2)\left(1500\ \frac{\text{strokes}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) \\ &= 89\,500\text{ J/s} \quad (89.5\text{ kW}) \end{aligned}$$

The answer is (B).

UNITS-39

What is the power required to transfer 97 000 C of charge through a potential rise of 50 V in 1 hr?

(A) 0.55 kW (B) 0.95 kW (C) 1.3 kW (D) 2.8 kW

$$W = qV$$

q is the charge, and V is the potential rise.

$$\begin{aligned} P &= \frac{W}{t} \\ &= \frac{qV}{t} \\ &= \frac{(97\,000\text{ C})(50\text{ V})}{3600\text{ s}}\left(\frac{1}{1000\ \frac{\text{W}}{\text{kW}}}\right) \\ &= 1.34\text{ kW} \quad (1.3\text{ kW}) \end{aligned}$$

The answer is (C).

UNITS–40

A current of 7 A passes through a 12 Ω resistor. What is the power dissipated in the resistor?

(A) 84 W (B) 0.59 hp (C) 0.79 hp (D) 7.5 hp

$$P = I^2R$$

I is the current, and R is the resistance.

$$P = (7\ \text{A})^2(12\ \Omega)\left(\frac{1\ \text{hp}}{745.7\ \text{W}}\right)$$
$$= 0.79\ \text{hp}$$

The answer is (C).

UNITS–41

If the average energy in a nuclear reaction is 200 MeV/fission, what is the power output of a reactor experiencing 2.34×10^{19} fissions/s?

(A) 550 W (B) 120 kW (C) 35 MW (D) 750 MW

The power output of the reactor is the energy per fission times the number of fissions per second.

$$P = \left(200\ \frac{\text{MeV}}{\text{fission}}\right)\left(2.34 \times 10^{19}\ \frac{\text{fissions}}{\text{s}}\right)\left(\frac{1 \times 10^{6}\ \text{eV}}{\text{MeV}}\right)\left(1.602 \times 10^{-19}\ \frac{\text{J}}{\text{eV}}\right)$$
$$= 750 \times 10^{6}\ \text{W} \quad (750\ \text{MW})$$

The answer is (D).

4 FLUID STATICS AND DYNAMICS

FLUIDS–1

Which statement is true for a fluid?

(A) It cannot sustain a shear force.
(B) It cannot sustain a shear force at rest.
(C) It is a liquid only.
(D) It has a very regular molecular structure.

A fluid is defined as a substance that deforms continuously under the application of a shear force. This means that it cannot sustain a shear force at rest. Therefore, option (B) is true.

The answer is (B).

FLUIDS–2

Which of the following is NOT a basic component of motion of a fluid element?

(A) translation (B) rotation (C) angular distortion (D) twist

The motion of a fluid element may be divided into three categories: translation, rotation, and distortion. Distortion can be further subdivided into angular and volume distortion. The only choice that is not a basic component of fluid element motion is twist.

The answer is (D).

FLUIDS–3

Which of the following must be satisfied by the flow of any fluid, real or ideal?

I. Newton's second law of motion
II. the continuity equation
III. the requirement of a uniform velocity distribution
IV. Newton's law of viscosity
V. the principle of conservation of energy

(A) I, II, and III (B) I, II, and IV (C) I, II, and V (D) I, II, III, and IV

Newton's second law, the continuity equation, and the principle of conservation of energy always apply for any fluid.

The answer is (C).

FLUIDS–4

What is the definition of pressure?

(A) $\frac{\text{area}}{\text{force}}$ (B) $\lim_{\text{force}\to 0} \frac{\text{force}}{\text{area}}$ (C) $\lim_{\text{area}\to 0} \frac{\text{force}}{\text{area}}$ (D) $\lim_{\text{force}\to 0} \frac{\text{area}}{\text{force}}$

The mathematical definition of pressure is

$$\lim_{\text{area}\to 0} \frac{\text{force}}{\text{area}}$$

The answer is (C).

FLUIDS–5

For a fluid, viscosity is defined as the constant of proportionality between shear stress and what other variable?

(A) time derivative of pressure
(B) time derivative of density
(C) spatial derivative of velocity
(D) spatial derivative of density

By definition,

$$\tau = \mu \frac{dv}{dy}$$

Thus, viscosity, μ, is the constant of proportionality between the shear stress, τ, and the gradient (spatial derivative) of the velocity.

The answer is (C).

FLUIDS–6

Surface tension has which of the following properties?

I. It has units of force per unit length.
II. It exists whenever there is a density discontinuity.
III. It is strongly affected by pressure.

(A) I only (B) II only (C) III only (D) I and II

III is incorrect because pressure only slightly affects surface tension. I and II are correct.

The answer is (D).

FLUIDS–7

A leak from a faucet comes out in separate drops. Which of the following is the main cause of this phenomenon?

(A) gravity (B) air resistance (C) viscosity of the fluid (D) surface tension

Surface tension is caused by the molecular cohesive forces in a fluid. It is the main cause of the formation of the drops of water.

The answer is (D).

FLUIDS–8

The surface tension of water in air is approximately 0.0756 N/m. If the atmospheric pressure is 101 kPa (abs), what is the pressure inside a droplet 0.254 mm in diameter?

(A) 99.83 kPa (abs) (B) 101.0 kPa (abs) (C) 101.5 kPa (abs) (D) 102.2 kPa (abs)

For a spherical droplet,

$$\Delta p = p_{\text{in}} - p_{\text{out}} = \frac{2\sigma}{r}$$

$$\begin{aligned} p_{\text{in}} &= p_{\text{out}} + \frac{2\sigma}{r} \\ &= 101\ \text{kPa} + \frac{(4)\left(7.56 \times 10^{-5}\ \frac{\text{kN}}{\text{m}}\right)}{25.4 \times 10^{-5}\ \text{m}} \\ &= 102.2\ \text{kPa (abs)} \end{aligned}$$

The answer is (D).

FLUIDS–9

Which of the following describes shear stress in a moving Newtonian fluid?

(A) It is proportional to the absolute viscosity.
(B) It is proportional to the velocity gradient at the point of interest.
(C) It is nonexistent.
(D) both A and B

$$\tau = \mu \frac{dv}{dy}$$

Shear stress is proportional to the velocity gradient at a point, as well as the absolute viscosity.

The answer is (D).

FLUIDS–10

If the shear stress in a fluid varies linearly with the velocity gradient, which of the following describes the fluid?

(A) It is inviscid.
(B) It is a perfect gas.
(C) It is a Newtonian fluid.
(D) It is at a constant temperature.

In order for shear stress to vary linearly with the velocity gradient, the fluid must be Newtonian.

The answer is (C).

FLUIDS–11

How are lines of constant pressure in a fluid related to the force field?

(A) They are parallel to the force field.
(B) They are perpendicular to the force field.
(C) They are at a 45° angle to the force field.
(D) They are perpendicular only to the force of gravity.

Lines of constant pressure are always perpendicular to the direction of the force field.

The answer is (B).

FLUIDS–12

Which of the following statements about a streamline is most accurate?

(A) It is a path of a fluid particle.
(B) It is a line normal to the velocity vector everywhere.
(C) It is fixed in space in steady flow.
(D) It is defined for nonuniform flow only.

Streamlines are tangent to the velocity vectors at every point in the field. Thus, for a steady flow $d\mathbf{v}/dt = 0$, a streamline is fixed in space.

The answer is (C).

FLUIDS–13

Which of the following describes a streamline?

I. It is a mathematical concept.
II. It cannot be crossed by the flow.
III. It is a line of constant entropy.

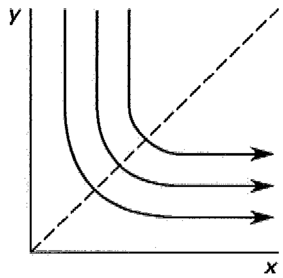
(A) I only (B) II only (C) I and II (D) I and III

A streamline is a mathematical concept that defines lines that are tangential to the velocity vector. Therefore, no flow can cross a streamline. Entropy is not related to streamlines.

The answer is (C).

FLUIDS–14

The following illustration shows several streamlines near the corner of two infinite plates. Which of the following could be the correct expression for the stream function, Ψ, of this potential flow?



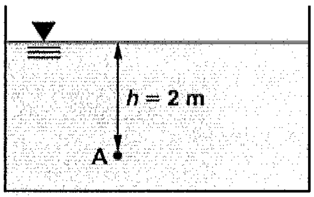
(A) $\Psi = x - y$ (B) $\Psi = 2xy$ (C) $\Psi = x$ (D) $\Psi = y$

Streamlines are graphs of constant values for the stream function. The graph shows hyperbolas that are of the form $axy = b$, where a and b are constants. Thus, of the choices shown, the stream function could only be $\Psi = 2xy$.

The answer is (B).

FLUIDS–15

What is most nearly the gage pressure at point A in the tank of water if $h = 2$ m?



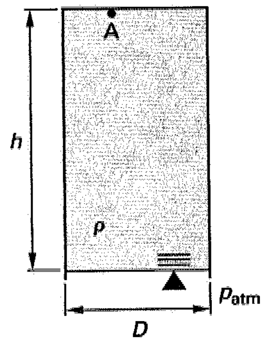
(A) 12 kPa (B) 13 kPa (C) 16 kPa (D) 20 kPa

$$\begin{aligned} p &= \rho g h \\ &= \left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(2\ \text{m}) \\ &= 19\,620\ \text{Pa} \quad (20\ \text{kPa}) \end{aligned}$$

The answer is (D).

FLUIDS–16

A drinking glass filled with a fluid of density ρ is quickly inverted. The top of the glass, which becomes the bottom after the glass is inverted, is open. What is the pressure at the closed end at point A?



(A) p_{atm} (B) $p_{atm} + \rho g h$ (C) $p_{atm} - \rho g h$ (D) $\rho g h$

The pressure at point A, p, plus the pressure exerted by the fluid equals the pressure outside the glass.

$$p + \rho g h = p_{atm}$$

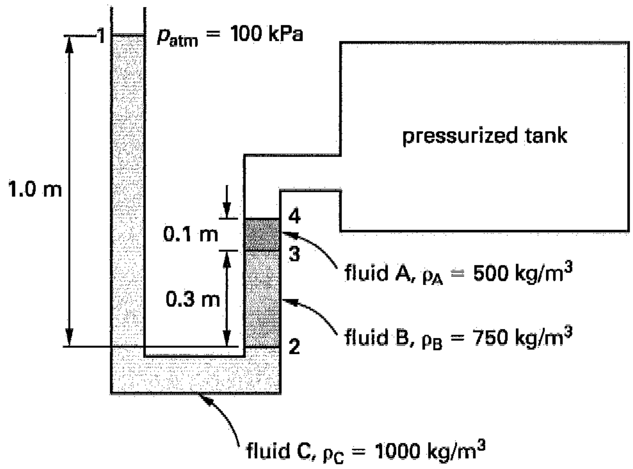
Therefore,

$$p = p_{atm} - \rho g h$$

The answer is (C).

FLUIDS–17

Find the pressure in the tank from the manometer readings shown.



(A) 102 kPa (B) 108 kPa (C) 112 kPa (D) 118 kPa

$$p_2 - p_1 = \rho_C g(z_1 - z_2)$$

$$p_3 - p_2 = \rho_B g(z_2 - z_3)$$

$$p_4 - p_3 = \rho_A g(z_3 - z_4)$$

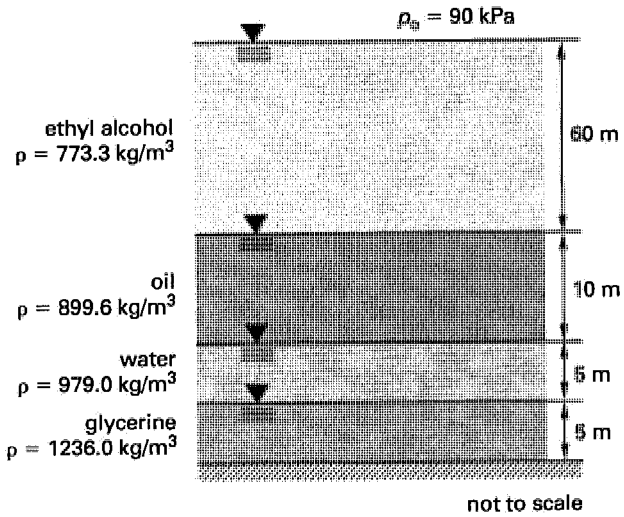
$$p_4 - p_1 = (p_4 - p_3) + (p_3 - p_2) + (p_2 - p_1)$$

$$\begin{aligned} p_4 &= p_1 + g\big(\rho_C(z_1 - z_2) + \rho_B(z_2 - z_3) + \rho_A(z_3 - z_4)\big) \\ &= 100\,000 \text{ Pa} + \left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)\left(\left(1000 \ \frac{\text{kg}}{\text{m}^3}\right)(1 \text{ m}) \right. \\ &\qquad \left. + \left(750 \ \frac{\text{kg}}{\text{m}^3}\right)(-0.3 \text{ m}) + \left(500 \ \frac{\text{kg}}{\text{m}^3}\right)(0.1 \text{ m})\right) \\ &= 108\,100 \text{ Pa} \quad (108 \text{ kPa}) \end{aligned}$$

The answer is (B).

FLUIDS–18

In which fluid will a pressure of 700 kPa occur?



(A) ethyl alcohol (B) oil (C) water (D) glycerin

Let p_i be the maximum pressure that can be measured in fluid level i. If $p_i \geq 700$ kPa, then a pressure of 700 kPa can be measured at that level.

$$p_0 = 90 \text{ kPa}$$

$$p_1 = p_0 + \rho_1 g z_1$$

$$= 90 \text{ kPa} + \frac{\left(773.3 \ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(60 \text{ m})}{\frac{1000 \ \frac{\text{N}}{\text{m}^2}}{1 \text{ kPa}}}$$

$$= 545.16 \text{ kPa}$$

$$p_1 < 700 \text{ kPa}$$

$$p_2 = p_1 + \rho_2 g z_2$$

$$= 545.56 \text{ kPa} + \frac{\left(899.6 \ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(10 \text{ m})}{\frac{1000 \ \frac{\text{N}}{\text{m}^3}}{1 \text{ kPa}}}$$

$$= 633.81 \text{ kPa}$$

$$p_2 < 700 \text{ kPa}$$

$$p_3 = p_2 + \rho_3 g z_3$$

$$= 633.81 \text{ kPa} + \frac{\left(979.0 \ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(5 \text{ m})}{\frac{1000 \ \frac{\text{N}}{\text{m}^2}}{1 \text{ kPa}}}$$

$$= 681.83 \text{ kPa}$$

$$p_3 < 700 \text{ kPa}$$

$$p_4 = p_3 + \rho_4 g z_4$$

$$= 681.83 \text{ kPa} + \frac{\left(1236 \ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(5 \text{ m})}{\frac{1000 \ \frac{\text{N}}{\text{m}^2}}{1 \text{ kPa}}}$$

$$= 742.46 \text{ kPa}$$

$$p_4 > 700 \text{ kPa}$$

Thus, a pressure of 700 kPa occurs in the glycerin level.

The answer is (D).

FLUIDS–19

The pressure drop across a turbine is 200 kPa. The flow rate is 0.25 m^3/min. What is most nearly the power output of the turbine?

(A) 0.41 kW (B) 0.83 kW (C) 0.95 kW (D) 1.3 kW

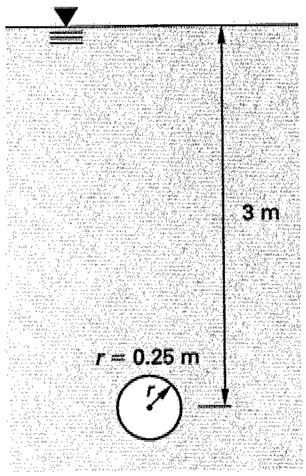
$$P = (\text{pressure drop})(\text{flow rate}) = (200 \text{ kPa})\left(\frac{0.25 \text{ m}^3}{\text{min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)$$

$$= 0.833 \text{ kW} \quad (0.83 \text{ kW})$$

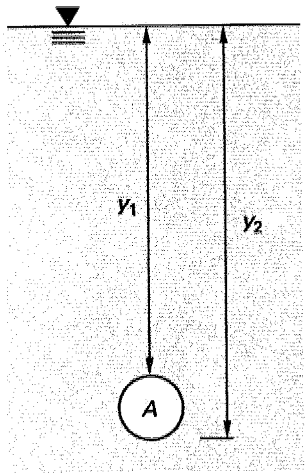
The answer is (B).

FLUIDS–20

A circular window with a radius of 0.25 m has its center 3 m below the water's surface. The window is vertical. What is most nearly the force acting on the window?



(A) 2.9 kN (B) 5.8 kN (C) 18 kN (D) 29 kN



$$F = \overline{p}A$$

$$\overline{p} = \left(\frac{y_1 + y_2}{2}\right)(\rho g \sin \alpha)$$

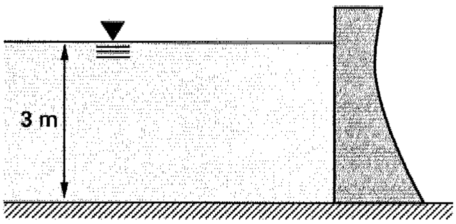
With $y_1 = 2.75$ m, $y_2 = 3.25$ m, the angle α between the surface of the water and the surface of the window $= \pi/2$, and $A = \pi r^2$,

$$\begin{aligned} F &= \left(\frac{y_1 + y_2}{2}\right)(\rho g \sin\alpha)A \\ &= \left(\frac{2.75 \text{ m} + 3.25 \text{ m}}{2}\right)\left(1000 \ \frac{\text{kg}}{\text{m}^3}\right)\left(\frac{9.81 \text{ m}}{\text{s}^2}\right)(1)\pi(0.25 \text{ m})^2 \\ &= 5780 \text{ N} \quad (5.78 \text{ kN}) \end{aligned}$$

The answer is (B).

FLUIDS-21

What is most nearly the overturning moment per unit width due to water acting on the dam shown?



(A) 15 kN·m (B) 30 kN·m (C) 44 kN·m (D) 72 kN·m

The hydrostatic force per unit width of dam is

$$F = \tfrac{1}{2}\rho g h A$$

$A = (3 \text{ m})(1 \text{ m}) = 3 \text{ m}^2$ per meter of width.

$$\begin{aligned} F &= \left(\frac{1}{2}\right)\left(1000 \ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(3 \text{ m})(3 \text{ m}) \\ &= 44\,145 \text{ N} \quad (44.145 \text{ kN·m/m}) \end{aligned}$$

This force acts one-third up from the base.

The overturning moment is

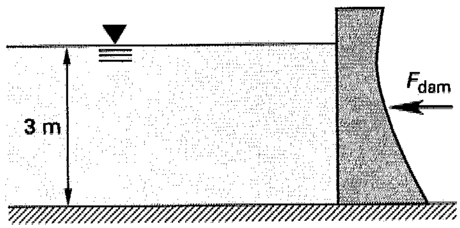
$$M_{\text{dam}} = Fy = \left(44.145\ \frac{\text{kN·m}}{\text{m}}\right)\left(\frac{3\ \text{m}}{3}\right)$$

$$= 44.1\ \text{kN·m} \quad (44\ \text{kN·m})$$

The answer is (C).

FLUIDS–22

What is most nearly the minimum required force per unit width, F_{dam}, to prevent the dam shown from sliding?



(A) 15 kN (B) 30 kN (C) 44 kN (D) 72 kN

$$F = \tfrac{1}{2}\rho g h A$$

$A = (3\ \text{m})(1\ \text{m}) = 3\ \text{m}^2$ per meter of width.

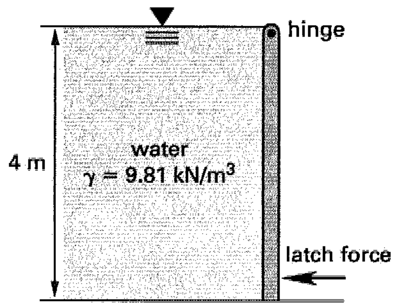
$$F = \left(\frac{1}{2}\right)\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(3\ \text{m})(3\ \text{m})$$

$$= 44\,145\ \text{N} \quad (44\ \text{kN})$$

The answer is (C).

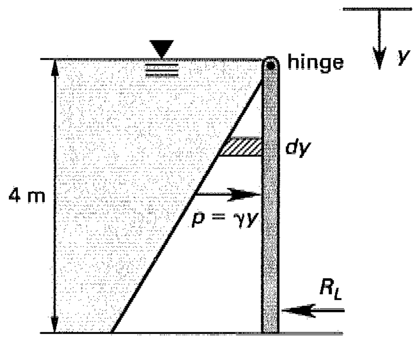
FLUIDS–23

Water is held in a tank by the sluice gate shown. What force per unit width of the dam must the latch supply to keep the gate closed?



(A) 25 kN/m (B) 34 kN/m (C) 52 kN/m (D) 74 kN/m

Draw a free-body diagram of the gate.



Use the coordinate system in the diagram. For the gate to stay in place, the sum of the moments around the hinge must be zero.

$$\sum M_{\text{hinge}} = 0$$
$$= (4\text{ m})R_L - \int_{0\text{ m}}^{4\text{ m}} \gamma y y dy$$

$$(4\text{ m})R_L = \int_{0\text{ m}}^{4\text{ m}} p y dy = \int_{0\text{ m}}^{4\text{ m}} \gamma y y dy$$

$$= \int_{0\text{ m}}^{4\text{ m}} \gamma y^2 dy$$

$$= \left.\frac{\left(9.81\ \frac{\text{kN}}{\text{m}^3}\right) y^3}{3}\right|_{0\text{ m}}^{4\text{ m}}$$

$$= 209.3\text{ kN}$$

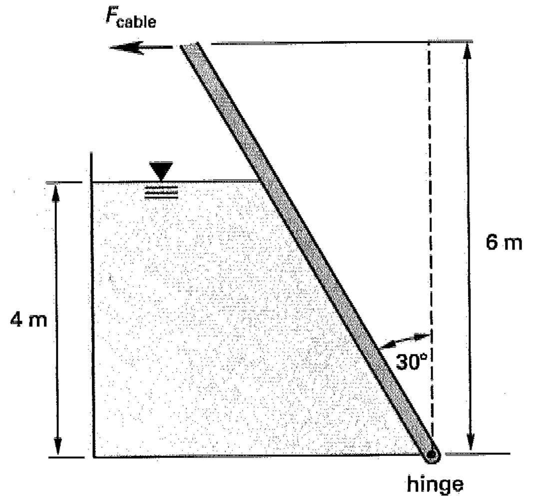
Rearranging to solve for R_L,

$$R_L = \frac{209.3\text{ kN}}{4\text{ m}}$$
$$= 52.3\text{ kN/m} \quad (52\text{ kN/m})$$

The answer is (C).

FLUIDS–24

A tank with one hinged wall is filled with water. The tank wall is held at a 30° angle by a horizontal cable. What is most nearly the tension in the cable per meter of the tank?



(A) 19 kN (B) 23 kN (C) 25 kN (D) 40 kN

The average pressure is

$$\begin{aligned}\overline{p} &= \overline{h}\gamma \\ &= \left(\frac{0 + 4\ \text{m}}{2}\right)\left(9.81\ \frac{\text{kN}}{\text{m}^3}\right) \\ &= 19.6\ \text{Pa}\end{aligned}$$

The length of the wetted inclined wall is

$$l = \frac{h}{\cos 30^\circ} = \frac{4\ \text{m}}{\cos 30^\circ} = 4.62\ \text{m}$$

The wall area per foot of wall is

$$A = lw = (4.62\ \text{m})(1\ \text{m}) = 4.62\ \text{m}^2$$

The resultant force is

$$R = \overline{p}A = (19.6\ \text{Pa})\,(4.62\ \text{m}^2) = 90.6\ \text{kN}$$

This resultant acts perpendicular to the wall at

$$\frac{2h}{3} = \frac{(2)(4\ \text{m})}{3} = 2.67\ \text{m} \quad \text{[vertical distance measured from surface]}$$

Taking moments about the hinge at the bottom,

$$\begin{aligned}\sum M = 0 &= \frac{(90.6\ \text{kN})(4\ \text{m} - 2.67\ \text{m})}{\cos 30^\circ} - \frac{(T_{\text{cable}})(\cos 30^\circ)(6\ \text{m})}{\cos 30^\circ} \\ &= 139.1\ \text{kN·m} - (6\ \text{m})T_{\text{cable}}\end{aligned}$$

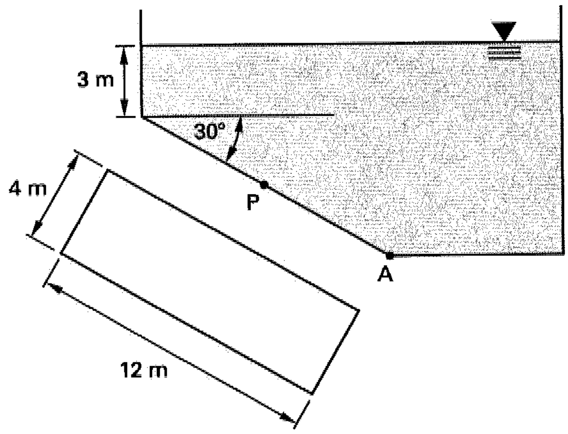
Rearranging to solve for T_{cable},

$$\begin{aligned}T_{\text{cable}} &= \frac{139.1\ \text{kN·m}}{6\ \text{m}} \\ &= 23.2\ \text{kN} \quad (23\ \text{kN})\end{aligned}$$

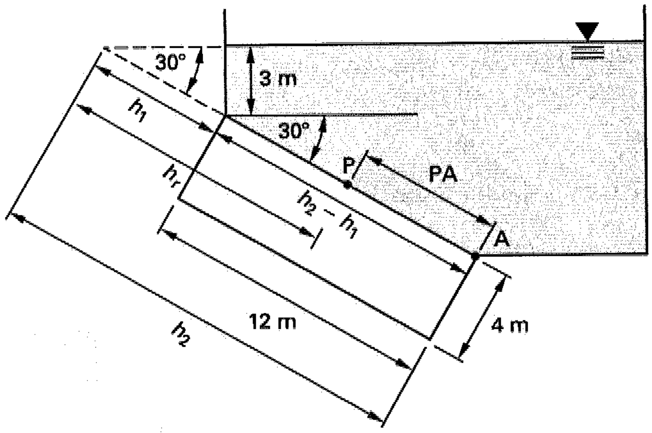
The answer is (B).

FLUIDS–25

A tank of water has a rectangular panel at its lower left side, as shown. The location of the center of pressure on the panel is at the point P. Describe the distance along the panel from the bottom of the tank to the center of pressure as PA. Determine the length of PA.



(A) 4 m (B) 5 m (C) 6 m (D) 7 m



The distance along the surface of an object from the surface of the fluid to the center of pressure, h_r, is given by

$$h_r = \frac{2}{3}\left(h_1 + h_2 - \frac{h_1 h_2}{h_1 + h_2}\right)$$

In the preceding equation, h_1 is the distance along the surface of the object from the surface of the fluid to the object's upper edge, and h_2 is the distance along the surface of the object from the surface of the fluid to the object's lower edge.

From the illustration,

$$\text{PA} = h_2 - h_r$$

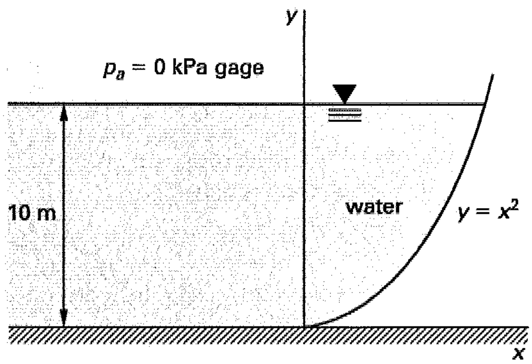
The plane is inclined at 30° below horizontal, has its upper edge at 3 m (vertically) below the surface of the fluid, and is 12 m long. Thus, the following can be determined.

$$\begin{aligned} h_1 &= \frac{3 \text{ m}}{\sin 30°} \\ &= 6 \text{ m} \\ h_2 - h_1 &= 12 \text{ m} \\ h_2 &= h_1 + 12 \text{ m} = 6 \text{ m} + 12 \text{ m} \\ &= 18 \text{ m} \\ h_r &= \left(\frac{2}{3}\right)\left(6 \text{ m} + 18 \text{ m} - \frac{(6 \text{ m})(18 \text{ m})}{6 \text{ m} + 18 \text{ m}}\right) \\ &= 13 \text{ m} \\ \text{PA} &= h_2 - h_r = 18 \text{ m} - 13 \text{ m} \\ &= 5 \text{ m} \end{aligned}$$

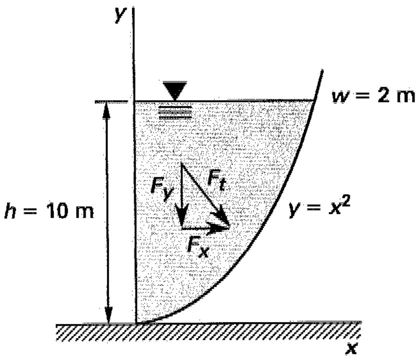
The answer is (B).

FLUIDS–26

What is most nearly the total force exerted on the curved surface described by the equation $y = x^2$? The width of the curved plate is 2 m, and the specific weight of water is 9.81 kN/m^3.



(A) 1020 kN (B) 1070 kN (C) 1260 kN (D) 1380 kN



w is the width of the tank.

$$F_t = \sqrt{F_x^2 + F_y^2}$$

The weight of the water in the portion of the tank above the curved surface, F_y, is

$$\begin{aligned}
F_y &= \gamma V \\
&= \gamma w \int_{0\text{ m}}^{\sqrt{10}\text{ m}} (10\text{ m} - y)dx \\
&= \gamma w \int_{0\text{ m}}^{\sqrt{10}\text{ m}} (10\text{ m} - x^2)dx \\
&= \gamma w \left(10x - \frac{x^3}{3}\right)\Big|_{0\text{ m}}^{\sqrt{10}\text{ m}} \\
&= \left(9.81\ \frac{\text{kN}}{\text{m}^3}\right)(2\text{ m})\left(10\text{ m}\sqrt{10}\text{ m} - \frac{10\text{ m}\sqrt{10}\text{ m}}{3}\right) \\
&= 413.6\text{ kN}
\end{aligned}$$

$$F_x = \overline{p}A_x$$

$$\overline{p} = \gamma\left(\frac{0+h}{2}\right)$$

The area of the tank perpendicular to the x-axis, A_x, is

$$\begin{aligned}
A_x &= wh \\
F_x &= \left(9.81\ \frac{\text{kN}}{\text{m}^3}\right)\left(\frac{10\text{ m}}{2}\right)((2\text{ m})(10\text{ m})) \\
&= 981.0\text{ kN} \\
F_t &= \sqrt{(413.6\text{ kN})^2 + (981.0\text{ kN})^2} \\
&= 1065\text{ kN} \quad (1070\text{ kN})
\end{aligned}$$

The answer is (B).

FLUIDS–27

The stream potential, Φ, of a flow is given by $\Phi = 2xy - y$. Determine the stream function, Ψ, for this potential.

(A) $\Psi = x^2 - y^2 + C$
(B) $\Psi = x - x^2 + y^2 + C$
(C) $\Psi = x + x^2 - y^2 + C$
(D) $\Psi = x^2 + y^2 + C$

$$u = \frac{dx}{dt} \quad \mathrm{v} = \frac{dy}{dt}$$

By definition,

$$\begin{aligned} u &= \frac{d\Phi}{dx} \\ &= \frac{d\Psi}{dy} \\ \mathrm{v} &= \frac{d\Phi}{dy} \\ &= -\frac{d\Psi}{dx} \end{aligned}$$

Substituting $u = 2y$,

$$\frac{d\Psi}{dy} = 2y$$

Rearranging,

$$\begin{aligned} \Psi &= \int 2y dy + f(x) \\ &= y^2 + f(x) \end{aligned}$$

Substituting $\mathrm{v} = 2x - 1$,

$$-\frac{df(x)}{dx} = 2x - 1$$

Rearranging,

$$f(x) = x - x^2 + C$$

Therefore,

$$\Psi = x - x^2 + y^2 + C$$

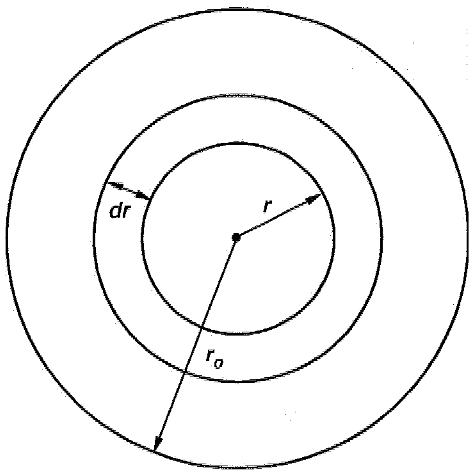
The answer is (B).

FLUIDS–28

Determine the average velocity through a circular section in which the velocity distribution is given as $v = v_{max}(1 - (r/r_o)^2)$. The distribution is symmetric with respect to the longitudinal axis, $r = 0$. r_o is the outer radius and v_{max} is the velocity along the longitudinal axis.

(A) $v_{max}/4$ (B) $v_{max}/3$ (C) $v_{max}/2$ (D) v_{max}

$$\begin{aligned}
v_{ave} &= \frac{1}{A}\int v\,dA \\
&= \frac{1}{\pi r_o^2}\int_0^{r_o} v_{max}\left(1-\left(\frac{r}{r_o}\right)^2\right)2\pi r\,dr \\
&= \frac{2\pi v_{max}}{\pi r_o^2}\int_0^{r_o} r\left(1-\left(\frac{r}{r_o}\right)^2\right)dr \\
&= \left(\frac{2v_{max}}{r_o^2}\right)\left(\frac{r^2}{2}-\frac{r^4}{4r_o^2}\right)\Bigg|_0^{r_o} \\
&= \left(\frac{2v_{max}}{r_o^2}\right)\left(\frac{r_o^2}{2}-\frac{r_o^2}{4}\right) \\
&= v_{max}/2
\end{aligned}$$



The answer is (C).

FLUIDS–29

Under what conditions is mass conserved in fluid flow?

(A) The fluid is baratropic.
(B) The flow is isentropic.
(C) The flow is adiabatic.
(D) It is always conserved.

Mass is always conserved in fluid flow.

The answer is (D).

FLUIDS–30

What is the absolute velocity of a real fluid at a surface?

(A) the same as the bulk fluid velocity
(B) the velocity of the surface
(C) zero
(D) proportional to the smoothness of the surface

For a real (nonzero viscosity) fluid there is no slip at the boundaries. In other words, the velocity of the surface is the same as the velocity of the fluid at the surface. Thus, option (B) is true.

Option (C) is true only if the velocity of the surface is zero.

The answer is (B).

FLUIDS–31

Which of the statements is true concerning the following continuity equation?

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho \mathrm{v})}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

ρ is density, u is velocity in the x direction, v is velocity in the y direction, and w is velocity in the z direction.

(A) It is valid only for incompressible flow.
(B) It is valid only for steady flow.
(C) It is derived from the principle of conservation of mass.
(D) It is derived from the principle of conservation of energy.

In essence, the continuity equation states that the mass flux entering a control volume is equal to the mass flux leaving the control volume plus the rate of accumulation of mass within the control volume. Thus, it is derived from the principle of conservation of mass. It is valid for all real and ideal fluids, and for all types of fluid flow.

The answer is (C).

FLUIDS–32

Which of the following sets of dimensional flow equations satisfies the continuity equation? (u, v, and w are the components of velocity in the x, y, and z directions, respectively.)

I. $u = x + 2y - t$
$\text{v} = t - 2y + z$
$w = t - 2x + z$

II. $u = y^2 - x^2$
$\text{v} = 2xy$
$w = 2tz$

III. $u = x^2 - y^2$
$\text{v} = -2xy + ty$
$w = -tz$

(A) I and II (B) I and III (C) II and III (D) I, II, and III

The continuity equation states that $\nabla \cdot \mathbf{V} = 0$. Check to see if this is true for each of the given flows.

$$\text{I.} \quad \frac{\partial u}{\partial x} + \frac{\partial \text{v}}{\partial y} + \frac{\partial w}{\partial z} = 1 + (-2) + 1 = 0$$

$$\text{II.} \quad \frac{\partial u}{\partial x} + \frac{\partial \text{v}}{\partial y} + \frac{\partial w}{\partial z} = -2x + 2x + 2t = 2t \neq 0$$

$$\text{III.} \quad \frac{\partial u}{\partial x} + \frac{\partial \text{v}}{\partial y} + \frac{\partial w}{\partial z} = 2x + (-2x + t) - t = 0$$

Thus, flows I and III both satisfy the continuity equation.

The answer is (B).

FLUIDS–33

A pipe has a diameter of 100 mm at section AA and a diameter of 50 mm at section BB. The velocity of an incompressible fluid is 0.3 m/s at section AA. What is the flow velocity at section BB?

(A) 0.95 m/s (B) 1.2 m/s (C) 2.1 m/s (D) 3.5 m/s

Use the continuity equation.

$$\text{mass through AA} = \text{mass through BB}$$
$$\rho A_1 \text{v}_1 = \rho A_2 \text{v}_2$$

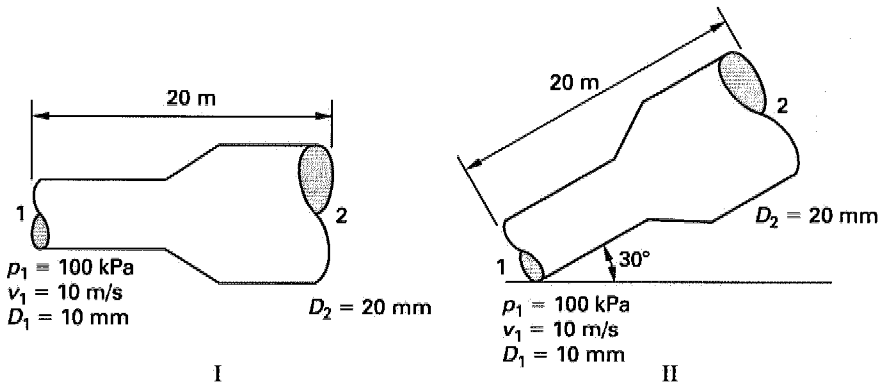
Rearranging to solve for v_2,

$$\begin{aligned} v_2 &= \frac{\rho A_1 v_1}{\rho A_2} \\ &= \left(\frac{A_1}{A_2}\right) v_1 \\ &= \left(\frac{\pi (0.10\ \text{m})^2}{\pi (0.05\ \text{m})^2}\right)\left(\frac{0.3\ \text{m}}{\text{s}}\right) \\ &= 1.2\ \text{m/s} \end{aligned}$$

The answer is (B).

FLUIDS–34

Consider the following two flows of water.



What is the relation between $v_2(\text{I})$ and $v_2(\text{II})$?

(A) $v_2(\text{I}) = v_2(\text{II})$ (B) $v_2(\text{I}) = \frac{v_2(\text{II})}{2}$

(C) $v_2(\text{I}) = 2v_2(\text{II})$ (D) $v_2(\text{I}) = 4\,v_2(\text{II})$

From the continuity equation,

$$A_1 \text{v}_1 = A_2 \text{v}_2$$

Rearranging to solve for v_2,

$$\text{v}_2 = \left(\frac{A_1}{A_2}\right) \text{v}_1 \quad \text{[independent of tilt angle]}$$

$$\text{v}_2(\text{I}) = \text{v}_2(\text{II})$$

The answer is (A).

FLUIDS–35

A constant-volume mixing tank mixes two inlet streams containing salt. The salt concentration in stream 1 is 5% by weight, and in stream 2 it is 15% by weight. Stream 1 flows at 25 kg/s, and stream 2 flows at 10 kg/s. There is only one exit stream. Find the salt concentration in the exit stream.

(A) 5.5% (B) 7.9% (C) 11% (D) 13%

$$\sum_{\text{inlet}} \dot{m}_{\text{salt}} = \sum_{\text{outlet}} \dot{m}_{\text{salt}}$$

$$(0.05)\left(25\ \frac{\text{kg}}{\text{s}}\right) + (0.15)\left(10\ \frac{\text{kg}}{\text{s}}\right) = x\left(35\ \frac{\text{kg}}{\text{s}}\right)$$

Rearranging to solve for x, the salt concentration in the exit stream,

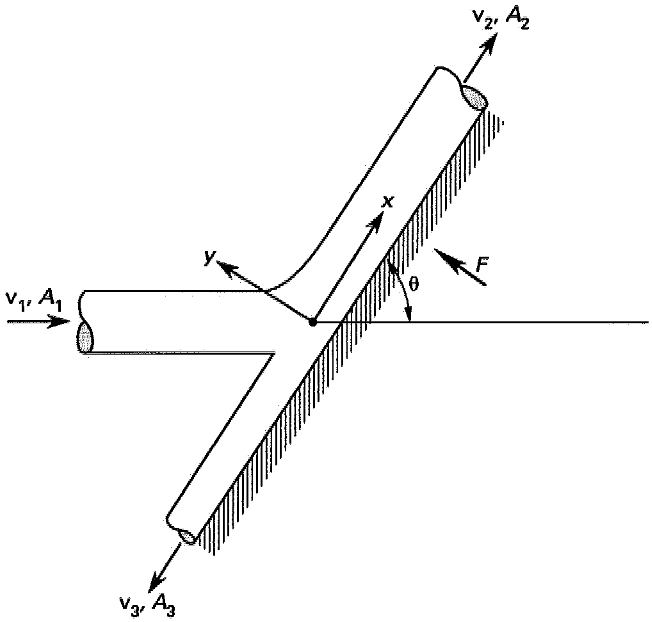
$$x = \frac{(0.05)\left(25\ \frac{\text{kg}}{\text{s}}\right) + (0.15)\left(10\ \frac{\text{kg}}{\text{s}}\right)}{35\ \frac{\text{kg}}{\text{s}}}$$

$$= 0.0786 \quad (7.9\%)$$

The answer is (B).

FLUIDS–36

Water flowing with a velocity of v_1 in a pipe is turned to flow in the x direction, as shown. What is the relation between the y component of the force of the water jet acting on the inclined plate and the inclination angle?



(A) $F_y = \rho A_1 v_1^2 \cos\theta$ (B) $F_y = \rho A_1 v_1 \sin\theta$
(C) $F_y = \rho A_1 v_1 \cos\theta$ (D) $F_y = \rho A_1 v_1^2 \sin\theta$

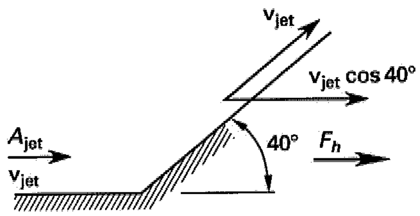
Evaluate momentum in the y direction. The y component of velocity v_1 is $v_y = v_1 \sin\theta$.

$$\begin{aligned} F_y &= \dot{m} v_y \\ &= (\rho A_1 v_1)(v_1 \sin\theta) \\ &= \rho A_1 v_1^2 \sin\theta \end{aligned}$$

The answer is (D).

FLUIDS–37

The vane shown deflects a jet of velocity v_{jet}, density ρ, and cross-sectional area A_{jet} through an angle of 40°. Calculate F_h, the horizontal force on the vane.



(A) $\rho A_{jet} v_{jet}^2$

(B) $\rho A_{jet} v_{jet}^2 \cos 40°$

(C) $\rho A_{jet} v_{jet}^2 (1 - \cos 40°)$

(D) $\rho A_{jet} v_{jet}^2 (1 - \sin 40°)$

Using the momentum equation, the rate of change of horizontal momentum, F_h, is

$$\begin{aligned} F_h &= \dot{m}(\text{horizontal velocity in} - \text{horizontal velocity out}) \\ &= \dot{m}(v_{jet} - v_{jet} \cos 40°) \end{aligned}$$

Substituting,

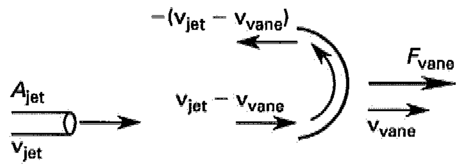
$$\dot{m} = \rho A_{jet} v_{jet}$$

$$F_h = \rho A_{jet} v_{jet} (v_{jet} - v_{jet} \cos 40°) = \rho A_{jet} v_{jet}^2 (1 - \cos 40°)$$

The answer is (C).

FLUIDS–38

A jet of velocity v_{jet}, cross-sectional area A_{jet}, and density ρ_{jet} impinges on a reversing vane and is turned through an angle of 180°. The vane is moving with velocity v_{vane} in the direction of the original jet. What is the force, F_{vane}, exerted on the vane by the water?



(A) $2\rho A_{jet} v_{jet}$
(B) $\rho A_{jet} v_{jet}$
(C) $2\rho A_{jet} v_{vane}$
(D) $2\rho A_{jet} \left(v_{jet} - v_{vane}\right)^2$

Use the momentum equation. The rate of change of momentum, F_{vane}, is

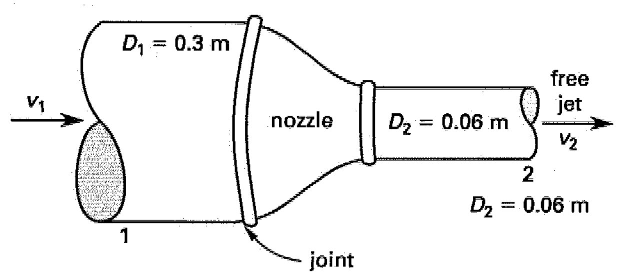
$$F_{vane} = \dot{m}\Delta v$$

$$\begin{aligned}\Delta v &= (v_{jet} - v_{vane}) - \big(-(v_{jet} - v_{vane})\big)\\ &= 2(v_{jet} - v_{vane})\\ &= \big(\rho A_{jet}(v_{jet} - v_{vane})\big)\big(2(v_{jet} - v_{vane})\big)\\ &= 2\rho A_{jet}(v_{jet} - v_{vane})^2\end{aligned}$$

The answer is (D).

FLUIDS–39

Oil (specific gravity = 0.8) at 3000 Pa flows at a constant rate of 1 m^3/s through the circular nozzle shown. What is most nearly the net force exerted by the joint to hold the nozzle in place?



(A) 140 kN (B) 190 kN (C) 240 kN (D) 270 kN

$$\mathrm{v} = \frac{Q}{A}$$

$$\mathrm{v}_1 = \frac{1\ \frac{\mathrm{m}^3}{\mathrm{s}}}{\pi\left(\frac{0.3\ \mathrm{m}}{2}\right)^2}$$

$$= 14.15\ \mathrm{m/s}$$

$$\mathrm{v}_2 = \frac{1\ \frac{\mathrm{m}^3}{\mathrm{s}}}{\pi\left(\frac{0.06\ \mathrm{m}}{2}\right)^2}$$

$$= 353.68\ \mathrm{m/s}$$

$$\dot{m} = \rho Q$$

$$= (0.8)\left(1000\ \frac{\mathrm{kg}}{\mathrm{m}^3}\right)\left(1\ \frac{\mathrm{m}^3}{\mathrm{s}}\right)$$

$$= 800\ \mathrm{kg/s}$$

$$\sum F_x = \dot{m}\Delta\mathrm{v}$$

$$F_h - F_1 = \dot{m}(\mathrm{v}_2 - \mathrm{v}_1)$$

Rearranging to solve for F_h, the horizontal force holding the nozzle in place,

$$F_h = F_1 + \dot{m}(\mathrm{v}_2 - \mathrm{v}_1)$$

The force exerted by the pressurized fluid, F_1, is

$$F_1 = pA_1 = \left(3000\ \frac{\mathrm{N}}{\mathrm{m}^2}\right)\left(\pi\left(\frac{0.3\ \mathrm{m}}{2}\right)^2\right)$$

$$= 212.06\ \mathrm{N}$$

$$F_h = 212.06\ \mathrm{N} + \left(800\ \frac{\mathrm{kg}}{\mathrm{s}}\right)\left(353.68\ \frac{\mathrm{m}}{\mathrm{s}} - 14.15\ \frac{\mathrm{m}}{\mathrm{s}}\right)$$

$$= 271\,800\ \mathrm{N} \quad (270\ \mathrm{kN})$$

The answer is (D).

FLUIDS–40

What is the origin of the energy conservation equation used in flow systems?

(A) Newton's first law of motion
(B) Newton's second law of motion
(C) the first law of thermodynamics
(D) the second law of thermodynamics

The energy equation for fluid flow is based on the first law of thermodynamics, which states that the heat input into the system added to the work done on the system is equal to the change in energy of the system.

The answer is (C).

FLUIDS–41

Which of the following is the basis for Bernoulli's law for fluid flow?

(A) the principle of conservation of mass
(B) the principle of conservation of energy
(C) the continuity equation
(D) the principle of conservation of momentum

Bernoulli's law is derived from the principle of conservation of energy.

The answer is (B).

FLUIDS–42

Under which of the following conditions is Bernoulli's equation valid?

(A) all points evaluated must be on the same streamline
(B) the fluid must be incompressible
(C) the fluid must be inviscid
(D) all of the above

Bernoulli's equation is valid only for incompressible, inviscid fluids. In order for Bernoulli's equation to be valid for two particular points, they must lie on the same streamline. Thus, options (A), (B), and (C) are all valid conditions for Bernoulli's equation.

The answer is (D).

FLUIDS–43

Under certain flow conditions, the expression for the first law of thermodynamics for a control volume reduces to Bernoulli's equation.

$$gz_1 + \frac{v_1^2}{2} + \frac{p_1}{\rho} = gz_2 + \frac{v_2^2}{2} + \frac{p_2}{\rho}$$

Which combination of the following conditions is necessary and sufficient to reduce the first law of thermodynamics for a control volume to Bernoulli's equation?

I. steady flow
II. incompressible fluid
III. no frictional losses of energy
IV. no heat transfer or change in internal energy

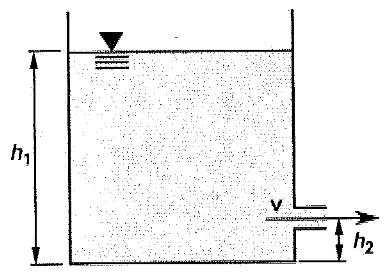
(A) I only (B) I and II (C) I and IV (D) I, II, III, and IV

Bernoulli's equation is essentially a statement of conservation of energy for steady flow of an inviscid, incompressible fluid. Bernoulli's equation does not account for any frictional losses or changes in internal energy of the fluid. For Bernoulli's equation to be valid, I, II, III, and IV must all describe the flow.

The answer is (D).

FLUIDS–44

Determine the velocity of the liquid at the exit, given that $h_1 = 1.5$ m and $h_2 = 0.3$ m.



(A) 1.9 m/s (B) 2.9 m/s (C) 3.9 m/s (D) 4.9 m/s

Use Bernoulli's equation. v_1 is essentially zero.

$$\rho g h_1 = \rho g h_2 - \frac{\rho v_2^2}{2}$$

$$v_2 = \sqrt{2g(h_1 - h_2)}$$

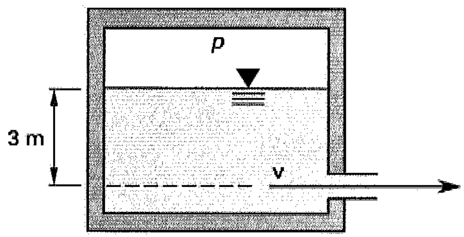
$$= \sqrt{(2)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1.5\text{m} - 0.3\text{m})}$$

$$= 4.9\ \text{m/s}$$

The answer is (D).

FLUIDS–45

A pressurized tank contains a fluid with a density of 1300 kg/m^3. The pressure in the air space above the fluid is 700 kPa. Fluid exits to the atmosphere from an opening 3 m below the fluid surface. What is most nearly the exit velocity, v?



(A) 11 m/s (B) 22 m/s (C) 31 m/s (D) 52 m/s

Apply Bernoulli's equation between the free surface and the exit.

$$\frac{p_{\text{tank}}}{\rho} + g z_1 + \frac{v_1^2}{2} = \frac{p_{\text{atm}}}{\rho} + g z_2 + \frac{v^2}{2}$$

$$v_1 = 0 \quad \text{[at the free surface]}$$

$$z_2 = 0 \quad \text{[at the exit]}$$

$$\frac{p_{\text{tank}}}{\rho} + g z_1 = \frac{p_{\text{atm}}}{\rho} + \frac{v_2^2}{2}$$

$$v_2 = \sqrt{2g\left(\frac{p_{\text{tank}} - p_{\text{atm}}}{\rho g} + z_1\right)}$$

Substituting,

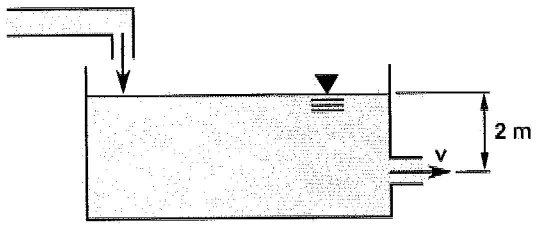
$$v_2 = \sqrt{(2)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{700\ \frac{\text{kN}}{\text{m}^2} - 101\ \frac{\text{kN}}{\text{m}^2}}{\left(1300\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{1}{1000\ \frac{\text{N}}{\text{kN}}}\right)} + 3\ \text{m}\right)}$$

$= 31.3\ \text{m/s} \quad (31\ \text{m/s})$

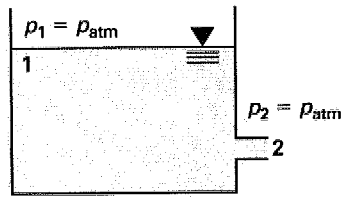
The answer is (C).

FLUIDS–46

Consider the holding tank shown. The tank volume remains constant. What is most nearly the velocity of the water exiting to the atmosphere?



(A) 3 m/s (B) 4 m/s (C) 5 m/s (D) 6 m/s



Apply Bernoulli's equation between the free surface (point 1) and the exit (point 2).

$$gz_1 + \frac{v_1^2}{2} + \frac{p_1}{\rho} = gz_2 + \frac{v_2^2}{2} + \frac{p_2}{\rho}$$

$$p_1 = p_2 \quad \text{[both are at atmospheric pressure]}$$

$$v_1 = 0 \quad \text{[the free surface is stationary]}$$

$$gz_1 = gz_2 + \frac{v_2^2}{2}$$

$$v_2 = \sqrt{2g(z_1 - z_2)}$$

$$= \sqrt{(2)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(2\ \text{m})}$$

$$= 6.3\ \text{m/s}$$

The answer is (D).

FLUIDS–47

Water is pumped at 1 m^3/s to an elevation of 5 m through a flexible hose using a 100% efficient pump rated at 100 kW. Using the same length of hose, what size motor is needed to pump 1 m^3/s of water to a tank with no elevation gain? Both ends of the hose are at atmospheric pressure. Neglect kinetic energy effects.

(A) 18 kW (B) 22 kW (C) 37 kW (D) 51 kW

From a mechanical power balance for the first case,

$$\dot{m}g\Delta z + \sum P_{\text{friction}} = P_{\text{motor}}$$

$$\sum P_{\text{friction}} = P_{\text{motor}} - \dot{m}g\Delta z$$

$$= P_{\text{motor}} - \rho Q g \Delta z$$

$$= 100\,000\ \text{W} - \left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(1\ \frac{\text{m}^3}{\text{s}}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(5\ \text{m})$$

$$= 51\ \text{kW}$$

In the second case, $\Delta z = 0$. Thus, a mechanical power balance yields the following.

$$\sum P_{\text{friction}} = P_{\text{motor}}$$

$$= 51\ \text{kW} \quad \text{[because the same hose is used]}$$

$$P_{\text{motor}} = 51\ \text{kW}$$

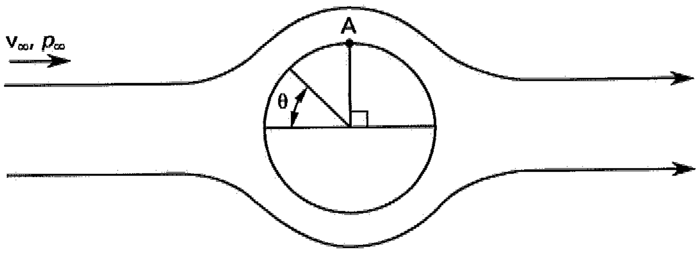
The answer is (D).

FLUIDS–48

The potential flow velocity distribution of atmospheric air around a cylinder is

$$v = 2v_\infty \sin\theta$$

The free-stream velocity is 30 m/s. The air density is approximately 1.202 kg/m^3. What is most nearly the pressure at point A?



(A) 64 kN/m^2 (B) 76 kN/m^2 (C) 80 kN/m^2 (D) 99 kN/m^2

Apply Bernoulli's equation between the free stream and point A.

$$p_\infty + \tfrac{1}{2}\rho_\infty v_\infty^2 = p_A + \tfrac{1}{2}\rho v_A^2$$
$$p_{atm} + \tfrac{1}{2}\rho_{air} v_\infty^2 = p_A + \tfrac{1}{2}\rho_{air} v_A^2$$
$$v_A = 2v_\infty \sin 90^\circ = 2v_\infty$$
$$p_A = p_{atm} + \rho_{air}\left(v_\infty^2 - 4v_\infty^2\right) = p_{atm} - \tfrac{3}{2}\rho_{air} v_\infty^2$$

Therefore,

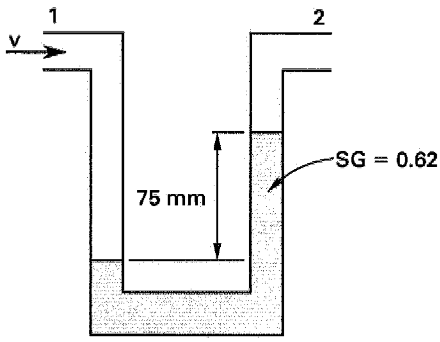
$$p_A = 101\,000\ \text{Pa} - \left(\frac{3}{2}\right)\left(1.202\ \frac{\text{kg}}{\text{m}^3}\right)\left(30\ \frac{\text{m}}{\text{s}}\right)^2 = 99\,377\ \text{Pa} \quad (99.4\ \text{kPa})$$

The answer is (D).

FLUIDS–49

Two tubes are mounted to the roof of a car. One tube points to the front of the car while the other points to the rear. The tubes are connected to a manometer filled with a fluid of specific gravity 0.62. The density of air is approximately 1.202 kg/m^3. When the height difference is 75 mm, what is the car's speed?

(A) 11 m/s (B) 15 m/s (C) 28 m/s (D) 96 m/s



Apply Bernoulli's equation between the front tube (point 1) and the tube facing the rear (point 2).

$$p_1 + \tfrac{1}{2}\rho_{\text{air}}\text{v}^2 = p_2 + \rho_f g h$$

$$p_1 = p_2 \quad \text{[both are at atmospheric pressure]}$$

The speed of the car, v, is

$$\text{v} = \sqrt{\frac{2\rho_f g h}{\rho_{\text{air}}}}$$

$$\rho_f = (\text{SG})(\rho_{\text{water}}) = (0.62)\left(1000\ \frac{\text{kg}}{\text{m}^3}\right) = 620\ \text{kg/m}^3$$

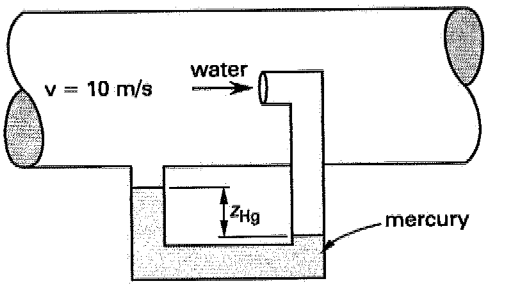
Therefore,

$$\text{v} = \sqrt{\frac{(2)\left(620\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(0.075\ \text{m})}{1.202\ \frac{\text{kg}}{\text{m}^3}}} = 27.6\ \text{m/s} \quad (28\ \text{m/s})$$

The answer is (C).

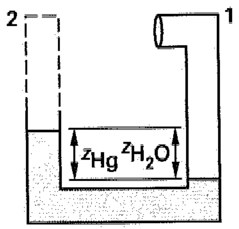
FLUIDS–50

Water is flowing through a pipe with a manometer as shown.



The density of mercury is 13 567 kg/m^3, and the velocity of the water is 10 m/s. Determine the height difference, z_{Hg}, in centimeters of mercury.

(A) 41 cm (B) 47 cm (C) 57 cm (D) 69 cm



From Bernoulli's equation,

$$gz_1 + \frac{v_1^2}{2} + \frac{p_1}{\rho} = gz_2 + \frac{v_2^2}{2} + \frac{p_2}{\rho}$$

$$\Delta z = 0$$

$$p_1 - p_2 = \left(\frac{v_2^2 - v_1^2}{2}\right)\rho$$

$$v_1 = 0$$

$$v_2 = 10 \text{ m/s}$$

$$p_1 - p_2 = \left(\frac{\left(10\ \frac{\text{m}}{\text{s}}\right)^2 - 0}{2}\right)\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)$$

$$= 50\,000 \text{ Pa}$$

$$= \rho_{Hg} g z_{Hg} - \rho_{H_2O} g z_{H_2O}$$

$$z_{Hg} = z_{H_2O}$$

$$50\,000 \text{ Pa} = (\rho_{Hg} - \rho_{H_2O}) g z_{Hg}$$

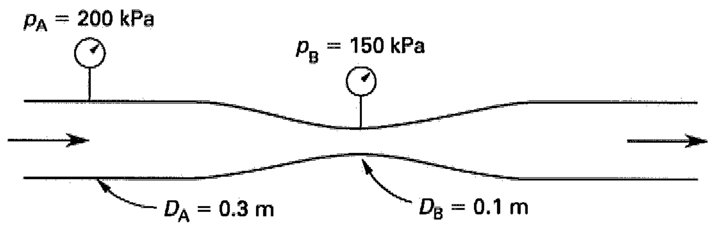
$$z_{Hg} = \frac{50\,000 \text{ Pa}}{\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)\left(13\,567 \ \frac{\text{kg}}{\text{m}^3} - 1000 \ \frac{\text{kg}}{\text{m}^3}\right)}$$

$$= 0.406 \text{ m} \quad (41 \text{ cm})$$

The answer is (A).

FLUIDS–51

Given the venturi meter and the two pressures shown, calculate the mass flow rate of water in the circular pipe.



(A) 52 kg/s (B) 61 kg/s (C) 65 kg/s (D) 79 kg/s

From the continuity equation,

$$A_A \text{v}_A = A_B \text{v}_B$$

$$\text{v}_A \pi (0.15 \text{ m})^2 = \text{v}_B \pi (0.05 \text{ m})^2$$

$$\text{v}_A = \left(\frac{(0.05 \text{ m})^2}{(0.15 \text{ m})^2}\right) \text{v}_B$$

$$= 0.111 \ \text{v}_B$$

Use Bernoulli's equation along the streamline in the center of the pipe.

$$g z_A + \frac{\text{v}_A^2}{2} + \frac{p_A}{\rho} = g z_B + \frac{\text{v}_B^2}{2} + \frac{p_B}{\rho}$$

$$\Delta z = 0$$

$$p_A - p_B = 200 \text{ kPa} - 150 \text{ kPa} = 50 \text{ kPa}$$

$$\frac{\text{v}_B^2 - \text{v}_A^2}{2} = \frac{50\,000 \text{ Pa}}{1000 \ \frac{\text{kg}}{\text{m}^3}}$$

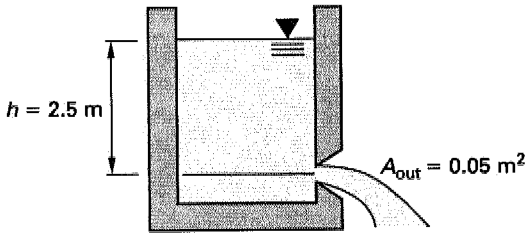
Rearranging,

$$\begin{aligned} v_B^2 - v_A^2 &= 100 \text{ m}^2/\text{s}^2 \\ v_B^2 - (0.111v_B)^2 &= 100 \ \frac{\text{m}^2}{\text{s}^2} \\ v_B^2\left(1 - (0.111)^2\right) &= 100 \ \frac{\text{m}^2}{\text{s}^2} \\ v_B &= 10.06 \text{ m/s} \\ \dot{m} &= \rho v_B A_B \\ &= \left(1000 \ \frac{\text{kg}}{\text{m}^3}\right)\left(10.06 \ \frac{\text{m}}{\text{s}}\right)\pi(0.05 \text{ m})^2 \\ &= 79 \text{ kg/s} \end{aligned}$$

The answer is (D).

FLUIDS–52

What is the volumetric discharge rate for the tank shown? The coefficient of contraction for the orifice is 0.61, and the coefficient of velocity is 0.98.



(A) 0.21 m^3/s (B) 0.33 m^3/s (C) 0.41 m^3/s (D) 0.52 m^3/s

$$\dot{V}_{\text{actual}} = C_c A_{\text{out}} v_{\text{out}}$$

In the preceding equation, C_c is the coefficient of contraction, and A_{out} is the area of the outlet.

$$v_{\text{out}} = C_v\sqrt{2gh}$$

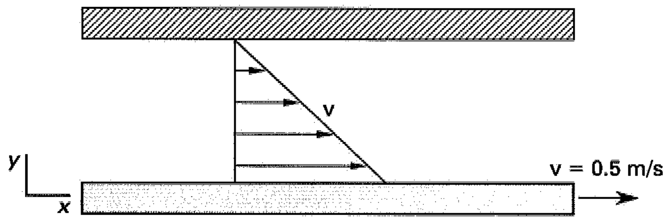
In the preceding equation, C_v is the coefficient of velocity, and h is the vertical distance from the exit to the fluid's surface.

$$\begin{aligned}\dot{V}_{\text{actual}} &= C_c C_v A_{\text{out}} \sqrt{2gh} \\ &= (0.61)(0.98)(0.05 \text{ m}^2)\sqrt{(2)\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(2.5 \text{ m})} \\ &= 0.21 \text{ m}^3/\text{s}\end{aligned}$$

The answer is (A).

FLUIDS–53

The upper plate illustrated is fixed, while the lower plate moves in the positive x direction at 0.5 m/s. The plate separation is 0.001 m, the fluid viscosity is 7×10^{-4} Pa·s, and the velocity profile is linear. Calculate the shear stress, τ_{xy}, in the moving fluid.



(A) 0.050 Pa (B) 0.15 Pa (C) 0.25 Pa (D) 0.35 Pa

$$\tau_{xy} = -\mu \frac{d\text{v}_x}{dy}$$

$$\mu = 0.7 \text{ cP} \quad (0.7 \text{ g/m}\cdot\text{s}^2)$$

$$\begin{aligned}\frac{d\text{v}_x}{dy} &= \frac{\Delta \text{v}_x}{\Delta y} \\ &= \frac{0.5 \ \frac{\text{m}}{\text{s}}}{0.001 \text{ m}} \\ &= 500 \text{ s}^{-1}\end{aligned}$$

$$\begin{aligned}\tau_{xy} &= (0.0007 \text{ Pa}\cdot\text{s})\left(500 \ \frac{1}{\text{s}}\right) \\ &= 0.35 \text{ Pa}\end{aligned}$$

The answer is (D).

FLUIDS–54

What are the units of Reynolds number for pipe flow?

(A) m/s (B) m^2/s (C) $kg/m{\cdot}s^2$ (D) none of the above

The Reynolds number is dimensionless.

The answer is (D).

FLUIDS–55

Which of the following ratios represents a physical interpretation of the Reynolds number?

(A) $\mathrm{Re} = \frac{\text{buoyant forces}}{\text{inertial forces}}$

(B) $\mathrm{Re} = \frac{\text{viscous forces}}{\text{inertial forces}}$

(C) $\mathrm{Re} = \frac{\text{drag forces}}{\text{viscous forces}}$

(D) $\mathrm{Re} = \frac{\text{inertial forces}}{\text{viscous forces}}$

$$\mathrm{Re} = \frac{\rho v D}{\mu}$$

By definition, the Reynolds number is the ratio of the inertial forces on an element of fluid to the viscous forces.

The answer is (D).

FLUIDS–56

Which of the following statements is FALSE?

(A) The Reynolds number is the ratio of the viscous force to the inertial force.

(B) Steady flows do not change with time at any point.

(C) The Navier-Stokes equation is the equation of motion for a viscous Newtonian fluid.

(D) Bernoulli's equation only holds on the same streamline.

The Reynolds number is the ratio of the inertial forces to the viscous forces.

The answer is (A).

FLUIDS–57

Calculate the Reynolds number for water at 20°C flowing in an open channel. The water is flowing at a volumetric rate of $0.8\ \text{m}^3/\text{s}$. The channel has a height of 1.2 m and a width of 2.5 m. At this temperature, water has a kinematic viscosity of $1.02 \times 10^{-6}\ \text{m}^2/\text{s}$.

(A) 6.5×10^5 (B) 8.5×10^5 (C) 9.2×10^5 (D) 1.2×10^6

$$\begin{aligned}\text{Re} &= \frac{\rho \text{v} D_e}{\mu} \\ &= \frac{\text{v} D_e}{\nu}\end{aligned}$$

$$\begin{aligned}D_e &= 4\left(\frac{\text{cross-sectional area}}{\text{wetted perimeter}}\right) \\ &= (4)\left(\frac{(1.2\ \text{m})(2.5\ \text{m})}{1.2\ \text{m} + 2.5\ \text{m} + 1.2\ \text{m}}\right) \\ &= 2.45\ \text{m}\end{aligned}$$

$$\dot{V} = \text{v}A$$

Rearranging,

$$\begin{aligned}\text{v} &= \frac{\dot{V}}{A} \\ &= \frac{0.8\ \frac{\text{m}^3}{\text{s}}}{(1.2\ \text{m})(2.5\ \text{m})} \\ &= 0.27\ \text{m/s}\end{aligned}$$

$$\begin{aligned}\text{Re} &= \frac{\left(0.27\ \frac{\text{m}}{\text{s}}\right)(2.45\ \text{m})}{1.02 \times 10^{-6}\ \frac{\text{m}^2}{\text{s}}} \\ &= 6.5 \times 10^5\end{aligned}$$

The answer is (A).

FLUIDS–58

A fluid with a kinematic viscosity of 2.5×10^{-6} m^2/s is flowing at 0.03 m/s from an orifice 75 mm in diameter. How can the fluid be described?

(A) The fluid is completely turbulent.
(B) The fluid is in the transition zone.
(C) The fluid is laminar.
(D) The fluid's turbulence cannot be calculated from the information given.

$$\begin{aligned} \text{Re} &= \frac{\text{v}D}{\nu} \\ &= \frac{\left(0.03\ \frac{\text{m}}{\text{s}}\right)(0.075\ \text{m})}{2.5 \times 10^{-6}\ \frac{\text{m}^2}{\text{s}}} \\ &= 900 \end{aligned}$$

A Reynolds number of 900 means that the flow is well within the laminar (Re < 2000) region.

The answer is (C).

FLUIDS–59

The Reynolds number of a sphere falling in air is 1×10^6. If the sphere's radius is 0.5 m, what is most nearly its velocity? ($\rho_{\text{air}} = 1.225\ \text{kg/m}^3$, $\mu_{\text{air}} = 1.789 \times 10^{-5}$ Pa·s)

(A) 2.5 m/s (B) 5.2 m/s (C) 11 m/s (D) 15 m/s

$$\begin{aligned} \text{Re} &= \frac{\text{v}D}{\nu} = \frac{\rho \text{v} D}{\mu} \\ \text{v} &= \frac{\mu \text{Re}}{\rho D} \\ &= \frac{(1.789 \times 10^{-5}\ \text{Pa·s})(1 \times 10^6)}{\left(1.225\ \frac{\text{kg}}{\text{m}^3}\right)(1\ \text{m})} \\ &= 14.6\ \text{m/s} \quad (15\ \text{m/s}) \end{aligned}$$

The answer is (D).

FLUIDS–60

Which of the following is NOT true regarding the Blasius boundary layer solution?

(A) It is valid only for potential flow.
(B) It is valid for laminar flow
(C) It is an approximate solution.
(D) It permits one to calculate the skin friction on a flat plate.

The Blasius solution is an approximate solution to the boundary layer equations and makes some simplifying assumptions. It is valid for laminar, viscous flow and permits the evaluation of shear stress and skin friction.

The Blasius solution or any other boundary layer concept has no meaning for potential flow.

The answer is (A).

FLUIDS–61

From the Blasius solution for laminar boundary layer flow, the average coefficient of skin friction is $C_f = 1.328/\sqrt{\text{Re}}$. If air ($\rho_{\text{air}} = 1.225$ kg/m^3 and $\mu_{\text{air}} = 1.789 \times 10^{-5}$ Pa·s) is flowing past a 10 m long flat plate at a velocity of 30 m/s, what is most nearly the force per unit width on the plate?

(A) 0.85 N (B) 1.0 N (C) 1.3 N (D) 1.6 N

$$\begin{aligned}
\text{Re}_L &= \frac{\rho_{\text{air}} \text{v}_{\text{air}} L_{\text{plate}}}{\mu_{\text{air}}} \\
&= \frac{\left(1.225\ \frac{\text{kg}}{\text{m}^3}\right)\left(30\ \frac{\text{m}}{\text{s}}\right)(10\ \text{m})}{1.789 \times 10^{-5}\ \text{Pa·s}} \\
&= 2.054 \times 10^7
\end{aligned}$$

$$\begin{aligned}
C_f &= \frac{1.328}{\sqrt{\text{Re}_L}} \\
&= \frac{1.328}{\sqrt{2.054 \times 10^7}} \\
&= 2.93 \times 10^{-4}
\end{aligned}$$

$$C_f = \frac{F}{\frac{1}{2}\rho_{\text{air}} \text{v}_{\text{air}}^2 L_{\text{plate}} w}$$

Rearranging to solve for F,

$$\begin{aligned}\frac{F}{w} &= \tfrac{1}{2}C_f \rho_{\text{air}} \text{v}_{\text{air}}^2 L_{\text{plate}} \\ &= \left(\frac{1}{2}\right)(2.93\times 10^{-4})\left(1.225\ \frac{\text{kg}}{\text{m}^3}\right)\left(30\ \frac{\text{m}}{\text{s}}\right)^2(10\ \text{m}) \\ &= 1.62\ \text{N/m} \quad (1.6\ \text{N per unit width of plate})\end{aligned}$$

The answer is (D).

FLUIDS–62

From what were the curves of the Moody friction factor diagram for pipe flow determined?

(A) calculations based on potential flow
(B) theoretical solutions of the Navier-Stokes equations
(C) experimental results for inviscid fluids
(D) experimental results for viscous fluids

The curves in the Moody diagram are experimental data plots. They are valid for viscous fluids.

The answer is (D).

FLUIDS–63

What is most nearly the friction factor for flow in a circular pipe where the Reynolds number is 1000?

(A) 0.008 (B) 0.06 (C) 0.08 (D) 0.1

For Re < 2000, the friction factor, f, is given by the following.

$$\begin{aligned}f &= \frac{64}{\text{Re}} \\ &= \frac{64}{1000} \\ &= 0.064 \quad (0.06)\end{aligned}$$

The answer is (B).

FLUIDS–64

For pipe flow in the laminar flow region, how is the friction factor related to the Reynolds number?

(A) $f \propto \frac{1}{\text{Re}}$ (B) $f \propto \left(\frac{1}{\text{Re}}\right)^2$ (C) $f \propto \text{Re}$ (D) $f \propto \text{Re}^2$

In the laminar region, $f = 64/\text{Re}$.

The answer is (A).

FLUIDS–65

Which of the following flow meters measure(s) the average fluid velocity rather than a point or local velocity in a pipe?

I. venturi meter
II. pitot tube
III. impact tube
IV. orifice meter
V. hot-wire anemometer

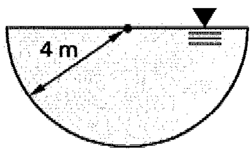
(A) I only (B) II only (C) I and IV (D) II and V

Of the four choices given, only venturi and orifice meters measure average velocity.

The answer is (C).

FLUIDS–66

What is the hydraulic radius of the semicircular channel shown?



(A) 2 m (B) 3 m (C) 4 m (D) 6 m

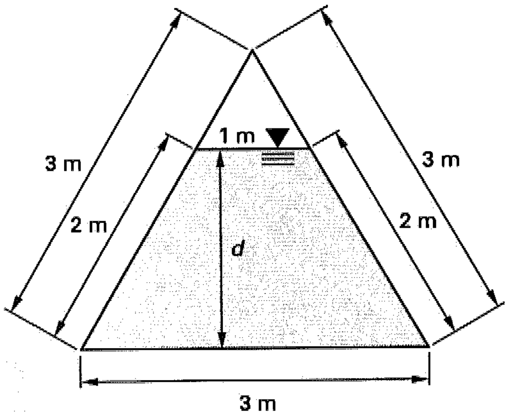
The hydraulic radius, r_h, is

$$\begin{aligned} r_h &= \frac{\text{cross-section area}}{\text{wetted perimeter}} \\ &= \frac{\frac{1}{2}(\pi r^2)}{\pi r} \\ &= \frac{r}{2} \\ &= \frac{4 \text{ m}}{2} \\ &= 2 \text{ m} \end{aligned}$$

The answer is (A).

FLUIDS–67

What is the hydraulic radius of the channel shown?



(A) 0.33 m (B) 0.43 m (C) 0.49 m (D) 1.5 m

$$d = \sqrt{(2 \text{ m})^2 - (1 \text{ m})^2} = \sqrt{3} \text{ m}$$

$$\begin{aligned} \text{cross-sectional area} &= \left(\frac{1 \text{ m} + 3 \text{ m}}{2}\right) d \\ &= (2 \text{ m})(\sqrt{3} \text{ m}) \\ &= 3.46 \text{ m}^2 \end{aligned}$$

Substituting,

$$r_h = \frac{\text{cross-sectional area}}{\text{wetted perimeter}}$$
$$= \frac{3.46 \text{ m}^2}{2 \text{ m} + 3 \text{ m} + 2 \text{ m}}$$
$$= 0.49 \text{ m}$$

The answer is (C).

FLUIDS–68

For fully developed laminar flow of fluids through circular pipes, the average velocity is what fraction of the maximum velocity?

(A) 1/8 (B) 1/4 (C) 1/2 (D) 3/4

For laminar flow in pipes,

$$\text{v}_{\text{ave}} = \frac{\text{v}_{\text{max}}}{2}$$

The answer is (C).

FLUIDS–69

The flow rate of water through a cast-iron pipe is 20 m^3/min. The diameter of the pipe is 0.3 m, and the coefficient of friction is $f = 0.0173$. What is most nearly the pressure drop over a 30 m length of pipe?

(A) 9.8 kPa (B) 13 kPa (C) 17 kPa (D) 19 kPa

$$\dot{V} = \frac{20 \ \frac{\text{m}^3}{\text{min}}}{60 \ \frac{\text{s}}{\text{min}}}$$
$$= 0.333 \text{ m}^3/\text{s}$$

$$\text{v} = \frac{\dot{V}}{A} = \frac{\dot{V}}{\pi r^2}$$
$$= \frac{0.333 \ \frac{\text{m}^3}{\text{s}}}{\pi \left(\frac{0.3 \text{ m}}{2}\right)^2}$$
$$= 4.71 \text{ m/s}$$

The head loss, Δh, is

$$\begin{aligned}\Delta h &= f\left(\frac{l}{d}\right)\left(\frac{v^2}{2g}\right)\\ &= (0.0173)\left(\frac{30\ \text{m}}{0.3\ \text{m}}\right)\left(\frac{\left(4.71\ \frac{\text{m}}{\text{s}}\right)^2}{(2)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}\right)\\ &= 1.96\ \text{m}\end{aligned}$$

Rearranging to solve for Δp,

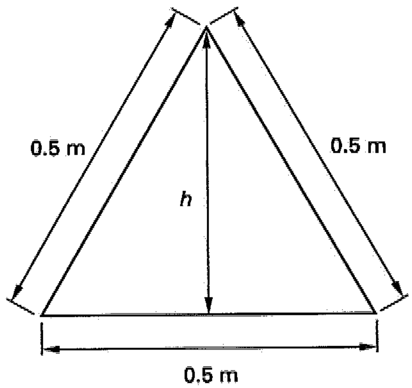
$$\begin{aligned}\Delta h &= \frac{\Delta p}{\rho g}\\ \Delta p &= \rho g \Delta h = \left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1.96\ \text{m})\\ &= 19\,200\ \text{Pa}\quad (19\ \text{kPa})\end{aligned}$$

The answer is (D).

FLUIDS–70

A completely full cast-iron pipe of equilateral triangular cross section (vertex up) and with side length of 0.5 m has water flowing through it. The flow rate is 22 m^3/min, and the friction factor for the pipe is 0.017. What is most nearly the pressure drop in a 30 m length of pipe?

(A) 6.8 kPa (B) 9.8 kPa (C) 10 kPa (D) 15 kPa



$$h = \sqrt{(0.5 \text{ m})^2 - \left(\frac{0.5 \text{ m}}{2}\right)^2}$$
$$= 0.433 \text{ m}$$

$$\text{cross-sectional area} = \tfrac{1}{2}bh$$
$$= \left(\frac{1}{2}\right)(0.5 \text{ m})(0.433 \text{ m})$$
$$= 0.108 \text{ m}^2$$

$$D_e = 4\left(\frac{\text{cross-sectional area}}{\text{wetted perimeter}}\right)$$
$$= (4)\left(\frac{0.108 \text{ m}^2}{0.5 \text{ m} + 0.5 \text{ m} + 0.5 \text{ m}}\right)$$
$$= 0.288 \text{ m}$$

$$\dot{V} = \frac{22 \ \frac{\text{m}^3}{\text{min}}}{60 \ \frac{\text{sec}}{\text{min}}}$$
$$= 0.367 \text{ m}^3/\text{s}$$

$$\text{v} = \frac{\dot{V}}{A}$$
$$= \frac{0.367 \ \frac{\text{m}^3}{\text{s}}}{0.108 \text{ m}^2}$$
$$= 3.40 \text{ m/s}$$

The head loss, Δh, is

$$\Delta h = f\left(\frac{L}{D_e}\right)\left(\frac{\text{v}^2}{2g}\right)$$
$$= (0.017)\left(\frac{30 \text{ m}}{0.288 \text{ m}}\right)\left(\frac{\left(3.40 \ \frac{\text{m}}{\text{s}}\right)^2}{(2)\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)}\right)$$
$$= 1.043 \text{ m}$$

$$\begin{aligned}\Delta p &= \rho g \Delta h = \gamma \Delta h \\ &= \left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1.043\ \text{m}) \\ &= \left(9.81\ \frac{\text{kN}}{\text{m}^3}\right)(1.043\ \text{m}) \\ &= 10.2\ \text{kN/m}^2 \quad (10\ \text{kPa})\end{aligned}$$

The answer is (C).

FLUIDS–71

A circular cylinder 4 m long and 3 m in diameter is in an air stream. The flow velocity is 5 m/s perpendicular to the longitudinal axis of the cylinder. Given that the coefficient of drag on the cylinder is 1.3, and the density of air is 1.225 kg/m^3, what is most nearly the drag force on the cylinder?

(A) 0.090 kN (B) 0.11 kN (C) 0.24 kN (D) 0.91 kN

A is the frontal area of the cylinder.

$$\begin{aligned}F_D &= \tfrac{1}{2} C_D \rho \text{v}^2 A \\ &= \left(\frac{1}{2}\right)(1.3)\left(1.225\ \frac{\text{kg}}{\text{m}^3}\right)\left(5\ \frac{\text{m}}{\text{s}}\right)^2 (3\ \text{m})(4\ \text{m}) \\ &= 238.9\ \text{N} \quad (0.24\ \text{kN})\end{aligned}$$

The answer is (C).

FLUIDS–72

Air flows past a 50 mm diameter sphere at 30 m/s. What is most nearly the drag force experienced by the sphere? The sphere has a coefficient of drag of 0.5. The density of the air is 1.225 kg/m^3.

(A) 0.26 N (B) 0.34 N (C) 0.54 N (D) 0.68 N

A is the frontal area of the sphere.

$$\begin{aligned} F_D &= \tfrac{1}{2}C_D\rho_{\text{air}}\text{v}_{\text{air}}^2A \\ &= \tfrac{1}{2}C_D\rho_{\text{air}}\text{v}_{\text{air}}^2\pi\left(\frac{d_{\text{sphere}}}{2}\right)^2 \\ &= \frac{C_D\rho_{\text{air}}\text{v}_{\text{air}}^2\pi d_{\text{sphere}}^2}{8} \\ &= \frac{(0.5)\left(1.225\ \frac{\text{kg}}{\text{m}^3}\right)\left(30\ \frac{\text{m}}{\text{s}}\right)^2\pi(0.05\ \text{m})^2}{8} \\ &= 0.541\ \text{N} \quad (0.54\ \text{N}) \end{aligned}$$

The answer is (C).

FLUIDS–73

A cylinder 10 m long and 2 m in diameter is suspended in air flowing at 8 m/s. The air flow is perpendicular to the longitudinal axis of the cylinder. The density of air is 1.225 kg/m^3, and the coefficient of drag of the cylinder is 1.3. What is most nearly the drag force on the cylinder?

(A) 0.31 kN (B) 0.85 kN (C) 1.0 kN (D) 2.3 kN

$$\begin{aligned} F_D &= \tfrac{1}{2}C_D\rho\text{v}^2A \\ &= \left(\frac{1}{2}\right)(1.3)\left(1.225\ \frac{\text{kg}}{\text{m}^3}\right)\left(8\ \frac{\text{m}}{\text{s}}\right)^2(10\ \text{m})(2\ \text{m}) \\ &= 1019.2\ \text{N} \quad (1.02\ \text{kN}) \end{aligned}$$

The answer is (C).

FLUIDS–74

What is most nearly the terminal velocity of a 50 mm diameter, solid aluminum sphere falling in air? The sphere has a coefficient of drag of 0.5, the density of aluminum, ρ_{alum}, is 2650 kg/m^3, and the density of air, ρ_{air}, is 1.225 kg/m^3.

(A) 25 m/s (B) 53 m/s (C) 88 m/s (D) 130 m/s

Let v_t be the terminal velocity. At terminal velocity, the drag force, F_D, equals the weight.

$$F_D = \tfrac{1}{2} C_D \rho_{\text{air}} \text{v}_t^2 A = mg$$

$$mg = \tfrac{4}{3}\pi \left(\frac{d}{2}\right)^3 \rho_{\text{alum}} g$$

$$A = \pi \left(\frac{d}{2}\right)^2$$

$$\left(\frac{1}{2}\right) C_D \rho_{\text{air}} \text{v}_t^2 \pi \left(\frac{d}{2}\right)^2 = \tfrac{4}{3}\pi \left(\frac{d}{2}\right)^3 \rho_{\text{alum}} g$$

$$\begin{aligned}\text{v}_t &= \sqrt{\frac{(2)(4\pi)\left(\frac{d}{2}\right)^3 \rho_{\text{alum}} g}{3 C_D \rho_{\text{air}} \pi \left(\frac{d}{2}\right)^2}} \\ &= \sqrt{\frac{4 d \rho_{\text{alum}} g}{3 C_D \rho_{\text{air}}}} \\ &= \sqrt{\frac{(4)(0.05\ \text{m})\left(2650\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}{(3)(0.5)\left(1.225\ \frac{\text{kg}}{\text{m}^3}\right)}} \\ &= 53.2\ \text{m/s}\end{aligned}$$

The answer is (B).

FLUIDS–75

In a flow of air ($\rho = 1.225\ \text{kg/m}^3$) around a cylinder, the circulation is calculated to be $3.97\ \text{m}^2/\text{s}$. If the free-stream velocity is 30 m/s, what is most nearly the lift generated per meter length of the cylinder?

(A) 150 N/m (B) 160 N/m (C) 170 N/m (D) 200 N/m

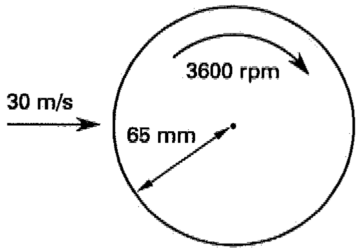
The Kutta-Joukowsky theorem states that

$$\begin{aligned}\frac{\text{lift}}{L} &= \rho_\infty \text{v}_\infty \Gamma \\ &= \left(1.225\ \frac{\text{kg}}{\text{m}^3}\right)\left(30\ \frac{\text{m}}{\text{s}}\right)\left(3.97\ \frac{\text{m}^2}{\text{s}}\right) \\ &= 146\ \text{N/m} \quad (150\ \text{N/m})\end{aligned}$$

The answer is (A).

FLUIDS–76

A 65 mm radius cylinder rotates at 3600 rpm. Air is flowing past the cylinder at 30 m/s. The density of air is 1.225 kg/m^3. Approximately how much lift is generated by the cylinder per unit length?



(A) 190 N/m (B) 220 N/m (C) 290 N/m (D) 370 N/m

From the Kutta-Joukowsky theorem,

$$\frac{\text{lift}}{L} = \rho v_\infty \Gamma$$

$$\Gamma = \oint \mathbf{V} \cdot d\ell$$

$$= \int_0^{2\pi} (r\omega)(r d\theta)$$

$$= 2\pi r^2 \omega$$

$$= 2\pi (0.065 \text{ m})^2 \left(\left(3600 \ \frac{\text{rev}}{\text{min}}\right) \left(2\pi \ \frac{\text{rad}}{\text{rev}}\right) \left(\frac{1 \text{ min}}{60 \text{ s}}\right) \right)$$

$$= 10 \text{ m}^2/\text{s}$$

$$\frac{\text{lift}}{L} = \rho v \Gamma = \left(1.225 \ \frac{\text{kg}}{\text{m}^3}\right) \left(30 \ \frac{\text{m}}{\text{s}}\right) \left(10 \ \frac{\text{m}^2}{\text{s}}\right)$$

$$= 367.5 \text{ N/m}$$

The answer is (D).

FLUIDS–77

A pump produces a head of 10 m. The volumetric flow rate through the pump is 6.3×10^{-4} m^3/s. The fluid pumped is oil with a specific gravity of 0.83. Approximately how much energy does the pump consume in one hour?

(A) 8.7 kJ (B) 17 kJ (C) 180 kJ (D) 200 kJ

$$P = \Delta p\dot{V}$$

The change in pressure, Δp, is

$$\begin{aligned}\Delta p &= \rho g h\\ &= (0.83)\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(10\ \text{m})\\ &= 81\,423\ \text{Pa}\\ E &= Pt\\ &= \Delta p\dot{V}t\\ &= (81\,423\ \text{Pa})\left(6.3\times 10^{-4}\ \frac{\text{m}^3}{\text{s}}\right)(3600\ \text{s})\\ &= 184\,667\ \text{N·m}\quad (180\ \text{kJ})\end{aligned}$$

The answer is (C).

FLUIDS–78

A pump has an efficiency of 65%. It is driven by a 550 W motor. The pump produces a pressure rise of 120 Pa in water. What is the required flow rate?

(A) $3.0\ \text{m}^3/\text{s}$ (B) $3.4\ \text{m}^3/\text{s}$ (C) $4.6\ \text{m}^3/\text{s}$ (D) $4.8\ \text{m}^3/\text{s}$

The power supplied by the pump to the water, P_r, is

$$P_r = \eta P_i$$

In the preceding equation, η is efficiency and P_i is ideal power.

$$P_r = \Delta p\dot{V}$$

In the preceding equation, Δp is pressure rise and $\dot{V}$ is the volumetric flow rate.

$$\Delta p\dot{V} = \eta P_i$$

Rearranging,

$$\dot{V} = \frac{\eta P_i}{\Delta p}$$

$$P_i = 550\ \text{W}$$

Therefore,

$$\begin{aligned}\dot{V} &= \frac{(0.65)(550\ \text{W})}{120\ \text{Pa}}\\ &= 2.98\ \text{m}^3/\text{s}\quad (3.0\ \text{m}^3/\text{s})\end{aligned}$$

The answer is (A).

5 THERMODYNAMICS

THERMODYNAMICS–1

Which of the following are intensive properties?

I. temperature
II. pressure
III. composition
IV. mass

(A) I only (B) IV only (C) I and II (D) I, II, and III

An intensive property does not depend on the amount of material present. This is true for temperature, pressure, and composition.

The answer is (D).

THERMODYNAMICS–2

How many independent properties are required to completely fix the equilibrium state of a pure gaseous compound?

(A) 0 (B) 1 (C) 2 (D) 3

The number of independently variable properties needed to fix the state of a gaseous compound, f, is given by the Gibbs phase rule.

$$f = n - p + 2$$

In the preceding equation, n is the number of components and p is the number of phases.

For a pure gas,

$$n = p = 1$$
$$f = 1 - 1 + 2$$
$$= 2$$

The answer is (C).

THERMODYNAMICS–3

Which of the following thermodynamic relations is INCORRECT?

(A) $TdS = dU + pdV$ (B) $TdS = dH - Vdp$
(C) $U = Q - W$ (D) $H = U + pV$

$U = Q + W$. Therefore, the relation in option (C) is incorrect.

The answer is (C).

THERMODYNAMICS–4

If air is at a pressure, p, of 135 Pa, and at a temperature, T, of 440K, what is most nearly the specific volume, v? (Air's specific gas constant is R = 88.81 J/kg·K, and air can be modeled as an ideal gas.)

(A) 110 m^3/kg (B) 130 m^3/kg (C) 290 m^3/kg (D) 300 m^3/kg

$$pv = RT$$
$$v = \frac{RT}{p}$$
$$= \frac{\left(88.81 \ \frac{\text{J}}{\text{kg·K}}\right)(440\text{K})}{135 \text{ Pa}}$$
$$= 289 \ \text{m}^3/\text{kg} \quad (290 \ \text{m}^3/\text{kg})$$

The answer is (C).

THERMODYNAMICS–5

Which of the following relationships defines enthalpy?

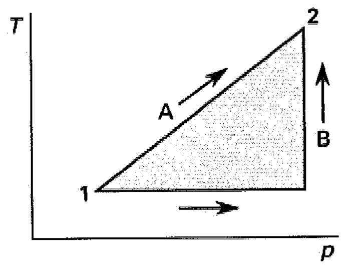
(A) $h = u + \frac{p}{T}$ (B) $h = u + pv$ (C) $h = u + \frac{p}{v}$ (D) $h = pv + T$

Enthalpy is given by $h = u + pv$.

The answer is (B).

THERMODYNAMICS–6

In a certain constant mass system, the conditions change from point 1 to point 2. How does the change in enthalpy for path A differ from the enthalpy change for path B in going from point 1 to point 2?



(A) $\Delta H_A > \Delta H_B$

(B) $\Delta H_A = \Delta H_B$

(C) $\Delta H_A < \Delta H_B$

(D) $\Delta H_B \rightarrow \infty$

Enthalpy is a state function. Therefore, its value depends only on the initial and final states, and not on the path taken between the two states. Thus, $\Delta H_A = \Delta H_B$.

The answer is (B).

THERMODYNAMICS–7

Steam at 416 Pa and 166K has a specific volume of 0.41 m^3/kg and a specific enthalpy of 29.4 kJ/kg. Find the internal energy per kilogram of steam.

(A) 28.5 kJ/kg (B) 29.2 kJ/kg (C) 30.2 kJ/kg (D) 30.4 kJ/kg

$$h = u + pv$$

$$\begin{aligned} u &= h - pv \\ &= 29.4\ \frac{\text{kJ}}{\text{kg}} - (416\ \text{Pa})\left(0.41\ \frac{\text{m}^3}{\text{kg}}\right) \\ &= 29.2\ \text{kJ/kg} \end{aligned}$$

The answer is (B).

THERMODYNAMICS–8

Which of the following is true for water at a reference temperature where enthalpy is zero?

(A) Internal energy is negative.
(B) Entropy is nonzero.
(C) Specific volume is zero.
(D) Vapor pressure is zero.

Typically, the saturation temperature (0°C for water) is chosen as the enthalpic reference temperature. At that temperature, the water has a distinct (vapor) pressure and volume. Therefore, options (C) and (D) are false. Although there is no thermodynamic relationship between entropy and enthalpy, the values of enthalpy and entropy are commonly referenced to the same temperature. Thus, by convention, entropy is zero when enthalpy is zero. Therefore, option (B) is also false.

The definition of enthalpy is the sum of internal energy and flow energy.

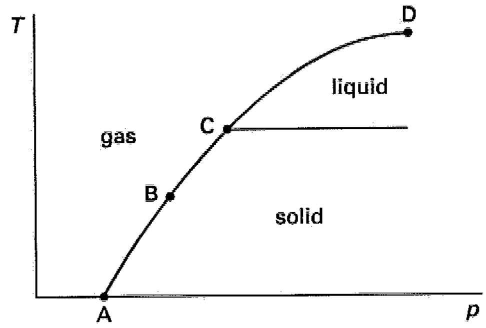
$$h = u + pv$$

If enthalpy is zero and the flow energy, pv, is nonzero, then the internal energy must be negative.

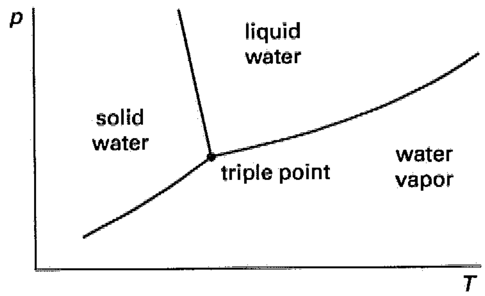
The answer is (A).

THERMODYNAMICS–9

Which of the following is the triple point for the phase diagram given?



(A) A (B) B (C) C (D) D

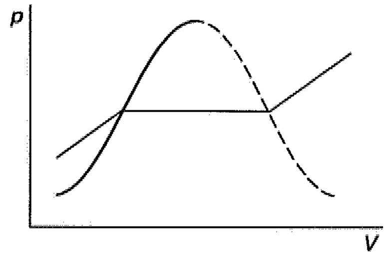


The triple point is the point at which the liquid, solid, and vapor states are all in equilibrium. Therefore, point C is the triple point.

The answer is (C).

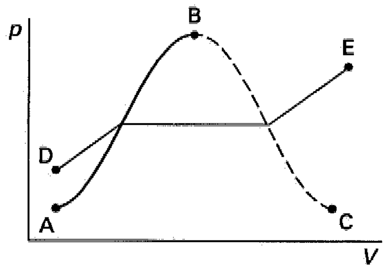
THERMODYNAMICS–10

What does the dashed curve in the figure represent?



(A) the solidus line
(B) an isotherm
(C) the saturated liquid line
(D) the saturated vapor line

The dashed line shown is the saturated vapor line.



AB is the saturated liquid line.
B is the critical point.
BC is the saturated vapor line.
DE represents a possible path of the system when heated.

The answer is (D).

THERMODYNAMICS–11

In an ideal gas mixture of constituents i and j, what is the mole fraction of component i, x_i, equal to?

(A) $\frac{T_i}{T_i + T_j}$ (B) $\frac{Z_i}{Z_i + Z_j}$ (C) $\frac{p_i}{p_i + p_j}$ (D) $\frac{m_i}{m_i + m_j}$

For an ideal gas, the mole fraction is equal to the partial pressure fraction.

$$x_i = \frac{p_i}{p_i + p_j}$$

The answer is (C).

THERMODYNAMICS–12

1.36 kg of air are held at 6.89 kPa and 38°C. Given that $R_{\text{air}} = 88.89$ J/kg·K, what is most nearly the volume of the container?

(A) 2.2 m^3 (B) 3.1 m^3 (C) 4.8 m^3 (D) 5.5 m^3

Use the ideal gas law.

$$T = (38°\text{C} + 273°) = 311\text{K}$$

$$pV = mRT$$

$$V = \frac{mRT}{p}$$

$$= \frac{(1.36\ \text{kg})\left(88.89\ \frac{\text{J}}{\text{kg·K}}\right)(311\text{K})}{6890\ \text{Pa}}$$

$$= 5.46\ \text{m}^3 \quad (5.5\ \text{m}^3)$$

The answer is (D).

THERMODYNAMICS–13

The compressibility factor, Z, is used for predicting the behavior of nonideal gases. How is the compressibility factor defined relative to an ideal gas? (Subscript "c" refers to critical value.)

(A) $Z = \frac{p}{p_c}$ (B) $Z = \frac{pV}{RT}$ (C) $Z = \frac{T}{T_c}$ (D) $Z = \left(\frac{T}{T_c}\right)\left(\frac{p_c}{p}\right)$

For real gases, the compressibility factor, Z, is a dimensionless constant given by $pV = ZRT$. Therefore, $Z = pV/RT$.

The answer is (B).

THERMODYNAMICS–14

On what plane is the Mollier diagram plotted?

(A) p-V (B) p-T (C) h-s (D) s-u

The axes for a Mollier diagram are enthalpy and entropy (h-s).

The answer is (C).

THERMODYNAMICS–15

How is the quality, x, of a liquid-vapor mixture defined?

(A) the fraction of the total volume that is saturated vapor
(B) the fraction of the total volume that is saturated liquid
(C) the fraction of the total mass that is saturated vapor
(D) the fraction of the total mass that is saturated liquid

The quality of the liquid-vapor mixture is defined as the fraction of the total mass that is saturated vapor.

The answer is (C).

THERMODYNAMICS–16

What is the expression for the heat of vaporization?

h_g = enthalpy of the saturated vapor

h_f = enthalpy of the saturated liquid

(A) $h_g + h_f$ (B) $h_f - h_g$ (C) $h_g - h_f$ (D) $h_g^2 - h_f^2$

The heat of vaporization, h_{fg}, is the difference between the enthalpy of the saturated vapor and the enthalpy of the saturated liquid. Thus, $h_{fg} = h_g - h_f$.

The answer is (C).

THERMODYNAMICS–17

From the steam tables, determine the average specific heat at constant pressure, c_p, of steam at 10 kPa and 45.8°C.

(A) 1.79 kJ/kg·°C (B) 10.3 kJ/kg·°C (C) 30.6 kJ/kg·°C (D) 100 kJ/kg·°C

$$\Delta h = c_p \Delta T$$

$$c_p = \frac{\Delta h}{\Delta T}$$

From the steam tables, for 10 kPa,

at 47.7°C: $h = 2588.1$ kJ/kg
at 43.8°C: $h = 2581.1$ kJ/kg

$$c_p = \frac{2588.1\ \text{kJ/kg} - 2581.1\ \text{kJ/kg}}{47.7°\text{C} - 43.8°\text{C}} = 1.79\ \text{kJ/kg·°C}$$

The answer is (A).

THERMODYNAMICS–18

A 10 m^3 vessel initially contains 5 m^3 of liquid water and 5 m^3 of saturated water vapor at 100 kPa. Calculate the internal energy of the system using the steam tables.

(A) 5×10^5 kJ (B) 8×10^5 kJ (C) 1×10^6 kJ (D) 2×10^6 kJ

From the steam tables,

$$v_f = 0.001043\ \text{m}^3/\text{kg}$$

$$v_g = 1.6940\ \text{m}^3/\text{kg}$$

$$u_f = 417.3\ \text{kJ/kg}$$

$$u_g = 2506\ \text{kJ/kg}$$

$$\begin{aligned} m_{\text{vap}} &= \frac{V_{\text{vap}}}{v_g} \\ &= \frac{5\ \text{m}^3}{1.694\ \frac{\text{m}^3}{\text{kg}}} \\ &= 2.95\ \text{kg} \end{aligned}$$

$$\begin{aligned} m_{\text{liq}} &= \frac{V_{\text{liq}}}{v_f} \\ &= \frac{5\ \text{m}^3}{0.001043\ \frac{\text{m}^3}{\text{kg}}} \\ &= 4794\ \text{kg} \end{aligned}$$

$$\begin{aligned} u &= u_f m_{\text{liq}} + u_g m_{\text{vap}} \\ &= \left(417.3\ \frac{\text{kJ}}{\text{kg}}\right)(4794\ \text{kg}) + \left(2506.1\ \frac{\text{kJ}}{\text{kg}}\right)(2.95\ \text{kg}) \\ &= 2.01\times10^6\ \text{kJ} \end{aligned}$$

The answer is (D).

THERMODYNAMICS–19

A vessel with a volume of 1 m^3 contains liquid water and water vapor in equilibrium at 600 kPa. The liquid water has a mass of 1 kg. Using the steam tables, determine the approximate mass of the water vapor.

(A) 0.99 kg (B) 1.6 kg (C) 1.9 kg (D) 3.2 kg

From the steam tables at 600 kPa,

$$\begin{aligned} v_f &= 0.001101\ \text{m}^3/\text{kg} \\ v_g &= 0.3157\ \text{m}^3/\text{kg} \\ V_{\text{total}} &= m_f v_f + m_g v_g \\ m_g &= \frac{V_{\text{total}} - m_f v_f}{v_g} \\ &= \frac{1\ \text{m}^3 - (1\ \text{kg})\left(0.001101\ \frac{\text{m}^3}{\text{kg}}\right)}{0.3157\ \frac{\text{m}^3}{\text{kg}}} \\ &= 3.16\ \text{kg} \quad (3.2\ \text{kg}) \end{aligned}$$

The answer is (D).

THERMODYNAMICS–20

What is most nearly the entropy of steam at 476 kPa with a quality of 0.6?

(A) 2.4 kJ/kg·K
(B) 3.8 kJ/kg·K
(C) 4.8 kJ/kg·K
(D) 5.7 kJ/kg·K

From the steam tables at 476 kPa,

$$\begin{aligned} s_f &= 1.8418\ \text{kJ/kg·K} \\ s_{fg} &= 4.9961\ \text{kJ/kg·K} \\ s &= s_f + x s_{fg} \end{aligned}$$

In the preceding equation, x is quality.

$$s = 1.8418\ \frac{\text{kJ}}{\text{kg·K}} + (0.6)\left(4.9961\ \frac{\text{kJ}}{\text{kg·K}}\right)$$
$$= 4.839\ \text{kJ/kg·K} \quad (4.8\ \text{kJ/kg·K})$$

The answer is (C).

THERMODYNAMICS–21

If 0.45 kg of steam at 101.3 kPa and 63% quality is heated isentropically, at approximately what pressure will it reach the saturated vapor state?

(A) 15 200 kPa (B) 16 300 kPa (C) 17 300 kPa (D) 17 800 kPa

Use the steam tables.

$$p_1 = 101.3\ \text{kPa}$$
$$s_{f1} = 1.307\ \text{kJ/kg·K}$$
$$s_{fg1} = 6.048\ \text{kJ/kg·K}$$
$$s_1 = s_{f1} + 0.63 s_{fg1}$$
$$= 1.307\ \frac{\text{kJ}}{\text{kg·K}} + (0.63)\left(6.048\ \frac{\text{kJ}}{\text{kg·K}}\right)$$
$$= 5.117\ \text{kJ/kg·K}$$

Now, find p_2 such that $s_{g2} = 5.117$ kJ/kg·K. Interpolating from the steam tables, $p_2 \approx 17\,800$ kPa.

The answer is (D).

THERMODYNAMICS–22

The first law of thermodynamics is based on which of the following principles?

(A) conservation of mass
(B) the enthalpy-entropy relationship
(C) action-reaction
(D) conservation of energy

The first law of thermodynamics is based on the principle of conservation of energy.

The answer is (D).

THERMODYNAMICS–23

The general energy equation for an open system involves the following five terms.

I. accumulation of energy

II. net energy transfer by work (standard sign convention)

III. net energy transfer by heat (standard sign convention)

IV. transfer of energy in by mass flow

V. transfer of energy out by mass flow

Using the standard sign conventions, what is the proper arrangement of these terms for the general energy equation satisfying the first law of thermodynamics?

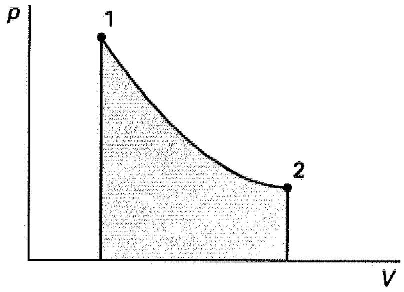
(A) I = −II + III + IV − V
(B) I = II + III + IV + V
(C) I = II + III + IV − V
(D) I = II − III − IV + V

The first law of thermodynamics states that the total change in energy (I) is equal to the energy in (IV) minus the energy out (V) minus the work done on the system (II) plus the heat transferred to the system (III). Thus, I = − II + III + IV − V.

The answer is (A).

THERMODYNAMICS–24

In a reversible process, the state of a system changes from state 1 to state 2 as shown on the p-V diagram. What does the shaded area on the diagram represent?



(A) free-energy change
(B) heat transfer
(C) enthalpy change
(D) work done by the system

For a reversible process, the work done by the system is given by the following.

$$W = \int_1^2 p dV$$

Therefore, the shaded area represents the work done by the system.

The answer is (D).

THERMODYNAMICS–25

What is the value of the work done for a closed, reversible, isometric system?

(A) zero
(B) positive
(C) negative
(D) positive or negative

$$W = \int p dV$$

An isometric system is a system which has a constant volume ($dV = 0$). Therefore, the work done by the system is zero.

The answer is (A).

THERMODYNAMICS–26

The expansion of a gas through a plug at a high pressure results in a temperature rise, while at lower pressures a temperature drop occurs. The Joule-Thompson coefficient, μ_{JT}, is defined as the ratio of the change in temperature to the change in pressure. The temperature at which μ_{JT} changes from positive to negative is called the inversion temperature. When μ_{JT} is negative, which of the following statements is true?

(A) Gases may be liquified by pressurization.
(B) No liquification is possible.
(C) Only trace liquification is possible.
(D) Liquification can be obtained only with a catalyst.

When $\mu_{JT} < 0$, then $\partial T/\partial p < 0$. Thus, a pressure rise is accompanied by a temperature drop. Therefore, a gas may be liquified by pressurization.

The answer is (A).

THERMODYNAMICS–27

A 5 m^3 vessel initially contains 50 kg of liquid water and saturated water vapor at a total internal energy of 27 300 kJ. Calculate the heat requirement to vaporize all of the liquid.

(A) 100 000 kJ (B) 200 000 kJ (C) 300 000 kJ (D) 400 000 kJ

An expression for the first law of thermodynamics is

$$Q = U_2 - U_1$$
$$U_1 = 27\,300 \text{ kJ}$$

Find U_2 in the steam tables at 100% vapor. $v_g = 5 \text{ m}^3/50 \text{ kg} = 0.10 \text{ m}^3/\text{kg}$.

The final state is at $p = 2.00$ MPa and $u_g = 2600$ kJ/kg.

$$\begin{aligned} U_2 &= mu_g \\ &= \left(2600 \ \frac{\text{kJ}}{\text{kg}}\right)(50 \text{ kg}) \\ &= 130\,000 \text{ kJ} \\ Q &= 130\,000 \text{ kJ} - 27\,300 \text{ kJ} \\ &= 103\,000 \text{ kJ} \quad (100\,000 \text{ kJ}) \end{aligned}$$

The answer is (A).

THERMODYNAMICS–28

What is most nearly the change in internal energy of 2.27 kg of oxygen gas when the temperature changes from 38°C to 49°C? ($c_v = 0.658$ kJ/kg·K)

(A) 16 kJ (B) 420 kJ (C) 470 kJ (D) 630 kJ

$$\begin{aligned} \Delta U &= mc_v\Delta T \\ \Delta T &= 49^\circ\text{C} - 38^\circ\text{C} \\ &= 11^\circ\text{C} \\ \Delta U &= (2.27 \text{ kg})\left(0.658 \ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(11^\circ\text{C}) \\ &= 16.4 \text{ kJ} \quad (16 \text{ kJ}) \end{aligned}$$

The answer is (A).

THERMODYNAMICS–29

Water (specific heat c_v = 4.2 kJ/kg·K) is being heated by a 1500 W heater. What is most nearly the temperature rate of change for 1 kg of water?

(A) 0.043K/s (B) 0.18K/s (C) 0.36K/s (D) 1.5K/s

$$\dot{Q} = mc_v(\dot{\Delta T})$$

$$\dot{\Delta T} = \frac{Q}{mc_v}$$

$$= \frac{1500 \text{ W}}{(1 \text{ kg})\left(4200 \frac{\text{J}}{\text{kg·K}}\right)}$$

$$= 0.357\text{K/s} \quad (0.36\text{K/s})$$

The answer is (C).

THERMODYNAMICS–30

1 kg of water (c_v = 4.2 kJ/kg·K) is heated by 316 kJ of energy. What is most nearly the change in temperature?

(A) 18K (B) 71K (C) 74K (D) 75K

$$mc_v\Delta T = Q$$

$$\Delta T = \frac{Q}{mc_v}$$

$$= \frac{316 \text{ kJ}}{(1 \text{ kg})\left(4.2 \frac{\text{kJ}}{\text{kg·K}}\right)}$$

$$= 75.2\text{K} \quad (75\text{K})$$

The answer is (D).

THERMODYNAMICS–31

What is most nearly the change in enthalpy per kg of nitrogen gas as its temperature changes from 260°C to 93°C ($c_p = 1.04$ kJ/kg·K)?

(A) −200 kJ (B) −170 kJ (C) 110 kJ (D) 170 kJ

$$\begin{aligned}\Delta h &= c_p \Delta T \\ &= \left(1.04\ \frac{\text{kJ}}{\text{kg·K}}\right)(93°\text{C} - 260°\text{C}) \\ &= -173.7\ \text{kJ} \quad (-170\ \text{kJ})\end{aligned}$$

The answer is (B).

THERMODYNAMICS–32

Calculate the change in enthalpy as 1 kg of nitrogen is heated from 1000K to 1500K, assuming the nitrogen is an ideal gas at a constant pressure. The temperature-dependent specific heat of nitrogen is

$$C_p = 39.06 - 512.79T^{-1.5} + 1072.7T^{-2} - 820.4T^{-3}$$

(C_p is in kJ/kmol·K, and T is in K.)

(A) 600 kJ (B) 700 kJ (C) 800 kJ (D) 900 kJ

$$C_p \equiv \left(\frac{\partial H}{\partial T}\right)_p$$

$$\begin{aligned}\partial H &= C_p \partial T \\ &= \int C_p dT \\ &= \int_{1000}^{1500} \left(39.06 - 512.79T^{-1.5} + 1072.7T^{-2} - 820.4T^{-3}\right) dT \\ &= \left| \left(39.06T + 1025.6T^{-0.5} + 1072.7T^{-1} + 410.2T^{-2}\right)\right|_{1000}^{1500} \\ &= 58\,617\ \frac{\text{kJ}}{\text{kmol}} - 39\,094\ \frac{\text{kJ}}{\text{kmol}} \\ &= 19\,523\ \text{kJ/kmol}\end{aligned}$$

$$\Delta H_{\text{total}} = m\Delta h = m\left(\frac{\Delta H}{\text{MW}}\right) = \left(19\,523\ \frac{\text{kJ}}{\text{kmol}}\right)(1\ \text{kg})\left(\frac{1\ \text{kmol}}{28\ \text{kg}}\right)$$

$$= 697.3\ \text{kJ} \quad (700\ \text{kJ})$$

The answer is (B).

THERMODYNAMICS–33

What is most nearly the resulting pressure when 400 g of air at 103.6 kPa and 93°C is heated at constant volume to 427°C?

(A) 160 kPa (B) 200 kPa (C) 250 kPa (D) 480 kPa

$$\frac{T_1}{p_1} = \frac{T_2}{p_2}$$

$$p_2 = \frac{p_1 T_2}{T_1}$$

$$= \frac{(103.6\ \text{kPa})(427^\circ\text{C} + 273^\circ)}{93^\circ\text{C} + 273^\circ}$$

$$= 198\ \text{kPa} \quad (200\ \text{kPa})$$

The answer is (B).

THERMODYNAMICS–34

Approximately how much power is required to isothermally compress 23 m^3/min of air from 101.5 kPa to 828.5 kPa?

(A) 64 kW (B) 82 kW (C) 92 kW (D) 98 kW

For an isothermal process,

$$W = -p_1 V_1 \ln\frac{p_1}{p_2}$$

$$= p_1 V_1 \ln\frac{p_2}{p_1}$$

$$= (101.5\ \text{kPa})(23\ \text{m}^3)\ln\left(\frac{828.5\ \text{kPa}}{101.5\ \text{kPa}}\right)$$

$$= 4901\ \text{kJ}$$

$$\begin{aligned} P &= \frac{dW}{dt} \\ &= \frac{4901 \text{ kJ}}{60 \text{ s}} \\ &= 81.7 \text{ kW} \quad (82 \text{ kW}) \end{aligned}$$

The answer is (B).

THERMODYNAMICS–35

What is most nearly the work done by a system in which 1 kmol of water completely evaporates at 100°C and 1 atm constant pressure?

(A) 1000 kJ (B) 2100 kJ (C) 2500 kJ (D) 3100 kJ

$$p = (1 \text{ atm})\left(101.3 \ \frac{\text{kPa}}{\text{atm}}\right) = 101.3 \text{ kPa}$$

From the steam tables,

$$\begin{aligned} v_f &= 0.001\,044 \text{ m}^3/\text{kg} \\ v_g &= 1.673 \text{ m}^3/\text{kg} \end{aligned}$$

The molecular weight of water is

$$\begin{aligned} \text{MW}_{\text{H}_2\text{O}} &= 18.016 \text{ kg/kmol} \\ V_1 &= v_f \text{MW}_{\text{H}_2\text{O}} m \\ &= \left(0.001\,044 \ \frac{\text{m}^3}{\text{kg}}\right)\left(18.016 \ \frac{\text{kg}}{\text{kmol}}\right)(1 \text{ kmol}) \\ &= 0.01881 \text{ m}^3 \\ V_2 &= v_g \text{MW}_{\text{H}_2\text{O}} m \\ &= \left(1.673 \ \frac{\text{m}^3}{\text{kg}}\right)\left(18.016 \ \frac{\text{kg}}{\text{kmol}}\right)(1 \text{ kmol}) \\ &= 30.141 \text{ m}^3 \end{aligned}$$

$$W = \int_1^2 pdV$$
$$= p(V_2 - V_1)$$
$$W = (101\,300 \text{ Pa})(30.141 \text{ m}^3 - 0.018\,81 \text{ m}^3)$$
$$= 3.05 \times 10^6 \text{ J} \quad (3100 \text{ kJ})$$

The answer is (D).

THERMODYNAMICS–36

5 mol of water vapor at 100°C and 1 atm pressure are compressed isobarically to form liquid at 100°C. The process is reversible, and the ideal gas laws apply. What is most nearly the initial volume of the vapor?

(A) 120 L (B) 130 L (C) 140 L (D) 150 L

Use the ideal gas law.

$$pV = nR^*T$$
$$V = \frac{nR^*T}{p}$$
$$= \frac{(5 \text{ mol})\left(0.0821 \ \frac{\text{atm·L}}{\text{mol·K}}\right)(100°\text{C} + 273°)}{1 \text{ atm}}$$
$$= 153 \text{ L} \quad (150 \text{ L})$$

The answer is (D).

THERMODYNAMICS–37

5 kmol of water vapor at 100°C and 1 atm pressure are compressed isobarically from an initial volume of 153 L to form liquid at 100°C. The process is reversible, and the ideal gas laws apply. What is most nearly the work done on the system?

(A) 6.0 kJ (B) 6.2 kJ (C) 6.0 MJ (D) 6.2 MJ

$$W = -p(V_2 - V_1)$$

From the steam tables,

$$\begin{aligned}
v_f &= 0.001\,044\ \text{m}^3/\text{kg} \\
\text{MW}_{\text{H}_2\text{O}} &= 18.016\ \text{kg/kmol} \\
V_1 &= (153\ \text{L})\left(0.001\ \frac{\text{m}^3}{\text{L}}\right) \\
&= 0.153\ \text{m}^3 \\
V_2 &= n\text{MW}_{\text{H}_2\text{O}}v_f \\
&= (5\ \text{kmol})\left(18.016\ \frac{\text{kg}}{\text{kmol}}\right)\left(0.001\,044\ \frac{\text{m}^3}{\text{kg}}\right) \\
&= 0.094\ \text{m}^3 \\
W &= -p\Delta\text{V} = -(101\,325\ \text{Pa})\left(0.094\ \text{m}^3 - 0.153\ \text{m}^3\right) \\
&= 5978\ \text{J} \quad (6.0\ \text{kJ})
\end{aligned}$$

The answer is (A).

THERMODYNAMICS–38

5 kmol of water vapor at 100°C and 1 atm pressure are compressed isobarically to form liquid at 100°C. The process is reversible, and the ideal gas laws apply. The heat of vaporization is 2257 kJ/kg. What is most nearly the heat required for condensation for the amount of water given?

(A) −200 MJ (B) −140 MJ (C) 200 MJ (D) 410 MJ

$$\begin{aligned}
h_{fg} &= 2257\ \text{kJ/kg} \\
Q &= \Delta H \\
&= m(-h_{fg}) = nM(-h_{fg}) \\
&= (5\ \text{kmol})\left(18.016\ \frac{\text{kg}}{\text{kmol}}\right)\left(-2257\ \frac{\text{kJ}}{\text{kg}}\right) \\
&= -203.3\ \text{MJ} \quad (-200\ \text{MJ})
\end{aligned}$$

The answer is (A).

THERMODYNAMICS–39

What is the equation for the work done by a constant temperature system?

(A) $W = mRT\ln(V_2 - V_1)$

(B) $W = mR(T_2 - T_1)\ln\dfrac{V_2}{V_1}$

(C) $W = mRT\ln\dfrac{V_2}{V_1}$

(D) $W = mRT\ln\dfrac{p_2}{p_1}$

$$W = \int_1^2 p dV$$

$$p = \frac{mRT}{V}$$

$$W = \int_1^2 \frac{mRT}{V} dV$$

$$= mRT\ln V\Big|_1^2$$

$$W = mRT\ln\frac{V_2}{V_1}$$

The answer is (C).

THERMODYNAMICS–40

20 g of oxygen gas (O_2) are compressed at a constant temperature of 30°C to 5% of their original volume. What work is done on the system?

(A) 820 cal (B) 920 cal (C) 950 cal (D) 1120 cal

$$W = -\int_{V_1}^{V_2} p dV \quad \left[\begin{array}{c}\text{negative to get work}\\ \text{done on system}\end{array}\right]$$

$$= -mRT\ln\frac{V_2}{V_1}$$

$$R = \frac{R^*}{\text{MW}} = \frac{1.98\ \frac{\text{cal}}{\text{mol·K}}}{32\ \frac{\text{g}}{\text{mol}}}$$

$$= 0.0619\ \text{cal/g·K}$$

$$T = 30°\text{C} + 273° = 303\text{K}$$

$$W = -(20\text{ g})\left(0.0619\ \frac{\text{cal}}{\text{g·K}}\right)(303\text{K})\ln\left(\frac{5\%}{100\%}\right)$$

$$= 1124\text{ cal} \quad (1120\text{ cal})$$

The answer is (D).

THERMODYNAMICS–41

Helium ($R^* = 0.6403$ kJ/kg·K) is compressed isothermally from 101.3 kPa and 20°C. The compression ratio is 4. What is most nearly the work done by the gas?

(A) −320 kJ/kg (B) −260 kJ/kg (C) 170 kJ/kg (D) 180 kJ/kg

$$W = \int_1^2 pdV$$

$$= RT\ln\frac{V_2}{V_1}$$

$$= \left(0.6403\ \frac{\text{kJ}}{\text{kg·K}}\right)(293\text{K})\ln\frac{1}{4}$$

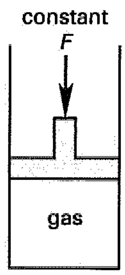
$$= -260\text{ kJ/kg}$$

The answer is (B).

THERMODYNAMICS–42

Gas is enclosed in a cylinder with a weighted piston as the top boundary. The gas is heated and expands from a volume of 0.04 m^3 to 0.10 m^3 at a constant pressure of 200 kPa. Calculate the work done by the system.

(A) 8.0 kJ (B) 10 kJ (C) 12 kJ (D) 14 kJ



At constant pressure,

$$
\begin{aligned}
W &= \int_1^2 p dV \\
&= p(V_2 - V_1) \\
&= (200\,000 \text{ Pa})(0.10 \text{ m}^3 - 0.04 \text{ m}^3) \\
&= 12\,000 \text{ J} \quad (12 \text{ kJ})
\end{aligned}
$$

The answer is (C).

THERMODYNAMICS–43

A piston-cylinder system contains a gas that expands under a constant pressure of 57 kPa. If the piston is displaced 0.3 m during the process, and the piston diameter is 0.6 m, what is the work done by the gas on the piston?

(A) 2.4 kJ (B) 3.2 kJ (C) 3.4 kJ (D) 4.8 kJ

The work is done at constant pressure.

$$
\begin{aligned}
W &= \int_1^2 p dV \\
&= p\Delta V \\
\Delta V &= A\Delta L \\
&= \pi(0.3 \text{ m})^2(0.3 \text{ m}) \\
&= 0.085 \text{ m}^3 \\
W &= (57 \text{ kPa})(0.085 \text{ m}^3) \\
&= 4.8 \text{ kJ}
\end{aligned}
$$

The answer is (D).

THERMODYNAMICS–44

Gas is enclosed in a cylinder with a weighted piston as the top boundary. The gas is heated and expands from a volume of 0.04 m^3 to 0.10 m^3. The pressure varies such that pV is constant, and the initial pressure is 200 kPa. What is most nearly the work done by the system?

(A) 6.8 kJ (B) 7.3 kJ (C) 10 kJ (D) 12 kJ

The work done by the system on the piston is given as follows.

$$W = \int_1^2 p dV$$

$$p_2 = \frac{p_1 V_1}{V_2}$$

$$W = p_1 V_1 \int_1^2 \frac{dV}{V}$$

$$= p_1 V_1 \ln \frac{V_2}{V_1}$$

$$= (200\,000 \text{ Pa})(0.04 \text{ m}^3) \ln \left(\frac{0.10 \text{ m}^3}{0.04 \text{ m}^3} \right)$$

$$= 7330 \text{ J} \quad (7.3 \text{ kJ})$$

The answer is (B).

THERMODYNAMICS–45

Steam flows into a turbine at a rate of 10 kg/s, and 10 kW of heat are lost from the turbine. Ignoring elevation and kinetic energy effects, what is most nearly the power output from the turbine?

	inlet conditions	exit conditions
pressure	2.0 MPa	0.1 MPa
temperature	350°C	–
quality	–	100%

(A) 4000 kW (B) 4400 kW (C) 4600 kW (D) 5000 kW

Use the first law of thermodynamics.

$$P = \dot{W}$$
$$= \dot{m}(h_i - h_e) + Q$$

From the steam tables,

$$h_i = 3137.0 \text{ kJ/kg}$$
$$h_e = 2675.5 \text{ kJ/kg}$$
$$\dot{W} = \left(10 \ \frac{\text{kg}}{\text{s}}\right)\left(3137.0 \ \frac{\text{kJ}}{\text{kg}} - 2675.5 \ \frac{\text{kJ}}{\text{kg}}\right) - 10 \text{ kW}$$
$$= 4605 \text{ kW} \quad (4600 \text{ kW})$$

The answer is (C).

THERMODYNAMICS–46

How does an adiabatic process compare to an isentropic process?

(A) adiabatic: heat transfer = 0; isentropic: heat transfer $\neq 0$
(B) adiabatic: heat transfer $\neq 0$; isentropic: heat transfer = 0
(C) adiabatic: reversible; isentropic: not reversible
(D) both: heat transfer = 0; isentropic: reversible

An adiabatic process is one in which there is no heat flow. It is not necessarily reversible. An isentropic process has no heat flow and is reversible.

The answer is (D).

THERMODYNAMICS–47

What is true about the polytropic exponent, n, for a perfect gas undergoing an isobaric process?

(A) $n > 0$ (B) $n < 0$ (C) $n \to \infty$ (D) $n = 0$

For an isobaric process,

$$p_1 = p_2 \quad \text{[I]}$$

For a polytropic process,

$$p_1 V_1^n = p_2 V_2^n \quad \text{[II]}$$

Equation I can be derived from II only if $n = 0$.

The answer is (D).

THERMODYNAMICS–48

In an isentropic compression, $p_1 = 2.14\ \text{N/cm}^2$, $p_2 = 4.28\ \text{N/cm}^2$, $V_1 = 164\ \text{cm}^3$, and the ratio of specific heats is $k = 1.4$. What is most nearly the value of V_2?

(A) $18\ \text{cm}^3$ (B) $21\ \text{cm}^3$ (C) $23\ \text{cm}^3$ (D) $100\ \text{cm}^3$

For an isentropic process,

$$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^k$$

$$V_2 = V_1 \left(\frac{p_1}{p_2}\right)^{1/k}$$

$$= (164\ \text{cm}^3) \left(\frac{2.14\ \frac{\text{N}}{\text{cm}^2}}{4.28\ \frac{\text{N}}{\text{cm}^2}}\right)^{1/1.4}$$

$$= 99.96\ \text{cm}^3 \quad (100\ \text{cm}^3)$$

The answer is (D).

THERMODYNAMICS–49

In an adiabatic, isentropic process, $p_1 = 4.28\ \text{N/cm}^2$, $p_2 = 6.42\ \text{N/cm}^2$, and $T_1 = 388\text{K}$. The ratio of specific heats is 1.4. What is most nearly the value of T_2?

(A) 270K (B) 390K (C) 430K (D) 440K

For an isentropic process,

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{(k-1)/k}$$

$$= (388\text{K}) \left(\frac{6.42\ \frac{\text{N}}{\text{cm}^2}}{4.28\ \frac{\text{N}}{\text{cm}^2}}\right)^{(1.4-1)/1.4}$$

$$= 436\text{K} \quad (440\text{K})$$

The answer is (D).

THERMODYNAMICS–50

Air undergoes an isentropic compression from $0.31\ \text{N/cm}^2$ to $3.87\ \text{N/cm}^2$. If the initial temperature is 20°C and the final temperature is 327.5°C, what is most nearly the work done by the gas?

(A) −320 kJ/kg (B) −220 kJ/kg (C) 120 kJ/kg (D) 230 kJ/kg

For air,

$$c_v = 0.718\ \text{kJ/kg·K}$$

$$W = c_v(T_1 - T_2)$$

$$= \left(0.718\ \frac{\text{kJ}}{\text{kg·K}}\right)(20°\text{C} - 327.5°\text{C})$$

$$= -221\ \text{kJ/kg} \quad (-220\ \text{kJ/kg})$$

The answer is (B).

THERMODYNAMICS–51

Nitrogen is expanded isentropically. Its temperature changes from 600K to 288K. Find the pressure ratio (p_1/p_2).

(A) 11 (B) 13 (C) 16 (D) 22

For an isentropic process,

$$\frac{p_1}{p_2} = \left(\frac{T_1}{T_2}\right)^{k/(k-1)}$$

$$k = 1.4$$

$$\frac{p_1}{p_2} = \left(\frac{600\text{K}}{288\text{K}}\right)^{1.4/(1.4-1)} = 13$$

The answer is (B).

THERMODYNAMICS–52

Nitrogen is expanded isentropically. Its temperature changes from 327°C to 15°C. The volumetric ratio is $V_2/V_1 = 6.22$, and the specific gas content for nitrogen is 0.1017 kJ/kg·K. What is most nearly the work done by the gas?

(A) −80 kJ/kg (B) −13 kJ/kg (C) 19 kJ/kg (D) 79 kJ/kg

For an isentropic process,

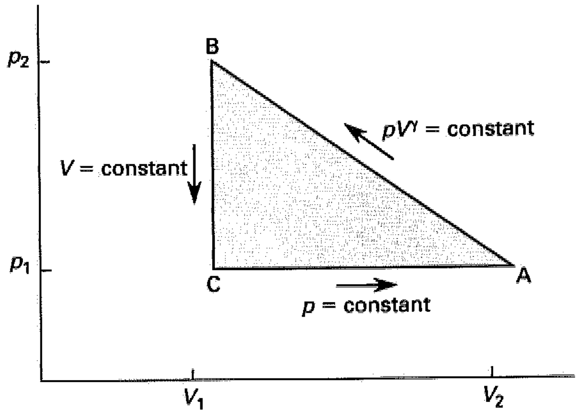
$$k = 1.4$$

$$W = \frac{R(T_1 - T_2)}{k-1} = \frac{\left(0.1017\ \frac{\text{kJ}}{\text{kg·K}}\right)(327^\circ\text{C} - 15^\circ\text{C})}{0.4} = 79.3\ \text{kJ/kg}\quad (79\ \text{kJ/kg})$$

The answer is (D).

THERMODYNAMICS–53

For the cycle shown, what is the work done on the system?



(A) 0
(B) area enclosed by the cycle in T-V space
(C) $R(T_B - T_C)$
(D) area enclosed by the cycle in p-V space

$$W = \oint p dV$$
$$= W_{\mathrm{A}}^{\mathrm{B}} - W_{\mathrm{C}}^{\mathrm{A}}$$
$$= \frac{R(T_{\mathrm{B}} - T_{\mathrm{A}})}{k-1} - \frac{R(T_{\mathrm{A}} - T_{\mathrm{C}})}{k-1}$$
$$= \left(\frac{R}{k-1}\right)(T_{\mathrm{B}} + T_{\mathrm{C}} - 2T_{\mathrm{A}})$$

Thus, options (A), (B), and (C) are incorrect. However, $\oint pdV$ is the area enclosed in p-V space. Therefore, option (D) is correct.

The answer is (D).

THERMODYNAMICS–54

An isobaric steam generating process starts with saturated liquid at 143 kPa. The change in entropy is equal to the initial entropy. Not all of the liquid is vaporized. What is most nearly the change in enthalpy during the process?

(A) 110 kJ/kg (B) 270 kJ/kg (C) 410 kJ/kg (D) 540 kJ/kg

$$\Delta h = x h_{fg}$$

In the preceding equation, x is quality.

$$
\begin{aligned}
h_{fg} &= 2230\ \text{kJ/kg} \\
s_{\text{initial}} &= s_f \\
&= 1.4185\ \text{kJ/kg·K} \quad [\text{at } 143\ \text{kPa}] \\
s_{\text{final}} &= s_{\text{initial}} + \Delta s \\
&= 2s_{\text{initial}} \\
&= (2)\left(1.4185\ \frac{\text{kJ}}{\text{kg·K}}\right) \\
&= 2.837\ \text{kJ/kg·K} \\
&= s_f + x s_{fg} \\
x &= \frac{s_{\text{final}} - s_f}{s_{fg}} \\
&= \frac{2.837\ \frac{\text{kJ}}{\text{kg·K}} - 1.4185\ \frac{\text{kJ}}{\text{kg·K}}}{5.8202\ \frac{\text{kJ}}{\text{kg·K}}} \\
&= 0.244 \\
\Delta h &= x h_{fg} = (0.244)\left(2230\ \frac{\text{kJ}}{\text{kg}}\right) \\
&= 544.1\ \text{kJ/kg} \quad (540\ \text{kJ/kg})
\end{aligned}
$$

The answer is (D).

THERMODYNAMICS–55

A cylinder and piston arrangement contains saturated water vapor at 110°C. The vapor is compressed in a reversible adiabatic process until the pressure is 1.6 MPa. What is most nearly the work done by the system?

(A) −640 kJ/kg (B) −510 kJ/kg (C) −430 kJ/kg (D) −330 kJ/kg

Use the first law of thermodynamics.

$$
\begin{aligned}
Q &= U_1 - U_2 + W \\
&= 0 \\
W &= U_1 - U_2
\end{aligned}
$$

State 1 is specified. Because the process is reversible and adiabatic, $s_1 = s_2$. Thus, it is possible to find state 2. To do so, use the steam tables.

$$\begin{aligned} \text{at } 110^\circ\text{C}: \quad s_1 &= 7.2387 \text{ kJ/kg·K} \\ u_1 &= 2518.1 \text{ kJ/kg} \\ \text{at } 1.6 \text{ MPa}: \quad s_2 &= 7.2374 \text{ kJ/kg·K} \\ u_2 &= 2950.1 \text{ kJ/kg} \\ T_2 &= 400^\circ\text{C} \\ W &= 2518.1 \text{ kJ/kg} - 2950.1 \text{ kJ/kg} \\ &= -432 \text{ kJ/kg} \quad (-430 \text{ kJ/kg}) \end{aligned}$$

The answer is (C).

THERMODYNAMICS–56

During an adiabatic, internally reversible process, what is true about the change in entropy?

(A) It is always zero.
(B) It is always less than zero.
(C) It is always greater than zero.
(D) It is infinite.

Since there is no heat flow, an adiabatic, reversible process has a zero change in entropy.

The answer is (A).

THERMODYNAMICS–57

For an irreversible process, what is true about the total change in entropy of the system and surroundings?

(A) $dS = \infty$ (B) $dS = 0$ (C) $dS > 0$ (D) $dS < 0$

For an irreversible process,

$$dS = dS_{\text{system}} + dS_{\text{surroundings}} > 0$$

The answer is (C).

THERMODYNAMICS–58

For which type of process is the equation $dQ = TdS$ valid?

(A) irreversible (B) isothermal (C) reversible (D) isobaric

$$\begin{aligned} TdS &= dH - Vdp \\ dH &= dU + pdV + Vdp \\ TdS &= dU + pdV \\ pdV &= dW \\ TdS &= dU + dW \end{aligned}$$

The first law of thermodynamics is

$$dQ = dU + dW$$

Therefore,

$$dQ = TdS \quad \text{[for a reversible process]}$$

The answer is (C).

THERMODYNAMICS–59

Which of the following is true for any process?

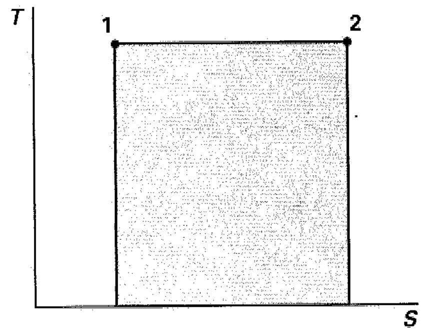
(A) $\Delta S_{\text{surroundings}} + \Delta S_{\text{system}} \geq 0$
(B) $\Delta S_{\text{surroundings}} + \Delta S_{\text{system}} \leq 0$
(C) $\Delta S_{\text{surroundings}} + \Delta S_{\text{system}} < 0$
(D) $\Delta S_{\text{surroundings}} + \Delta S_{\text{system}} > 0$

The total entropy either increases or, for a reversible process, remains the same. Therefore, the total change in entropy is always greater than or equal to zero.

The answer is (A).

THERMODYNAMICS–60

In a reversible process, the state of a system changes from state 1 to state 2, as shown on the T-S diagram. What does the shaded area of the diagram represent?



(A) free-energy change
(B) heat transfer
(C) enthalpy change
(D) work

For a reversible process, $Q = \int T dS$. Thus, the shaded area represents the heat transfer.

The answer is (B).

THERMODYNAMICS–61

Helium is compressed isothermally from 14.7 psia and 68°F. The compression ratio is 4. What is most nearly the change in entropy of the gas, given that the specific gas constant is $R_{He} = 0.6411$ kJ/kg·K?

(A) −0.97 kJ/kg·K (B) −0.89 kJ/kg·K (C) 0.45 kJ/kg·K (D) 0.89 kJ/kg·K

For an isothermal process,

$$\begin{aligned}\Delta s &= R_{\text{He}} \ln \frac{V_2}{V_1}\\ &= \left(0.6411 \ \frac{\text{kJ}}{\text{kg·K}}\right) \ln \frac{1}{4}\\ &= -0.889 \ \text{kJ/kg·K} \quad (-0.89 \ \text{kJ/kg·K})\end{aligned}$$

The answer is (B).

THERMODYNAMICS-62

For an ideal gas, what is most nearly the specific molar entropy change during an isothermal process in which the pressure changes from 200 kPa to 150 kPa?

(A) 2.0 J/mol·K (B) 2.4 J/mol·K (C) 2.8 J/mol·K (D) 3.1 J/mol·K

For an ideal gas,

$$\Delta S = c_p \ln \frac{T_2}{T_1} - R^* \ln \frac{p_2}{p_1}$$

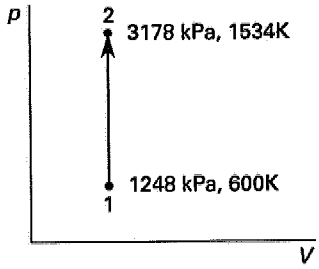
For an isothermal process,

$$\begin{aligned}T_1 &= T_2\\ \ln \frac{T_2}{T_1} &= 0\\ \Delta S &= -R^* \ln \frac{p_2}{p_1}\\ &= \left(-8.314 \ \frac{\text{J}}{\text{mol·K}}\right) \ln \left(\frac{150 \ \text{kPa}}{200 \ \text{kPa}}\right)\\ &= 2.39 \ \text{J/mol·K} \quad (2.4 \ \text{J/mol·K})\end{aligned}$$

The answer is (B).

THERMODYNAMICS–63

In the p-V diagram shown, heat addition occurs between points 1 and 2. Given that $c_v = 0.434$ kJ/kg·K, what is most nearly the entropy produced during this step?



(A) −0.41 kJ/kg·K (B) −0.23 kJ/kg·K (C) 0.23 kJ/kg·K (D) 0.41 kJ/kg·K

$$\Delta s = c_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$$

$$V_2 = V_1$$

$$\begin{aligned}\Delta s &= c_v \ln \frac{T_2}{T_1} \\ &= \left(0.434\ \frac{\text{kJ}}{\text{kg·K}}\right) \ln \left(\frac{1534\text{K}}{600\text{K}}\right) \\ &= 0.407\ \text{kJ/kg·K} \quad (0.41\ \text{kJ/kg·K})\end{aligned}$$

The answer is (D).

THERMODYNAMICS–64

200 g of water are heated from 5°C to 100°C and vaporized at a constant pressure. The heat of vaporization of water at 100°C is 539.2 cal/g. The heat capacity at constant pressure, c_p, is 1.0 cal/g·K. What is most nearly the total change in entropy?

(A) 250 cal/K (B) 300 cal/K (C) 350 cal/K (D) 400 cal/K

$$T_1 = 5°\text{C} + 273° = 278\text{K}$$

$$T_2 = 100°\text{C} + 273° = 373\text{K}$$

$$\Delta S = \Delta S_{\text{heat}} + \Delta S_{\text{vaporization}}$$

$$\Delta s_{\text{heat}} = s_2 - s_1$$

$$= \int_{T_1}^{T_2} \frac{c_p}{T} dT - R \ln \frac{p_2}{p_1}$$

$$p_1 = p_2$$

$$\Delta s_{\text{heat}} = \int_{T_1}^{T_2} \frac{c_p}{T} dT$$

$$= c_p \ln \frac{T_2}{T_1}$$

$$= \left(1 \ \frac{\text{cal}}{\text{g·K}}\right) \ln \left(\frac{373\text{K}}{278\text{K}}\right)$$

$$= 0.2940 \ \text{cal/g·K}$$

$$\Delta S_{\text{heat}} = m\Delta s_{\text{heat}}$$

$$= (200 \ \text{g}) \left(0.2940 \ \frac{\text{cal}}{\text{g·K}}\right)$$

$$= 58.8 \ \text{cal/K}$$

$$\Delta s_{\text{vaporization}} = \frac{Q}{T} = \frac{h_{fg}}{T_{\text{vap}}}$$

$$= \frac{539.2 \ \frac{\text{cal}}{\text{g}}}{373\text{K}}$$

$$= 1.446 \ \text{cal/g·K}$$

$$\Delta S_{\text{vaporization}} = m\Delta s_{\text{vaporization}}$$

$$= (200 \ \text{g}) \left(1.446 \ \frac{\text{cal}}{\text{g·K}}\right)$$

$$= 289.2 \ \text{cal/K}$$

$$\Delta S = 58.5 \ \frac{\text{cal}}{\text{K}} + 289.2 \ \frac{\text{cal}}{\text{K}}$$

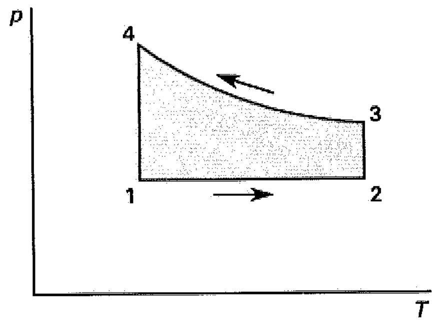
$$= 348 \ \text{cal/K} \quad (350 \ \text{cal/K})$$

The answer is (C).

6 POWER CYCLES

POWER CYCLES-1

What kind of process occurs between points 3 and 4?



(A) isentropic
(B) isobaric
(C) isothermal
(D) There is insufficient information to determine the process type.

The process between points 3 and 4 is indeterminate. All that can be said about the process is that it is neither isobaric nor isothermal.

The answer is (D).

POWER CYCLES–2

Which of the following thermodynamic cycles is the most efficient?

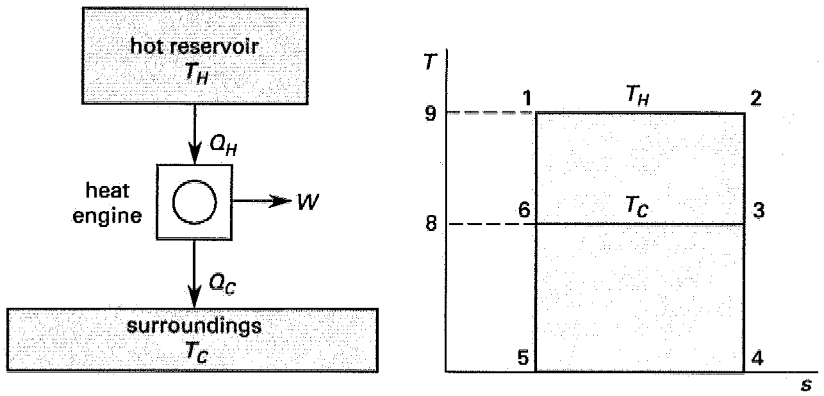
(A) Brayton
(B) Rankine
(C) Carnot
(D) combined Brayton-Rankine

No cycle is more efficient than the Carnot cycle, because it is completely reversible.

The answer is (C).

POWER CYCLES–3

For the reversible heat engine shown, which area on the corresponding T-S diagram represents the work done by the system?



(A) work = 0 (B) 1–2–4–5 (C) 6–3–4–5 (D) 1–2–3–6

Use the first law of thermodynamics.

$$\begin{aligned} W &= Q_H - Q_C \\ &= T_H \Delta S - T_C \Delta S \\ &= (1-2-4-5) - (3-4-5-6) \\ &= 1-2-3-6 \end{aligned}$$

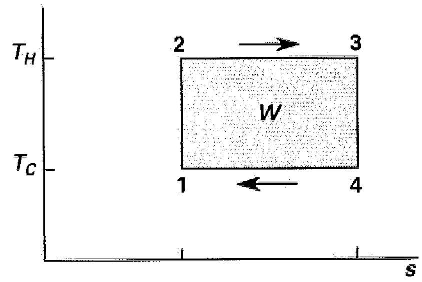
The answer is (D).

POWER CYCLES–4

The ideal reversible Carnot cycle involves several basic processes. What type of processes are they?

(A) all adiabatic
(B) all isentropic
(C) two adiabatic and two isentropic
(D) two isothermal and two isentropic

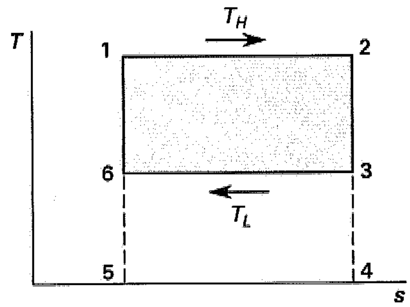
By definition, a Carnot cycle consists of two isothermal processes and two isentropic processes.



The answer is (D).

POWER CYCLES–5

An ideal reversible Carnot cycle is represented on the T-S diagram shown. The efficiency of the cycle is represented by which of the following ratios of areas?



(A) $\dfrac{1-2-3-6}{1-2-4-5}$ (B) $\dfrac{1-2-4-5}{1-2-3-6}$

(C) $\dfrac{3-4-5-6}{1-2-4-5}$ (D) $\dfrac{1-2-4-5}{3-4-5-6}$

The efficiency, η, is defined as follows.

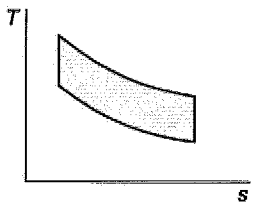
$$
\begin{aligned}
\eta &= \frac{W_{\text{net}}}{Q_H} \\
W_{\text{net}} &= Q_{\text{net}} \\
&= 1-2-3-6 \\
Q_H &= 1-2-4-5 \\
\eta &= \frac{1-2-3-6}{1-2-4-5}
\end{aligned}
$$

> The answer is (A).

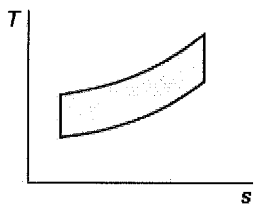
POWER CYCLES–6

Which of the following T-S diagrams may be that of a Carnot cycle?

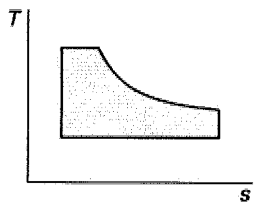
(A)



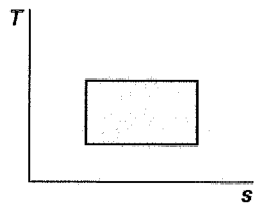
(B)



(C)



(D)

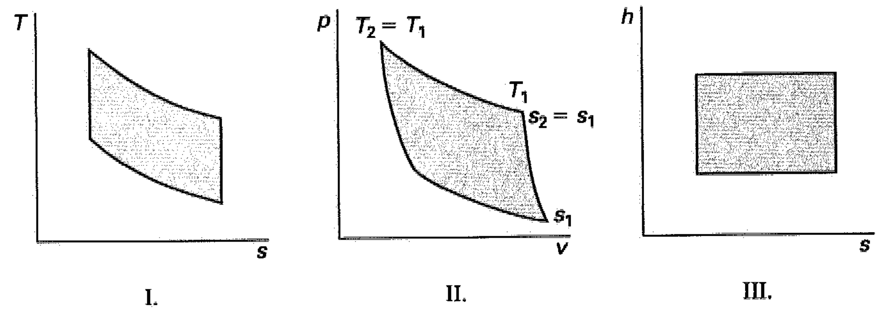


A Carnot cycle has two isothermal processes (horizontal lines on a T-S diagram) and two isentropic processes (vertical lines on a T-S diagram). The only diagram that has both of these properties is the diagram represented in option (D).

> The answer is (D).

POWER CYCLES–7

Which of the following is/are representations of a Carnot cycle?



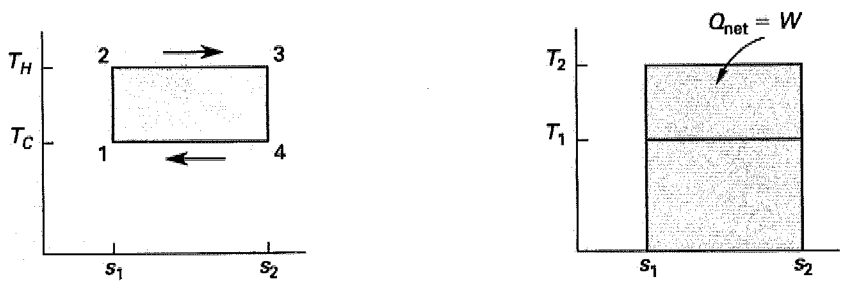
(A) I only (B) II only (C) I and II (D) II and III

A Carnot cycle has two isentropic processes and two isothermal processes. Diagram I has two isentropic processes, but no isothermal processes. Diagram II has two isentropic processes and two isothermal processes. When h is constant, T is constant. Diagram III also has two isentropic processes and two isothermal processes. Thus, II and III both represent a Carnot cycle.

The answer is (D).

POWER CYCLES–8

Consider the T-S diagram of a Carnot cycle in the figure. What is the amount of total work done in one cycle?



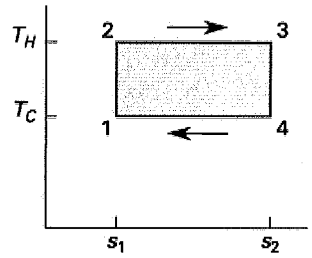
(A) $(S_2 - S_1)(T_2 - T_1)$ (B) $(S_2 - S_1)T_2$
(C) $(S_1 - S_2)T_1$ (D) $(S_2 - S_1)T_2 - T_1$

$$\begin{aligned} W &= Q_{\text{net}} - \Delta U \\ &= Q_{\text{H}} - Q_C - 0 \\ &= T_H \Delta S_H - T_C \Delta S_C \\ &= T_2(S_2 - S_1) - T_1(S_2 - S_1) \\ &= (S_2 - S_1)(T_2 - T_1) \end{aligned}$$

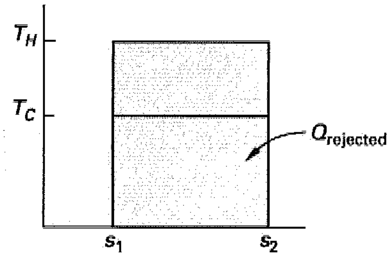
The answer is (A).

POWER CYCLES–9

Consider the T-S diagram of a Carnot cycle shown. What amount of heat is rejected to the surroundings?



(A) $T_H(S_3 - S_2)$ (B) $T_H(S_2 - S_1)$ (C) $T_C(S_3 - S_2)$ (D) $T_C(S_2 - S_1)$



The rejected heat is equal to the area under the lower branch of the cycle.

$$Q_{\text{rejected}} = T_C(S_2 - S_1)$$

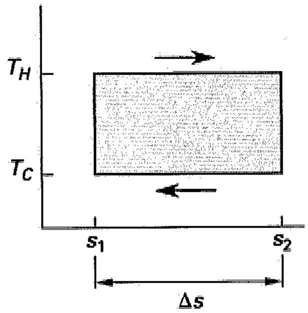
The answer is (D).

POWER CYCLES–10

What is the temperature difference of the cycle if the entropy difference is ΔS and the work done is W?

(A) $W - \Delta S$ (B) $\frac{W}{\Delta S}$ (C) $\frac{\Delta S}{W}$ (D) $W(\Delta S)$

In the figure, the work done, W, is represented by the shaded area.



$$W = (\Delta T)(\Delta S)$$

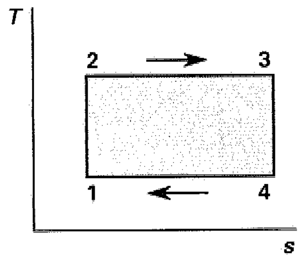
Rearranging,

$$\Delta T = \frac{W}{\Delta S}$$

The answer is (B).

POWER CYCLES–11

In the Carnot cycle shown, the net amount of heat put into the system is equal to the total amount of work done by the system. However, it cannot be stated that the heat put into the system between states 1 and 2 is equal to the work done between states 1 and 2. What is the reason for this?



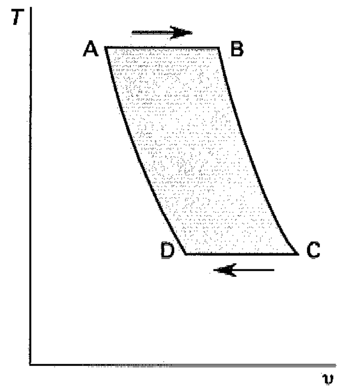
(A) The process is adiabatic.
(B) The process is not adiabatic.
(C) The second law states that the amount of energy put into the system is equal to the amount taken out of the system.
(D) The first law states that $dQ = dU + dW$. Since $dU \neq 0$, $dQ \neq dW$.

The first law states that $dQ = dU + dW$. Between states 1 and 2, $\Delta U \neq 0$. Therefore, $dQ_{12} \neq dW_{12}$.

The answer is (D).

POWER CYCLES–12

For the Carnot cycle shown, helium is the gas used with a specific heat ratio, k, of 5/3. Given that $V_B/V_A = 2$ and $T_A/T_D = 1.9$, calculate p_C/p_A.



(A) 0.0633 (B) 0.100 (C) 0.180 (D) 0.262

$$\frac{T_A}{T_D} = \frac{T_B}{T_C} = 1.9$$

For a constant temperature,

$$\frac{p_B}{p_A} = \frac{V_A}{V_B} = 1/2$$

For a Carnot cycle, stage B to C is isentropic. The relation for ideal gases undergoing a constant entropy process is

$$\frac{p_C}{p_B} = \left(\frac{T_C}{T_B}\right)^{k/(k-1)}$$

$$\frac{p_C}{p_A} = \left(\frac{p_C}{p_B}\right)\left(\frac{p_B}{p_A}\right)$$

$$= \left(\frac{1}{1.9}\right)^{5/2}\left(\frac{1}{2}\right)$$

$$= 0.100$$

The answer is (B).

POWER CYCLES–13

A Carnot engine operates between 444K and 555K. What is its thermal efficiency?

(A) 20% (B) 30% (C) 40% (D) 50%

$$\eta_t = 1 - \frac{T_C}{T_H}$$

$$= 1 - \frac{800\text{K}}{1000\text{K}}$$

$$= 0.2 \quad (20\%)$$

The answer is (A).

POWER CYCLES–14

For a heat engine operating between two temperatures ($T_2 > T_1$), what is the maximum efficiency attainable?

(A) $1 - \frac{T_2}{T_1}$ (B) $1 - \frac{T_1}{T_2}$ (C) $\frac{T_1}{T_2}$ (D) $1 - \left(\frac{T_1}{T_2}\right)^k$

The maximum efficiency attainable is the Carnot efficiency.

$$\eta = \frac{W}{Q_{\text{in}}}$$

$$= \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}}$$

$$\eta_{\text{Carnot}} = 1 - \frac{T_1}{T_2}$$

The answer is (B).

POWER CYCLES–15

Which of the following is NOT an advantage of a superheated, closed Rankine cycle over an open Rankine cycle?

(A) increased efficiency
(B) increased turbine work output
(C) increased turbine life
(D) increased boiler life

Option (D) is not an advantage because a superheated Rankine cycle has higher boiler heat temperatures that decrease boiler life.

The answer is (D).

POWER CYCLES–16

Which of the following statements regarding Rankine cycles is FALSE?

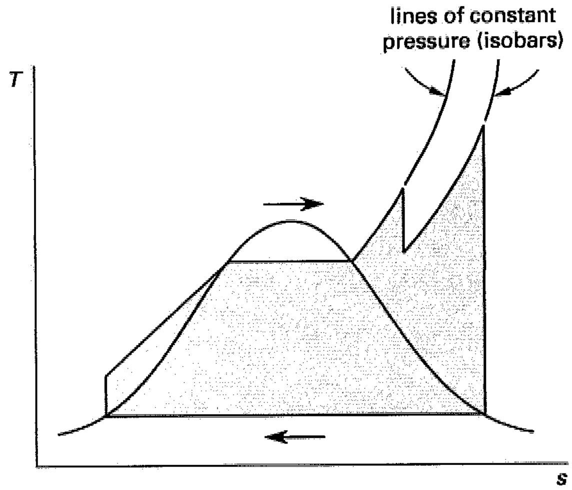
(A) Use of a condensible vapor in the cycle increases the efficiency of the cycle.
(B) The temperatures at which energy is transferred to and from the working liquid are less separated than in a Carnot cycle.
(C) Superheating increases the efficiency of a Rankine cycle.
(D) In practical terms, the susceptibility of the engine materials to corrosion is not a key limitation on the operating efficiency.

Corrosion is a principal limitation on the use of higher temperatures for this type of engine. Thus, the susceptibility of engine materials to corrosion does limit operating efficiency.

The answer is (D).

POWER CYCLES–17

What type of power cycle does the following diagram illustrate?



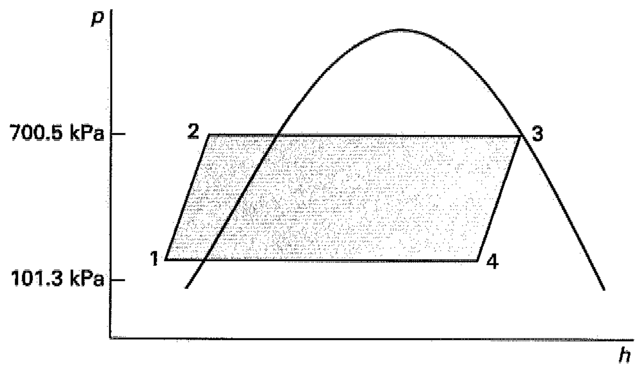
(A) a Carnot cycle
(B) an idealized Rankine cycle with superheat and reheat
(C) an idealized diesel cycle
(D) an idealized Stirling cycle

The diagram shows an idealized Rankine cycle with reheat.

The answer is (B).

POWER CYCLES–18

For the steam Rankine cycle shown, determine the approximate enthalpy at state 3.



(A) 2000 kJ/kg (B) 2500 kJ/kg (C) 2680 kJ/kg (D) 2760 kJ/kg

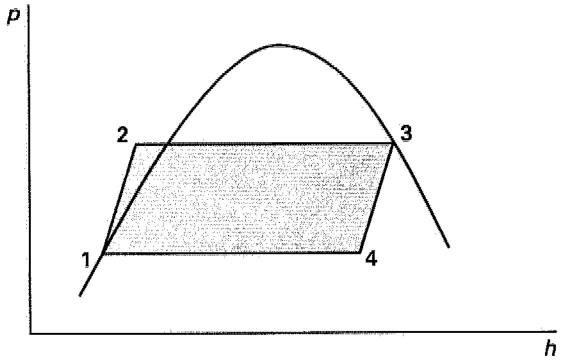
State 3 is saturated steam at 700.5 kPa. From the steam tables,

$$h_3 = h_g$$
$$= 2763.5 \text{ kJ/kg} \quad (2760 \text{ kJ/kg})$$

The answer is (D).

POWER CYCLES–19

In a Rankine cycle, state 3 is saturated steam at 1.398 MPa. Assuming that the turbine is isentropic and $p_4 = 101.35$ kPa, find the approximate enthalpy at state 4.



(A) 420 kJ/kg (B) 2260 kJ/kg (C) 2350 kJ/kg (D) 2680 kJ/kg

$$p_3 = 1.398 \text{ MPa}$$

From the steam tables,

$$\begin{aligned} s_3 &= s_g \\ &= 6.4698 \text{ kJ/kg·K} \\ s_4 &= s_3 \\ &= 6.4698 \text{ kJ/kg·K} \\ s_4 &= s_f + x s_{fg} \\ &= 1.3069 \; \frac{\text{kJ}}{\text{kg·K}} + x\left(6.0480 \; \frac{\text{kJ}}{\text{kg·K}}\right) \end{aligned}$$

Rearrange to solve for quality x.

$$x = \frac{s_4 - s_f}{s_{fg}} = \frac{6.4698 \; \frac{\text{kJ}}{\text{kg·K}} - 1.3069 \; \frac{\text{kJ}}{\text{kg·K}}}{6.0480 \; \frac{\text{kJ}}{\text{kg·K}}}$$

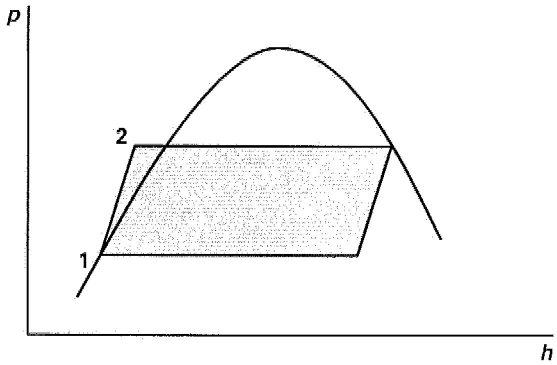
$$= 0.854$$

$$h_4 = h_f + xh_{fg}$$
$$= 419.04 \ \frac{\text{kJ}}{\text{kg}} + (0.854)\left(2257.0 \ \frac{\text{kJ}}{\text{kg}}\right)$$
$$= 2346.5 \ \text{kJ/kg} \quad (2350 \ \text{kJ/kg})$$

The answer is (C).

POWER CYCLES–20

In a steam Rankine cycle, state 1 is saturated liquid at 101.35 kPa. State 2 is high-pressure liquid at 0.7005 MPa. Approximately how much work is required to pump 0.45 kg of water from state 1 to state 2?



(A) 125 kJ (B) 136 J (C) 280 J (D) 19.5 kJ

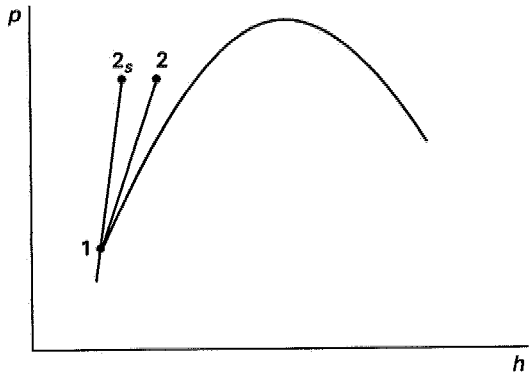
$$p_1 = 101.35 \ \text{kPa}$$
$$v_f = 0.001\,044 \ \text{m}^3/\text{kg}$$
$$W = m(h_2 - h_1)$$
$$= mv\Delta p$$
$$= (0.45 \ \text{kg})\left(1.044 \times 10^{-3} \ \frac{\text{m}^3}{\text{kg}}\right)(700.5 \ \text{kPa} - 101.35 \ \text{kPa})\left(1000 \ \frac{\text{Pa}}{\text{kPa}}\right)$$
$$= 281.5 \ \text{J} \quad (280 \ \text{J})$$

Note: State 2 is not a saturated state.

The answer is (C).

POWER CYCLES–21

In a steam Rankine cycle, saturated liquid at 101.35 kPa is pumped to 1.398 MPa. If the pump were isentropic, the enthalpy of state 2 would be 420.2 kJ/kg. The isentropic efficiency of the pump is 60%. What is most nearly the enthalpy of state 2?



(A) 252.1 kJ/kg (B) 417.5 kJ/kg (C) 418.9 kJ/kg (D) 421.0 kJ/kg

$$\begin{aligned}\eta &= \frac{W_s}{W}\\ &= \frac{m\Delta h_s}{m\Delta h}\\ &= \frac{\Delta h_s}{\Delta h}\\ &= \frac{(h_2 - h_1)_s}{h_2 - h_1}\\ h_2 - h_1 &= \frac{(h_2 - h_1)_s}{\eta}\\ h_2 &= h_1 + \frac{(h_2 - h_1)_s}{\eta}\end{aligned}$$

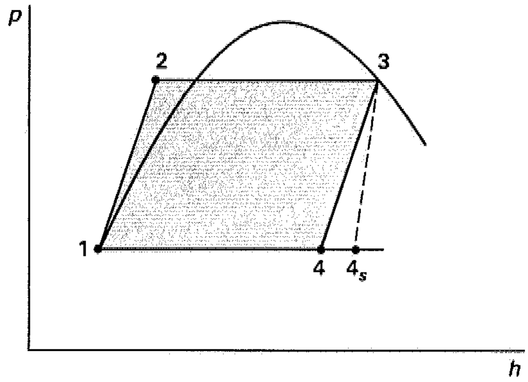
From the steam tables,

$$\begin{aligned}h_1 &= 419.04\ \text{kJ/kg}\\ h_2 &= 419.04\ \frac{\text{kJ}}{\text{kg}} + \frac{420.2\ \frac{\text{kJ}}{\text{kg}} - 419.04\ \frac{\text{kJ}}{\text{kg}}}{0.6}\\ &= 421.0\ \text{kJ/kg}\end{aligned}$$

The answer is (D).

POWER CYCLES–22

Steam in a Rankine cycle is expanded from a 1.398 MPa saturated vapor state to 0.1433 MPa. The turbine has an efficiency of 0.8. What is most nearly the enthalpy of state 4?



(A) 2233 kJ/kg (B) 2294 kJ/kg (C) 2393 kJ/kg (D) 2476 kJ/kg

Call the isentropic state 4_s.

$$p_3 = 1.398 \text{ MPa} \quad \text{[saturated vapor]}$$

$$s_3 = s_{f,4s} + x s_{fg,4s}$$

$$6.4698 \ \frac{\text{kJ}}{\text{kg·K}} = 1.4185 \ \frac{\text{kJ}}{\text{kg·K}} + x\left(5.8202 \ \frac{\text{kJ}}{\text{kg·K}}\right)$$

$$x = 0.8679$$

$$\begin{aligned} h_{4s} &= h_{h,4s} + x h_{fg,4s} \\ &= 461.30 \ \frac{\text{kJ}}{\text{kg}} + (0.8679)\left(2230.2 \ \frac{\text{kJ}}{\text{kg}}\right) \\ &= 2396.89 \text{ kJ/kg} \end{aligned}$$

$$\eta = \frac{h_s - h_4}{h_3 - h_{4s}}$$

$$\begin{aligned} h_4 &= h_3 - \eta(h_3 - h_{4s}) \\ &= 2790.0 \ \frac{\text{kJ}}{\text{kg}} - (0.8)\left(2790.0 \ \frac{\text{kJ}}{\text{kg}} - 2396.89 \ \frac{\text{kJ}}{\text{kg}}\right) \\ &= 2476 \text{ kJ/kg} \end{aligned}$$

The answer is (D).

POWER CYCLES–23

Which of the following sets of reversible processes describes an ideal Otto cycle?

I. adiabatic compression, constant volume heat addition, adiabatic expansion, constant volume heat rejection

II. isothermal compression, isobaric heat addition, isothermal expansion, isobaric heat rejection

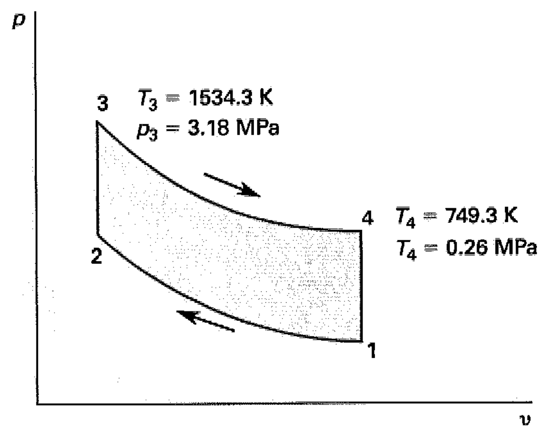
(A) I only
(B) II only
(C) I and II in succession
(D) II and I in succession

An Otto cycle is defined by the set of reversible processes in I.

The answer is (A).

POWER CYCLES–24

In the power stroke (3 to 4) of the ideal Otto cycle shown, what is the entropy change?



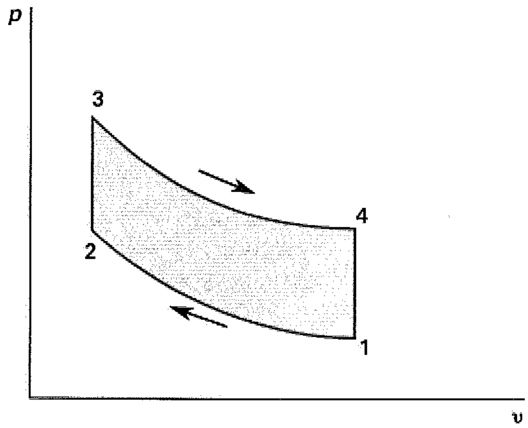
(A) −0.536 kJ/kg·K (B) 0.00 kJ/kg·K (C) 0.536 kJ/kg·K (D) 0.749 kJ/kg·K

In the ideal Otto cycle, the expansion process is reversible and adiabatic (isentropic). For an isentropic process, $\Delta S = 0$.

The answer is (B).

POWER CYCLES–25

The compression ratio of an ideal air Otto cycle is 6:1. p_1 is 101.35 kPa, and T_1 is 20°C. What is the temperature at state 2?



(A) 159.4°C
(B) 332.6°C
(C) 600.0°C
(D) 600.0K

The process from state 1 to state 2 is an isentropic compression, with $k = 1.4$ for air.

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^k$$

$$= (6)^{1.4}$$

$$p_2 = p_1(6)^{1.4}$$

$$= (101.35 \text{ kPa})(6)^{1.4}$$

$$= 1245.19 \text{ kPa}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1}$$

$$= (6)^{1.4-1}$$

$$T_2 = T_1(6)^{0.4}$$
$$= (20°C + 273°)(6)^{0.4}$$
$$= 599.97\text{K} \quad (600.0\text{K})$$

The answer is (D).

POWER CYCLES–26

What is the ideal efficiency of an Otto cycle with a compression ratio of 6:1? The gas used is air.

(A) 0.167 (B) 0.191 (C) 0.488 (D) 0.512

By definition,

$$\eta = 1 - r^{1-k}$$

In this equation, r is the compression ratio.

For air,

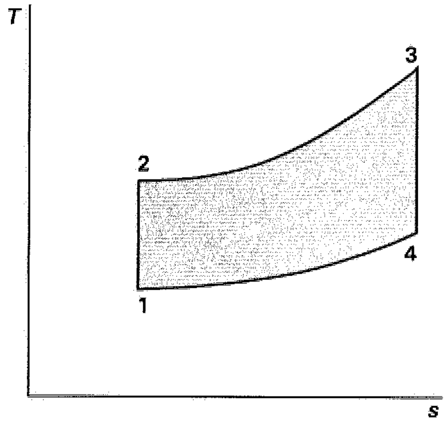
$$k = 1.4$$
$$\eta = 1 - (6)^{1-1.4}$$
$$= 0.512$$

The answer is (D).

POWER CYCLES–27

The cycle shown in the diagram can be described as follows.

The process from 1 to 2 is adiabatic, isentropic compression.
The process from 2 to 3 is isobaric heat addition.
The process from 3 to 4 is adiabatic, isentropic expansion.
The process from 4 to 1 is constant volume heat rejection.



Which of the following is the name of this cycle?

(A) Otto (B) Carnot (C) diesel (D) Rankine

The cycle described above is a diesel cycle.

The answer is (C).

POWER CYCLES–28

A device produces 37.5 J per power stroke. There is one power stroke per revolution. Approximate the power output, P_{out}, if the device is run at 45 rpm.

(A) 4.7 W (B) 14 W (C) 28 W (D) 280 W

$$\begin{aligned} P_{out} &= \frac{W}{t} \\ &= \left(37.5\ \frac{\text{J}}{\text{power stroke}}\right)\left(1\ \frac{\text{power stroke}}{\text{revolution}}\right)\left(45\ \frac{\text{revolutions}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) \\ &= 28.125\ \text{W} \quad (28\ \text{W}) \end{aligned}$$

The answer is (C).

POWER CYCLES–29

A steam generator produces saturated steam at 700.5 kPa from saturated liquid at 101.35 kPa. If the heat source is a bath at 171°C that provides 1860.8 kJ/kg, which of the following is true?

(A) The device violates the first law of thermodynamics only.
(B) The device violates the second law of thermodynamics only.
(C) The device violates both the first and second laws of thermodynamics.
(D) No thermodynamic laws are violated.

For the heat output,

$$\begin{aligned}\Delta Q &= h_2 - h_1\\ &= 2763.5\ \frac{\text{kJ}}{\text{kg}} - 419.04\ \frac{\text{kJ}}{\text{kg}}\\ &= 2344.5\ \text{kJ/kg}\end{aligned}$$

However, it is given that the heat input is 1860.8 kJ/kg. This would mean that the heat output is greater than the heat input. Therefore, the first law of thermodynamics is violated.

$$\begin{aligned}\Delta s &= s_2 - s_1 - \frac{Q}{T}\\ &= \left(6.7078\ \frac{\text{kJ}}{\text{kg·K}} - 1.3069\ \frac{\text{kJ}}{\text{kg·K}}\right) - \frac{1860.8\ \frac{\text{kJ}}{\text{kg}}}{171^\circ\text{C} + 273\text{K}}\\ &= 5.4009\ \frac{\text{kJ}}{\text{kg·K}} - 4.1910\ \frac{\text{kJ}}{\text{kg·K}}\\ &= 1.2099\ \text{kJ/kg·K}\end{aligned}$$

Since $\Delta s > 0$, the second law of thermodynamics is not violated.

The answer is (A).

POWER CYCLES–30

A device that is meant to extract power from waste process steam starts with steam of 75% quality at 700.5 kPa. The exit conditions of the steam are 70% quality at 101.35 kPa. Which of the following statements are true?

(A) This device violates the first law of thermodynamics.
(B) This device violates the second law of thermodynamics.
(C) The device generates positive net power.
(D) The device generates no net power.

$$W = -(h_{\text{final}} - h_{\text{initial}})$$
$$h = h_f + xh_{fg}$$

h_f and h_{fg} can be found in the steam tables.

$$\begin{aligned} h_{\text{final}} &= 419.04\ \frac{\text{kJ}}{\text{kg}} + (0.7)\left(2257.0\ \frac{\text{kJ}}{\text{kg}}\right) \\ &= 1998.94\ \text{kJ/kg} \\ h_{\text{initial}} &= 697.34\ \frac{\text{kJ}}{\text{kg}} + (0.75)\left(2066.2\ \frac{\text{kJ}}{\text{kg}}\right) \\ &= 2246.99\ \text{kJ/kg} \\ W &= h_{\text{initial}} - h_{\text{final}} \\ &= 2246.99\ \frac{\text{kJ}}{\text{kg}} - 1998.94\ \frac{\text{kJ}}{\text{kg}} \\ &= 248.05\ \text{kJ/kg} \end{aligned}$$

Thus, the device generates positive net power, and the first law of thermodynamics is not violated.

$$\Delta s = s_{\text{final}} - s_{\text{initial}}$$
$$s = s_f + xs_{fg}$$

s_f and s_{fg} can be found in the steam tables.

$$\begin{aligned} s_{\text{final}} &= 1.3069\ \frac{\text{kJ}}{\text{kg·K}} + (0.7)\left(6.0480\ \frac{\text{kJ}}{\text{kg·K}}\right) \\ &= 5.5405\ \text{kJ/kg·K} \\ s_{\text{initial}} &= 1.9925\ \frac{\text{kJ}}{\text{kg·K}} + (0.75)\left(4.7153\ \frac{\text{kJ}}{\text{kg·K}}\right) \\ &= 5.5290\ \text{kJ/kg·K} \\ \Delta s &= 5.5405\ \frac{\text{kJ}}{\text{kg·K}} - 5.5290\ \frac{\text{kJ}}{\text{kg·K}} \\ &= 0.0115\ \text{kJ/kg·K} \end{aligned}$$

Since $\Delta s > 0$, the second law of thermodynamics is not violated. Therefore, options (A), (B), and (D) are all false. Only option (C) is true.

The answer is (C).

POWER CYCLES–31

An engineer devises a scheme for extracting some power from waste process steam. The engineer claims that the steam enters the device at 700.5 kPa and quality 75%, and the steam exits at 101.35 kPa and 65% quality. Which of the following statements are true?

I. The device produces 316 kJ/kg of work.
II. The device violates the second law of thermodynamics.
III. The device violates the first law of thermodynamics.

(A) I only (B) II only (C) III only (D) I and II

$$h = h_f + xh_{fg}$$

h_f and h_{fg} can be found in the steam tables.

$$\begin{aligned} h_{\text{initial}} &= 697.34\ \frac{\text{kJ}}{\text{kg}} + (0.75)\left(2066.2\ \frac{\text{kJ}}{\text{kg}}\right) \\ &= 2246.99\ \text{kJ/kg} \\ h_{\text{final}} &= 419.04\ \frac{\text{kJ}}{\text{kg}} + (0.65)\left(2257.0\ \frac{\text{kJ}}{\text{kg}}\right) \\ &= 1886.09\ \text{kJ/kg} \\ W &= 2246.99\ \frac{\text{kJ}}{\text{kg}} - 1886.09\ \frac{\text{kJ}}{\text{kg}} \\ &= 360.9\ \text{kJ/kg} \end{aligned}$$

Thus, the device produces 316 kJ/kg of work without violating the first law of thermodynamics.

$$\Delta s = s_{\text{final}} - s_{\text{initial}}$$
$$s = s_f + xs_{fg}$$

s_f and s_{fg} can be found in the steam tables.

$$\begin{aligned} s_{\text{initial}} &= 1.9925\ \frac{\text{kJ}}{\text{kg·K}} + (0.75)\left(4.7153\ \frac{\text{kJ}}{\text{kg·K}}\right) \\ &= 5.529\ \text{kJ/kg·K} \\ s_{\text{final}} &= 1.3069\ \frac{\text{kJ}}{\text{kg·K}} + (0.65)\,(6.0480\ \text{kg·K}) \\ &= 5.2381\ \text{kJ/kg·K} \\ \Delta s &= 5.2381\ \frac{\text{kJ}}{\text{kg·K}} - 5.529\ \frac{\text{kJ}}{\text{kg·K}} \\ &= -0.2909\ \text{kJ/kg·K} \end{aligned}$$

Since $\Delta s < 0$, the device violates the second law of thermodynamics. Thus, I and II are true, but III is false.

The answer is (D).

POWER CYCLES–32

An engine burns a liter of fuel each 12 min. The fuel has a specific gravity of 0.8 and a heating value of 45 MJ/kg. The engine has an efficiency of 25%. What is most nearly the brake horsepower of the engine?

(A) 12.5 hp (B) 15.6 hp (C) 16.8 hp (D) 21.0 hp

The fuel flow rate is

$$\begin{aligned} Q &= \left(\frac{1\ \text{L}}{12\ \text{min}}\right)\left(\frac{1\ \text{m}^3}{1000\ \text{L}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) \\ &= 1.39 \times 10^{-6}\ \text{m}^3/\text{s} \end{aligned}$$

For the power input to the engine,

$$\begin{aligned} P_i &= \dot{m}(\text{heating value}) \\ &= \rho Q(\text{heating value}) \\ &= (0.8)\left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(1.389 \times 10^{-6}\ \frac{\text{m}^3}{\text{s}}\right)\left(45\ \frac{\text{MJ}}{\text{kg}}\right) \\ &= 0.0500\ \text{MJ/s} \quad (50.0\ \text{kW}) \end{aligned}$$

For the power output (actual power),

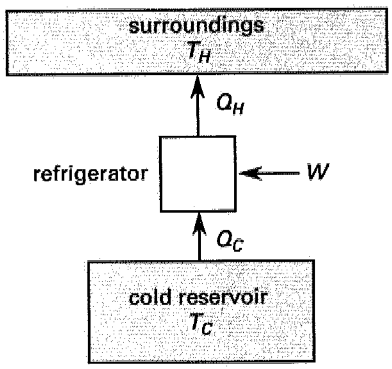
$$\begin{aligned} P_{\text{out}} &= \eta P_i \\ &= (0.25)(50.0\ \text{kW})\left(\frac{1\ \text{hp}}{0.746\ \text{kW}}\right) \\ &= 16.8\ \text{hp} \end{aligned}$$

The answer is (C).

POWER CYCLES–33

A Carnot refrigerator operates between two reservoirs. One reservoir is at a higher temperature, T_H, and the other is at a cooler temperature, T_C. What is the coefficient of performance, COP, of the refrigerator?

(A) $T_H - \frac{T_C}{T_H}$ (B) $1 - \frac{T_C}{T_H}$ (C) $\frac{T_H}{T_C} - T_C$ (D) $\frac{T_C}{T_H - T_C}$



$$\begin{aligned}\text{COP} &= \frac{Q_C}{W}\\ &= \frac{Q_C}{Q_H - Q_C}\\ &= \frac{T_C}{T_H - T_C}\end{aligned}$$

The answer is (D).

POWER CYCLES–34

A refrigeration system produces 348.9 kJ/kg of cooling. In order to have a rating of 1 ton of refrigeration, what must be the mass flow rate of the vapor? (1 ton of refrigeration = 12.66 MJ/hr, approximately the rate required to freeze 2000 lbm of ice in a day.)

(A) 0.998 kg/hr (B) 6.800 kg/hr (C) 36.29 kg/hr (D) 163.3 kg/hr

$$\text{refrigeration rate} = \dot{m}(\text{cooling capacity})$$

Therefore, the mass flow rate, $\dot{m}$, is

$$\begin{aligned}\dot{m} &= \frac{\text{refrigeration rate}}{\text{cooling capacity}} \\ &= \frac{12.66\ \frac{\text{MJ}}{\text{hr}}}{348.9\ \frac{\text{kJ}}{\text{kg}}} \\ &= 36.29\ \text{kg/hr}\end{aligned}$$

The answer is (C).

POWER CYCLES–35

What is the ideal compression ratio of an Otto cycle that uses air as the gas and has an efficiency of 50%?

(A) 4.3:1 (B) 5.0:1 (C) 5.7:1 (D) 6.2:1

By definition,

$$\eta_{\text{Otto}} = 1 - r^{1-k}$$

r is the compression ratio.

$$\begin{aligned}k_{\text{air}} &= 1.4 \\ r &= (1 - \eta_{\text{Otto}})^{1/1-k} \\ &= (1 - 0.5)^{1/-0.4} \\ &= 5.7\end{aligned}$$

Therefore, the ratio is 5.7:1.

The answer is (C).

7 CHEMISTRY

CHEMISTRY–1

The mole is a basic unit of measurement in chemistry. Which of the following is NOT equal to or the same as 1 mol of the substance indicated?

(A) 22.4 L of nitrogen (N_2) gas at STP
(B) 6.02×10^{23} oxygen (O_2) molecules
(C) 12 g of carbon atoms
(D) 16 g of oxygen (O_2) molecules

Oxygen has a molar mass of 16 g/mol. Therefore, 1 mol of O_2 has a mass of 32 g.

The answer is (D).

CHEMISTRY–2

Which one of the following is standard temperature and pressure (STP)?

(A) 0K and one atmosphere pressure
(B) 0°F and zero pressure
(C) 32°F and zero pressure
(D) 0°C and one atmosphere pressure

By definition, standard temperature and pressure is 0°C and 1 atm pressure.

The answer is (D).

CHEMISTRY–3

An ideal gas at 0.60 atm and 87°C occupies 0.450 L. The gas constant is $R^* = 0.0821$ L·atm/mol·K. How many moles are in the sample?

(A) 0.000 20 mol (B) 0.0091 mol (C) 0.0120 mol (D) 0.038 mol

Use the ideal gas law.

$$pV = nR^*T$$

$$n = \frac{pV}{R^*T}$$

$$= \frac{(0.60 \text{ atm})(0.45 \text{ L})}{\left(0.0821 \ \frac{\text{L·atm}}{\text{mol·K}}\right)(87°\text{C} + 273°)}$$

$$= 0.0091 \text{ mol}$$

The answer is (B).

CHEMISTRY–4

A gas occupies 0.213 L at STP. How many moles are there in this sample of gas?

(A) 0.0089 mol (B) 0.0095 mol (C) 0.089 mol (D) 0.095 mol

$$pV = nR^*T$$

$$n = \frac{pV}{R^*T}$$

At STP,

$$p = 1 \text{ atm}$$

$$T = 273\text{K}$$

$$n = \frac{(1.0 \text{ atm})(0.213 \text{ L})}{\left(0.0821 \ \frac{\text{atm·L}}{\text{mol·K}}\right)(273\text{K})}$$

$$= 0.009\,50 \text{ mol}$$

The answer is (B).

CHEMISTRY–5

An ideal gas is contained in a vessel of unknown volume at a pressure of 1 atmosphere. The gas is released and allowed to expand into a previously evacuated vessel whose volume is 0.500 L. Once equilibrium has been reached, the temperature remains the same while the pressure is recorded as 500 mm of mercury. What is the unknown volume, V, of the first vessel?

(A) 0.853 L (B) 0.962 L (C) 1.07 L (D) 1.18 L

For an ideal gas at a constant temperature,

$$p_1V_1 = p_2V_2$$

$$p_1 = 1.0 \text{ atm} = 760 \text{ mm Hg}$$

$$(760 \text{ mm Hg})V_1 = (500 \text{ mm Hg})(0.5 \text{ L} + V_1)$$

$$V_1 = 0.962 \text{ L}$$

The answer is (B).

CHEMISTRY–6

What is most nearly the combined volume of 1.0 g of hydrogen gas (H_2) and 10.0 g of helium gas (He) when confined at 20°C and 5 atm?

(A) 10 L (B) 12 L (C) 14 L (D) 16 L

Use the ideal gas law.

$$pV = nR^*T$$

$$V = \frac{n_{\text{total}}R^*T}{p}$$

$$\begin{aligned} n_{\text{total}} &= n_{H_2} + n_{He} \\ &= (1.0 \text{ g})\left(\frac{1 \text{ mol } H_2}{2 \text{ g } H_2}\right) + (10.0 \text{ g})\left(\frac{1 \text{ mol He}}{4 \text{ g He}}\right) \\ &= 3.0 \text{ mol} \end{aligned}$$

$$\begin{aligned} V &= \frac{(3.0 \text{ mol})\left(0.0821 \frac{\text{atm}\cdot\text{L}}{\text{mol}\cdot\text{K}}\right)(293\text{K})}{5 \text{ atm}} \\ &= 14 \text{ L} \end{aligned}$$

The answer is (C).

CHEMISTRY–7

The valve between a 9 L tank containing gas at 5 atm and a 6 L tank containing gas at 10 atm is opened. What is the equilibrium pressure obtained in the two tanks at constant temperature? Assume ideal gas behavior.

(A) 5 atm (B) 6 atm (C) 7 atm (D) 8 atm

$$p_{\text{total}} = p_1 + p_2$$

$$p_n = \text{partial pressure of gas } n$$

For an ideal gas at a constant temperature,

$$p_i V_i = p_f V_f$$

$$p_f = p_i \left(\frac{V_i}{V_f} \right)$$

$$p_1 = (5 \text{ atm}) \left(\frac{9 \text{ L}}{15 \text{ L}} \right) = 3 \text{ atm}$$

$$p_2 = (10 \text{ atm}) \left(\frac{6 \text{ L}}{15 \text{ L}} \right) = 4 \text{ atm}$$

$$p_{\text{total}} = 3 \text{ atm} + 4 \text{ atm} = 7 \text{ atm}$$

The answer is (C).

CHEMISTRY–8

A bicycle tire has a volume of 600 cm^3. It is inflated with CO_2 to a pressure of 5.4 atm at 20°C. Approximately how many grams of CO_2 are contained in the tire?

(A) 3.8 g (B) 4.8 g (C) 6.0 g (D) 6.4 g

$$pV = nR^*T$$

$$n = \frac{pV}{R^*T}$$

$$V = (600 \text{ cm}^3) \left(\frac{1 \text{ L}}{1000 \text{ cm}^3} \right) = 0.6 \text{ L}$$

$$T = 20°\text{C} + 273° = 293\text{K}$$

$$n = \frac{(5.4 \text{ atm})(0.6 \text{ L})}{\left(0.0821 \ \frac{\text{atm·L}}{\text{mol·K}}\right)(293\text{K})}$$

$$= 0.135 \text{ mol}$$

The molecular weight of CO_2, MW_{CO_2}, is

$$MW_{CO_2} = 12 \ \frac{\text{g}}{\text{mol}} + \left(16 \ \frac{\text{g}}{\text{mol}}\right)(2)$$

$$= 44 \text{ g/mol}$$

The mass of CO_2 in the tire, m, is

$$m = n(MW_{CO_2})$$

$$= (0.135 \text{ mol})\left(44 \ \frac{\text{g}}{\text{mol}}\right)$$

$$= 5.94 \text{ g} \quad (6.0 \text{ g})$$

The answer is (C).

CHEMISTRY–9

On a hot day, the temperature rises from 13°C early in the morning to 37°C in the afternoon. What is the ratio of the concentration (in mol/L) of helium in a spherical balloon in the afternoon to the concentration of helium in the balloon in the morning?

(A) 0.51 (B) 0.69 (C) 0.92 (D) 1.1

$$pV = nR^*T$$

The concentration, C, is

$$C = \frac{n}{V}$$

$$= \frac{p}{R^*T}$$

Determine the ratio by dividing the concentration of helium in the balloon in the afternoon, C_2, by the concentration of helium in the balloon in the morning, C_1.

$$\begin{aligned} \frac{C_2}{C_1} &= \frac{T_1}{T_2} \\ &= \frac{13°\text{C} + 273°}{37°\text{C} + 273°} \\ &= 0.92 \end{aligned}$$

The answer is (C).

CHEMISTRY–10

When 0.5 g of a liquid is completely evaporated and collected in a 1 L manometer, the pressure is 0.25 atm and the temperature is 27°C. Assuming ideal gas behavior, what is most nearly the molecular weight? The universal gas constant is $R^* = 0.0821$ L·atm/mol·K.

(A) 10 g/mol (B) 12 g/mol (C) 30 g/mol (D) 49 g/mol

$$\begin{aligned} n &= \frac{m}{\text{MW}} \\ pV &= nR^*T \\ \text{MW} &= \frac{mR^*T}{pV} \\ &= \frac{(0.5\text{ g})\left(0.0821\ \frac{\text{atm}\cdot\text{L}}{\text{mol}\cdot\text{K}}\right)(300\text{K})}{(0.25\text{ atm})(1.0\text{ L})} \\ &= 49.3\text{ g/mol} \end{aligned}$$

In the preceding equation, MW is molecular weight.

The answer is (D).

CHEMISTRY–11

200 mL of oxygen gas (O_2) are collected over water at 23°C and a pressure of 1 atm. What volume would the oxygen occupy dry at 273K and 1 atm?

(A) 179 mL (B) 184 mL (C) 190 mL (D) 194 mL

At 23°C, the vapor pressure of water is 0.0277 atm. Find the pressure of the oxygen assuming ideal gas behavior.

$$p_{\text{total}} = \sum \text{partial pressures}$$

$$= p_{O_2} + p_{\text{water vapor}}$$

$$p_{O_2} = 1.000 \text{ atm} - 0.0277 \text{ atm}$$

$$= 0.9723 \text{ atm}$$

$$= p_1$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$V_2 = \left(\frac{p_1}{p_2}\right)\left(\frac{T_2}{T_1}\right) V_1$$

$$= \left(\frac{0.9723 \text{ atm}}{1.000 \text{ atm}}\right)\left(\frac{273\text{K}}{296\text{K}}\right)(200 \text{ mL})$$

$$= 179 \text{ mL}$$

The answer is (A).

CHEMISTRY–12

8 g of Ag_2O (solid) are heated to produce oxygen gas (O_2) as follows.

$$2Ag_2O \longrightarrow 4Ag + O_2$$

The oxygen gas is collected at 35°C over water. The water vapor pressure at 35°C is 0.0555 atm. Given that the barometric pressure is 1 atm, what (wet) volume of O_2 is collected?

(A) 415 mL (B) 425 mL (C) 434 mL (D) 455 mL

The number of moles of Ag_2O, n_{Ag_2O}, is

$$n_{Ag_2O} = \frac{\text{weight of substance}}{\text{MW}}$$

$$= (8\text{ g})\left(\frac{1\text{ mol}}{(2\text{ mol})\left(108\ \frac{\text{g}}{\text{mol}}\right) + \left(16\ \frac{\text{g}}{\text{mol}}\right)(1\text{ mol})}\right)$$

$$= 0.034\text{ mol}$$

Since 2 mol of Ag_2O produce 1 mol of O_2,

$$n_{O_2} = \left(\frac{1}{2}\right)(0.034\text{ mol})$$

$$= 0.017\text{ mol}$$

$$T = 35°\text{C} + 273°$$

$$= 308\text{K}$$

$$p_{O_2} = p_{\text{total}} - p_{H_2O}$$

$$= 1\text{ atm} - 0.0555\text{ atm}$$

$$= 0.945\text{ atm}$$

$$pV = nR^*T$$

$$V_{O_2} = \frac{n_{O_2}R^*T}{p_{O_2}}$$

$$= \frac{(0.017\text{ mol})\left(0.0821\ \frac{\text{atm}\cdot\text{L}}{\text{mol}\cdot\text{K}}\right)(308\text{K})}{0.945\text{ atm}}$$

$$= 0.455\text{ L} \quad (455\text{ mL})$$

The answer is (D).

CHEMISTRY–13

A total of 0.1 g of water is held in a closed container at 40°C. The container holds 500 cm^3. The pressure in the container is atmospheric pressure, and the vapor pressure of water at 40°C is 55.3 torr. Most nearly how much water is in liquid form at equilibrium?

(A) There is no liquid present.

(B) 1/2 of the water is liquid.

(C) 2/3 of the water is liquid.

(D) 3/4 of the water is liquid.

Use the ideal gas law to determine how much of the water is vapor.

$$\begin{aligned} pV &= nR^*T \\ &= \frac{mR^*T}{\text{MW}} \end{aligned}$$

MW is the molecular weight of the water.

$$\begin{aligned} m &= \frac{pV(\text{MW})}{R^*T} \\ \text{MW} &= 2\text{ g} + 16\text{ g} \\ &= 18\text{ g} \\ p &= \frac{55.3\text{ torr}}{760\ \frac{\text{torr}}{\text{atm}}} \\ &= 0.073\text{ atm} \\ V &= \frac{500\text{ mL}}{1000\text{ L}} \\ &= 0.5\text{ L} \\ T &= 40°\text{C} + 273° = 313\text{K} \\ m_{\text{water}} &= \frac{(18\text{ g})(0.073\text{ atm})(0.5\text{ L})}{\left(0.0821\ \frac{\text{atm·L}}{\text{mol·K}}\right)(313\text{K})} \\ &= 0.026\text{ g} \end{aligned}$$

The remainder of the H_2O is liquid.

$$m_{\text{liquid}} = m_{\text{total}} - m_{\text{water}} = 0.1 \text{ g} - 0.026 \text{ g}$$
$$= 0.074 \text{ g}$$

$$\text{fraction that is liquid} = \frac{m_{\text{liquid}}}{m_{\text{total}}}$$
$$= \frac{0.074 \text{ g}}{0.1 \text{ g}}$$
$$= 0.74 \quad (3/4)$$

The answer is (D).

CHEMISTRY–14

Which of the following statements is FALSE for an ideal gas?

(A) The molecules behave as solid spheres of finite radius.

(B) $pV = nR^*T$

(C) Collisions between gas molecules are perfectly elastic and result in no decrease in kinetic energy.

(D) No attractive forces exist between the molecules.

The volume of molecules in an ideal gas is not considered. Real gases consist of molecules of finite volume.

The answer is (A).

CHEMISTRY–15

The following statements are made with regard to the boiling point of a liquid. Which statement is FALSE?

(A) A nonvolatile substance having zero vapor pressure in solution (e.g., sugars or salts) has no true boiling point.

(B) The boiling point is the temperature at which the vapor pressure of a liquid equals the applied pressure on the liquid.

(C) Combinations of liquids having different boiling points can be separated by slowly raising the temperature to draw off each fraction (i.e., by fractional distillation).

(D) At high elevations, water boils at a lower temperature because of a reduction in the surface tension of the water.

A liquid boils when its vapor pressure is equal to the pressure of the surroundings. The lower boiling temperature at high elevations is due to the reduced atmospheric pressure, not to a change in the surface tension of a liquid.

The answer is (D).

CHEMISTRY–16

The critical point for a mixture occurs for which of the following cases?

(A) The vapor and liquid exist in a single form.
(B) The liquid has no absorbed gas.
(C) The vapor phase is stable.
(D) The liquid is completely vaporized.

The critical point for a mixture occurs when the vapor and the liquid have a form that is stable for a "critical temperature and critical pressure." It is both a liquid and a vapor with no boundaries and a uniform composition (a single form). A few substances have a triple point at which a solid, a liquid, and a gas are in equilibrium.

The answer is (A).

CHEMISTRY–17

How is "molality" defined?

(A) the number of moles of solute in 1000 g of solvent
(B) the number of moles of solute in 1 L of solution
(C) the number of gram-formula weights of solute per liter
(D) the number of gram-equivalent weights of solute in 1 L of solution

Molality is defined as the number of moles of solute per 1000 g of solvent. Option (B) is the definition of molarity, option (C) is the definition of formality, and option (D) is the definition of normality.

The answer is (A).

CHEMISTRY–18

L is a nonvolatile, nonelectrolytic liquid. A solid, S, is added to L to form a solution that just boils at 1 atm pressure. The vapor pressure of pure L is 850 torr. What is the mole fraction of liquid L in the solution?

(A) 64.3% (B) 79.4% (C) 85.7% (D) 89.4%

A liquid boils when its vapor pressure equals the pressure of its surroundings. Thus, the vapor pressure of the solution is 760 torr. From Raoult's law,

$$
\begin{aligned}
p_{\text{solution}} &= p_{\text{solvent}}(\text{mol\% of solvent}) \\
760\ \text{torr} &= (850\ \text{torr})(\text{mol\% of L}) \\
\text{mol\% of L} &= \frac{760\ \text{torr}}{850\ \text{torr}} \\
&= 0.894 \quad (89.4\%)
\end{aligned}
$$

The answer is (D).

CHEMISTRY–19

Which of the following postulates does Bohr's model of the hydrogen atom involve?

(A) The electron in an atom has an infinite range of motion allowed to it.

(B) When an atom changes from a low energy state to a high energy state, it emits a quantum of radiation whose energy is equal to the difference in energy between the two states.

(C) In any of its energy states, the electron moves in a circular orbit about the nucleus.

(D) The states of allowed electron motion are those in which the angular momentum of the electron is an integral multiple of $\hbar/\pi$.

Bohr's model of the hydrogen atom involves the following postulates.

1. Each atom has only certain definite stationary states of motion allowed to it.
2. A quantum of energy is emitted when an atom changes from a higher energy state to a lower energy state.
3. The states of allowed electron motion are those in which the angular momentum of the electron is an integral multiple of $\hbar/2\pi$.

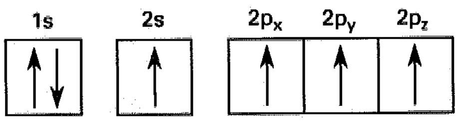
Thus, the only choice that is correct is option (C).

The answer is (C).

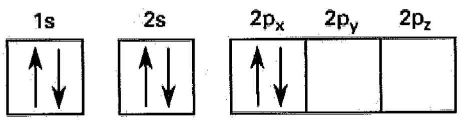
CHEMISTRY–20

Which of the following diagrams best depicts the electron configuration of carbon?

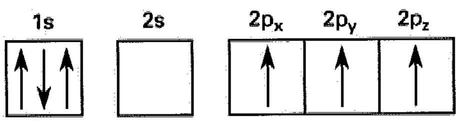
(A)



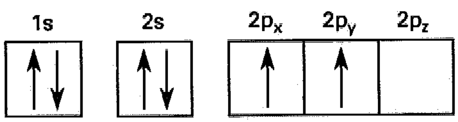
(B)



(C)



(D)



Carbon has a total of six electrons. Electrons position themselves in orbitals according to the following rules.

1. There is a maximum of two electrons per orbital.
2. Electrons in the same orbital have different spins ($\pm^1/_2$).
3. Electrons usually fill up empty orbitals before moving into the same orbital as another electron.

Thus, option (D) gives the correct electron configuration of carbon.

The answer is (D).

CHEMISTRY–21

Which of the following elements and compounds is reactive in its pure form?

(A) sodium (Na)
(B) helium (He)
(C) carbon dioxide (CO_2)
(D) hydrochloric acid (HCl)

Helium is an inert gas and, therefore, is not very reactive. Hydrochloric acid and carbon dioxide have all of their valence orbitals filled. Thus, they are also not very reactive. Sodium has only one valence electron that is easily ionizable. Therefore, it is very reactive.

The answer is (A).

CHEMISTRY–22

Two major types of chemical bonds are observed in chemical bonding: ionic and covalent. Which of the following has a bond that is the least ionic in character?

(A) NaCl (B) CH_4 (C) H_2 (D) H_2O

The electronegativity difference between two similar atoms is zero. Therefore, the H_2 bond is completely covalent. It has no ionic bond characteristics.

The answer is (C).

CHEMISTRY–23

Which of the following statements is FALSE?

(A) It is not possible for bonds between a pair of atoms to be different (e.g., different bond lengths or bond energies) in different compounds.

(B) The bond length for a pair of atoms is the point of lowest energy.

(C) The electrostatic repulsion between two nuclei increases as the atoms are brought together.

(D) The repulsion between two nuclei increases as their charge increases.

It is possible for bonds between a pair of atoms to be different in different compounds. For example, there is more than one type of carbon-carbon bond.

> The answer is (A).

CHEMISTRY–24

Which of the following statements is FALSE?

(A) For a diatomic molecule, the bond dissociation energy is the change in the enthalpy of the reaction when the diatomic molecule is separated into atoms.

(B) The average bond energy is the approximate energy required to break a bond in any compound in which it occurs.

(C) The energy released when a gaseous molecule is formed from its gaseous atoms can be estimated using average bond energies.

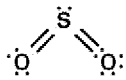
(D) The change in enthalpy is negative when energy is absorbed in the formation of a compound from its elements.

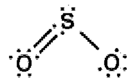
The change in enthalpy is negative for the formation of a compound from elements when energy is released in the process.

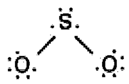
> The answer is (D).

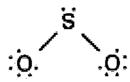
CHEMISTRY–25

Which of the following is the correct Lewis structure for sulfur dioxide?

(A) 

(B) 

(C) 

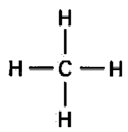
(D) 

Sulfur and oxygen each have six valence electrons. Thus, there are a total of 18 valence electrons in SO_2. Therefore, there is one single S–O bond and one double S=O bond. The Lewis structure of sulfur dioxide is as follows.

The answer is (B).

CHEMISTRY–26

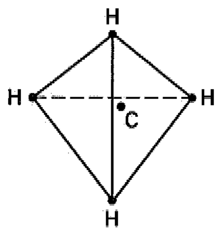
The molecule methane, CH_4, is often represented by the following structural formula.



What is the actual geometric shape of the molecule?

(A) linear
(B) square planar
(C) planar, but not 90° bond angles
(D) tetrahedral

The structure of methane is as follows.

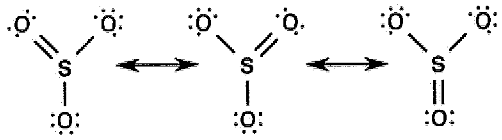


In the tetrahedral structure, bond angles are maximized and repulsions minimized, with bond angles of 109°.

The answer is (D).

CHEMISTRY–27

SO_3 has a structural formula represented as a resonance hybrid.



Which of the following is a true statement about the meaning of such a structure?

(A) One-third of the SO_3 molecules exists as each of the three structures shown.

(B) The true structure is a combination of the three with each S ⩵ O bond identical to another.

(C) The molecule fluctuates between the three structures.

(D) The arrows indicate equilibrium where an actual chemical reaction is taking place.

The true structure is a combination with each bond identical, somewhere between a single and a double bond.

The answer is (B).

CHEMISTRY–28

Which of the following chemical equations is incorrect?

(A) $S + Fe \rightarrow FeS$
(B) $ZnSO_4 + Na_2S \rightarrow ZnS + Na_2SO_4$
(C) $H_2SO_4 + ZnS \rightarrow ZnSO_4 + H_2S$
(D) $ZnS + O_2 \rightarrow SO_2 + ZnO$

The equation in option (D) does not balance. It needs $^3/_2O_2$ on the left side. The equation $ZnS + {}^3/_2O_2 \rightarrow SO_2 + ZnO$ would be correct.

The answer is (D).

CHEMISTRY–29

Na_2CO_3 reacts with HCl, but not by the stoichiometry implied in the following unbalanced chemical equation.

$$Na_2CO_3 + HCl \longrightarrow NaCl + H_2O + CO_2$$

What is the smallest possible whole-number coefficient for Na_2CO_3 in the balanced equation?

(A) 1 (B) 2 (C) 4 (D) 5

The simplest balanced equation is

$$Na_2CO_3 + HCl \longrightarrow 2NaCl + H_2O + CO_2$$

The smallest whole-number coefficient for Na_2CO_3 is 1.

The answer is (A).

CHEMISTRY–30

Which of the following is the result of the reaction given?

$$5SO_2 + 2KMnO_4 + 2H_2O \longrightarrow ?$$

(A) $2MnSO_4 + K_2SO_4 + 2H_2SO_4$
(B) $2MnSO_4 + K_2SO_2 + HSO_4 + H_2O$
(C) $2MnSO_4 + K_2SO_4 + H_2SO_4$
(D) $MnSO_4 + 2K_2SO_4 + 2H_2SO_4$

Only the products listed in option (A) would balance the elements on the right and left sides of the equation.

The answer is (A).

CHEMISTRY–31

What is the balanced form of the equation given?

$$H_2O + P_4 + OH^- \longrightarrow PH_3 + H_2PO_2^-$$

(A) $4OH^- + 4P_4 + H_2O \longrightarrow 6H_2PO_2^- + 2PH_3$
(B) $P_4 + H_2O \longrightarrow H_2PO_2^- + 3PH_3$
(C) $8OH^- + 2P_4 + 2H_2O \longrightarrow H_2PO_2^- + PH_3$
(D) $3OH^- + P_4 + 3H_2O \longrightarrow 3H_2PO_2^- + PH_3$

The two half reactions are

$$8OH^- + P_4 \longrightarrow 4H_2PO_2^- + 4E^-$$
$$12E^- + 12H_2O + P_4 \longrightarrow 4PH_3 + 12OH^-$$

Multiplying the top equation by 3 and adding the two equations together yields

$$12OH^- + 4P_4 + 12H_2O \longrightarrow 12H_2PO_2^- + 4PH_3$$

In order to reduce the equation to the lowest whole number coefficients, divide by 4.

$$3OH^- + P_4 + 3H_2O \longrightarrow 3H_2PO_2^- + PH_3$$

The answer is (D).

CHEMISTRY–32

Which of the following chemical reactions relates to the softening procedure in water purification?

(A) $CO_2 + Ca(OH)_2 \longrightarrow CaCO_3 + H_2O$
(B) $Ca(HCO_3)_2 + Ca(OH)_2 \longrightarrow 2CaCO_3 + 2H_2O$
(C) $2H_2O + O_2 \longrightarrow 2H_2O_2$
(D) $NaOH + HCl \longrightarrow NaCl + H_2O$

Option (B) gives the chemical reaction for adding lime to hard water in order to remove calcium salts. The resulting calcium carbonate precipitate can be removed by sedimentation.

The answer is (B).

CHEMISTRY–33

A substance is oxidized when which of the following occurs?

(A) It turns red.
(B) It becomes more negative.
(C) It loses electrons.
(D) It gives off heat.

By definition, a substance is oxidized when it loses electrons.

The answer is (C).

CHEMISTRY–34

In order to assign oxidation states in polyatomic molecules, which of the following rules is followed?

(A) The oxidation of all elements in any allotropic form is zero.

(B) The oxidation state of oxygen is always -2.

(C) The oxidation state of hydrogen is always $+1$.

(D) All other oxidation states are chosen such that the algebraic sum of the oxidation states for the ion or molecule is zero.

Option (B) is false because it does not take into account the peroxides in which the oxidation state of O is -1. Option (C) is false because it does not account for hydrogen combined with metals, where its oxidation state is -1. Option (D) is wrong because the sum of the oxidation states should equal the net charge on the ion or molecule. Thus, only option (A) is correct.

The answer is (A).

CHEMISTRY–35

What is the oxidation state of nitrogen in NO_3^-?

(A) −1 (B) +1 (C) +3 (D) +5

The oxidation state of O is −2, and the net charge on the ion is −1. The oxidation state of nitrogen is given as follows.

$$\begin{aligned} 3(\text{oxidation state of O}) + (\text{oxidation state of N}) &= -1 \\ (3)(-2) + (\text{oxidation state of N}) &= -1 \\ \text{oxidation state of N} &= +5 \end{aligned}$$

The answer is (D).

CHEMISTRY–36

What is the oxidation number of Cr in the dichromate ion $(Cr_2O_7)^{-2}$?

(A) −1 (B) 0 (C) 3 (D) 6

The oxidation number of O is −2. Therefore, the oxidation number of O_7 is −14. The charge on the ion is −2, so the charge on Cr_2 is 12. Thus, the oxidation number of Cr is 6.

The answer is (D).

CHEMISTRY–37

Given the following information, determine the oxidation state of nitrogen in nitric acid, HNO_3.

oxidation state	formula	name
1	$HClO$	hypochlorous acid
3	$HClO_2$	chlorous acid
5	$HClO_3$	chloric acid
7	$HClO_4$	perchloric acid
3	HNO_2	nitrous acid

(A) 2 (B) 3 (C) 4 (D) 5

The outer shell of oxygen is 2 electrons short of being full (inert gases have a full shell). Thus, the oxidation number of oxygen is 2 for both ions. By adding another oxygen atom to nitrous acid, the oxidation level is increased by 2. This situation compares directly with that of $HClO_2$ and $HClO_3$. Thus, the oxidation state of nitric acid is 5.

The answer is (D).

CHEMISTRY–38

Which are the oxidizing and reducing agents in the following reaction?

$$2CCl_4 + K_2CrO_4 \longrightarrow 2Cl_2CO + CrO_2Cl_2 + 2KCl$$

(A) oxidizing agent: chromium; reducing agent: chlorine
(B) oxidizing agent: oxygen; reducing agent: chlorine
(C) oxidizing agent: chromium; reducing agent: oxygen
(D) There are no oxidizing or reducing agents in this reaction.

The oxidation state of chromium is 6 in each compound. Carbon remains with a +4 oxidation state throughout the reaction. The oxidation states of both chlorine and oxygen remain the same throughout this reaction. Thus, nothing is oxidized or reduced in the reaction.

The answer is (D).

CHEMISTRY–39

A volumetric analysis of a gaseous mixture is as follows.

CO_2	12%
O_2	4%
N_2	82%
CO	2%

What is the percentage of CO on a mass basis?

(A) 0.5% (B) 0.8% (C) 1% (D) 2%

name	vol. (%)	mole frac. (mol%)		mol. wt.		mass (g)
CO_2	12	0.12	×	44	=	5.3
O_2	4	0.04	×	32	=	1
N_2	82	0.82	×	28	=	23
CO	2	0.02	×	28	=	0.6
						30.0

The total mass of the mixture is 30.08 kg. Thus, the mass percentage of CO is given as follows.

$$\text{mass \% of CO} = \frac{0.6\ \text{g}}{30.0\ \text{g}} = 0.02 \quad (2\%)$$

The answer is (D).

CHEMISTRY–40

What is the empirical formula for a compound that has the following composition by mass?

element	mass %
Si	30.2
O	8.59
F	61.2

(A) $SiOF_4$ (B) Si_2OF_4 (C) Si_2OF_6 (D) Si_3OF_6

element	mass (%)	mass (g, based on 100 g)	moles	mole (mol%)
Si	30.2	30.2	1.075	22.2
O	8.59	8.59	0.537	11.1
F	61.2	61.2	3.221	66.6

Find the smallest whole-number ratio of the mole percentage of each element to that of oxygen.

$$\frac{\text{Si}}{\text{O}} = \frac{22.2\ \text{g}}{11.1\ \text{g}} = 2$$

$$\frac{\text{F}}{\text{O}} = \frac{66.6\ \text{g}}{11.1\ \text{g}} = 6$$

$$\frac{\text{O}}{\text{O}} = \frac{11.1\ \text{g}}{11.1\ \text{g}} = 1$$

Therefore, the simplest formula is Si_2OF_6.

The answer is (C).

CHEMISTRY–41

The following equation describes the decomposition of potassium chlorate to produce oxygen gas.

$$2KClO_3 \longrightarrow 2KCl\ (solid) + 3O_2\ (gas)$$

Approximately how many grams of $KClO_3$ must be used to produce 4.00 L of O_2 (*gas*) measured at 7400 torr and 30°C?

(A) 110 g (B) 120 g (C) 130 g (D) 140 g

$$p = \frac{7400\ \text{torr}}{760\ \frac{\text{torr}}{\text{atm}}} = 9.74\ \text{atm}$$

$$V = 4\ \text{L}$$

$$T = 30°\text{C} + 273° = 303\text{K}$$

$$pV = nR^*T$$

$$n = \frac{pV}{R^*T}$$

$$= \frac{(9.74\ \text{atm})(4.00\ \text{L})}{\left(0.0821\ \frac{\text{atm}\cdot\text{L}}{\text{mol}\cdot\text{K}}\right)(303\text{K})}$$

$$= 1.57\ \text{mol}$$

$$\frac{\text{no. of moles KClO}_3}{\text{no. of moles of O}_2} = 2\ \text{mol}/3\ \text{mol}$$

$$\text{no. of moles KClO}_3\ \text{needed} = \left(\frac{2\ \text{mol}}{3\ \text{mol}}\right)(1.57\ \text{mol})$$

$$= 1.05\ \text{mol}$$

$$\text{MW}_{\text{KClO}_3} = 39.1\ \frac{\text{g}}{\text{mol}} + 35.5\ \frac{\text{g}}{\text{mol}} + \left(16\ \frac{\text{g}}{\text{mol}}\right)(3\ \text{mol})$$
$$= 123\ \text{g/mol}$$

$$\text{no. of grams KClO}_3 = (1.05\ \text{mol})\left(123\ \frac{\text{g}}{\text{mol}}\right)$$

$$= 129\ \text{g} \quad (130\ \text{g})$$

The answer is (C).

CHEMISTRY–42

Determine which of the statements is true, given the following facts.

1. A 40 L sample of H_2 (gas) at 10°C and 740 torr is added to a 75 L sample of O_2 (gas) at 20°C and 730 torr.
2. The mixture is ignited to produce water.

(A) There is an excess of O_2 greater than 0.2 mol.
(B) There is an excess of H_2 greater than 0.2 mol.
(C) There is H_2O only.
(D) There is an excess of H_2 less than 0.2 mol.

The stoichiometric equation is

$$H_2 + \tfrac{1}{2}O_2 \longrightarrow H_2O$$

The number of moles of each gas initially present is

$$\begin{aligned} n_{H_2} &= \frac{pV}{R^*T} \\ &= \frac{\left(\dfrac{740\ \text{torr}}{760\ \dfrac{\text{torr}}{\text{atm}}}\right)(40\ \text{L})}{(10^\circ\text{C} + 273^\circ)\left(0.0821\ \dfrac{\text{atm}\cdot\text{L}}{\text{mol}\cdot\text{K}}\right)} \\ &= 1.7\ \text{mol} \end{aligned}$$

$$\begin{aligned} n_{O_2} &= \frac{pV}{R^*T} \\ &= \frac{\left(\dfrac{730\ \text{torr}}{760\ \dfrac{\text{torr}}{\text{atm}}}\right)(75\ \text{L})}{(20^\circ\text{C} + 273^\circ)\left(0.082\ \dfrac{\text{atm}\cdot\text{L}}{\text{mol}\cdot\text{K}}\right)} \\ &= 3.0\ \text{mol} \end{aligned}$$

For each mole of H_2O formed, 0.5 mol of O_2 and 1 mol of H_2 are required. The oxygen necessary to completely react with 1.7 mol of H_2 is given by

$$\begin{aligned} n &= \frac{1.7\ \text{mol}}{2} \\ &= 0.85\ \text{mol} \end{aligned}$$

Therefore, there is an excess of O_2. The amount of O_2 extra is

$$3.0 \text{ mol} - 0.85 \text{ mol} = 2.15 \text{ mol}$$

The answer is (A).

CHEMISTRY–43

If 2.25 g of pure calcium metal are converted to 3.13 g of pure CaO, what is the atomic weight of calcium? The atomic weight of oxygen is 16 g/mol.

(A) 28 g/mol (B) 33 g/mol (C) 37 g/mol (D) 41 g/mol

The stoichiometric equation is

$$\text{Ca} + \text{O} \longrightarrow \text{CaO}$$

One mol of oxygen and 1 mol of calcium are required to make 1 mol of CaO.

$$\begin{aligned} n_\text{O} &= \frac{3.13 \text{ g} - 2.25 \text{ g}}{16 \ \frac{\text{g}}{\text{mol}}} \\ &= \frac{0.88 \text{ g}}{16 \ \frac{\text{g}}{\text{mol}}} \\ &= 0.055 \text{ mol} \end{aligned}$$

$$\begin{aligned} n_\text{Ca} &= 0.055 \text{ mol} \\ &= \frac{2.25 \text{ g}}{\text{atomic weight of Ca}} \end{aligned}$$

$$\begin{aligned} \text{atomic weight of Ca} &= \frac{2.25 \text{ g}}{0.055 \text{ mol}} \\ &= 41 \text{ g/mol} \end{aligned}$$

The answer is (D).

CHEMISTRY–44

Methane, CH_4, burns to form CO_2 and H_2O according to the equation

$$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$$

How many grams of CO_2 will theoretically be formed when a mixture of 50 g of CH_4 and 100 g of O_2 is ignited?

(A) 34 g (B) 68 g (C) 69 g (D) 72 g

$$n = \frac{m}{\text{MW}}$$

In the preceding equation, m is the mass of compound and MW is the molecular weight of compound.

$$\begin{aligned} n_{CH_4} &= \frac{50 \text{ g}}{12 \frac{\text{g}}{\text{mol}} + \left(4 \frac{\text{g}}{\text{mol}}\right)\left(1 \frac{\text{g}}{\text{mol}}\right)} \\ &= 3.125 \text{ mol} \\ n_{O_2} &= \frac{100 \text{ g}}{\left(2 \frac{\text{g}}{\text{mol}}\right)\left(16 \frac{\text{g}}{\text{mol}}\right)} \\ &= 3.125 \text{ mol} \end{aligned}$$

Since 1 mol of CH_4 and 2 mol of O_2 are needed for each mole of CO_2 formed, O_2 is the limiting reactant.

$$\frac{\text{no. of moles } CO_2 \text{ formed}}{\text{no. of moles } O_2 \text{ ignited}} = 1 \text{ mol}/2 \text{ mol}$$

$$\begin{aligned} n_{CO_2} &= (3.125 \text{ mol})\left(\frac{1 \text{ mol}}{2 \text{ mol}}\right) \\ &= 1.563 \text{ mol} \\ m_{CO_2} &= n_{CO_2}\text{MW}_{CO_2} \\ &= (1.563 \text{ mol})\left(44 \frac{\text{g}}{\text{mol}}\right) \\ &= 69 \text{ g} \end{aligned}$$

The answer is (C).

CHEMISTRY–45

Determine the mole percent of CO_2 in the products of combustion of C_8H_{18} when 200% theoretical air is used.

(A) 5.5% (B) 6.5% (C) 7.5% (D) 8.5%

The formula for theoretical air is $O_2 + 3.76N_2$. For 200% theoretical air, the stoichiometric equation is

$$C_8H_{18} + 25(O_2 + 3.76N_2) \longrightarrow 8CO_2 + 9H_2O + 12.5O_2 + 94N_2$$

The mole percent of CO_2 is given by the ratio of the number of moles of CO_2 formed to the total number of moles formed.

$$\begin{aligned}\%CO_2 &= \frac{8 \text{ mol}}{8 \text{ mol} + 9 \text{ mol} + 12.5 \text{ mol} + 94 \text{ mol}} \\ &= 6.5\%\end{aligned}$$

The answer is (B).

CHEMISTRY–46

Approximately what volume of O_2 at 298K and 1 atm is required for complete combustion of 10 L of C_2H_6 (gas) at 500K and 1 atm? The combustion equation is

$$7O_2 + 2C_2H_6 \longrightarrow 6H_2O + 4CO_2$$

(A) 16 L (B) 19 L (C) 21 L (D) 22 L

Assume ideal gas behavior.

$$\begin{aligned}n_{C_2H_6} &= \frac{pV}{R^*T} \\ &= \frac{(1.0 \text{ atm})(10 \text{ L})}{\left(0.0821 \ \frac{\text{atm·L}}{\text{mol·K}}\right)(500\text{K})} \\ &= 0.24 \text{ mol}\end{aligned}$$

$$\frac{n_{O_2}}{n_{C_2H_6}} = 7 \text{ mol}/2 \text{ mol}$$

$$\begin{aligned}n_{O_2} &= \left(\frac{7 \text{ mol}}{2 \text{ mol}}\right)(0.24 \text{ mol}) \\ &= 0.84 \text{ mol}\end{aligned}$$

The volume of 1 mol of ideal gas at STP (standard temperature and pressure) is 22.4 L. Therefore, the volume of O_2 required at 298K is

$$V_{O_2} = n_{O_2}\left(\frac{V_{298}}{V_{\text{STP}}}\right)V_{\text{STP}}$$

$$\frac{V_{298\text{K}}}{V_{\text{STP}}} = \frac{T_{298}}{T_{\text{STP}}} = \frac{298\text{K}}{273\text{K}} = 1.09$$

$$V_{O_2} = (0.84 \text{ mol})(1.09)\left(22.4 \ \frac{\text{L}}{\text{mol}}\right) = 20.6 \text{ L}$$

The answer is (C).

CHEMISTRY–47

One gram of gas made up of carbon and hydrogen is ignited in excess oxygen to produce 3.30 g of CO_2 and 1.125 g of H_2O. What is the empirical formula of the compound?

(A) CH (B) CH_3 (C) C_2H_3 (D) C_3H_5

The stoichiometric equation is

$$C_xH_y + \left(x + \frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$$

$$\text{MW}_{CO_2} = 44 \text{ g/mol}$$

$$\text{MW}_{H_2O} = 18 \text{ g/mol}$$

$$x = \text{moles of C} = \text{moles of } CO_2 = \frac{3.3 \text{ g}}{44 \ \frac{\text{g}}{\text{mol}}} = 0.0750 \text{ mol}$$

$$y = \text{moles of H} = 2(\text{moles of } H_2O) = (2)\left(\frac{1.125 \text{ g}}{18 \ \frac{\text{g}}{\text{mol}}}\right) = 0.125 \text{ mol}$$

$$\frac{x}{y} = \frac{\text{C atoms}}{\text{H atoms}}$$
$$= \frac{\text{moles of C}}{\text{moles of H}}$$
$$= \frac{0.075 \text{ mol}}{0.125 \text{ mol}}$$
$$= 3/5$$

Thus, the empirical formula of the gas is C_3H_5.

The answer is (D).

CHEMISTRY–48

Complete combustion of 13.02 g of a compound (C_xH_y) produces 40.94 g CO_2 and 16.72 g of H_2O. Determine the empirical formula of the compound.

(A) CH (B) CH_2 (C) CH_4 (D) CH_2O

The stoichiometric equation for combustion is

$$C_xH_y + \left(x + \frac{y}{4}\right) O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$$
$$n = \frac{m}{\text{MW}}$$

In the preceding equation, m is the mass of compound and MW is the molecular weight.

$$n_{CO_2} = \frac{40.94 \text{ g}}{44 \ \frac{\text{g}}{\text{mol}}}$$
$$= 0.93 \text{ mol}$$
$$= n_C$$

Therefore,

$$n_C = 0.93 \text{ mol}$$
$$n_{H_2O} = \frac{16.72 \text{ g}}{18 \ \frac{\text{g}}{\text{mol}}}$$
$$= 0.93 \text{ mol}$$
$$= \frac{n_H}{2}$$

$$n_H = 1.86 \text{ mol}$$

$$\frac{n_C}{n_H} = \frac{0.93 \text{ mol}}{1.86 \text{ mol}}$$

$$= 1/2$$

Therefore, the empirical formula for the compound is CH_2.

The answer is (B).

CHEMISTRY–49

When 0.01 mol of a substance consisting of O, H, and C is burned, the following products are obtained.

1. 896 cm^3 of CO_2 at standard temperature and pressure (STP)
2. 0.72 g of water

It is also found that the ratio of oxygen mass to the mass of H plus C in the substance is $^4/_7$. What is the chemical formula of the substance? 1 mol of CO_2 has a volume of 22 400 cm^3 at STP.

(A) CHO_2 (B) $C_4H_6O_2$ (C) CH_2O_2 (D) $C_4H_8O_2$

The stoichiometric equation is

$$C_xH_yO_z + \left(x + \frac{y}{4} - \frac{z}{2}\right) O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$$

$$\frac{n_1}{V_2} = \frac{n_1}{V_1}$$

$$n_C = V_C \left(\frac{n_{STP}}{V_{STP}}\right)$$

$$= (896 \text{ cm}^3)\left(\frac{1.0 \text{ mol}}{22\,400 \text{ cm}^3}\right)$$

$$= 0.04 \text{ mol}$$

$$n_{H_2O} = \frac{m_{H_2O}}{MW_{H_2O}}$$

$$= \frac{0.72 \text{ g}}{18 \frac{\text{g}}{\text{mol}}}$$

$$= 0.04 \text{ mol}$$

$$= \frac{n_H}{2}$$

$$n_H = 0.08 \text{ mol}$$

Thus, there are 0.04 mol C and 0.08 mol H in 0.01 mol of the substance $C_xH_yO_z$. For 1 mol of $C_xH_yO_z$, there are $x = 0.04/0.01 = 4$ mol of C and $y = 0.08/0.01 = 8$ mol of H.

$$\frac{\text{mass of O}}{\text{mass of H} + \text{mass of C}} = \frac{\left(16\ \frac{\text{g}}{\text{mol}}\right)(z\ \text{mol})}{(8\ \text{mol})\left(1\ \frac{\text{g}}{\text{mol}}\right) + (4\ \text{mol})\left(12\ \frac{\text{g}}{\text{mol}}\right)} = 4/7$$

$$\frac{16z}{56} = 4/7$$

$$z = 2\ \text{mol}$$

Thus, the formula is $C_4H_8O_2$.

The answer is (D).

CHEMISTRY–50

What is most nearly the melting point of sodium chloride, given that the heat of melting is 30 kJ/mol, and the associated entropy change is 28 J/mol·K?

(A) 370K (B) 880K (C) 930K (D) 1100K

For the phase change,

$$\Delta G = \Delta H - T_m \Delta S = 0$$

$$T_m = \frac{\Delta H}{\Delta S} = \frac{30\,000\ \frac{\text{J}}{\text{mol}}}{28\ \frac{\text{J}}{\text{mol·K}}} = 1071\text{K} \quad (1100\text{K})$$

The answer is (D).

CHEMISTRY–51

The temperature of 100 g of liquid water at 0°C is raised by 1°C. The number of calories consumed is most nearly

(A) 1.2 cal (B) 4.2 cal (C) 99 cal (D) 100 cal

By definition, 1 cal is the energy needed to heat 1 g of liquid water by 1°C. Therefore, the heat needed to heat 100 g of water by 1°C is

$$q = mc_p\Delta T = (100\ \text{g})\left(1\ \frac{\text{cal}}{\text{g}\cdot{}^\circ\text{C}}\right)(1^\circ\text{C} - 0^\circ\text{C}) = 100\ \text{cal}$$

The answer is (D).

CHEMISTRY–52

Ice with a volume of 50 cm^3 and at a temperature of 0°C is added to 100 g of water at 20°C. Assume that there is no spurious heat loss. The density of ice is 0.92 g/cm^3, and the heat of fusion of ice is 1.44 kcal/mol at 0°C. Approximately how much ice is left unmelted after the mixture reaches thermal equilibrium?

(A) 13 cm^3 (B) 19 cm^3 (C) 23 cm^3 (D) 39 cm^3

The ice will melt until the temperature of the water reaches 0°C. The heat necessary to lower the water temperature from 20°C to 0°C is

$$\begin{aligned} q &= mc_p\Delta T \\ &= (100\ \text{g})\left(1\ \frac{\text{cal}}{\text{g}\cdot{}^\circ\text{C}}\right)(20^\circ\text{C}) \\ &= 2000\ \text{cal} \end{aligned}$$

The heat necessary to melt all the ice is

$$\begin{aligned} q &= \rho V h_{sf} = \frac{\rho V H_{sf}}{\text{MW}} \\ &= \frac{\left(0.92\ \frac{\text{g}}{\text{cm}^3}\right)(50\ \text{cm}^3)\left(1440\ \frac{\text{cal}}{\text{mol}}\right)}{18\ \frac{\text{g}}{\text{mol}}} \\ &= 3680\ \text{cal} \end{aligned}$$

The amount of unmelted ice is

$$V = (50 \text{ cm}^3)\left(\frac{3680 \text{ cal} - 2000 \text{ cal}}{3680 \text{ cal}}\right)$$
$$= 22.83 \text{ cm}^3 \quad (23 \text{ cm}^3)$$

The answer is (C).

CHEMISTRY-53

What is most nearly the final temperature when 10 g of copper and 20 g of lead at −100°C are added to 50 g of H_2O at 50°C? Disregard spurious heat losses. The atomic weight of copper is 63.55 g/mol, and the specific heat of lead is 0.032 cal/g·°C (0.134 J/g·°C).

(A) 33°C (B) 38°C (C) 39°C (D) 45°C

The law of Dulong and Petit is

$$\begin{aligned}&(\text{atomic weight}) \quad [\text{in g/mol}]\\ &\quad \times (\text{specific heat}) \quad [\text{in cal/g·°C}]\\ &= \left(6.4 \ \frac{\text{cal}}{\text{mol·°C}}\right)\left(4.184 \ \frac{\text{J}}{\text{cal}}\right)\\ &= 26.8 \text{ J/mol·°C}\end{aligned}$$

Since there are no spurious heat losses, the heat loss by the water equals the heat gained by the copper and lead.

$$q = mc_p\Delta T$$

In the preceding equation, m is the mass (in grams), c_p is the specific heat capacity, and ΔT is the change in temperature.

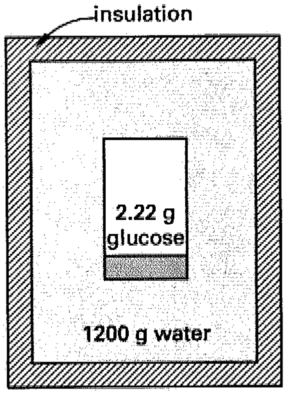
$$(50\text{ g})\left(1\ \frac{\text{cal}}{\text{g}\cdot{}^\circ\text{C}}\right)(50^\circ\text{C} - T_f) = \left((10\text{ g})\left(\frac{6.4\ \frac{\text{cal}}{\text{mol}\cdot{}^\circ\text{C}}}{63.55\ \frac{\text{g}}{\text{mol}}}\right) + (20\text{ g})\left(0.032\ \frac{\text{cal}}{\text{g}\cdot{}^\circ\text{C}}\right)\right) \times (T_f - (-100^\circ\text{C}))$$

$$T_f = 45.21^\circ\text{C} \quad (45^\circ\text{C})$$

The answer is (D).

CHEMISTRY–54

A bomb calorimeter is used to determine thermal properties. What is most nearly the enthalpy of reaction (in kcal/mol) of the combustion of glucose (molecular weight = 180 g/mol) when 2.22 g of glucose are ignited, and the water in the well-insulated calorimeter rises in temperature from 18.00°C to 23.19°C? Assume that the water absorbs all of the heat given off.



(A) 100 kcal/mol
(B) 320 kcal/mol
(C) 510 kcal/mol
(D) 730 kcal/mol

$$q = mc_p\Delta T$$

$$q_{H_2O} = (1200\text{ g})\left(1\ \frac{\text{cal}}{\text{g}\cdot{}^\circ\text{C}}\right)(23.19^\circ\text{C} - 18.00^\circ\text{C})$$

$$= 6228\text{ cal}$$

$$q_{\text{glucose}} = 6228\text{ cal}$$

$$n = \frac{m}{\text{MW}}$$

$$n_{\text{glucose}} = \frac{2.22\text{ g}}{180\ \frac{\text{g}}{\text{mol}}}$$

$$= 0.0123\text{ mol}$$

$$\text{molar enthalpy} = \frac{6228\text{ cal}}{0.0123\text{ mol}}$$

$$= 506.34\text{ kcal/mol} \quad (510\text{ kcal/mol}$$

The answer is (C).

CHEMISTRY-55

What is most nearly the standard heat of reaction, $\Delta\hat{H}^\circ$, per mole of C_6H_6 for the following reaction?

$$C_6H_6(g) \longrightarrow 3C_2H_2(g)$$

The enthalpy of reaction for C_2H_2 is 226 757 J/gmol; for C_6H_6, it is 82 923 J/gmol.

(A) −650 kJ (B) −600 kJ (C) 600 kJ (D) 650 kJ

$$n_{C_6H_6} = 1\text{ mol}$$

$$n_{C_2H_2} = 3\text{ mol}$$

$$\hat{H}_R^\circ = \sum_i n_i\hat{h}_i^0\Big|_{\text{reactants}}$$

$$= 82\,923\text{ J/mol} \quad (82.9\text{ kJ})$$

$$\hat{H}_P^\circ = \sum_i n_i\hat{h}_i^0\Big|_{\text{products}}$$

$$= (3\text{ mol})\left(226\,757\ \frac{\text{J}}{\text{mol}}\right)$$

$$= 680\,300\text{ J} \quad (680.3\text{ kJ})$$

$$\Delta\hat{H}^\circ = \hat{H}_P^0 - \hat{H}_R^0$$
$$= 680.3 \text{ kJ} - 82.9 \text{ kJ}$$
$$= 597.4 \text{ kJ} \quad (600 \text{ kJ})$$

The answer is (C).

CHEMISTRY–56

The heats of reaction for three equations are as follows.

$$\begin{array}{lllr} \text{I.} & -2C_2H_2 - 5O_2 + 4CO_2 + 2H_2O & & = -620\,000 \text{ cal} \\ \text{II.} & \quad\; - O_2 + CO_2 & - C & = -\;96\,960 \text{ cal} \\ \text{III.} & \quad\; - O_2 + 2H_2O - 2H_2O & - 2H_2 & = -136\,800 \text{ cal} \end{array}$$

What is the heat of formation of C_2H_2?

(A) 4.14 kcal/mol (B) 45.7 kcal/mol (C) 47.7 kcal/mol (D) 95.7 kcal/mol

Adding (−Eq. I) + 4(Eq. II) + (Eq. III) gives the formation of 2 mol of C_2H_2.

$$\begin{array}{llllr} 2C_2H_2+5O_2-4CO_2-2H_2O & & & = & 620\,000 \text{ cal} \\ \quad -4O_2+4CO_2 & & -4C & = & -387\,840 \text{ cal} \\ \quad -\;O_2 & +2H_2O & -2H_2 & = & -136\,800 \text{ cal} \end{array}$$

Therefore,

$$2C_2H_2 - 4C - 2H_2 = 95\,360 \text{ cal}$$

Because H_2 and C are at the standard reference state, $\hat{h}_C = 0$, and $\hat{h}_{H_2} = 0$. Therefore,

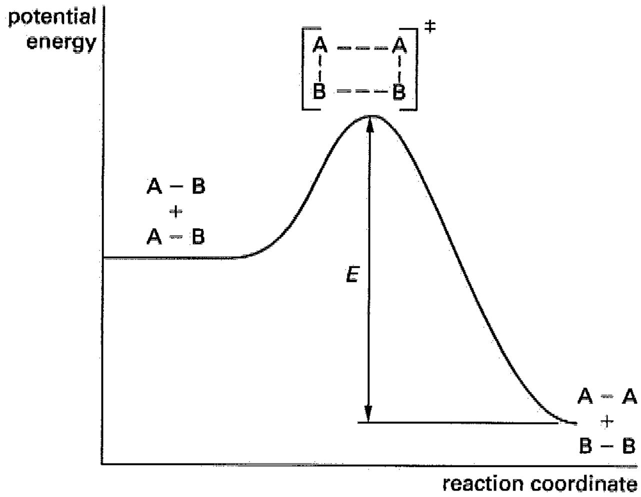
$$2\hat{h}_{C_2H_2} = 95\,360 \text{ cal}$$
$$\hat{h}_{C_2H_2} = 47\,680 \text{ cal/mol} \quad (47.7 \text{ kcal/mol})$$

The answer is (C).

CHEMISTRY–57

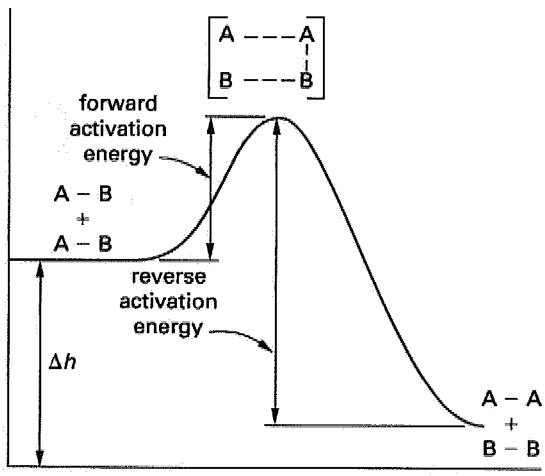
A chemical reaction involving the collision of two molecules of A and B goes through the following energy profile.

$$A_2 + B_2 \longrightarrow 2AB$$



The energy, E, shown on the diagram represents which of the following?

(A) entropy of reaction
(B) enthalpy of reaction
(C) forward activation energy
(D) reverse activation energy



E is the energy required for the reverse reaction (A–A + B–B $\longrightarrow$ A–B + A–B) to proceed. Thus, E is the reverse activation energy.

The answer is (D).

CHEMISTRY–58

Reactions generally proceed faster at higher temperatures because of which of the following?

(A) The molecules collide more frequently.
(B) The activation energy is less.
(C) The molecules are less energetic.
(D) Both options (A) and (B).

At higher temperatures, the molecules travel faster and, therefore, have a higher kinetic energy. This means that the molecules will collide more frequently and that the activation energy for a chemical reaction will be smaller.

The answer is (D).

CHEMISTRY–59

Which of the following statements is FALSE?

(A) In general, as reaction products are formed, they react with each other and re-form reactants.
(B) The net rate at which a reaction proceeds from left to right is equal to the forward rate minus the reverse rate.
(C) At equilibrium, the net reaction rate is zero.
(D) The differential rate law is the mathematical expression that shows how the rate of a reaction depends on volume.

The differential rate law is the mathematical expression that shows how the rate of a reaction depends on concentration, not volume.

The answer is (D).

CHEMISTRY–60

For the reaction $3A + 2B \longrightarrow C + D$, the differential rate law is

$$\left(\frac{1}{3}\right)\frac{dA}{dr} = \frac{dC}{dt} = k[A]^n[B]^m$$

Which of the following statements is FALSE?

(A) The order of the reaction with respect to A is called n.

(B) The sum of $n + m$ is called the overall order of the reaction.

(C) The exponents of [A] and [B], n and m, are not necessarily equal to the stoichiometric coefficients of A and B in the net reaction.

(D) The overall order for the reaction can be predicted by or deduced from the equation for the reaction.

The order for the reaction must be found experimentally and cannot be determined from the equation for the reaction.

The answer is (D).

CHEMISTRY–61

Which of the following statements is FALSE?

(A) When the temperature is raised, the rate of any reaction is always increased.

(B) In general, when any two compounds are unmixed, a large number of reactions may be possible, but those which proceed the fastest are the ones observed.

(C) It is possible to influence the products of a chemical change by controlling the factors which affect reaction rates.

(D) Heterogeneous reactions are the reactions that take place at the boundary surface between two faces.

When temperature is increased, the rates of most reactions increase. However, the rates of some reactions do decrease.

The answer is (A).

CHEMISTRY–62

The following rate expression was found to accurately represent the kinetics of a chemical reaction.

$$r = kC_A^2 C_B$$

If C represents concentration in units of mol/L, what are the units of the rate constant, k?

(A) unitless (B) s^{-1} (C) L/mol·s (D) $L^2/mol^2{\cdot}s$

The reaction rate always has units of mol/L·s. The units of k may be found as follows.

$$\frac{\text{mol}}{\text{L·s}} = k\left(\frac{\text{mol}}{\text{L}}\right)^2\left(\frac{\text{mol}}{\text{L}}\right)$$

$$k = \left(\frac{\text{mol}}{\text{L·s}}\right)\left(\frac{\text{L}}{\text{mol}}\right)^3$$

$$= \frac{\text{L}^2}{\text{mol}^2\text{·s}}$$

The answer is (D).

CHEMISTRY–63

Let C represent the concentration of a reagent. For a first-order reaction, what would a plot of $\ln C$ versus t yield?

(A) a straight line whose slope is $-k$
(B) a straight line whose slope is k
(C) a logarithmic curve approaching a value of k
(D) an exponential curve approaching a value of k

For a first-order reaction,

$$-\frac{dC}{dt} = kC$$

In the preceding equation, k is the rate constant and c is the concentration.

$$-\frac{dC}{C} = kdt$$

$$-\int_{C_0}^{C} \frac{dC}{C} = k\int_0^t dt$$

$$-\ln\frac{C}{C_0} = kt$$

$$\ln\frac{C}{C_0} = -kt$$

$$\ln C = -kt + \ln C_0$$

This is of the form $y = ax + y_0$. Therefore, the graph is a straight line with a slope of $-k$.

The answer is (A).

CHEMISTRY–64

The following kinetic data were collected for a specific chemical reaction. What is the rate constant for the reaction?

reaction: $A + B \longrightarrow$

experiment	C_A (mol/L)	C_B (mol/L)	initial rate (mol A/L·s)
1	0.10	0.10	0.0010
2	0.20	0.10	0.0020
3	0.30	0.10	0.0030
4	0.10	0.20	0.0010
5	0.10	0.30	0.0010

(A) 0.01 s^{-1}
(B) 0.02 s^{-1}
(C) 0.02 L/mol·s
(D) 0.03 L/mol·s

First, determine the rate law. Experiments 4 and 5 show that the rate is not a function of c_B. Experiments 1, 2, and 3 show that the rate is directly proportional to c_A. Therefore,

$$r = kC_A$$

$$k = \frac{r}{C_A}$$

Use the data from experiment 1 to determine k.

$$k = \frac{0.0010\ \frac{\text{mol}}{\text{L·s}}}{0.10\ \frac{\text{mol}}{\text{L}}}$$

$$= 0.01\ \text{s}^{-1}$$

The answer is (A).

CHEMISTRY–65

A certain temperature-dependent reaction proceeds 10 times faster at 500K than it does at 300K. Approximately how much faster will it react at 1000K than it does at 300K?

(A) 10 (B) 20 (C) 30 (D) 60

$$r = kf(C_i)$$

$$k = A\left(\exp\left(-\frac{E_A}{R^*T}\right)\right)$$

In the preceding equation, E_A is the activation energy.

Therefore,

$$r \propto \exp\left(-\frac{E_A}{R^*T}\right)$$

$$\frac{r_1}{r_2} = \frac{\exp\left(\frac{-E_A}{R^*T_2}\right)}{\exp\left(\frac{-E_A}{R^*T_1}\right)}$$

$$= \exp\left(\frac{E_A}{R^*}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right)$$

$$\ln 10 = \frac{E_A}{R^*}\left(\frac{1}{300\text{K}} - \frac{1}{500\text{K}}\right)$$

$$\frac{E_A}{R^*} = 1726.94$$

$$\frac{r_{1000}}{r_{300}} = \exp\left((1727)\left(\frac{1}{300\text{K}} - \frac{1}{1000\text{K}}\right)\right)$$

$$= 56 \quad (60)$$

The answer is (D).

CHEMISTRY–66

A reaction rate is observed to triple as the result of raising the temperature from 0°C to 20°C. What is most nearly the activation energy of the reaction? (R^* is the universal gas constant.)

(A) $3900R^*$ (B) $4400R^*$ (C) $4700R^*$ (D) $5100R^*$

$$r = C\exp\left(-\frac{E_A}{R^*T}\right)$$

In the preceding equation, C is a constant, and E_A is the activation energy.

$$\frac{r_1}{r_2} = \left(\exp\left(\frac{E_A}{R^*}\right)\right)\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\frac{1}{3} = \left(\exp\left(\frac{E_A}{R^*}\right)\right)\left(\frac{1}{0°\text{C} + 273°} - \frac{1}{20°\text{C} + 273°}\right)$$

$$\ln\frac{1}{3} = -\frac{E_A}{R^*}\left(\frac{1}{273\text{K}} - \frac{1}{293\text{K}}\right)$$

$$E_A = -\frac{R^*\ln\frac{1}{3}}{\frac{1}{273\text{K}} - \frac{1}{293\text{K}}}$$

$$= 4394R^* \quad (4400R^*)$$

The answer is (B).

CHEMISTRY–67

In the troposphere, ozone is produced during the day and consumed during the night. Determine the half-life of ozone if it is depleted to 10% of its initial value after 10 h of darkness.

(A) 3.0 h (B) 3.5 h (C) 4.0 h (D) 4.5 h

$$C = C_0e^{-kt}$$

$$C_{10} = 0.1C_0$$

$$0.1 = e^{-10k}$$

$$-k = \frac{\ln 0.1}{10}$$

$$= -0.2303\ \text{h}^{-1}$$

In the preceding equation, $t_{1/2}$ is the half-life.

$$C_{t_{1/2}} = 0.5C_0$$
$$0.5 = e^{-0.2303t_{1/2}}$$
$$t_{1/2} = \frac{\ln 0.5}{-0.2303\ \text{h}^{-1}}$$
$$= 3.0\ \text{h}$$

The answer is (A).

CHEMISTRY–68

Which of the following statements is FALSE?

(A) In considering chemical equilibrium, the relative stabilities of products and reactants are important.

(B) In considering chemical equilibrium, the pathway from the initial state to the final state is important.

(C) In treating reaction rates, the rate at which reactants are converted to products is important.

(D) In treating reaction rates, the sequence of physical processes by which reactants are converted to products is important.

Considerations of chemical equilibrium do not take into account the pathway from initial to final states.

The answer is (B).

CHEMISTRY–69

Consider the following reaction at equilibrium.

$$3H_2\ (gas) + N_2\ (gas) \rightleftharpoons 2NH_3\ (gas) \qquad \Delta\hat{H} = -11.0\ \text{kcal/mol}$$

Which single change in conditions will cause a shift in equilibrium toward an increase in production of NH_3?

(A) removal of hydrogen gas
(B) increase in temperature
(C) increase in volume of the system
(D) increase in pressure on the system

According to Le Châtelier's principle, the effects of each change in condition are as follows.

removal of hydrogen	shifts equilibrium to the reactants
increase in temperature	shifts equilibruim to the reactants
increase in volume	shifts equilibrium to the reactants
increase in pressure	shifts equilibrium to the products

The answer is (D).

CHEMISTRY–70

Consider the following reaction at equilibrium.

$$\tfrac{1}{2}N_2 + \tfrac{3}{2}H_2 \rightleftharpoons NH_3 \qquad \Delta\hat{H} = -11.0 \text{ kcal/mol}$$

What would be the expected effect on the amount of NH_3 produced under each of the following conditions?

I. raise the temperature
II. compress the mixture
III. add additional H_2

(A) I: decrease, II: increase, III: increase
(B) I: increase, II: increase, III: decrease
(C) I: increase, II: decrease, III: decrease
(D) I: decrease, II: increase, III: decrease

According to Le Châtelier's principle, each change has the following effects.

I. raise the temperature: shifts equilibrium to the reactants0. because the reaction is exothermic
II. compress the mixture: shifts equilibrium to the products because products contain a smaller number of moles
III. add hydrogen gas: shifts equilibrium to the products because adding addtional reactants will force the formation of more products

The answer is (A).

CHEMISTRY–71

Consider the following reaction.

$$MgSO_4\ (solid) \rightleftharpoons MgO\ (solid) + SO_3\ (gas)$$

What is the equilibrium constant for the given reaction?

(A) $k = \frac{[Mg][SO_3]}{2[MgSO_4]}$ (B) $k = \frac{[MgSO_4]}{[MgO][SO_3]}$
(C) $k = [MgO][SO_3]$ (D) $k = [SO_3]$

Solids have a concentration of 1. Therefore, $k = [SO_3]$.

The answer is (D).

CHEMISTRY–72

The solubility of barium sulfate, $BaSO_4$, is 0.0091 g/L at 25°C. The molecular weight of barium sulfate is 233 g/mol. What is the value of the solubility product constant k_{sp} for $BaSO_4$?

(A) 1.52×10^{-9} mol²/L²
(B) 4.24×10^{-8} mol²/L²
(C) 8.63×10^{-7} mol²/L²
(D) 2.98×10^{-6} mol²/L²

$$BaSO_4(s) \rightleftharpoons Ba^{2+}(aq) + SO_4^{2-}(aq)$$

$$k_{sp} = [Ba^{2+}][SO_4^{2-}]$$

$$[BaSO_4] = \left(0.0091\ \frac{g}{L}\right)\left(\frac{1\ mol}{233\ g}\right)$$

$$= 3.9 \times 10^{-5}\ mol/L$$

$$[Ba^{2+}] = [SO_4^{2-}] = [BaSO_4]$$

$$k_{sp} = \left(3.9 \times 10^{-5}\ \frac{mol}{L}\right)^2$$

$$= 1.52 \times 10^{-9}\ mol^2/L^2$$

The answer is (A).

CHEMISTRY–73

Consider the reaction shown.

$$H_2\ (gas) + I_2\ (gas) \rightleftharpoons 2HI\ (gas)$$

With $k_{eq} = 25$, determine the number of moles of H_2 remaining when 1 mol each of H_2 and I_2 reach equilibrium in a 1 L vessel.

(A) 1/6 mol (B) 2/7 mol (C) 5/7 mol (D) 5/6 mol

	$H_2\ (gas)$	+	$I_2\ (gas)$	$\rightleftharpoons$	$2HI\ (gas)$
initial moles	1		1		0
final moles	$1 - x$		$1 - x$		$2x$

$$k = \frac{\left[HI\ \frac{mol}{L}\right]^2}{\left[H_2\ \frac{mol}{L}\right]\left[I_2\ \frac{mol}{L}\right]}$$

$$= \frac{(2x)^2}{(1-x)(1-x)}$$

$$= 25$$

$$\frac{4x^2}{(1-x)^2} = 25$$

$$4x^2 = (25)(1 - 2x + x^2)$$

$$21x^2 - 50x + 25 = 0$$

$$x = \frac{+50 \pm \sqrt{(50)^2 - (4)(25)(21)}}{(2)(21)}$$

$$= 5/3 \text{ mol or } 5/7 \text{ mol}$$

Only the second value for x makes sense, because the first value is greater than the initial amounts of H_2 and I_2. Thus, the remaining number of moles of H_2 at equilibrium is $1 - {}^5/_7 = {}^2/_7$ mol.

The answer is (B).

CHEMISTRY–74

Consider the reaction.

$$A + B \rightleftharpoons 2C$$

With $k_{eq} = 50$, what is most nearly the final concentration of C when 1 mol of both A and B are added to a 1 L container containing 0.1 mol of C?

(A) 0.77 mol (B) 0.95 mol (C) 1.5 mol (D) 1.6 mol

	A (mol)	+	B (mol)	$\rightleftharpoons$	2C (mol)
initial moles	1		1		0.1
final moles	$1 - x$		$1 - x$		$2x + 0.1$

$$k_{\text{eq}} = \frac{\left[C\ \frac{\text{mol}}{\text{L}}\right]^2}{\left[A\ \frac{\text{mol}}{\text{L}}\right]\left[B\ \frac{\text{mol}}{\text{L}}\right]}$$

$$= \frac{(2x + 0.1\ \text{mol})^2}{(1\ \text{mol} - x)(1\ \text{mol} - x)} = 50$$

$$4x^2 + 0.4x + 0.01 = 50x^2 - 100x + 50$$

$$46x^2 - 100.4x + 49.99 = 0$$

$$x = 0.7685\ \text{mol or } 1.4141\ \text{mol}$$

However, x cannot be 1.4141 mol because this is greater than the initial amounts of A and B, and this value would make $1-x$ negative. Therefore, $x = 0.7685$ mol. The final number of moles of C is $2x = (2)(0.7685\ \text{mol}) = 1.537$ mol.

$$2x + 0.1 = (2)(0.7685\ \text{mol}) + 0.1 = 1.637\ \text{mol} \quad (1.6\ \text{mol})$$

The answer is (D).

CHEMISTRY–75

The voltage of a galvanic cell does NOT depend on which of the following parameters?

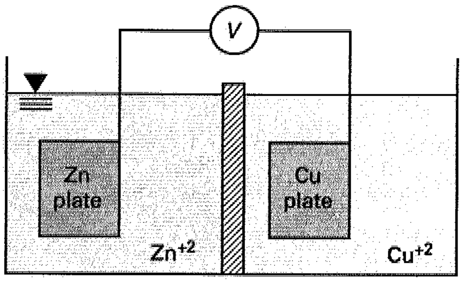
(A) concentration of solutions
(B) temperature of solutions
(C) pressure of solutions
(D) volume of solutions

The cell potential is dependent on all of the above except volume.

The answer is (D).

CHEMISTRY–76

Given the electrochemical cell shown, what is the reaction at the anode?



(A) $Cu \longrightarrow Cu^{2+} + 2e^-$
(B) $Cu^{2+} + 2e^- \longrightarrow Cu$
(C) $Zn \longrightarrow Zn^{2+} + 2e^-$
(D) $Zn^{2+} + 2e^- \longrightarrow Zn$

Zinc has a higher potential and will, therefore, act as the anode. By definition, the anode is where electrons are lost. Thus, the reaction at the anode of the electrochemical cell is $Zn \longrightarrow Zn^{2+} + 2e^-$.

The answer is (C).

CHEMISTRY–77

Which of the following statements regarding a galvanic cell is FALSE?

(A) A negative value of cell voltage $\Delta\mathcal{E}$ means that the reaction in the cell proceeds spontaneously from right to left.

(B) If the standard potential of a cell is zero, a concentration difference alone is sufficient to generate a voltage.

(C) When a current I flows through the voltage difference $\Delta\mathcal{E}$ for a time, t, the electrical work performed is $(\Delta\mathcal{E})It$.

(D) The cell voltage, $\Delta\mathcal{E}$, is totally independent of the number of electrons transferred in a given reaction.

For the reaction $a\text{A} + b\text{B} \longrightarrow c\text{C} + d\text{D}$, the Nernst equation states

$$\Delta\mathcal{E} = \Delta\mathcal{E}^0 - \frac{0.059}{n}\log\frac{[\text{C}]^c[\text{D}]^d}{[\text{A}]^a[\text{B}]^b}$$

Here, n is the number of moles of electrons transferred in the reaction. Therefore, the cell voltage does depend on the number of electrons transferred in a given reaction.

The answer is (D).

CHEMISTRY–78

Consider the Nernst equation.

$$\Delta\mathcal{E} = \Delta\mathcal{E}^0 - \frac{0.059}{n}\log\frac{[\text{C}]^c[\text{D}]^d}{[\text{A}]^a[\text{B}]^b}$$

Which of the following statements is FALSE?

(A) n is the number of moles of electrons transferred in the reaction.

(B) The cell must be operating at a temperature of 25°C.

(C) The equation holds for the reaction $a\text{A} + b\text{B} \longrightarrow c\text{C} + d\text{D}$.

(D) The factor of 0.059 is common to all cells, regardless of temperature.

The factor of 0.059 applies only to cells with an operating temperature of 25°C.

The answer is (D).

CHEMISTRY–79

A zinc-copper standard cell with $\Delta\mathcal{E}^0 = 1.10$ V is connected to an independent variable voltage supply such that the variable voltage opposes the cell voltage. Given the following reaction, what happens?

$$Zn + Cu^{2+} \rightleftharpoons Cu + Zn^{2+} \qquad \Delta\mathcal{E}^0 = 1.10 \text{ V}$$

(A) When the variable voltage is below 1.10 V, the cell reaction $Cu + Zn^{2+} \longrightarrow Cu^{2+} + Zn$ predominates.

(B) When the variable voltage is above 1.10 V, the cell reaction $Cu + Zn^{2+} \longrightarrow Cu^{2+} + Zn$ predominates.

(C) When the variable voltage is above 1.10 V, the cell reaction $Zn + Cu^{2+} \longrightarrow Cu + Zn^{2+}$ predominates.

(D) When the variable voltage is equal to 1.10 V, the cell reaction $Cu + Zn^{2+} \longrightarrow Cu^{2+} + Zn$ predominates.

When the variable voltage is below 1.10 V, the reaction $Zn + Cu^{2+} \longrightarrow Cu + Zn^{2+}$ predominates. When it is equal to 1.10 V, no net reaction occurs. When the variable voltage is above 1.10 V, the reverse reaction, $Cu + Zn^{2+} \longrightarrow Zn + Cu^{2+}$ predominates.

The answer is (B).

CHEMISTRY–80

Given that $\Delta\mathcal{E}^0 = 0.03$ V, [Ni] = 1 M, [Co] = 0.1 M, and $T = 25°C$, calculate the cell voltage for the following equation.

$$Co + Ni^{2+} \longrightarrow Co^{2+} + Ni$$

(A) 0.01 V (B) 0.03 V (C) 0.06 V (D) 0.09 V

Use the Nernst equation.

$$\Delta\mathcal{E} = \Delta\mathcal{E}^0 - \frac{0.059}{n} \log \frac{[Co^{2+}]}{[Ni^{2+}]}$$

$$n = 2$$

$$\begin{aligned}\Delta\mathcal{E} &= 0.03 \text{ V} - \frac{0.059 \text{ V}}{2} \log \frac{0.1}{1.0} \\ &= 0.03 \text{ V} + 0.03 \text{ V} \\ &= 0.06 \text{ V}\end{aligned}$$

The answer is (C).

CHEMISTRY–81

In organic chemistry, which compound families are associated with the following bonds?

I. C – C
II. C = C
III. C ≡ C

(A) I: alkene, II: alkyne, III: alkane
(B) I: alkyne, II: alkane, III: alkene
(C) I: alkane, II: alkene, III: alkyne
(D) I: alkane, II: alkyne, III: alkene

An alkane is a saturated organic compound. Thus, the carbons may only have single bonds. In an alkene, the carbon atoms may have double bonds. In alkynes, the carbon atoms may have triple bonds.

The answer is (C).

CHEMISTRY–82

Which one of the following statements regarding organic substances is FALSE?

(A) Organic matter is generally stable at very high temperatures.
(B) All organic matter contains carbon.
(C) Organic substances generally do not dissolve in water.
(D) Organic substances generally dissolve in high-concentration acids.

Organic matter contains carbon, is generally insoluble in water, soluble in high-concentration acids, not easily ionizable, and unstable at high temperatures.

The answer is (A).

CHEMISTRY–83

Which one of the following is most likely to prove that a substance is inorganic?

(A) The substance is heated together with copper oxide and the resulting gases are found to have no effect on limestone.

(B) The substance evaporates in room temperature and pressure.

(C) Analysis shows that the substance contains hydrogen.

(D) The substance floats in water.

The carbon from organic matter generally reacts with copper oxide to produce carbon dixoide. This gas darkens limestone.

The answer is (A).

CHEMISTRY–84

Which of the following organic chemicals is most soluble in water?

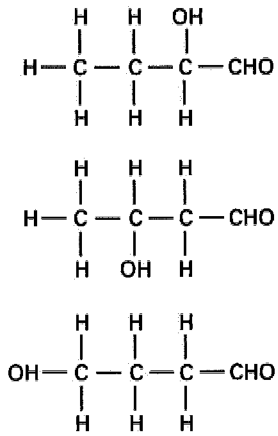
(A) CH_3CH_3
(B) CH_3OH
(C) CCl_4
(D) $CH_3\text{-}(CH_2)_n\text{-}CH_3$

Water is a polar molecule. Thus, a polar substance is more likely to dissolve in water than a nonpolar substance. Methanol (CH_3OH) is polar and, therefore, very miscible in water. All of the other molecules are nonpolar.

The answer is (B).

CHEMISTRY–85

Which statement describes all of the following three chemical structural formulas?



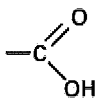
(A) They are isotopes of a certain substance.
(B) They are the only possible forms of $C_4H_2O_2$.
(C) They are incorrectly written.
(D) They are isomers.

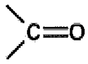
When a compound has one chemical formula, but different possible physical structures, the different structures are called isomers. The three formulas are all possible structures of $C_4H_8O_2$. Therefore, they are isomers.

The answer is (D).

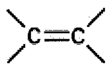
CHEMISTRY–86

What structures do both aldehydes and ketones contain?

(A) the carboxyl group 

(B) the carbonyl group 

(C) the hydroxyl group —OH

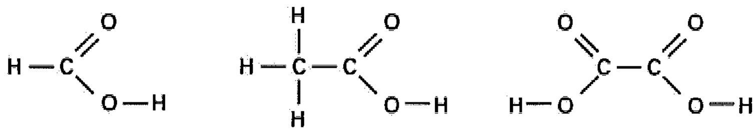
(D) the carbon-carbon double bond 

Aldehydes and ketones both contain the carbonyl group.

The answer is (B).

CHEMISTRY–87

Identify the following acid structures.



I. II. III.

(A) I: formic acid, II: oxalic acid, III: acetic acid
(B) I: oxalic acid, II: acetic acid, III: formic acid
(C) I: acetic acid, II: formic acid, III: oxalic acid
(D) I: formic acid, II: acetic acid, III: oxalic acid

HCOOH is formic acid. CH_3COOH is acetic acid. $C_2H_2O_4$ is oxalic acid.

The answer is (D).

CHEMISTRY–88

According to the Bohr model of the hydrogen atom, which of the following statements are true?

I. As the electron orbits the proton, it constantly radiates light with a frequency equal to its frequency of revolution.
II. The electron orbits the proton in certain orbits that can be found by assuming that its angular momentum is quantized.
III. Because of the quantization of angular momentum, calculations using the Bohr model and those based on classical physics can never give the same results.
IV. When an electron orbiting a proton changes states to a lower energy level, the frequency of the radiation given off is proportional to the change in energy. This accounts for the hydrogen spectrum.

(A) I and II (B) II and IV (C) I, II, and III (D) II, III, and IV

The Bohr postulates are

1. The electron moves in a certain orbit without radiating.
2. The frequency of the emitted photon is proportional to the change in energy of the electron.
3. The correspondence principle states that, in the limit as energies and orbits become large, quantum calculations must agree with classical calculations.

Thus, only II and IV are true.

> The answer is (B).

CHEMISTRY–89

In a three-level laser, electrons in atoms are excited into an energy state, E_3, then decay spontaneously to an energy E_2, which is a metastable state. The atoms are struck by photons of a specific frequency, and make stimulated emissions to the ground state, E_1. If the photons cause all of the emissions to be of the same frequency, the light will be amplified. What frequency must the photons be for this to occur?

(A) $f = \dfrac{E_2 - E_1}{\hbar}$

(B) $f = \dfrac{E_3 - E_1}{\hbar}$

(C) $f = \dfrac{E_3 - E_1}{\hbar} + \dfrac{E_2 - E_1}{\hbar}$

(D) $f = \dfrac{E_3 - E_1}{\hbar} - \dfrac{E_2 - E_1}{\hbar}$

The transition that must be amplified is the E_2 to E_1 transition. Thus, the frequency of the radiated light is $(E_2 - E_1)/\hbar$. Photons of this frequency will be more likely to cause this transition. Therefore, if photons of this frequency are used, more transitions will take place, and the light will be amplified.

> The answer is (A).

CHEMISTRY–90

In an atom such as sodium, there is one electron in the outermost shell (in this case, $n = 3$). Which of the following statements is true regarding the energy required to excite an electron in the $n = 1$ shell compared to that required to excite an electron in the $n = 2$ shell?

(A) It is greater because the electron is closer to the proton, and thus the Coulomb attractive force is much stronger.

(B) It is greater because the shell next to it is full. Thus, by the Pauli exclusion principle, it must jump to the first shell that is not full, in this case, the $n = 3$ shell.

(C) It is greater because an electron must first jump to the $n = 3$ shell from the $n = 2$ shell. Then the electron from the $n = 1$ shell can jump to the $n = 2$ shell.

(D) It is equal to the energy required to excite an electron in the $n = 2$ shell because in both cases the electron makes a jump to the next shell.

In an atom, an excited electron will jump to the next highest unfilled shell (in this case, the $n = 3$ shell). So the electrons in both the $n = 1$ shell and the $n = 2$ shell will jump to the $n = 3$ shell. However, the energy difference between the $n = 1$ and the $n = 3$ shell is greater than the energy difference between the $n = 2$ shell and the $n = 3$ shell. Thus, option (B) is correct.

The answer is (B).

CHEMISTRY–91

A state of energy E_1 with a lifetime of t_1 decays into the state of energy E_2. The state of E_2 then decays with a lifetime of t_2 into the state of E_3. The decay constants are related by $\tau_1 = 2\tau_2$. Initially, all of the atoms (quantity N_0) are in the E_1 state. Calculate the number of atoms N_2 that are in the state E_2 at any instant in time, t.

(A) $N_0\left(e^{-t/\tau_1} - e^{-2t/\tau_1}\right)$

(B) $N_0\left(e^{-t/\tau_1} + e^{-2t/\tau_1}\right)$

(C) $2N_0\left(e^{-t/\tau_1} - e^{-2t/\tau_1}\right)$

(D) $2N_0\left(e^{-t/\tau_1} + e^{-2t/\tau_1}\right)$

The number of atoms that decay from E_1 to E_2 is

$$N_1 = N_0 e^{-t/\tau_1}$$

The number of atoms in state E_2 equals the number of atoms coming from E_1 to E_2 minus the number of atoms decaying from E_2 to E_3. Thus, the number of atoms in state E_2 at a given time is

$$\begin{aligned} N_2 &= N_0 e^{-t/\tau_1} - N_{2,0} e^{-t/\tau_2} \\ &= N_0 e^{-t/\tau_1} - N_{2,0} e^{-2t/\tau_1} \end{aligned}$$

Since there are no atoms in state E_2 at $t = 0$, the initial conditions are

$$\begin{aligned} N_2(0) &= 0 \\ 0 &= N_0 e^0 - N_{2,\,0} e^0 \\ N_0 &= N_{2,\,0} \\ N_2 &= N_0 \left(e^{-t/\tau_1} - e^{-2t/\tau_1} \right) \end{aligned}$$

The answer is (A).

CHEMISTRY–92

A source of radiation has a mean nucleus life of $\tau = 35.8$ s. There are initially $N_0 = 5.37 \times 10^{10}$ nuclei in the source. Which of the following statements are true?

I. The decay constant is $\lambda = 0.0279\ \text{s}^{-1}$.
II. The half-life is $t_{1/2} = 24.8$ s.
III. If a rate counter with an 80% efficiency is placed near the source, it will show a rate of 4.2×10^7 after 2 min.
IV. The sample will essentially have all decayed (0.01% remaining) in 5.5 min.

(A) I and III (B) II and III (C) I, III, and IV (D) I, II, III, and IV

First, find the decay constant.

$$\begin{aligned} \lambda &= \frac{1}{\tau} \\ &= \frac{1}{35.8\ \text{s}} \\ &= 0.0279\ \text{s}^{-1} \end{aligned}$$

Therefore, statement I is true.

Next, find the half-life.

$$\frac{1}{2} = e^{t_{1/2}/\tau}$$

$$t_{1/2} = \tau \ln 2$$

$$= (35.8 \text{ s}) \ln 2$$

$$= 24.8 \text{ s}$$

Thus, statement II is true.

Next, find the count rate after 2 min. The count rate at time t (in seconds) is

$$R = R_0 \text{e}^{-\lambda t}$$

The initial count rate, R_0, is

$$R_0 = \lambda N_0$$

Since the detector is only 80% efficient, the rate shown by the detector after 2 min, R_d, is

$$R_d = 0.80R = 0.8\ \lambda N_0 e^{-\lambda t}$$

$$= (0.8)\left(0.0279\ \frac{1}{\text{s}}\right)(5.37 \times 10^{10} e^{-\lambda t})$$

$$= (0.8)\left(1.50 \times 10^9\ \frac{1}{\text{s}}\right) e^{-(0.0279\ 1/\text{s})(120\ \text{s})}$$

$$= 4.2 \times 10^7 \text{ decays s}^{-1}$$

Therefore, statement III is true.

The time it takes for the sample to decay to an amount N is

$$t = \frac{-1}{\lambda} \ln \frac{N}{N_0}$$

$$N = 1 \times 10^{-4} N_0$$

$$\frac{N}{N_0} = 1 \times 10^{-4}$$

$$t = \frac{\left(\dfrac{-1}{0.0279\ \dfrac{1}{\text{s}}}\right) \ln(1 \times 10^{-4})}{60\ \dfrac{\text{s}}{\text{min}}}$$

$$= \frac{330 \text{ s}}{60\ \dfrac{\text{s}}{\text{min}}}$$

$$= 5.5 \text{ min}$$

Thus, IV is also true. Statements I, II, III, and IV are all true.

The answer is (D).

CHEMISTRY–93

A fossil fern containing 49.9 g of carbon is carbon dated to determine its age. The decay rate of C^{14} in the fossil is 191 decays/min. How old is the fern? (The half-life of C^{14} is 5730 years, and the rate of decay of C^{14} in a living organism per g of carbon is 15.0 decays/min-g.)

(A) 7290 yr (B) 11 300 yr (C) 14 100 yr (D) 23 800 yr

The rate of decay of C^{14} in a dead organism is given by

$$R = R_0 e^{-\lambda t}$$

In the preceding equation, λ is the decay constant, R_0 is the initial decay rate, and t is the time elapsed.

R_0 is simply the decay rate of carbon-14 in a living organism, because up to the point it dies, it replenishes its carbon. Thus, until the organism's death, the decay rate is fairly constant.

$$R_0 = \left(15.0\ \frac{\text{decays}}{\text{min-g}}\right)(49.9\ \text{g})$$

$$= 749\ \text{decays/min}$$

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

$$= \frac{\ln 2}{5730\ \text{yr}}$$

$$= 1.21 \times 10^{-4}\ \text{yr}^{-1}$$

$$t = -\frac{1}{\lambda} \ln \frac{R}{R_0}$$

$$= -\left(\frac{1}{1.21 \times 10^{-4}\ \text{yr}}\right) \ln \left(\frac{191\ \frac{\text{decays}}{\text{min}}}{749\ \frac{\text{decays}}{\text{min}}}\right)$$

$$= 11\,300\ \text{yr}$$

The answer is (B).

CHEMISTRY–94

What is the total relativistic energy of a particle if its mass is equal to 1 kg when it is traveling at a speed of $\sqrt{2}c$?

(A) $c/2$ (B) $c^2 - 1$ (C) c^2 (D) $c^2 + 1$

Regardless of the particle's speed, the total relativistic energy is

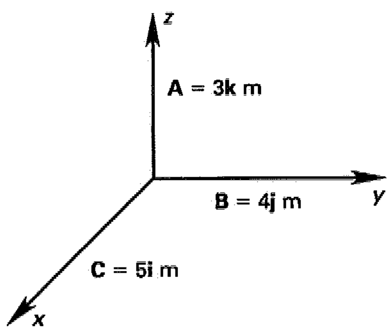
$$\begin{aligned} E_{\text{total}} &= mc^2 \\ &= (1 \text{ kg})(c^2) \\ &= c^2 \end{aligned}$$

The answer is (C).

8 STATICS

STATICS–1

What is the length of the vector $\mathbf{A}+\mathbf{B}+\mathbf{C}$, the sum of three orthogonal vectors?



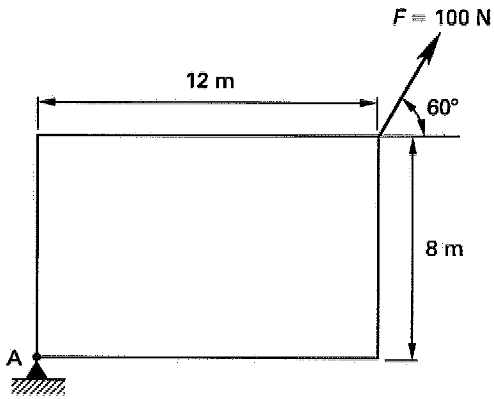
(A) 3.5 m (B) 4.3 m (C) 7.1 m (D) 10 m

$$|\mathbf{A}+\mathbf{B}+\mathbf{C}| = \sqrt{A^2+B^2+C^2}$$
$$= \sqrt{(3 \text{ m})^2+(4 \text{ m})^2+(5 \text{ m})^2}$$
$$= 7.07 \text{ m} \quad (7.1 \text{ m})$$

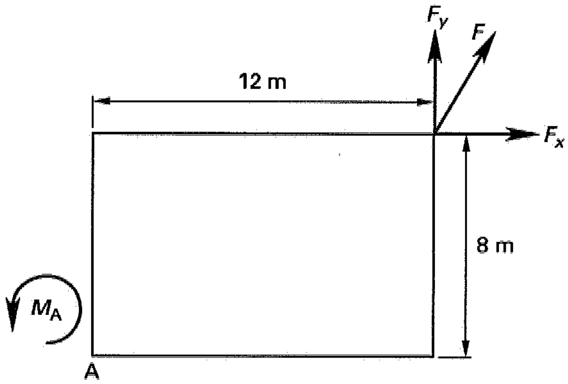
The answer is (C).

STATICS–2

Determine the magnitude of the moment of the force F about the corner A.



(A) 120 N·m (B) 240 N·m (C) 320 N·m (D) 640 N·m



$$F_x = (100 \text{ N}) \cos 60° = 50.0 \text{ N}$$

$$F_y = (100 \text{ N}) \sin 60° = 86.6 \text{ N}$$

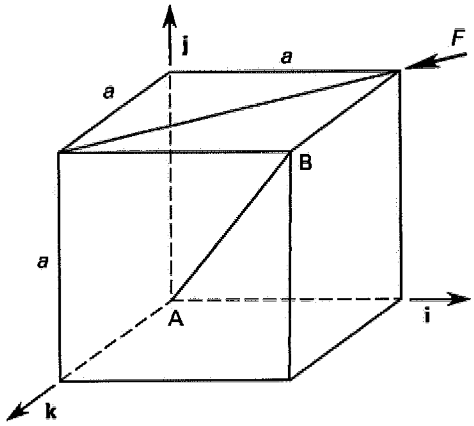
Taking counterclockwise moments as positive,

$$\begin{aligned} \sum M_{\text{A}} &= -yF_x + xF_y \\ &= -(8 \text{ m})(50.0 \text{ N}) + (12 \text{ m})(86.6 \text{ N}) \\ &= 640 \text{ N·m} \end{aligned}$$

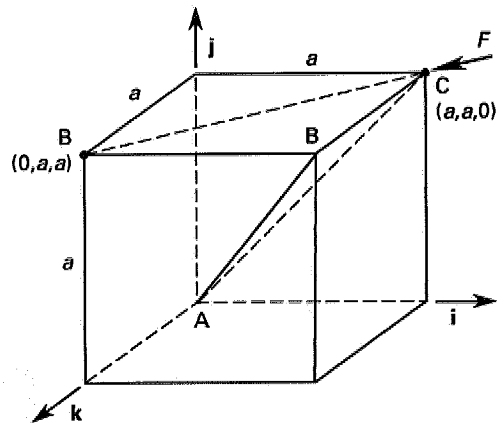
The answer is (D).

STATICS–3

A cube of side length a is acted upon by a force F as shown. Determine the magnitude of the moment of F about the diagonal AB.



(A) $\frac{aF}{\sqrt{8}}$ (B) $\frac{aF}{\sqrt{6}}$ (C) $\frac{aF}{\sqrt{4}}$ (D) $\frac{aF}{\sqrt{3}}$



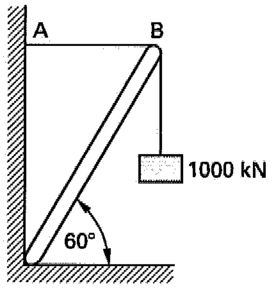
$$
\begin{aligned}
M_A &= \mathbf{r}_{AC} \times \mathbf{F} \\
&= a(\mathbf{i}+\mathbf{j}) \times \frac{F}{\sqrt{2}}(-\mathbf{i}+\mathbf{k}) \\
&= \frac{aF}{\sqrt{2}}(\mathbf{i}-\mathbf{j}+\mathbf{k})
\end{aligned}
$$

$$U_{AB} = \frac{1}{\sqrt{3}}(\mathbf{i}+\mathbf{j}+\mathbf{k})$$
$$M_{AB} = U_{AB} \cdot M_A$$
$$= \left(\frac{1}{\sqrt{3}}(\mathbf{i}+\mathbf{j}+\mathbf{k})\right) \cdot \left(\frac{aF}{\sqrt{2}}(\mathbf{i}+\mathbf{j}+\mathbf{k})\right)$$
$$= \frac{aF}{\sqrt{6}}$$

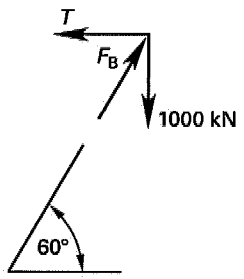
The answer is (B).

STATICS–4

The boom shown has negligible weight, but it has sufficient strength to support the 1000 kN load without buckling. What is most nearly the tension in the supporting cable between points A and B?



(A) 200 kN (B) 430 kN (C) 580 kN (D) 870 kN



For equilibrium, $\sum F_x = 0$ and $\sum F_y = 0$.

$$\sum F_y = 0 = F_\text{B} \sin 60° - 1000 \text{ kN} = 0$$

$$F_\text{B} = 1154.7 \text{ kN}$$

$$\sum F_x = 0$$

$$F_\text{B} \cos 60° - T = 0$$

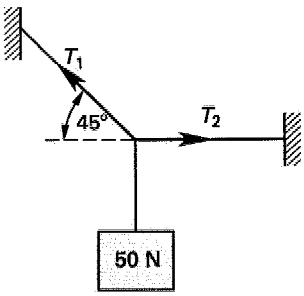
The tension in the cable, T, is

$$\begin{aligned} T &= F_\text{B} \cos 60° \\ &= (1154.7 \text{ kN}) \cos 60° \\ &= 577.4 \text{ kN} \quad (580 \text{ kN}) \end{aligned}$$

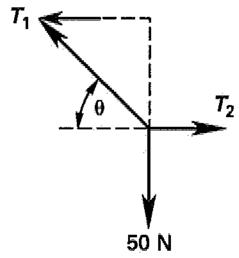
The answer is (C).

STATICS–5

Find the tensions, T_1 and T_2, in the ropes shown so that the system is in equilibrium.



(A) $T_1 = 50.0$ N, $T_2 = 0.0$ N
(B) $T_1 = 50.0$ N, $T_2 = 50.0$ N
(C) $T_1 = 70.7$ N, $T_2 = 50.0$ N
(D) $T_1 = 70.7$ N, $T_2 = 70.7$ N



$$\sum F_y = 0 = T_1 \sin 45° - 50 \text{ N} = 0$$

$$T_1 \sin 45° = 50 \text{ N}$$

$$T_1 = 70.7 \text{ N}$$

$$\sum F_x = 0$$

$$T_1 \cos 45° - T_2 = 0$$

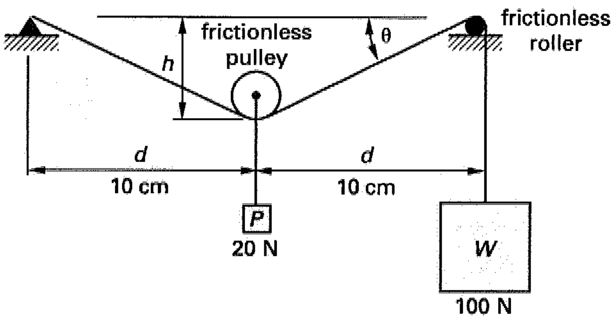
$$T_2 = T_1 \cos 45°$$

$$= 50 \text{ N}$$

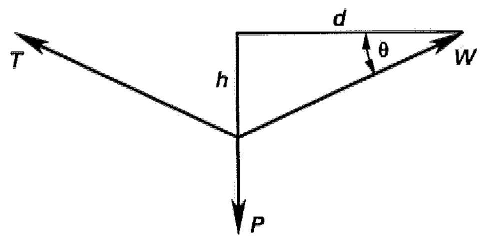
The answer is (C).

STATICS–6

For the system illustrated, determine the value of h that puts the system in equilibrium.



(A) 0.50 cm (B) 1.0 cm (C) 1.5 cm (D) 2.1 cm



$$\sum F_y = 0$$

$$0 = -P + W\sin\theta + T\sin\theta$$

$$(T + W)\sin\theta = P$$

$$\sum F_x = 0$$

$$0 = W\cos\theta - T\cos\theta$$

$$T = W$$

$$= 100 \text{ N}$$

$$2W\sin\theta = P$$

$$\sin\theta = \frac{P}{2W}$$

$$\frac{h}{\sqrt{h^2 + d^2}} = \frac{P}{2W} = \frac{20 \text{ N}}{(2)(100 \text{ N})}$$

$$= 0.1$$

$$h^2 = 0.01(h^2 + d^2)$$

$$0.99h^2 = 0.01d^2$$

$$h = d\sqrt{\frac{0.01}{0.99}}$$

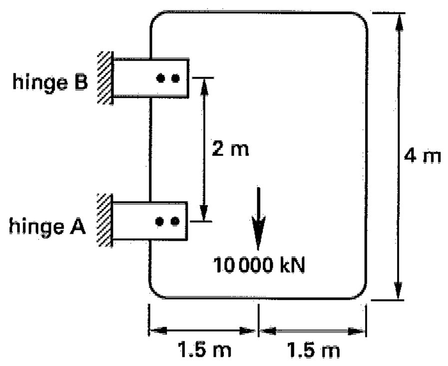
$$= 10 \text{ cm}\sqrt{\frac{0.01}{0.99}}$$

$$= 1.0 \text{ cm}$$

The answer is (B).

STATICS–7

Hinges A and B support a 10 000 kN bank vault door as shown. Determine the horizontal force in the hinge pin at A.



(A) 1500 kN (B) 2500 kN (C) 5500 kN (D) 7500 kN

Sum the moments around joint B.

$$\sum M_{\text{B}} = 0$$

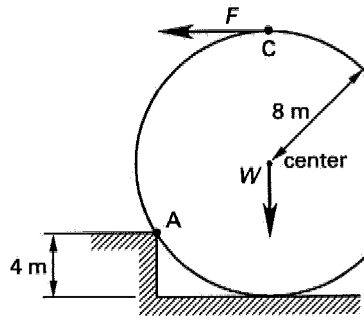
$$0 = (-10\,000 \text{ kN})(1.5 \text{ m}) + (2 \text{ m})R_{\text{A}x}$$

$$R_{\text{A}x} = 7500 \text{ kN}$$

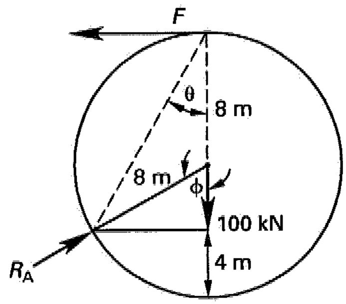
The answer is (D).

STATICS–8

A cylindrical tank is at rest as shown. The tank has a weight of 100 kN. Approximately what horizontal force should be applied horizontally at point C to raise the tank?



(A) 25 kN (B) 58 kN (C) 67 kN (D) 110 kN



In order to raise the tank, $\sum M_A \geq 0$. Therefore, the minimum force that must be applied at point C can be found as follows.

$$\sum M_A = 0$$

$$F(12 \text{ m}) - W(8 \text{ m})\sin\phi = 0$$

$$= \frac{2W\sin\phi}{3}$$

$$\cos\phi = \frac{4 \text{ m}}{8 \text{ m}} = 0.5$$

$$\phi = 60°$$

$$F = \frac{2W\sin 60°}{3}$$

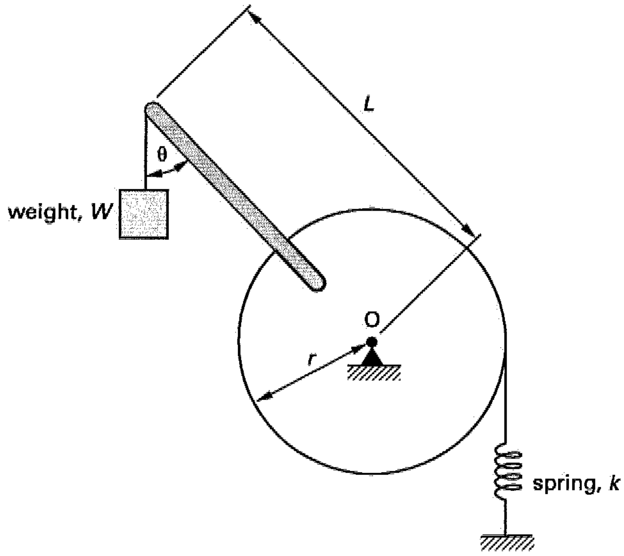
$$= \frac{(2)(100 \text{ kN})\sin 60°}{3}$$

$$= 57.7 \text{ kN} \quad (58 \text{ kN})$$

The answer is (B).

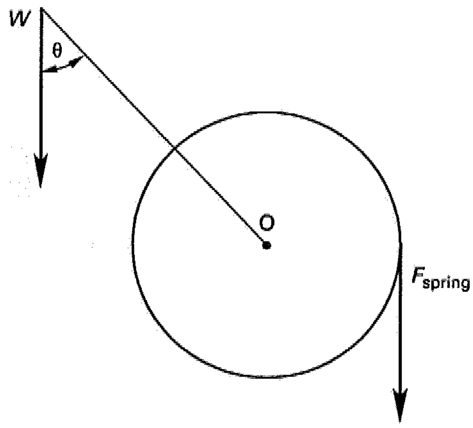
STATICS–9

A weight is attached to a lever as shown. Determine the expression for θ when the system is at equilibrium. The spring constant is k, the length of the lever is L, the radius of the wheel is r, and the magnitude of the weight is W.



(A) $\theta = \dfrac{WL\sin\theta}{kr}$ (B) $\theta = \dfrac{WL\sin\theta}{kr^2}$

(C) $\theta = \dfrac{Wr}{kL}$ (D) $\theta = \dfrac{WL\cos\theta}{kr}$



At equilibrium, the sum of the moments about the center of the wheel (point O) must equal zero.

$$\sum M_O = 0$$

$$= rF_{\text{spring}} - WL\sin\theta$$

$$rF_{\text{spring}} = WL\sin\theta$$

$$F_{\text{spring}} = kr\theta$$

$$r^2k\theta = WL\sin\theta$$

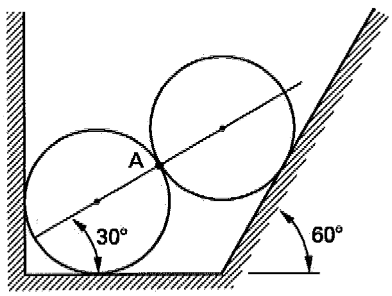
$$\theta = \frac{WL\sin\theta}{kr^2}$$

Successive iterations are necessary to solve for θ.

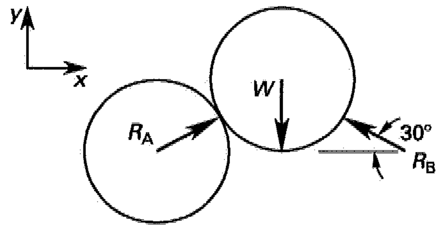
The answer is (B).

STATICS–10

Two identical spheres weighing 100 N each are placed as shown. The line connecting the two centers of the spheres makes an angle of 30° to the horizontal surface. All walls are smooth and frictionless. What is most nearly the reaction force at A?



(A) 33 N (B) 67 N (C) 75 N (D) 100 N



$$W = 100 \text{ N}$$

$$\sum F_x = 0$$

$$R_A \cos 30° - R_B \cos 30° = 0$$

$$R_A = R_B$$

$$\sum F_y = 0$$

$$-W + R_A \sin 30° + R_B \sin 30° = 0$$

$$R_A + R_B = \frac{W}{\sin 30°}$$

$$2R_A = \frac{W}{\sin 30°}$$

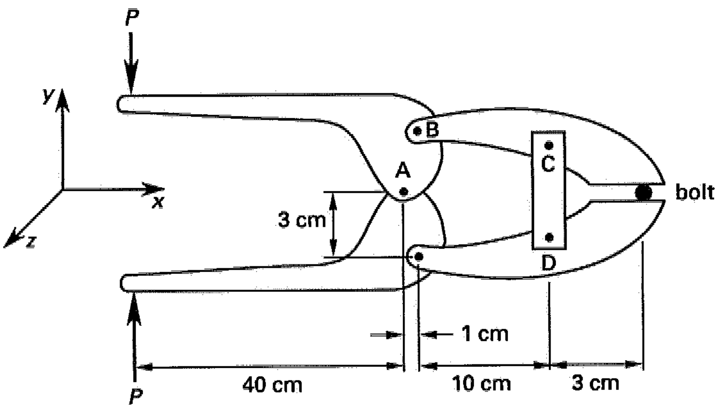
$$R_A = W$$

$$= 100 \text{ N}$$

The answer is (D).

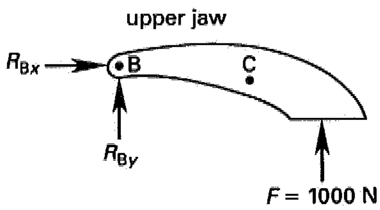
STATICS–11

What is most nearly the force, P, that must be exerted on the handles of the bolt cutter shown if the force on the bolt is 1000 N?



(A) 7.5 N (B) 30 N (C) 53 N (D) 260 N

First, consider the upper jaw.



$$\sum F_x = 0$$

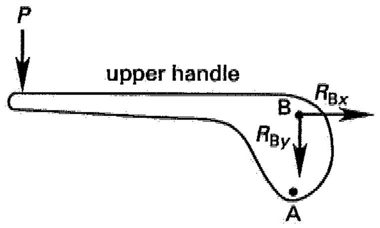
$$R_{Bx} = 0$$

$$\sum M_C = 0$$

$$-(R_{By})(10\text{ cm}) + F(3\text{ cm}) = 0$$

$$\begin{aligned} R_{By} &= \left(\frac{3\text{ cm}}{10\text{ cm}}\right)F \\ &= \left(\frac{3\text{ cm}}{10\text{ cm}}\right)(1000\text{ N}) \\ &= 300\text{ N} \end{aligned}$$

Now, consider the upper handle.



$$\sum M_A = 0$$

$$-R_{By}(1\ \text{cm}) + P(40\ \text{cm}) = 0$$

$$P = \left(\frac{1\ \text{cm}}{40\ \text{cm}}\right) R_{B_y}$$

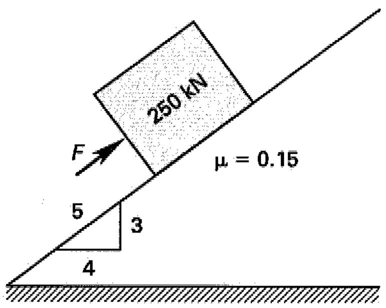
$$= \left(\frac{1\ \text{cm}}{40\ \text{cm}}\right)(300\ \text{N})$$

$$= 7.5\ \text{N}$$

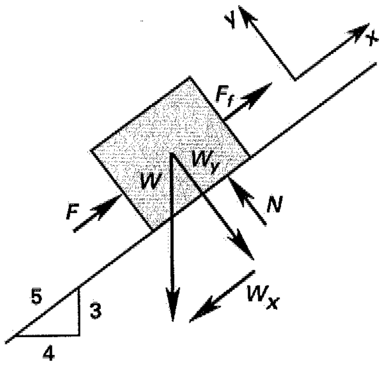
The answer is (A).

STATICS–12

Determine the force, F, required to keep the package from sliding down the plane shown.



(A) 15 kN (B) 35 kN (C) 65 kN (D) 120 kN



$$\sum F_y = 0$$

$$W_y - N = 0$$

$$W_y = \tfrac{4}{5}W$$

$$N = \tfrac{4}{5}W$$

$$= 200 \text{ kN}$$

$$F_f = \mu N$$

$$= (0.15)(200 \text{ kN})$$

$$= 30 \text{ kN}$$

$$\sum F_x = 0$$

$$F - W_x + F_f = 0$$

$$F = W_x - F_f$$

$$W_x = \tfrac{3}{5}W$$

$$= 150 \text{ kN}$$

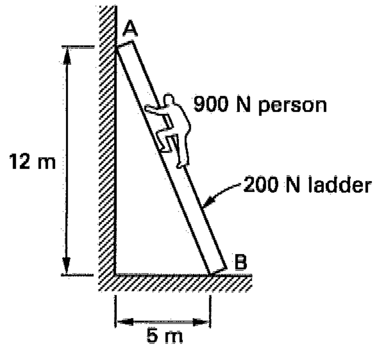
$$F = 150 \text{ kN} - 30 \text{ kN}$$

$$= 120 \text{ kN}$$

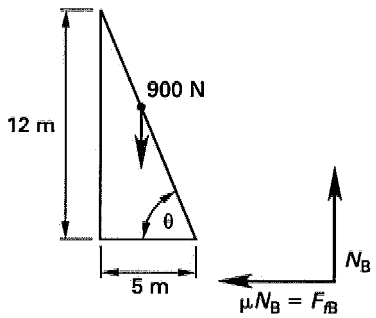
The answer is (D).

STATICS–13

Determine the minimum coefficient of friction at point B required for a person to use the ladder shown. Assume there is no friction at point A.



(A) 0.18 (B) 0.28 (C) 0.42 (D) 0.56



The total reaction force at B ($N_B + F_{fB}$) must point along the ladder. Therefore,

$$\frac{F_{fB}}{N_B} = \frac{\mu N_B}{N_B} = \cot\theta$$

$$\mu = \cot\theta$$

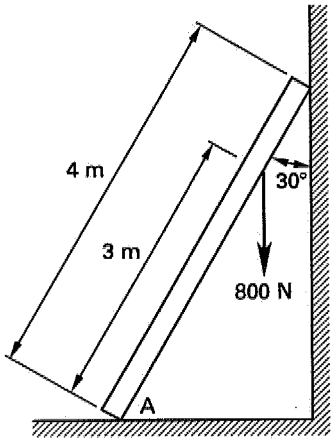
$$= \frac{5 \text{ m}}{12 \text{ m}}$$

$$= 0.42$$

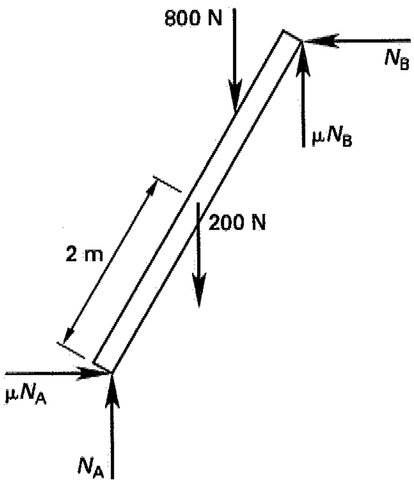
The answer is (C).

STATICS–14

A 4 m ladder weighing 200 N is placed as shown. When an 800 N person climbs 3 m above the lower end, the ladder is just about to slip. Determine the friction coefficient between the ladder and the floor. The coefficient of friction between the ladder and the wall is 0.20.



(A) 0.19 (B) 0.29 (C) 0.39 (D) 0.49



$$\sum M_A = 0$$

$$0 = (-200 \text{ N})(2 \text{ m}) \sin 30° - (800 \text{ N})(3 \text{ m}) \sin 30° + N_B(4 \text{ m}) \cos 30° + 0.2N_B(4 \text{ m}) \sin 30°$$

$$N_B = 362 \text{ N}$$

$$\sum F_y = 0$$

$$0 = N_A + \mu_{\text{wall}} N_B - 800 \text{ N} - 200 \text{ N}$$
$$= N_A + (0.20)(362 \text{ N}) - 800 \text{ N} - 200 \text{ N}$$

$$N_A = 928 \text{ N}$$

$$\sum F_x = 0$$

$$0 = \mu_{\text{floor}} N_A - N_B$$

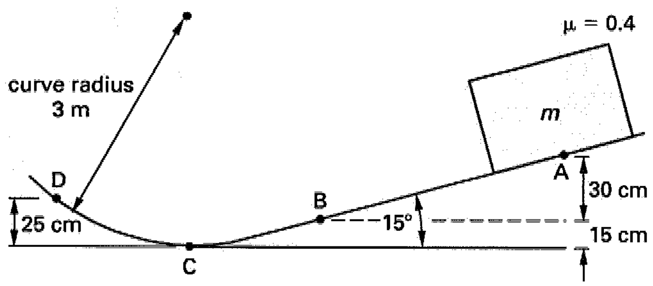
$$\mu_{\text{floor}} N_A = N_B$$

$$\mu_{\text{floor}} = \frac{N_B}{N_A} = \frac{362 \text{ N}}{928 \text{ N}} = 0.39$$

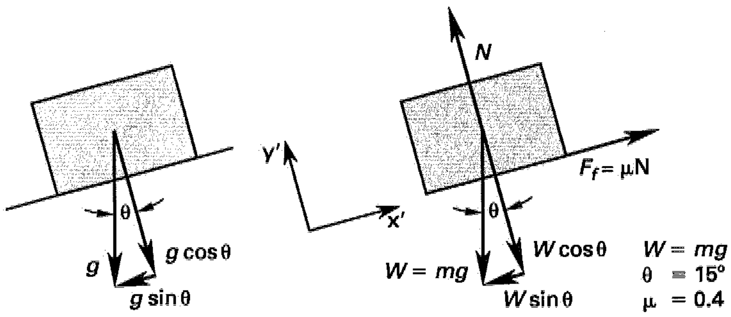
The answer is (C).

STATICS–15

A 2 kg block is released at point A on an inclined plane that is tangent to a circular arc. The plane is tilted 15° from horizontal, and the coefficient of friction is 0.4. Which point is the equilibrium position of the block?



(A) A (B) B (C) C (D) D



$$\begin{aligned}\sum F_{x'} &= -mg\sin\theta + F_f \\ &= -mg\sin\theta + \mu mg\cos\theta\end{aligned}$$

For the block to slide downhill, $\sum F_x < 0$.

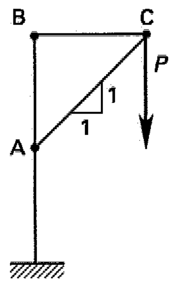
$$\begin{aligned}-mg\sin\theta + \mu mg\cos\theta &< 0 \\ \mu mg\cos\theta &< mg\sin\theta \\ \mu &< \tan\theta \\ &< \tan 15° \\ &< 0.27\end{aligned}$$

Thus, for the block to move, $\mu < 0.27$. However, $\mu = 0.4$. Therefore, the block never moves. It stays at point A.

The answer is (A).

STATICS–16

Determine the force in member AC in terms of force P.



(A) P (B) $\frac{4}{3}P$ (C) $\sqrt{2}P$ (D) $\frac{\sqrt{3}}{2}P$

$$\text{AC} = \text{force in member AC}$$

$$\sum F_y = 0$$

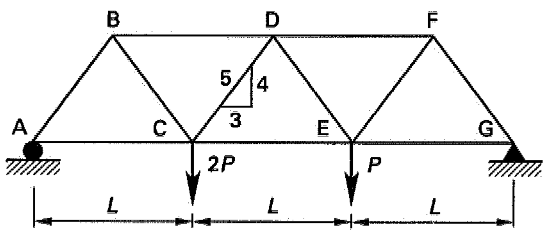
$$0 = (\text{AC})\cos 45° - P$$

$$\text{AC} = \frac{P}{\cos 45°} = \sqrt{2}P$$

The answer is (C).

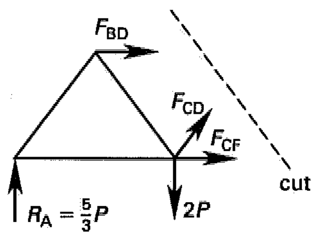
STATICS–17

Determine the force in member CD.



(A) $\frac{1}{12}P$ (B) $\frac{1}{3}P$ (C) $\frac{5}{12}P$ (D) P

Use the method of sections.



Only CD can support a vertical force.

$$\sum F_y = 0$$

$$0 = R_A - 2P + CD_y$$

$$CD_y = \frac{P}{3}$$

$$CD = \tfrac{5}{4}CD_y$$

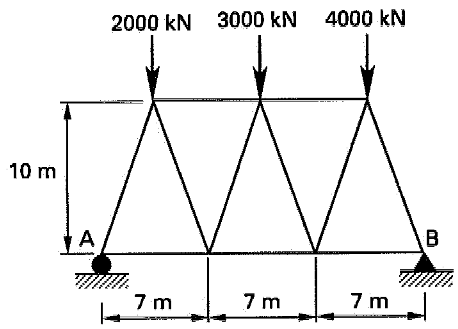
$$= \left(\frac{5}{4}\right)\left(\frac{P}{3}\right)$$

$$= \frac{5P}{12}$$

The answer is (C).

STATICS–18

A truss is subjected to three loads. The truss is supported by a roller at A and by a pin joint at B. What is most nearly the reaction force at A?



(A) 3800 kN (B) 4400 kN (C) 4900 kN (D) 5000 kN

The rolling support at A can only support a vertical reaction force. R_A is the reaction force at A.

$$\sum M_B = 0$$

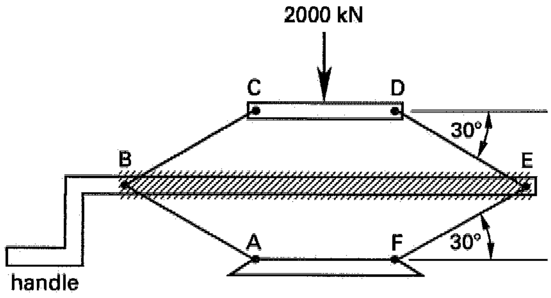
$$0 = -R_A(21 \text{ m}) + (2000 \text{ kN})(17.5 \text{ m}) + (3000 \text{ kN})(10.5 \text{ m}) + (4000 \text{ kN})(3.5 \text{ m})$$

$$R_A = 3833 \text{ kN} \quad (3800 \text{ kN})$$

The answer is (A).

STATICS–19

A scissor jack is used to raise a car. If the jack supports a weight of 2000 kN, determine the force in member BE.



(A) $\sqrt{3} \times 100$ kN (B) $\sqrt{3} \times 500$ kN (C) $\sqrt{3} \times 1000$ kN (D) $\sqrt{3} \times 2000$ kN

CB and DE equally share the 2000 kN load. Therefore, $BC_y = DE_y = 2000\text{ kN}/2 = 1000$ kN, and $DE = DE_y/\sin 30° = 1000\text{ kN}/0.5 = 2000$ kN. Use the method of joints at E.

$$\sum F_y = 0$$

$$0 = -FE_y + DE_y$$

$$FE_y = 1000\text{ kN}$$

$$FE = \frac{FE_y}{\sin 30°} = \frac{1000\text{ kN}}{0.5} = 2000\text{ kN}$$

$$\sum F_x = 0$$

Therefore,

$$BE = FE_x + DE_x$$

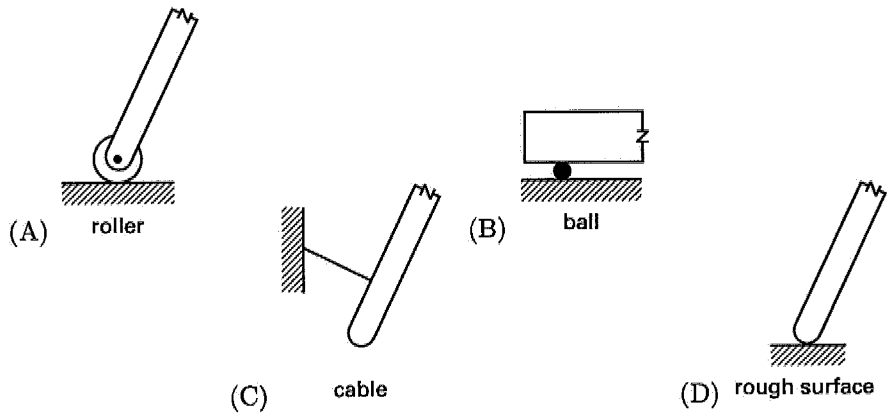
$$FE\cos 30° + DE\cos 30° = (2)(2000\text{ kN})\left(\frac{\sqrt{3}}{2}\right)$$

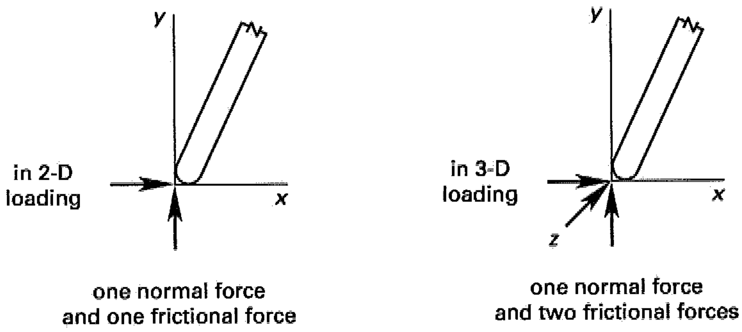
$$= \sqrt{3} \times 2000\text{ kN}$$

The answer is (D).

STATICS–20

When loaded, which support has a reaction involving more than a single force?



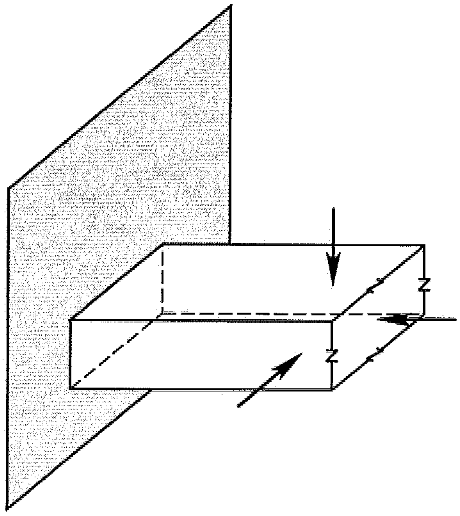


Options (A) and (B) have a normal force. Option (C) has a single force along the cable. Only option (D) can support two reaction forces.

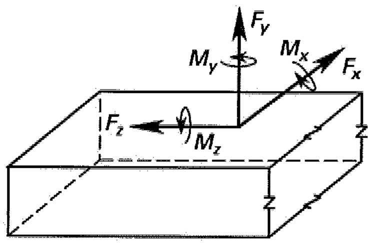
The answer is (D).

STATICS–21

A beam, securely fastened to a wall, is subjected to three-dimensional loading. How many components of reaction are possible?



(A) two (B) three (C) four (D) six

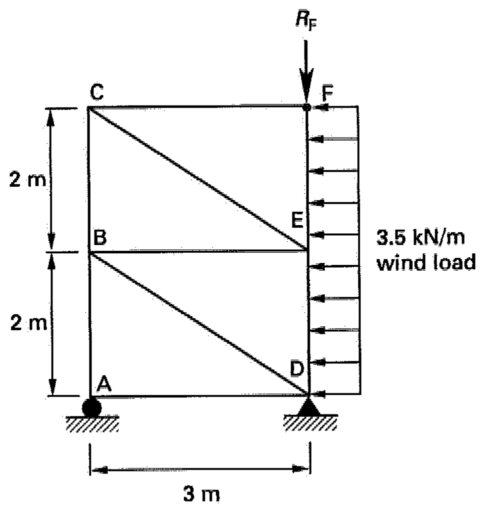


There are six components of reaction possible: three force components and three moment components.

The answer is (D).

STATICS–22

What is most nearly the vertical force at point F necessary to resist the wind load of 3.5 kN/m on DEF?



(A) 2.2 kN (B) 4.3 kN (C) 9.3 kN (D) 10 kN

$$\sum M_A = 0$$

$$0 = \left(3.5\ \frac{\text{kN}}{\text{m}}\right)(4\text{ m})(2\text{ m}) - (3\text{ m})R_F$$

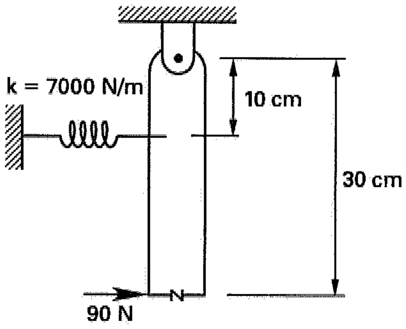
Rearranging to solve for R_F,

$$R_F = \frac{28\ \text{kN}\cdot\text{m}}{3\ \text{m}} = 9.33\ \text{kN} \quad (9.3\ \text{kN})$$

The answer is (C).

STATICS–23

Compute the equilibrium displacement of the spring shown.



(A) 12 mm (B) 32 mm (C) 38 mm (D) 75 mm

Sum the moments around the hinge.

$$\sum M_{\text{hinge}} = 0 = (90 \text{ N})(30 \text{ cm}) - F_{\text{spring}}(10 \text{ cm})$$

$$F_{\text{spring}} = 270 \text{ N} = k\delta_{\text{spring}}$$

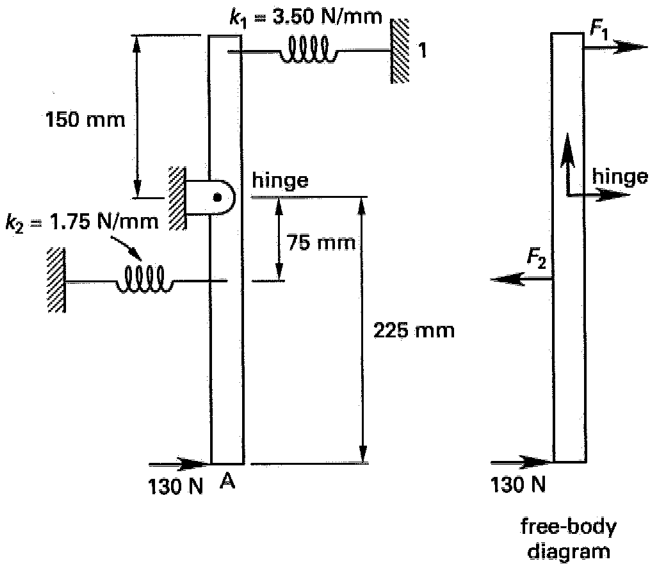
$$\delta_{\text{spring}} = -\frac{F_{\text{spring}}}{k} = \frac{270 \text{ N}}{7000 \frac{\text{N}}{\text{m}}}$$

$$= 0.038 \text{ m} \quad (38 \text{ mm})$$

The answer is (C).

STATICS–24

The system shown is in equilibrium prior to the application of the 130 N force at A. After equilibrium is reestablished, what is most nearly the displacement at A?



(A) 25 mm (B) 30 mm (C) 37 mm (D) 74 mm

Sum the moments around the hinge and use $F = -k\delta$ to find the spring forces.

$$\sum M_{\text{hinge}} = 0$$

$$0 = -k_1\delta_1(150 \text{ mm}) + (130 \text{ N})(225 \text{ mm}) - k_2\delta_2(75 \text{ mm})$$

The ratio of the deflection of the bar at a point to that point's distance from the hinge is equal to the angular displacement of the bar. Since the bar does not bend, this ratio is the same for any point on the bar. Therefore,

$$\frac{\delta_1}{150 \text{ mm}} = \frac{\delta_2}{75 \text{ mm}}$$

$$\delta_1 = 2\delta_2$$

Substitute for δ_1 in the equation for the moment.

$$0 = -\left(3.5 \ \frac{\text{N}}{\text{mm}}\right) 2\delta_2(150 \text{ mm}) + (130 \text{ N})(225 \text{ mm}) - \left(1.75 \ \frac{\text{N}}{\text{mm}}\right) \delta_2(75 \text{ mm})$$

Rearranging to solve for δ_2,

$$\delta_2 = \frac{(130\ \text{N})(225\ \text{mm})}{1181.3\ \text{N}} = 24.8\ \text{mm}$$

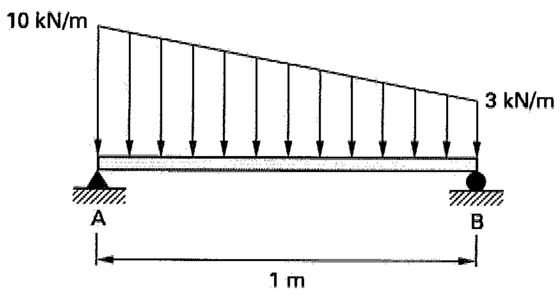
$$\frac{\delta_A}{225\ \text{mm}} = \frac{\delta_2}{75\ \text{mm}}$$

$$\delta_A = 3\delta_2 = 74.4\ \text{mm} \quad (74\ \text{mm})$$

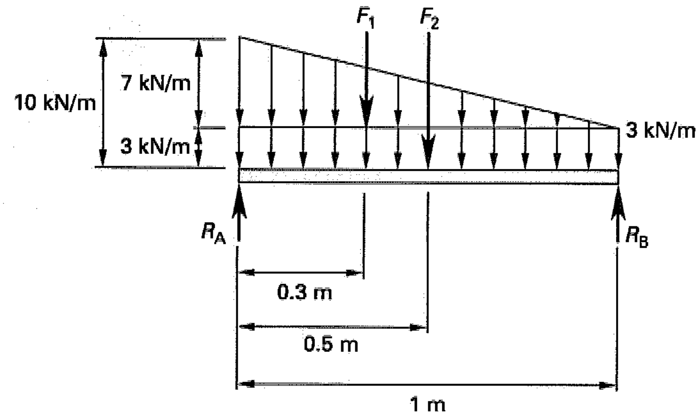
The answer is (D).

STATICS–25

What is most nearly the reaction force at support B on the simply supported beam with a linearly varying load?



(A) 1.5 kN (B) 2.3 kN (C) 2.6 kN (D) 3.5 kN



$$F_1 = \tfrac{1}{2}Lh = \left(\frac{1}{2}\right)(1\ \text{m})\left(7\ \frac{\text{kN}}{\text{m}}\right)$$
$$= 3.5\ \text{kN}$$
$$F_2 = Lh = (1\ \text{m})\left(3\ \frac{\text{kN}}{\text{m}}\right)$$
$$= 3\ \text{kN}$$

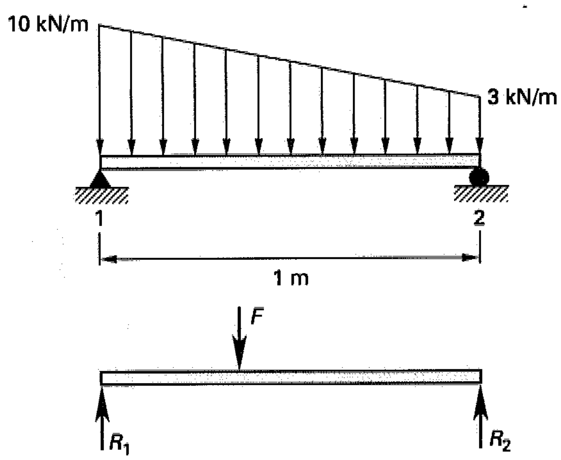
Sum the moments around support A.

$$\sum M_A = 0 = R_B(1\ \text{m}) - F_1(0.3\ \text{m}) - F_2(0.5\ \text{m})$$
$$= R_B(1\ \text{m}) - (3.5\ \text{kN})(0.3\ \text{m}) - (3\ \text{kN})(0.5\ \text{m})$$
$$R_B = 2.55\ \text{kN} \quad (2.6\ \text{kN})$$

The answer is (C).

STATICS–26

For the simply supported beam with the linearly varying load shown, what is the sum of the reactions at the supports?



free-body diagram

(A) 3.5 kN (B) 6.5 kN (C) 9.2 kN (D) 13 kN

$$\sum F_y = 0$$

$$R_1 + R_2 - F = 0$$

$$R_1 + R_2 = F$$

The area of the trapezoidal load distribution, F, is

$$F = \tfrac{1}{2}Lh = \left(\frac{10\ \frac{\text{kN}}{\text{m}} + 3\ \frac{\text{kN}}{\text{m}}}{2}\right)(1\ \text{m})$$

$$= 6.5\ \text{kN}$$

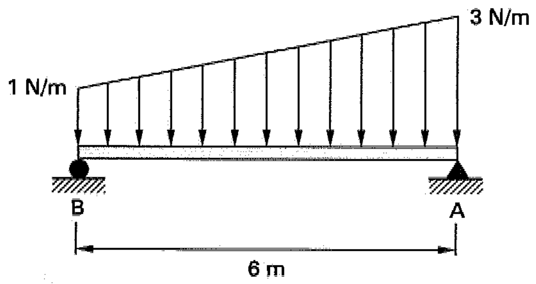
Therefore,

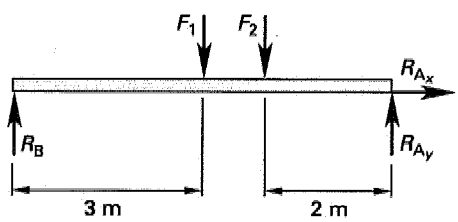
$$R_1 + R_2 = 6.5\ \text{kN}$$

The answer is (B).

STATICS–27

A beam is subjected to a distributed load as shown. Determine the reactions at the right support, A.





free-body diagram

(A) 2.2 kN (B) 4.3 kN (C) 5.5 kN (D) 7.0 kN

$$\sum F_x = 0$$

$$R_{Ax} = 0$$

$$F_1 = Lh = \left(1\ \frac{\text{N}}{\text{m}}\right)(6\ \text{m}) = 6\ \text{N}$$

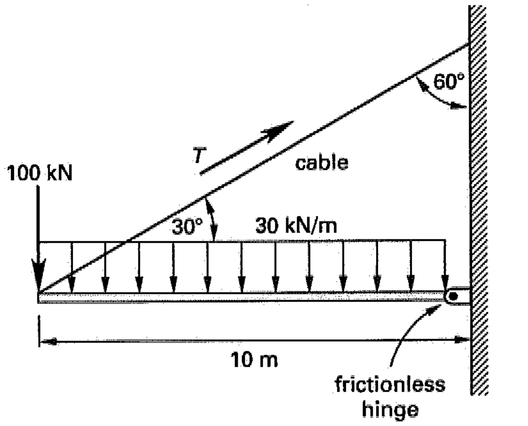
$$F_2 = \tfrac{1}{2}Lh = \left(\frac{1}{2}\right)\left(2\ \frac{\text{N}}{\text{m}}\right)(6\ \text{m}) = 6\ \text{N}$$

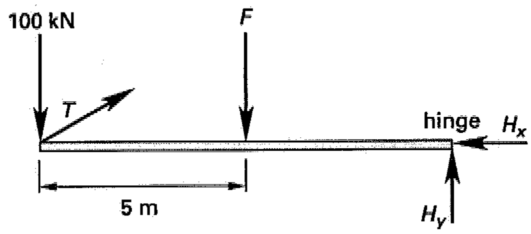
Since $R_{Ax} = 0$, $R_{Ay} = R_A$. Thus, $R_A = 7.0$ kN.

The answer is (D).

STATICS–28

A beam is hinged at a wall and loaded as shown. What is the tension in the cable?





free-body diagram

(A) 200 kN (B) 250 kN (C) 430 kN (D) 500 kN

$$\sum m_{\text{hinge}} = 0$$

$$F(5\text{ m}) + (100\text{ kN})(10\text{ m}) - T\sin 30°(10\text{ m}) = 0$$

$$F = Lh = (10\text{ m})\left(30\ \frac{\text{kN}}{\text{m}}\right)$$
$$= 300\text{ kN}$$

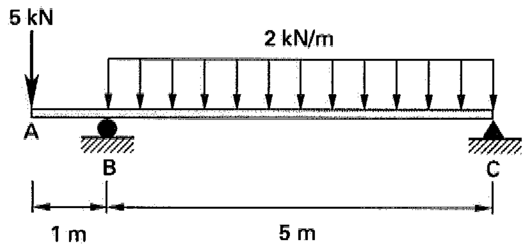
Substituting for F and rearranging to solve for T,

$$T = \frac{(300\text{ kN})(5\text{ m}) + (100\text{ kN})(10\text{ m})}{(10\text{ m})(\sin 30°)}$$
$$= 500.0\text{ kN}$$

The answer is (D).

STATICS-29

Determine the position of maximum moment in the beam ABC.



(A) at point A (B) at point B
(C) at point C (D) 2 m left of point C

$$\sum M_B = 0$$

$$0 = (5\text{ kN})(1\text{ m}) - \int_0^5 2x\,dx + (R_{C_y})(5\text{ m})$$
$$= 5\text{ kN·m} - x^2\Big|_0^5 + R_{C_y}(5\text{ m})$$
$$= 5\text{ kN·m} - (25 - 0) + R_{C_y}(5\text{ m})$$
$$R_{C_y} = 4\text{ kN}$$

$$\sum F_y = 0 = -5\text{ kN} - \int_0^5 2dx + 4\text{ kN} + R_\text{B}$$
$$= -5\text{ kN} - 2x\Big|_0^5 + 4\text{ kN} + R_\text{B}$$
$$= -5\text{ kN} - (10 - 0) + 4\text{ kN} + R_\text{B}$$
$$= -5\text{ kN} - 10\text{ kN} + 4\text{ kN} + R_\text{B}$$
$$R_\text{B} = 11\text{ kN}$$

At point A,

$$M_\text{A} = 0$$

At point B,

$$M_\text{B} = (5\text{ kN})(1\text{ m}) = 5\text{ kN·m}$$

From C to B,

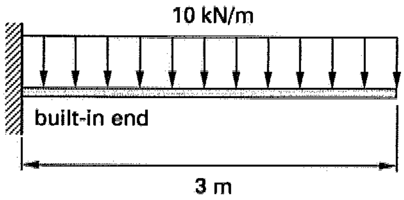
$$M_x = R_{\text{C}y}x - \int_0^x 2xdx$$
$$= \left(4x - \frac{2x^2}{2}\right)$$
$$= \left(4x - x^2\right)$$
$$\frac{dM_x}{dx} = 4 - 2x$$
$$= 0 \quad [\text{where } M_x \text{ is a max}]$$
$$0 = (4\text{ kN}) - \left(2\ \frac{\text{kN}}{\text{m}}\right)x$$
$$x = \frac{4\text{ kN}}{2\ \frac{\text{kN}}{\text{m}}}$$
$$= 2\text{ m}$$
$$M_{x,\text{max}} = (4\text{ kN})(2\text{ m}) - 4\text{ kN·m}$$
$$= 4\text{ kN·m} < M_\text{B}$$

Thus, the maximum moment is 5 kN·m, and it occurs at point B.

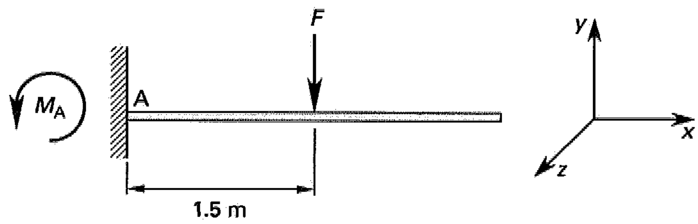
The answer is (B).

STATICS–30

For the cantilever beam with the distributed load shown, what is the moment at the built-in end?



(A) 5 kN·m (B) 10 kN·m (C) 20 kN·m (D) 45 kN·m

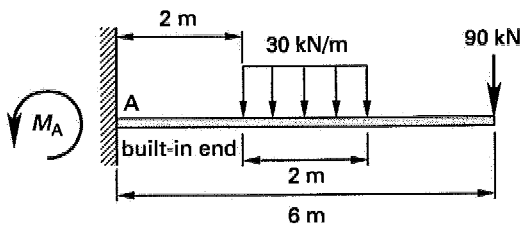


$$F = \left(10\ \frac{\text{kN}}{\text{m}}\right)(3\ \text{m})$$
$$= 30\ \text{kN}$$
$$\sum M_A = 0 = M_A - F(1.5\ \text{m})$$
$$M_A = (30\ \text{kN})(1.5\ \text{m})$$
$$= 45\ \text{kN·m}$$

The answer is (D).

STATICS–31

For the cantilever beam shown, what is the moment acting at the built-in end?



(A) 270 kN·m (B) 310 kN·m (C) 540 kN·m (D) 720 kN·m

Sum the moments around the support, A.

$$\sum M_A = 0 = M_A - (90\ \text{kN})(6\ \text{m}) - \left(30\ \frac{\text{kN}}{\text{m}}\right)(2\ \text{m})\left(\frac{6\ \text{m}}{2}\right)$$

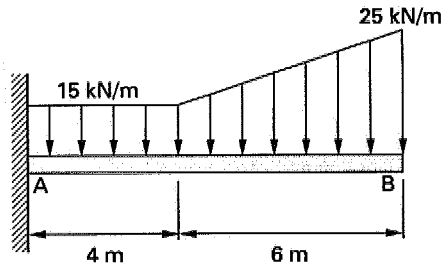
Rearranging to solve for M_A,

$$M_A = 720\ \text{kN·m}$$

The answer is (D).

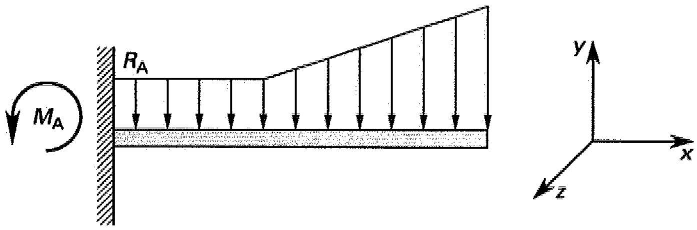
STATICS–32

Determine the resultant moment at the built-in end, A, for the cantilever beam shown.



(A) 660 kN·m (B) 990 kN·m (C) 1100 kN·m (D) 1200 kN·m

Divide the beam into two sections: from 0 m to 4 m where $F_1 = 15$ kN·m, and from 4 m to 10 m where $F_2 = 5(x+5)/3$ kN/m. Sum the moments around point A.



$$\sum M_A = 0$$

$$0 = M_A - \int_0^4 F_1(x)x\,dx - \int_4^{10} F_2(x)x\,dx$$

$$M_A = \int_0^4 15x\,dx + \int_4^{10} \frac{5(x+5)}{3}x\,dx$$

$$= \frac{15x^2}{2}\Big|_0^4 + \frac{5x^3}{9} + \frac{25x^2}{6}\Big|_4^{10}$$

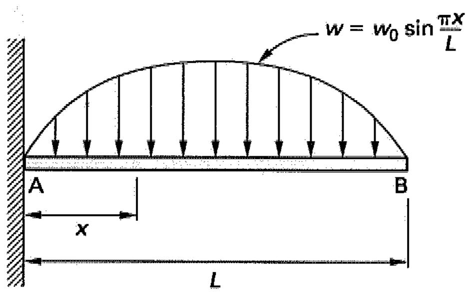
$$= (15)(8) + \frac{(5)\left(10^3 - (4)^3\right)}{9} + \frac{(25)\left(10^2 - (4)^2\right)}{6}$$

$$= 990 \text{ kN·m}$$

The answer is (B).

STATICS–33

Determine the moment at the built-in end, A, for the beam shown.



(A) $\frac{w_0 L^2}{\pi}$ (B) $w_0 L$ (C) $\frac{2w_0 L^2}{\pi}$ (D) $\frac{w_0 L^2}{2\pi}$

$$\sum M_A = 0$$

$$0 = M_A - \int_0^L \left(w_0 \sin \frac{\pi x}{L}\right) x dx$$

$$M_A = \int_0^L \left(w_0 \sin \frac{\pi x}{L}\right) x dx$$

Integrate by parts.

$$u = x$$

$$dv = \left(w_0 \sin \frac{\pi x}{L}\right) dx$$

$$du = dx$$

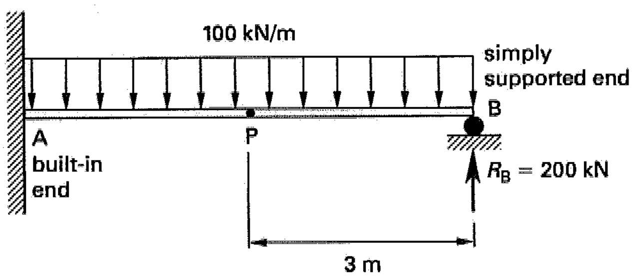
$$v = -\frac{w_0 L}{\pi} \cos \frac{\pi x}{L}$$

$$\begin{aligned} M_A &= -\frac{x w_0 L}{\pi} \cos \frac{\pi x}{L}\Big|_0^L + \frac{w_0 L}{\pi} \int_0^L \left(\cos \frac{\pi x}{L}\right) dx \\ &= -\left(\frac{L^2 w_0}{\pi} \cos \pi - 0\right) + \left(\left(\frac{w_0 L^2}{\pi^2}\right) \sin \frac{\pi x}{L}\Big|_0^L\right) \\ &= -\left(\frac{L^2 w_0}{\pi}\right)(-1 - 0) + \left(\frac{w_0 L^2}{\pi^2}\right)(0 - 0) \\ &= \frac{L^2 w_0}{\pi} \end{aligned}$$

The answer is (A).

STATICS–34

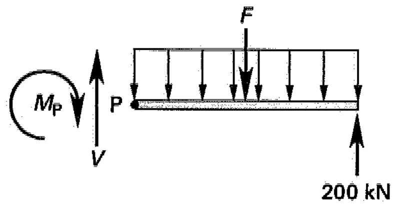
The beam shown is statically indeterminate, but the reaction force at B is known to be 200 kN. What is the bending moment at point P?



(A) 90 kN·m (B) 150 kN·m (C) 240 kN·m (D) 350 kN·m

Measure x from point B.

The moment at point P is



$$\sum M_P = 0 = -M_P - F(1.5\text{ m}) + (200\text{ kN})(3\text{ m})$$

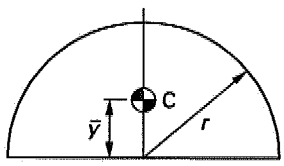
Rearranging to solve for M_P,

$$\begin{aligned} M_P &= (200\text{ kN})(3\text{ m}) - \left(100\ \frac{\text{kN}}{\text{m}}\right)(3\text{ m})(1.5\text{ m}) \\ &= 150\text{ kN·m} \end{aligned}$$

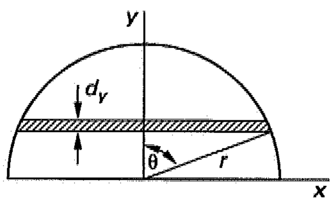
The answer is (B).

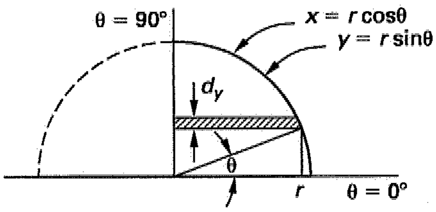
STATICS–35

Determine the height of the centroid, $\bar{y}$, of the semicircle with radius r shown.



(A) $\frac{2r}{3}$ (B) $\frac{2\pi r}{5}$ (C) $\frac{4r}{3\pi}$ (D) $\frac{3r}{4}$





$$A = \tfrac{1}{2}\pi r^2 \quad \text{[for the semicircle]}$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$dy = d(r\sin\theta) = r\cos\theta d\theta$$

$$dA = xdy = r\cos\theta r\cos\theta d\theta$$

Since the area is symmetrical, area for $0 \leq \theta \leq 90°$ is half the total area.

$$\begin{aligned}\int ydA &= 2\int_0^{\pi/2} (r\sin\theta)(r\cos\theta)(r\cos\theta)d\theta \\ &= 2\int_0^{\pi/2} r^3 \sin\theta \cos^2\theta d\theta \\ &= \frac{-2r^3\cos^3\theta}{3}\bigg|_0^{\pi/2} \\ &= \frac{2r^3}{3}\end{aligned}$$

By definition,

$$\bar{y}A = \int_A ydA$$

$$\bar{y} = \frac{\int_A^y dA}{A}$$

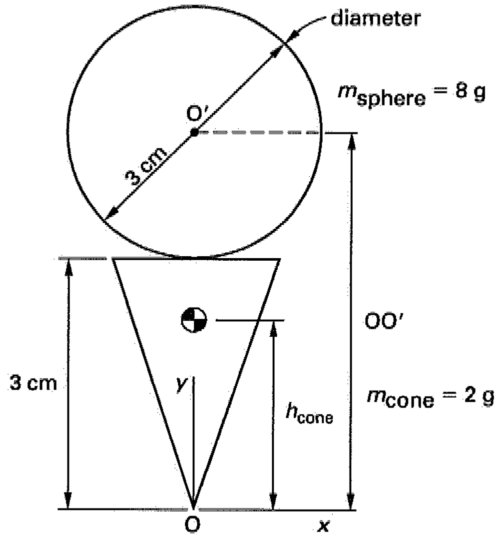
$$\bar{y}A = \frac{2r^3}{3}$$

$$\bar{y} = \frac{\frac{2r^3}{3}}{\frac{\pi r^2}{2}} = \frac{4r}{3\pi}$$

The answer is (C).

STATICS–36

What is the height of the center of mass of the cone-sphere system shown?



(A) 2 cm (B) 3 cm (C) 4 cm (D) 5 cm

By symmetry, the center of mass of the system is on the y-axis. Find the height of the center of mass for each part of the system.

First, find the height of the sphere's center of mass, OO′.

$$\begin{aligned} \text{OO}' &= \text{height of cone} + r_{\text{sphere}} \\ &= 3 \text{ cm} + \frac{3 \text{ cm}}{2} \\ &= 4.5 \text{ cm} \end{aligned}$$

Next, find the height of the cone's center of mass, h_c.

$$\begin{aligned} h_c &= (\text{height of cone})\left(\frac{2}{3}\right) \\ &= (3 \text{ cm})\left(\frac{2}{3}\right) \\ &= 2 \text{ cm} \end{aligned}$$

Finally, find the height of the cone-sphere system's center of mass.

$$h = \frac{m_{\text{sphere}}\text{OO}' + m_{\text{cone}}h_c}{m_{\text{sphere}} + m_{\text{cone}}}$$

$$= \frac{(8\ \text{g})(4.5\ \text{cm}) + (2\ \text{g})(2\ \text{cm})}{8\ \text{g} + 2\ \text{g}}$$

$$= 4\ \text{cm}$$

The answer is (C).

STATICS–37

Which statement about area moments of inertia is FALSE?

(A) $I = \int d^2 dA$

(B) The parallel axis theorem is used to calculate moments of inertia about a parallel displaced axis.

(C) The moment of inertia of a large area is equal to the summation of the inertia of the smaller areas within the large area.

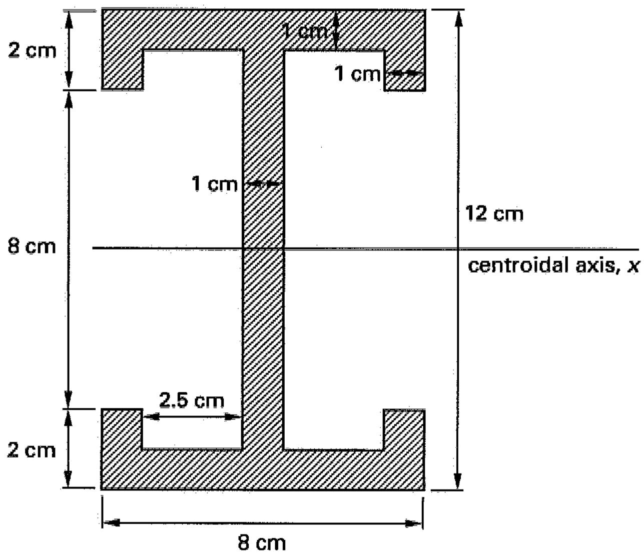
(D) The areas closest to the axis of interest contribute most to the moment of inertia.

Area moment of inertia is defined as $I = \int d^2 dA$, where d is the distance from the axis to the area element. Thus, the areas farthest from the axis have the largest contributions.

The answer is (D).

STATICS–38

Determine the moment of inertia around the centroidal axis of the following beam.

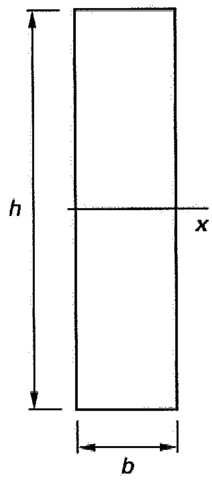


(A) 420 cm^4 (B) 650 cm^4 (C) 730 cm^4 (D) 950 cm^4

The moment of inertia of the beam is equivalent to the moment of inertia of a solid beam with the same dimensions (a height of 12 cm and a width of 8 cm) minus the moments of inertia of the missing sections.

The moment of inertia about the centroidal axis of rectangular sections is

$$I_x = \frac{bh^3}{12}$$



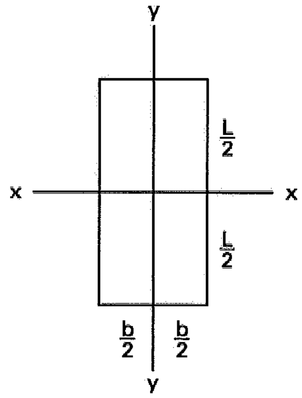
This equation is applied as follows for the solid section (8 cm × 12 cm), the two removed sections (2.5 cm × 10 cm each), and two other removed sections (1 cm × 8 cm each).

$$\begin{aligned} I_x &= \left(\frac{1}{12}\right)(8\text{ cm})(12\text{ cm})^3 - (2)\left(\left(\frac{1}{12}\right)(2.5\text{ cm})(10\text{ cm})^3\right) \\ &\quad - (2)\left(\left(\frac{1}{12}\right)(1\text{ cm})(8\text{ cm})^3\right) \\ &= 650\text{ cm}^4 \end{aligned}$$

The answer is (B).

STATICS–39

I_{xx} is the moment of inertia of the plane area about its centroidal x axis. How can I_{xx} be expressed?



(A) $\frac{bL^3}{96}$ (B) $\frac{bL^3}{16}$ (C) $\frac{bL^3}{12}$ (D) $\frac{bL^3}{8}$

$$
\begin{aligned}
I_{xx} &= \int y^2 dA \\
&= \int_{-\frac{L}{2}}^{\frac{L}{2}} y^2 b\, dy \\
&= b \cdot \frac{y^3}{3} \Bigg|_{-\frac{L}{2}}^{\frac{L}{2}} \\
&= b \left(\frac{\left(\frac{L}{2}\right)^3}{3} - \frac{\left(\frac{-L}{2}\right)}{3} \right) \\
&= \frac{b}{3} \left(\frac{2L^3}{8} \right) \\
&= \frac{bL^3}{12}
\end{aligned}
$$

The answer is (C).

9 MATERIALS SCIENCE

MATERIALS SCIENCE–1

Which of the following affects most of the electrical and thermal properties of materials?

(A) the atomic weight expressed in grams per gram-atom
(B) the electrons, particularly the outermost ones
(C) the magnitude of electrical charge of the protons
(D) the weight of the atoms

The outermost electrons are responsible for determining most of the material's properties.

The answer is (B).

MATERIALS SCIENCE–2

The atomic weight of hydrogen is 1 g/mol. What is most nearly the mass of a hydrogen atom?

(A) 1.7×10^{-24} g/atom (B) 6.0×10^{-23} g/atom (C) 1.0×10^{-10} g/atom (D) 1.0 g/atom

By definition, the mass of an atom is its atomic weight divided by Avogadro's number.

$$W = \frac{1\ \frac{\text{g}}{\text{mol}}}{6.02 \times 10^{23}\ \frac{\text{atoms}}{\text{mol}}} = 1.66 \times 10^{-24}\ \text{g/atom} \quad (1.7 \times 10^{-24}\ \text{g/atom})$$

The answer is (A).

MATERIALS SCIENCE–3

What are valence electrons?

(A) the outer-shell electrons
(B) electrons with positive charge
(C) the electrons of complete quantum shells
(D) the K-quantum shell electrons

By definition, the outermost electrons are the valence electrons.

The answer is (A).

MATERIALS SCIENCE–4

What is the strong bond between hydrogen atoms called?

(A) the ionic bond
(B) the metallic bond
(C) ionic and metallic bonds
(D) the covalent bond

Covalent bonds provide the strongest attractive forces between atoms.

The answer is (D).

MATERIALS SCIENCE–5

What are van der Waals forces?

(A) weak secondary bonds between atoms
(B) primary bonds between atoms
(C) forces between electrons and protons
(D) forces not present in liquids

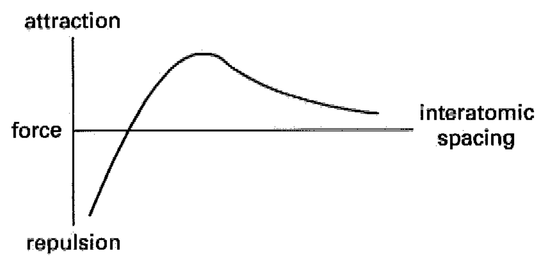
By definition, van der Waals forces are weak attractive forces between atoms or molecules.

The answer is (A).

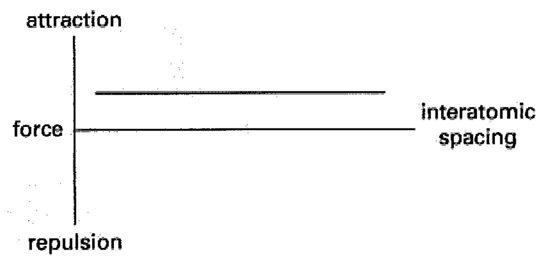
MATERIALS SCIENCE–6

Which of the following curves best illustrates the relationship between interatomic forces and interatomic spacing?

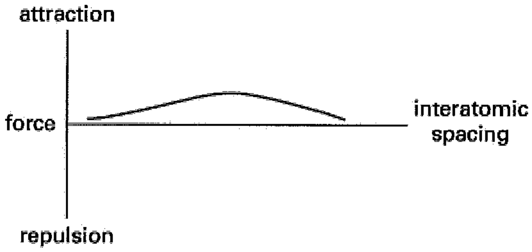
(A)



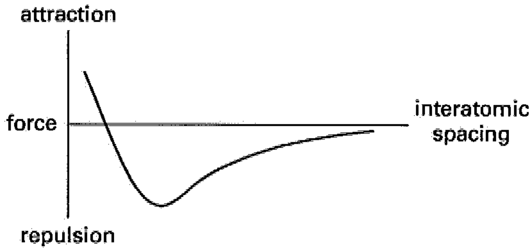
(B)



(C)



(D)



The interatomic force changes from repulsion to attraction as spacing between atoms increases.

The answer is (A).

MATERIALS SCIENCE-7

Compare the metallic iron atom Fe and the ferrous and ferric ions Fe^{2+} and Fe^{3+} at the same temperature. Which has the smallest atomic radius?

(A) Fe
(B) Fe^{2+}
(C) Fe^{3+}
(D) They have the same radii.

Ionizing removes valence electrons, causing the remaining electrons to be pulled in closer to the nucleus. Further reduction in spacing occurs with the removal of more electrons.

The answer is (C).

MATERIALS SCIENCE–8

Cesium (Cs) and sodium (Na) both have the same valence (+1), yet with chlorine (Cl), cesium has a coordination number of 8 in CsCl, while sodium has a coordination number of only 6 in NaCl. What is the main reason for this difference?

(A) The atomic weight of Cs is larger than the weight of Na.
(B) Cs forms covalent bonds in CsCl.
(C) Cs contains more electrons than Na.
(D) Cs is too large to be coordinated by only 6 chloride ions.

Since the Cl atoms are of constant size, the larger coordination number for Cs means that more Cl atoms are needed to fit around a Cs atom than around a Na atom. Therefore, the Cs atom is larger than the Na atom.

The answer is (D).

MATERIALS SCIENCE–9

Which of the following statements is FALSE?

(A) Ceramics are inorganic, nonmetallic solids that are processed or used at high temperatures.
(B) Metals are chemical elements that form substances that are opaque, lustrous, and good conductors of heat and electricity.
(C) Oxides, carbides, and nitrides are considered to be within the class of materials known as glasses.
(D) Most metals are strong, ductile, and malleable. In general, they are heavier than most other substances.

The classes of materials are ceramics, metals, and polymers. Oxides, carbides, nitrides, and glasses are all ceramics.

The answer is (C).

MATERIALS SCIENCE–10

Which of the following materials is NOT a viscoelastic material?

(A) plastic (B) metal (C) rubber (D) glass

A material that is viscoelastic exhibits time-dependent elastic strain. Of the choices, only metal does not fit this description. Metal is considered to be an elastoplastic material.

The answer is (B).

MATERIALS SCIENCE–11

In molecules of the same composition, what are variations of atomic arrangements known as?

(A) polymers
(B) noncrystalline structures
(C) monomers
(D) isomers

Isomers are molecules that have the same composition but different atomic arrangements.

The answer is (D).

MATERIALS SCIENCE–12

Which of the following accurately describes differences between crystalline polymers and simple crystals?

I. Crystalline polymers, unlike simple crystals, are made of folded chains of atoms.
II. Crystal size can be increased by raising the crystallization temperature only in polymers.
III. While a simple crystal may be totally crystallized, a polymer can reach only partial crystallization.

(A) I only (B) II only (C) III only (D) I and III

Only crystalline polymers are composed of folded chains and, at best, exhibit partial crystallization. The crystal size of both simple crystals and polymers can be increased by raising temperature.

The answer is (D).

MATERIALS SCIENCE–13

Polymers that favor crystallization are least likely to have which of the following?

(A) an atactic configuration of side groups
(B) small side groups
(C) only one repeating unit
(D) small chain lengths

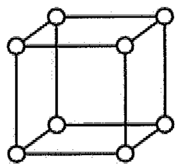
In order for crystallization to be favored, the molecules must be able to arrange themselves into an orderly structure. "Atactic" refers to a random configuration of side groups in the polymer; such a configuration would hinder crystallization.

The answer is (A).

MATERIALS SCIENCE–14

What is the atomic packing factor (APF) for a simple cubic crystal?

(A) 0.48 (B) 0.52 (C) 1.0 (D) 1.1



For a simple cubic crystal, there is one complete atom of radius r per cell. The cell has edges of length $2r$. By definition,

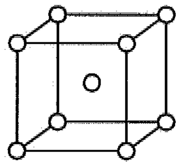
$$\begin{aligned} \text{APF} &= \frac{\text{volume of atoms}}{\text{volume of unit cell}} \\ &= \frac{\frac{4\pi r^3}{3}}{(2r)^3} \\ &= 0.52 \end{aligned}$$

The answer is (B).

MATERIALS SCIENCE–15

How many atoms are in the unit cell of a body-centered cubic structure?

(A) one (B) two (C) three (D) four



There is one atom at the center position and $^1/_8$ of an atom at each of the corners of the cube, since the atom present at each corner is shared by the adjoining unit cells. Therefore,

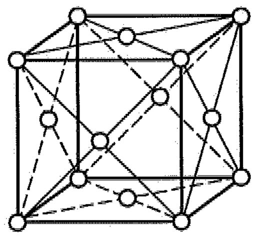
$$\begin{aligned} \text{total no. of atoms} &= \text{no. of atoms at center} \\ &\quad + (\text{no. of atoms at each corner})(\text{no. of corners}) \\ &= 1 + (\tfrac{1}{8})(8) = 2 \end{aligned}$$

The answer is (B).

MATERIALS SCIENCE–16

How many atoms are there per unit cell for a face-centered cubic structure?

(A) one (B) two (C) three (D) four



Like a body-centered cubic structure, there is $^1/_8$ of an atom at each corner of the cube. There is also $^1/_2$ of an atom at the center of each of the six faces, since each atom here is shared by the neighboring unit cell.

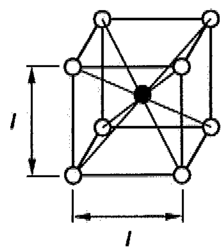
$$\begin{aligned}\text{total no. of atoms} &= (\text{no. of atoms at each corner})(\text{no. of corners}) \\ &\quad + (\text{no. of atoms at center of each face})(\text{no. of faces}) \\ &= \left(\frac{1}{8}\right)(8) + \left(\frac{1}{2}\right)(6) = 4\end{aligned}$$

The answer is (D).

MATERIALS SCIENCE–17

What is the first coordination number of a body-centered cubic structure?

(A) 4 (B) 6 (C) 8 (D) 10



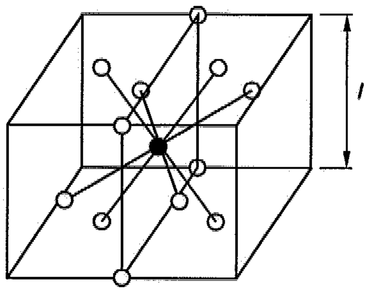
The first coordination number is the number of nearest neighbor atoms. In a body-centered cubic cell of edge length l, the minimum distance between atoms is $(\sqrt{3}/2)l$. By inspection of the figure, there are eight neighboring atoms at this distance.

The answer is (C).

MATERIALS SCIENCE–18

What is the first coordination number of a face-centered cubic structure?

(A) 2 (B) 4 (C) 8 (D) 12



The closest atoms in a face-centered cubic cell of edge length l are $l/\sqrt{2}$ apart. Each atom in the center of a face has 12 such neighboring atoms. The atoms are: $(\pm(1/2)l, \pm(1/2)l, 0)$, $(\pm(1/2)l, 0, \pm(1/2)l)$, and $(0, \pm(1/2)l, \pm(1/2)l)$.

The answer is (D).

MATERIALS SCIENCE–19

Which of the following statements is FALSE?

(A) Both copper and aluminum have a face-centered cubic crystal structure.
(B) Both magnesium and zinc have a hexagonal close-packed crystal structure.
(C) Iron can have either a face-centered or a body-centered cubic crystal structure.
(D) Both lead and cadmium have a hexagonal close-packed crystal structure.

Lead does not have a hexagonal close-packed structure. Its structure is face-centered cubic.

The answer is (D).

MATERIALS SCIENCE–20

Which of the following statements is FALSE?

(A) The coordinates of the unique lattice points for a body-centered cubic unit cell are: (0 0 0) and ($\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$).

(B) The coordinates of the unique lattice points for a face-centered cubic unit cell are: (0 0 0); ($\frac{1}{2}$ $\frac{1}{2}$ 0); ($\frac{1}{2}$ 0 $\frac{1}{2}$); and (0 $\frac{1}{2}$ $\frac{1}{2}$).

(C) The coordinates of the unique lattice points for a simple cubic unit cell are: (0 0 0).

(D) The coordinates of the unique lattice points for a rhombohedral unit cell are: ($\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$).

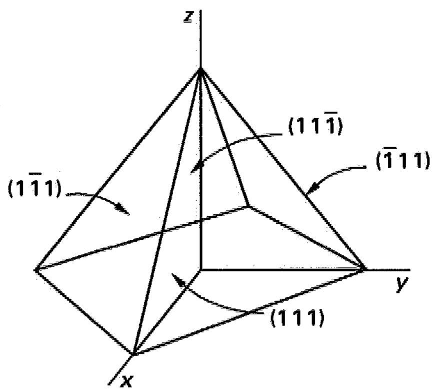
The rhombohedral Bravais lattice is a primitive cell and has only the point (0 0 0).

The answer is (D).

MATERIALS SCIENCE–21

How are the close-packed planes in a face-centered cubic metal designated?

(A) (1 0 0) (B) (2 0 0) (C) (1 1 0) (D) (1 1 1)



The close-packed planes are as shown.

The answer is (D).

MATERIALS SCIENCE–22

Which crystal structure possesses the highest number of close-packed planes and close-packed directions?

(A) simple cubic
(B) body-centered cubic
(C) face-centered cubic
(D) close-packed hexagonal

The face-centered cubic structure has four close-packed planes: (1 1 1), $(\bar{1}\ 1\ 1)$, $(1\ \bar{1}\ 1)$, and $(1\ 1\ \bar{1})$. Each plane has three close-packed directions.

The answer is (C).

MATERIALS SCIENCE–23

What are the most common slip planes for face-centered cubic and body-centered cubic structures, respectively?

(A) face-centered: (1 1 1); body-centered: (1 1 0)
(B) face-centered: (1 0 0); body-centered: (1 1 0)
(C) face-centered: (1 1 0); body-centered: (1 1 1)
(D) face-centered: (1 1 1); body-centered: (1 0 0)

Slip planes are usually the most closely packed planes, since they have the largest spacing. The close-packed planes are (1 1 1) and (1 1 0) for the respective crystal structures.

The answer is (A).

MATERIALS SCIENCE–24

Comparing the face-centered cubic lattice with the hexagonal close-packed lattice, which of the following features describes the hexagonal close-packed structure only?

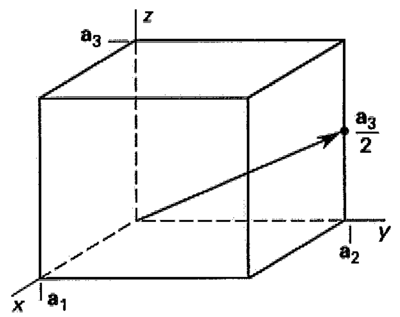
(A) It has the closest packed lattice structure.
(B) Its coordination number is 12.
(C) Its deformation properties are more directional.
(D) Its stacking order is ABCABC.

Options (A) and (B) are true for both face-centered cubic and hexagonal close-packed structures, while option (D) is true for the face-centered cubic lattice. Option (C) applies to the hexagonal close-packed lattice only.

The answer is (C).

MATERIALS SCIENCE–25

In the following unit cell, what direction is indicated by the arrow?



(A) (0 1 2) (B) $(0\ 1\ \frac{1}{2})$ (C) (2 1 0) (D) $(0\ \frac{1}{2}\ 1)$

Direction is given by the intercepts, as ratios of the lattice dimension.

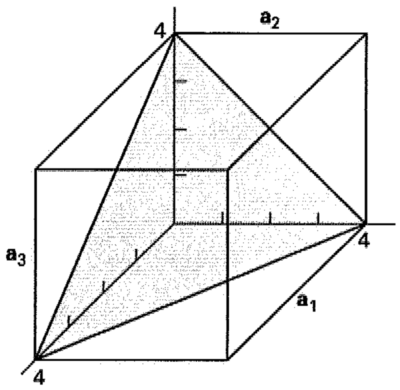
$$\left(\frac{x}{a_1}\ \frac{y}{a_2}\ \frac{z}{a_3}\right)$$

$$\left(\frac{0}{a_1}\ \frac{a_2}{a_2}\ \frac{\frac{a_3}{2}}{a_3}\right) = (0\ 1\ \tfrac{1}{2})$$

The answer is (B).

MATERIALS SCIENCE–26

What are the Miller indices of the given plane?



(A) (4 4 4) (B) (1 1 1) (C) $(\frac{1}{4}\ \frac{1}{4}\ \frac{1}{4})$ (D) (2 2 2)

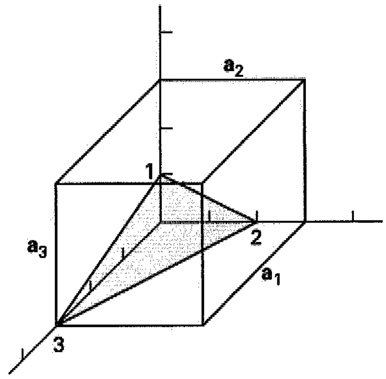
The x, y, and z intercepts are a_1, a_2, and a_3, respectively, since $a_1 = a_2 = a_3 = 4$. The Miller indices are

$$\left(\left(\frac{x}{a_1}\right)^{-1} \left(\frac{y}{a_2}\right)^{-1} \left(\frac{z}{a_3}\right)^{-1}\right) = \left(\left(\frac{4}{a_1}\right)^{-1} \left(\frac{4}{a_2}\right)^{-1} \left(\frac{4}{a_3}\right)^{-1}\right) = (1\ 1\ 1)$$

The answer is (B).

MATERIALS SCIENCE–27

What are the Miller indices of the given plane?



(A) (3 2 1) (B) $(\frac{1}{3}\ \frac{1}{2}\ 1)$ (C) $(\frac{2}{6}\ \frac{3}{6}\ \frac{6}{6})$ (D) (2 3 6)

The intercepts for this plane are $\mathbf{a}_1$, $2\mathbf{a}_2/3$, and $\mathbf{a}_3/3$. The Miller indices are

$$\left(\left(\frac{\mathbf{a}_1}{\mathbf{a}_1}\right)^{-1}\left(\frac{\frac{2\mathbf{a}_2}{3}}{\mathbf{a}_2}\right)^{-1}\left(\frac{\frac{\mathbf{a}_3}{3}}{\mathbf{a}_3}\right)^{-1}\right) = (1\ \tfrac{3}{2}\ 3) = (2\ 3\ 6)$$

The answer is (D).

MATERIALS SCIENCE–28

A plane intercepts the coordinate axis at $x = 1$, $y = 3$, and $z = 2$. What are the Miller indices of the plane?

(A) (1 3 2) (B) (1 2 3) (C) (6 2 3) (D) (3 2 6)

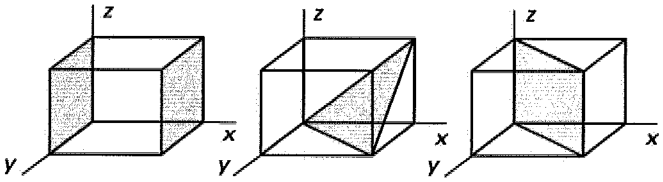
The Miller indices are computed by taking the reciprocal of each intercept and converting to whole numbers of the same ratio.

$$(1\ \tfrac{1}{3}\ \tfrac{1}{2}) = (6\ 2\ 3)$$

The answer is (C).

MATERIALS SCIENCE–29

Which of the following gives the correct designations for the planes shown?



I. II. III.

(A) I: (1 1 1), II: (2 0 0), III: (1 1 0)
(B) I: (1 0 0), II: (1 1 0), III: (1 1 1)
(C) I: (1 0 0), II: (1 1 1), III: (1 0 2)
(D) I: (1 0 0), II: (1 1 1), III: (1 1 0)

I. Since the plane passes through the origin, move the plane to $x = 1$, $y = \infty$, $z = \infty$. Take reciprocals of the intercepts.

$$\frac{1}{x} = 1, \ \frac{1}{y} = 0, \ \frac{1}{z} = 0 \quad (1\ 0\ 0)$$

II. Since the plane passes through the origin, move the plane to $x = -1$, $y = 1$, $z = 1$. Take reciprocals of the intercepts.

$$\frac{1}{x} = -1, \ \frac{1}{y} = 1, \ \frac{1}{z} = 1 \quad (-1\ 1\ 1)$$

(−1 1 1), but (−1 1 1) is identical to (1 1 1).

III. Since the plane passes through the origin, move the plane to $x = -1$, $y = 1$, $z = \infty$. Take reciprocals of the intercepts.

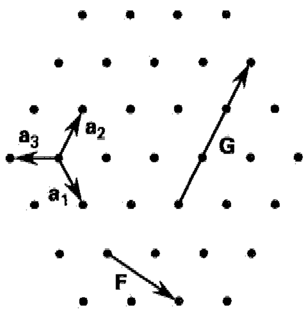
$$\frac{1}{x} = -1, \ \frac{1}{y} = 1, \ \frac{1}{z} = 0 \quad (-1\ 1\ 0)$$

(−1 1 0) is identical to (1 1 0).

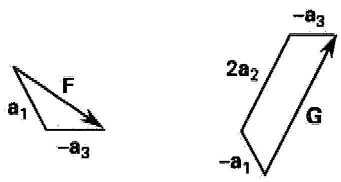
The answer is (D).

MATERIALS SCIENCE–30

Using the four-index scheme for a hexagonal crystal system, how would the directions **F** and **G** shown be defined?



(A) $\mathbf{F} = [\bar{1}\ 0\ 1\ 0]$, $\mathbf{G} = [1\ \bar{2}\ 1\ 0]$
(B) $\mathbf{F} = [0\ 1\ 1\ 0]$, $\mathbf{G} = [0\ 3\ 0\ 0]$
(C) $\mathbf{F} = [0\ \bar{1}\ 1\ 0]$, $\mathbf{G} = [\bar{1}\ 3\ \bar{2}\ 0]$
(D) $\mathbf{F} = [1\ 0\ \bar{1}\ 0]$, $\mathbf{G} = [\bar{1}\ 2\ \bar{1}\ 0]$



The **F** vector is the sum of one unit in the positive $\mathbf{a}_1$ direction and one unit in the negative $\mathbf{a}_3$ direction. The **G** vector is the sum of one unit in the negative $\mathbf{a}_1$ direction, two units in the positive $\mathbf{a}_2$ direction, and one unit in the negative $\mathbf{a}_3$ direction.

The answer is (D).

MATERIALS SCIENCE–31

Given that **a** is a lattice constant and that h, k, and l are the Miller indices, which of the following equations describes the interplanar distance d in a cubic crystal?

(A) $d = \dfrac{2\mathbf{a}}{\sqrt{\left(\frac{1}{h}\right)^2 + \left(\frac{1}{k}\right)^2 + \left(\frac{1}{l}\right)^2}}$

(B) $d = \mathbf{a}\left(\frac{1}{h} + \frac{1}{k} + \frac{1}{l}\right)$

(C) $d = \left(\frac{\mathbf{a}}{2}\right)\sqrt{h^2 + k^2 + l^2}$

(D) $d = \dfrac{\mathbf{a}}{\sqrt{h^2 + k^2 + l^2}}$

Geometrically, $1/d^2 = (h^2 + k^2 + l^2)/\mathbf{a}^2$. Therefore,

$$d = \frac{\mathbf{a}}{\sqrt{h^2 + k^2 + l^2}}$$

The answer is (D).

MATERIALS SCIENCE–32

The atomic weight of copper is 63.5 g/mol. Calculate the theoretical density of copper given that the unit cell is face-centered cubic and the lattice parameter is 3.61 Å.

(A) 4.5 g/cm^3 (B) 7.9 g/cm^3 (C) 8.8 g/cm^3 (D) 9.0 g/cm^3

There are four atoms per unit cell for a face-centered cubic structure. By definition,

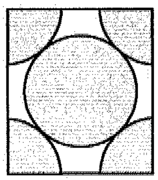
$$\begin{aligned}\rho &= \frac{m}{V}\\ &= \frac{\left(4\ \frac{\text{atoms}}{\text{unit cell}}\right)\left(63.5\ \frac{\text{g}}{\text{mol}}\right)\left(\frac{1\ \text{mol}}{6.02\times 10^{23}\ \text{atoms}}\right)}{\left((3.61\ \text{Å})\left(\frac{1\times 10^{-8}\ \text{cm}}{\text{Å}}\right)\right)^3}\\ &= 8.97\ \text{g/cm}^3 \quad (9.0\ \text{g/cm}^3)\end{aligned}$$

The answer is (D).

MATERIALS SCIENCE–33

Determine the planar density of copper atoms in a (1 0 0) plane given that the unit cell is face-centered cubic and the lattice parameter is 3.61 Å.

(A) 7.68×10^{18} atoms/m^2
(B) 1.53×10^{19} atoms/m^2
(C) 2.30×10^{19} atoms/m^2
(D) 3.84×10^{19} atoms/m^2



There are two atoms total in the (1 0 0) plane. The planar density is, therefore,

$$\text{planar density} = \frac{\text{no. atoms per face}}{\text{area of face}}$$

$$= \frac{2 \text{ atoms}}{(3.61 \times 10^{-10} \text{ m})^2} = 1.53 \times 10^{19} \text{ atoms/m}^2$$

The answer is (B).

MATERIALS SCIENCE–34

Which of the following statements is FALSE regarding X-ray diffraction?

(A) The geometrical structure factor F(hkl) is the ratio of the amplitude of the X-ray reflected from a plane in a crystal to the amplitude of the X-ray scattered from a single electron.

(B) X-ray diffraction is only useful for studying simpler crystals such as the body-centered cubic structure, rather than more complex crystals like the hexagonal close-packed structure.

(C) X-ray diffraction can be used to determine the grain size of a specimen.

(D) Bragg's law states that $n\lambda/2d = \sin\theta$ (n is an integer, λ is the wavelength of the X-ray, d is the interplanar spacing, and θ is the scattering angle).

X-ray diffraction is used to study all types of crystals. It is not limited to simple crystals.

The answer is (B).

MATERIALS SCIENCE–35

A sample of face-centered cubic nickel (Ni) was placed in an X-ray beam of wavelength $\lambda = 0.154$ nm. If the lattice parameter for Ni is $a_0 = 0.352$ nm, what is the first-order angle of diffraction most nearly?

(A) 5.7° (B) 7.0° (C) 13° (D) 19°

Using Bragg's law, with $n = 1$, $\lambda = 0.154$ nm, and $d = 0.352$ nm,

$$\lambda = 2d \sin\theta$$

$$\theta = \sin^{-1}\frac{\lambda}{2d} = \sin^{-1}\frac{0.154 \text{ nm}}{(2)(0.352 \text{ nm})}$$

$$= 12.6°$$

The answer is (C).

MATERIALS SCIENCE–36

In a crystal structure, what is an interstitial atom?

(A) an extra atom sitting at a nonlattice point
(B) an atom missing at a lattice point
(C) a different element at a lattice point
(D) a line defect

An interstitial atom is an extra atom lodged within the crystal structure; it is a point defect.

The answer is (A).

MATERIALS SCIENCE–37

Which of the following is a line defect in a lattice crystal structure?

(A) tilt boundary
(B) screw dislocation
(C) vacancy
(D) Schottky imperfection

The most common type of line defect is a dislocation.

The answer is (B).

MATERIALS SCIENCE–38

It is often desired to know the number of atoms, n, in a crystal structure that possess more than a specified amount of energy, E. Which of the following equations gives n, given that N is the total number of atoms present, M is a constant, k is the Boltzmann constant, and T is the temperature of the specimen?

(A) $n = \frac{M}{N}e^{-kE/T}$
(B) $n = \frac{EM}{N}e^{-kT}$
(C) $n = MNe^{-E/kT}$
(D) $n = MNe^{-kT/E}$

The equation in option (C) is the correct relationship for thermal energy distribution within a specimen.

The answer is (C).

MATERIALS SCIENCE–39

Which of the following statements regarding diffusion in a crystal structure is true?

(A) Solid interstitial atoms cannot diffuse through structures that lack vacancies.
(B) It occurs only in alloys, never in pure crystals.
(C) It often uses an exchange or vacancy mechanism.
(D) It occurs primarily as a result of mechanical work.

Diffusion is the movement of a defect from one point to another.

The answer is (C).

MATERIALS SCIENCE–40

What is Fick's first law for one-dimensional, steady-state diffusion? C is the volume concentration of atoms, x is the distance along which diffusion occurs, D is the diffusion coefficient, and J is the flux or current density.

(A) $J = -D\frac{\partial C}{\partial x}$ (B) $J = C\frac{\partial D}{\partial x}$ (C) $J = \left(-\frac{1}{D}\right)\frac{\partial C}{\partial x}$ (D) $J = 2D\frac{\partial C}{\partial x}$

The flux is proportional to the diffusion constant and the concentration gradient $\partial C/\partial x$.

The answer is (A).

MATERIALS SCIENCE–41

Which of the following are true about Fick's first law for diffusion?

I. It is only applicable to gases and liquids, not solids.
II. The law states that the flux moves from high to low concentration.
III. J, the flux, may be in units of $cm^3/cm^2{\cdot}s$.

(A) I only (B) II only (C) III only (D) II and III

Fick's law says that the flux of diffusion is proportional to the negative volume concentration gradient; the negative sign indicates that the flux is in the down-gradient direction. It applies to diffusion in a crystal. Flux, by definition, is the amount of volume moving across a unit surface area in unit time.

The answer is (D).

MATERIALS SCIENCE–42

What is the Arrhenius equation for the rate of a thermally activated process? A is the reaction constant, T is the absolute temperature, R is the gas constant, and Q is the activation energy.

(A) rate $= Ae^{-Q/RT}$ (B) rate $= Ae^{-QRT}$ (C) rate $= Ae^{Q/RT}$ (D) rate $= Ae^{QRT}$

The rate increases as the thermal energy increases.

The answer is (A).

MATERIALS SCIENCE–43

Which of the following statements is FALSE?

(A) The surface energy of a liquid tends toward a minimum.

(B) The surface energy is the work required to create a unit area of additional space.

(C) The energy of an interior atom is greater than the energy of an atom on the surface of a liquid.

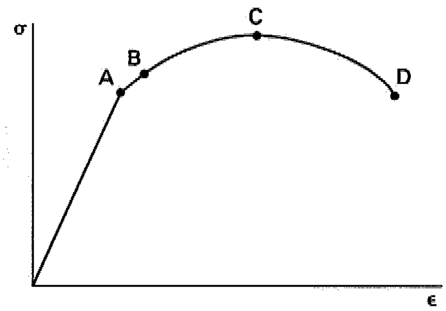
(D) Total surface energy is directly proportional to the surface area.

In a liquid, the energy of a surface atom is greater than the surface energy of an interior atom. Note: Although surface energy and surface tension have the same numerical value, they have different units.

The answer is (C).

MATERIALS SCIENCE–44

Which point on the stress-strain curve shown gives the ultimate stress?



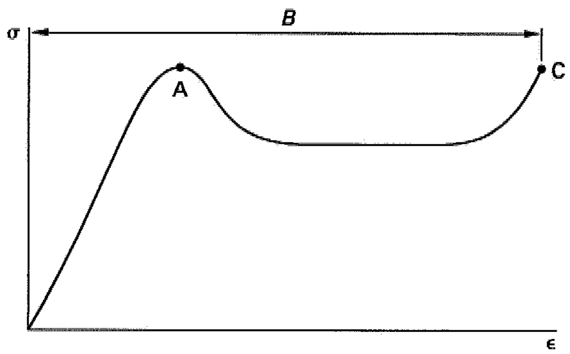
(A) A (B) B (C) C (D) D

The ultimate stress corresponds to the point of maximum load, beyond which further strain is accompanied by a reduction in load.

The answer is (C).

MATERIALS SCIENCE–45

A stress-strain diagram for a polymer is shown. Identify items A, *B*, and C.



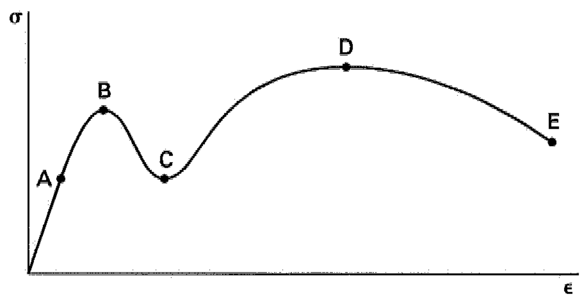
(A) A = lower yield point; *B* = plastic deformation; C = upper yield point
(B) A = lower yield point; *B* = proportional limit; C = upper yield point
(C) A = yield point; *B* = elastic deformation; C = elastic limit
(D) A = yield point; *B* = elongation at fracture; C = fracture point

Beginning at the yield point, considerable elongation occurs with no noticeable increase in tensile stress. Eventually, fracture occurs. The total strain at fracture is known as the elongation.

The answer is (D).

MATERIALS SCIENCE–46

Which statement is true for the stress-strain relationship for the metal shown?



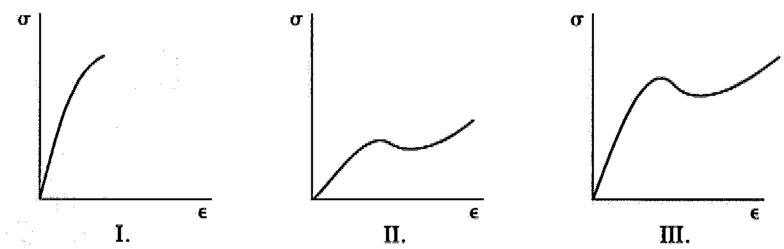
(A) Point A is the lower yield point.
(B) Point D is the fracture stress point.
(C) Point B is the upper yield point.
(D) The range from point C to point D is known as the elastic range.

Point A is the elastic limit, and point D is the ultimate stress point. The region between points C and D is not the elastic region but the plastic region. Only option (C) is true.

The answer is (C).

MATERIALS SCIENCE–47

Identify the properties of the materials whose stress-strain diagrams are shown.



(A) I: soft and weak; II: soft and tough; III: hard and brittle
(B) I: hard and brittle; II: soft and weak; III: hard and tough
(C) I: soft and tough; II: hard and brittle; III: hard and strong
(D) I: hard and strong; II: soft and brittle; III: soft and tough

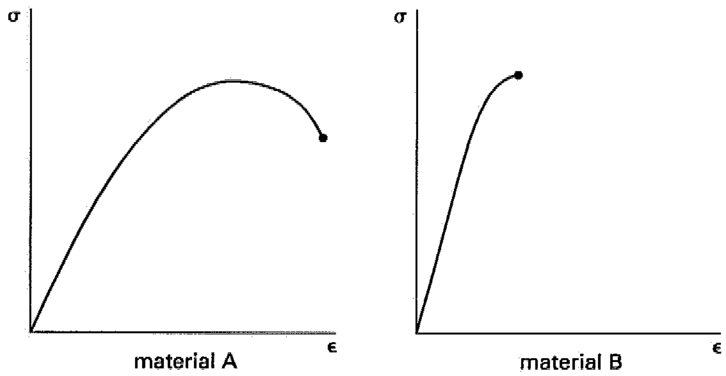
The properties and their relationships to the stress-strain diagrams are given in the following table.

	elastic modulus	yield point	elongation at fracture	ultimate strength
hard and brittle	high	undefined	low	moderate to high
soft and weak	low	low	moderate	low
hard and tough	high	high	high	high

The answer is (B).

MATERIALS SCIENCE–48

Which statement is most accurate regarding the two materials represented in the given stress-strain diagrams?



(A) Material B is more ductile and has a lower modulus of elasticity than material A.

(B) Material B would require more total energy to fracture than material A.

(C) Material A will withstand more stress before plastically deforming than material B.

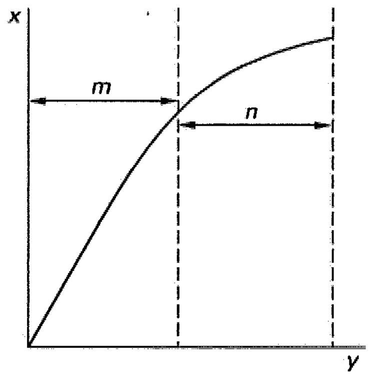
(D) Material B will withstand a higher load than material A but is more likely to fracture suddenly.

From the graphs, the modulus of elasticity of material B is greater than that of material A. This means that material A is more ductile, that is, it can undergo more strain before fracturing. However, material B can withstand higher loads than material A. Only option (D) is correct.

The answer is (D).

MATERIALS SCIENCE-49

If the diagram below represents deformation of rigid bodies, what do x, y, m, and n refer to?



(A) x = stress, y = strain, m = plastic deformation, n = elastic deformation
(B) x = strain, y = stress, m = plastic deformation, n = elastic deformation
(C) x = stress, y = strain, m = elastic deformation, n = plastic deformation
(D) x = strain, y = stress, m = elastic deformation, n = plastic deformation

Option (C) is the only choice that fits the graph.

The answer is (C).

MATERIALS SCIENCE–50

Which of the following best describes the 0.2% offset yield stress?

(A) It is the elastic limit after which a measurable plastic strain has occurred.
(B) It is the stress at which the material plastically strains 0.2%.
(C) It is the stress at which the material elastically strains 0.2%.
(D) It is 0.2% below the fracture point of the material.

By definition, the offset yield stress is where the material undergoes a 0.2% plastic strain.

The answer is (B).

MATERIALS SCIENCE–51

Which of the following is true regarding the ductile-to-brittle transition temperature?

I. It is important for structures used in cold environments.
II. It is the point at which the size of the shear lip or tearing rim goes to zero.
III. It is the temperature at which 20 J of energy causes failure in a Charpy v-notch specimen of standard dimensions.

(A) I only (B) I and II (C) I and III (D) II and III

II is the only choice that is false. A test piece broken at 20 J of energy usually has a small shear lip.

The answer is (C).

MATERIALS SCIENCE–52

Which of the following are true regarding creep?

I. It is caused by the diffusion of vacancies to edge dislocations, permitting dislocation climb.
II. It involves the plastic deformation of materials at loads below the yield stress.
III. It may involve whole grain sliding.

(A) I only (B) II only (C) I and III (D) I, II, and III

All are true.

The answer is (D).

MATERIALS SCIENCE–53

Under conditions of very slow deformation and high temperature, it is possible to have plastic flow in a crystal at shear stresses lower than the critical shear stress. What is this phenomenon called?

(A) slip (B) twinning (C) creep (D) bending

Creep involves the flow of material.

The answer is (C).

MATERIALS SCIENCE–54

What does the Charpy impact test measure?

I. the energy required to break a test sample
II. the strength of a test sample
III. the ductile to brittle transition temperature of metals

(A) I only (B) II only (C) III only (D) I and III

The Charpy test measures toughness, the energy required to break a sample. By conducting the test at different temperatures, the brittle transition temperature can be determined.

The answer is (D).

MATERIALS SCIENCE–55

A shaft made of good quality steel breaks in half due to fatigue. What would the surface of the fracture site look like?

(A) like a cup and cone

(B) quite smooth to the unaided eye, with ripples apparent under low-power magnification

(C) smooth over most of the surface, with tearing at the location of fracture

(D) very jagged and rough

Typically, the surface is mostly smooth. Where final fracture took place however, the surface is torn.

The answer is (C).

MATERIALS SCIENCE–56

To which of the following can the large discrepancy between the actual and theoretical strengths of metals mainly be attributed?

(A) heat (B) dislocations (C) low density (D) stress direction

Although point defects do contribute to the discrepancy in strengths, the major reason for the difference is the presence of dislocations.

The answer is (B).

MATERIALS SCIENCE–57

The ease with which dislocations are able to move through a crystal under stress accounts for which of the following?

I. ductility
II. lower yield strength
III. hardness

(A) I only (B) II only (C) III only (D) I and II

The ease with which dislocations move through a crystal accounts for its ductility and lower yield strength.

The answer is (D).

MATERIALS SCIENCE–58

As the amount of slip increases, additional deformation becomes more difficult and decreases until the plastic flow finally stops. Slip may begin again only if a larger stress is applied. What is this phenomenon known as?

(A) cooling (B) crowding (C) strain hardening (D) twinning

This is known as strain hardening.

The answer is (C).

MATERIALS SCIENCE–59

Which word combination best completes the following sentence?

"Plastic deformation of a single crystal occurs either by ______ or by ______, but ______ is the more common method."

(A) bending; compression; bending
(B) shearing; compression; compression
(C) slip; twinning; slip
(D) twinning; slip; twinning

Bending, compression, and shear are elastic phenomena. Slip is a more common method of plastic deformation than twinning.

The answer is (C).

MATERIALS SCIENCE–60

Which one of these statements is true for twinning?

(A) It occurs at lower shear stresses than slip.
(B) It is the most significant form of plastic deformation.
(C) It cannot be caused by impact or thermal treatment.
(D) It frequently occurs in hexagonal close-packed structures.

Options (A), (B), and (C) are false. Twinning requires a relatively high shear stress, is much less common than slip, and can be caused by impact or thermal treatment. It occurs in hexagonal close-packed crystal structures.

The answer is (D).

MATERIALS SCIENCE–61

Which of the following does NOT produce vacancies, interstitial defects, or impurity defects in a material?

(A) plastic deformation
(B) slow equilibrium cooling
(C) quenching
(D) increasing the temperature (which increases atomic energy)

Slow equilibrium cooling is used to reduce variations in the material.

The answer is (B).

MATERIALS SCIENCE–62

Which of the following are true statements about the modulus of elasticity, E?

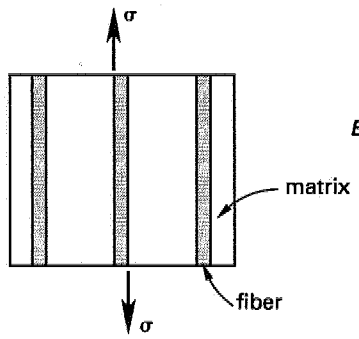
(A) It is the same as the rupture modulus.
(B) It is the slope of the stress-strain diagram in the linearly elastic region.
(C) It is the ratio of stress to volumetric strain.
(D) Its value depends only on the temperature of the material.

The modulus of elasticity is equal to the ratio of stress to strain for a particular material. It is the slope of the stress-strain diagram in the linearly elastic region.

The answer is (B).

MATERIALS SCIENCE–63

What is the modulus of elasticity, E, for a composite material in which the fibers take up 20% of the total volume and the load is applied parallel to the fibers as shown?



(A) 2.76×10^{10} Pa (B) 2.95×10^{10} Pa (C) 1.38×10^{11} Pa (D) 3.45×10^{11} Pa

The matrix and fibers experience the same strain, ϵ. The total stress, σ, is the sum of the stresses carried by the fibers and the matrix.

$$\sigma = E_f \epsilon V_f + E_m \epsilon (1 - V_f)$$

V_f is the fraction of the total volume taken up by the fibers. Thus,

$$\begin{aligned} E &= \frac{\sigma}{\epsilon} = E_f V_f + E_m(1 - V_f) \\ &= (1.38 \times 10^{11} \text{ Pa})(0.2) + (3.45 \times 10^6 \text{ Pa})(1 - 0.2) \\ &= 2.76 \times 10^{10} \text{ Pa} \end{aligned}$$

The answer is (A).

MATERIALS SCIENCE–64

What is the proper relationship between the modulus of elasticity, E, the Poisson ratio, ν, and the bulk modulus of elasticity, K?

(A) $E = K(1 - 2\nu)$ (B) $E = K(1 - \nu)$

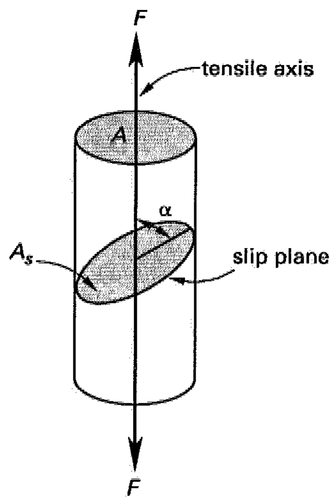
(C) $E = \dfrac{3K}{1 - 2\nu}$ (D) $E = 3K(1 - 2\nu)$

For an element in triaxial stress, the unit volume change can be obtained from Hooke's law. The resultant equation is given by option (D).

The answer is (D).

MATERIALS SCIENCE–65

A crystal is subjected to a tensile load acting along its axis. α is the angle between the tensile axis and the slip plane as shown. At what value of α will the shear stress in the slip plane be a maximum?



(A) 0° (B) 30° (C) 45° (D) 60°

The component of force along the shear surface is equal to $F\cos\alpha$. The area of the shear surface, A_s, is related to the cross-sectional area, A, by $A_s = A/\sin\alpha$.

$$\tau = \frac{F\cos\alpha}{\frac{A}{\sin\alpha}} = \left(\frac{F}{A}\right)\sin\alpha\cos\alpha$$

Taking the first derivative and setting it equal to zero,

$$\frac{\partial\tau}{\partial\alpha} = \left(\frac{F}{A}\right)\left(\cos^2\alpha - \sin^2\alpha\right) = 0$$

$$\cos^2\alpha - \sin^2\alpha = 0$$

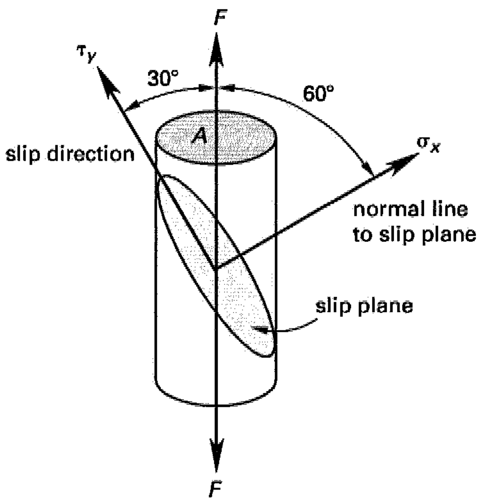
$$\cos\alpha = \sin\alpha$$

$$\alpha = 45°$$

The answer is (C).

MATERIALS SCIENCE–66

An axial stress $\sigma_x = F/A$ is applied as shown. Calculate the resolved shear stress, τ_y, along the slip plane.



(A) $\tau_y = \frac{1}{4}\sigma_x$ (B) $\tau_y = \frac{1}{2}\sigma_x$ (C) $\tau_y = \frac{\sqrt{2}}{3}\sigma_x$ (D) $\tau_y = \frac{3}{4}\sigma_x$

$$\tau_y = \left(\frac{F}{A}\right)\sin 60°\cos 30°$$

$$= \sigma_x\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \tfrac{3}{4}\sigma_x$$

The answer is (D).

MATERIALS SCIENCE–67

If G is the shear modulus, b is the magnitude of the Burgers vector, and r is half the distance between particles, what is the local stress, τ, required to bend dislocations around a particle?

(A) $\frac{Gb}{r}$ (B) Gbr (C) $\frac{br}{G}$ (D) $\frac{Gr}{b}$

Line tension is given by $\tau = 2T/bl$. $T = Gb^2$ and $l = 2r$. Therefore, $\tau = Gb/r$.

The answer is (A).

MATERIALS SCIENCE–68

Given that d is the distance between dislocations and b is the magnitude of the Burgers vector, what is the expression for the misorientation angle θ of a tilt boundary?

(A) $\sin\theta = \frac{d}{b}$ (B) $\tan\theta = \frac{b}{d}$ (C) $\theta = \frac{b}{d}$ (D) $\theta = \frac{d}{b}$

By definition, $\tan\theta = b/d$.

The answer is (B).

MATERIALS SCIENCE–69

In general, what are the effects of cold working a metal?

(A) increased strength and ductility
(B) increased strength, decreased ductility
(C) decreased strength and ductility
(D) decreased strength, increased ductility

The strength of the metal will increase at the expense of a loss in ductility.

The answer is (B).

MATERIALS SCIENCE–70

Which of the following does cold working a metal cause?

(A) elongation of grains in the flow direction, an increase in dislocation density, and an overall increase in energy of the metal

(B) elongation of grains in the flow direction, a decrease in dislocation density, and an overall decrease in energy of the metal

(C) elongation of grains in the flow direction, a decrease in dislocation density, and an overall increase in energy of the metal

(D) shortening of grains in the flow direction, a decrease in dislocation density, and an overall decrease in the energy of the metal

Cold working a metal produces elongations of grains coupled with increases in both dislocation density and energy.

The answer is (A).

MATERIALS SCIENCE–71

Which of the following statements is FALSE?

(A) The amount or percentage of cold work cannot be obtained from information about change in the area or thickness of a metal.

(B) The process of applying force to a metal at temperatures below the temperature of crystallization in order to plastically deform the metal is called cold working.

(C) Annealing eliminates most of the defects caused by the cold working of a metal.

(D) Annealing reduces the hardness of the metal.

The percentage of cold work can be calculated directly from the reduction in thickness or area of the metal.

The answer is (A).

MATERIALS SCIENCE–72

Which of the following statements is FALSE?

(A) There is a considerable increase in the hardness and the strength of a cold-worked metal.

(B) Cold working a metal significantly reduces its ductility.

(C) Cold working causes a slight decrease in the density and electrical conductivity of a metal.

(D) Cold work decreases the yield point of metal.

Cold working increases the yield point as well as the strength and hardness of metal.

The answer is (D).

MATERIALS SCIENCE–73

Which of the following statements is FALSE?

(A) Hot working can be regarded as the simultaneous combination of cold working and annealing.

(B) Hot working increases the density of the metal.

(C) One of the primary goals of hot working is to produce a fine-grained product.

(D) Hot working causes much strain hardening of the metal.

In hot working, the high temperature immediately releases any strain hardening that could occur in the deformation of the metal.

The answer is (D).

MATERIALS SCIENCE–74

Which of the following is FALSE?

(A) Grain size is of minor importance in considering the properties of polycrystalline materials.

(B) Fine-grained materials usually exhibit greater yield stresses than coarse-grained materials at low temperatures.

(C) At high temperatures, grain boundaries become weak, and sliding occurs.

(D) Grain boundary sliding is the relative movement of two grains by a shear movement parallel to the grain boundary between them.

Grain size is an important factor to consider in understanding the properties of polycrystalline materials because it affects the area and length of the grain boundaries.

The answer is (A).

MATERIALS SCIENCE–75

Which of the following correctly describes atoms located at grain boundaries?

(A) They are subjected to the same type of interatomic forces that are present in the interior atoms of the crystal.

(B) They are located primarily in highly strained and distorted positions.

(C) They have a higher free energy than atoms in the undisturbed part of the crystal lattice.

(D) All of the above are correct.

All are correct statements regarding atoms at the grain boundary.

The answer is (D).

MATERIALS SCIENCE–76

What causes the vinyl interiors of automobiles to crack when subjected to prolonged direct sunlight?

(A) the volatilization (evaporation) of plasticizers
(B) repetitive expansion and contraction of the plastic
(C) oxidation of the plastic by sunlight and oxygen
(D) all of the above

All of the statements are true.

The answer is (D).

MATERIALS SCIENCE–77

Low-density polyethylene undergoes extensive (over 100%) elongation prior to rupture, while polystyrene undergoes only 1–2% elongation. What is the main reason for this difference?

(A) The polyethylene is less dense.
(B) The large styrene groups in the polystyrene prevent slippage.
(C) More cross-linking occurs in the polystyrene.
(D) Polyethylene is less crystalline.

Polystyrene has large styrene groups on the side of its carbon chain. These prevent slippage, making the polystyrene brittle.

> The answer is (B).

MATERIALS SCIENCE–78

Which of the following describe(s) the modulus of elasticity of an elastomer?

I. It is directly proportional to the number of cross links in the elastomer.
II. Its value increases with temperature.
III. It is directly proportional to the number of double bonds in the chemical structure.

(A) I only (B) II only (C) III only (D) I and II

Choice III is false, since a double bond prevents rotation along the bond, inhibiting elasticity.

> The answer is (D).

MATERIALS SCIENCE–79

Which statement(s) describe(s) the glass transition temperature?

I. It is the temperature at which the rate of volume contraction decreases abruptly.
II. It is the temperature at which residual stresses in the glass can be relieved.
III. It is the point where the material behaves more like a solid than a viscous liquid.

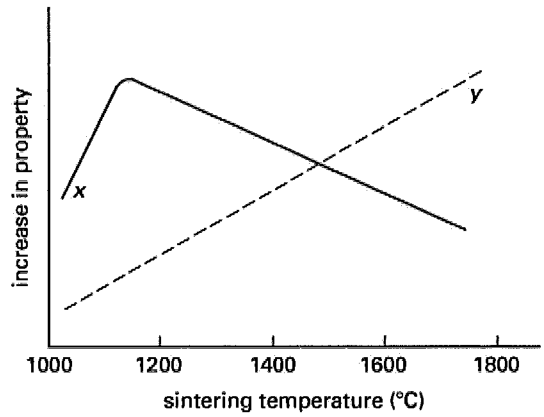
(A) I only (B) I and II (C) I and III (D) II and III

The glass transition temperature is the point at which the free movement of the glass molecules past each other becomes difficult. The glass begins to act like a solid, increasing in specific volume.

The answer is (C).

MATERIALS SCIENCE–80

If the following diagram represents the sintering of the ceramic MgO, what could the curves x and y refer to?



(A) x = grain size; y = porosity
(B) x = grain size; y = strength
(C) x = porosity; y = grain size
(D) x = strength; y = grain size

As the sintering temperature increases, the strength of a ceramic will increase first and then drop abruptly. The grain size will increase linearly with rising temperature.

The answer is (D).

MATERIALS SCIENCE–81

Of the following inorganic glasses, which have tetrahedral lattice structures?

$$SiO_2,\ B_2O_3,\ BeF_2,\ GeO_2$$

(A) SiO_2 and B_2O_3
(B) SiO_2 and BeF_2
(C) SiO_2, B_2O_3, and BeF_2
(D) SiO_2, BeF_2, and GeO_2

SiO_2, BeF_2, and GeO_2 have tetrahedral structures. B_2O_3 has an almost triangular structure.

The answer is (D).

MATERIALS SCIENCE–82

Which of the following is NOT an important criterion for forming a complete binary solid solution?

(A) The difference in radii should be less than 15%.
(B) The constituent elements must have the same crystal structure.
(C) The atoms should be close to one another in the periodic table.
(D) The difference in atomic numbers should be small.

All choices except option (D) are criteria for a binary solid solution.

The answer is (D).

MATERIALS SCIENCE–83

How can an ordered solid solution be distinguished from a compound?

(A) In an ordered solid solution, the solute atoms occupy interstitial positions within the lattice.
(B) The solute atoms in an ordered solid solution substitute for atoms in the parent lattice.
(C) The atoms in an ordered solid solution form layers in the lattice structure.
(D) When heated, an ordered solid solution becomes disordered before melting.

Unlike a compound, an ordered solid solution becomes disordered when heated.

The answer is (D).

MATERIALS SCIENCE–84

What is transformed in a eutectoid reaction?

(A) One liquid is transformed into two solids of different composition.
(B) A solid becomes a liquid at the eutectic temperature.
(C) A liquid becomes a solid at the solidus temperature.
(D) A solid becomes a liquid at the liquidus temperature.

In a eutectoid reaction, one liquid is transformed into two different solids.

The answer is (A).

MATERIALS SCIENCE–85

Which of the following is the correct representation of a eutectic cooling reaction? (The subscripts denote different compositions.)

(A) $(\text{liquid}) \longrightarrow (\text{solid})_1 + (\text{solid})_2$
(B) $(\text{solid})_1 + (\text{liquid}) \longrightarrow (\text{solid})_2$
(C) $(\text{solid})_1 \longrightarrow (\text{solid})_2 + (\text{solid})_3$
(D) $(\text{solid})_1 + (\text{solid})_2 \longrightarrow (\text{solid})_3$

A eutectic reaction is the transformation from one liquid phase to two solid phases.

The answer is (A).

MATERIALS SCIENCE–86

Two pieces of copper are brazed together using a eutectic alloy of copper and silver. The braze material melts at 780°C. If a second braze is attempted in order to attach another piece of copper, which of the following is true?

(A) The first braze will melt if the braze temperature is again 780°C.
(B) The braze temperature must be lowered below 780°C.
(C) The first braze will partially melt, causing the parts to slide.
(D) The first braze will not melt at 780°C, but the second braze will.

All compositions of copper and silver other than the eutectic will have a melting point higher than the eutectic temperature. The alloy of the first braze will dissolve somewhat into the copper pieces, changing its composition. It will not melt again at the second braze temperature of 780°C.

The answer is (D).

MATERIALS SCIENCE–87

On an alloy phase diagram, what is the solidus temperature?

(A) The point at which all solids completely reach the liquid stage.

(B) The temperature of the liquid phase at which the first solid forms for a given overall composition.

(C) The temperature of the solid phase at which the first liquid forms for a given overall composition.

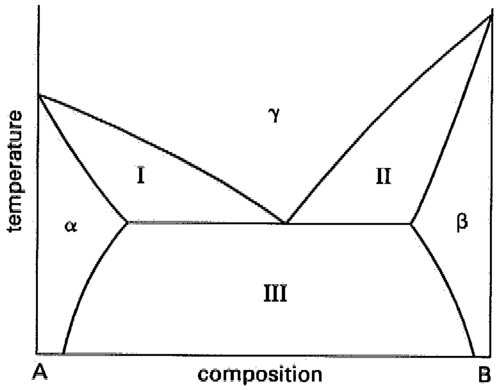
(D) The temperature at which the solid is at equilibrium.

The solidus temperature is the temperature at which liquid first forms.

The answer is (C).

MATERIALS SCIENCE–88

In this phase diagram, what can be said about the phases present in regions I, II, and III?



(A) α, β, and γ are present in region I.

(B) β and γ are present in region II.

(C) α, β, and γ are present in region III.

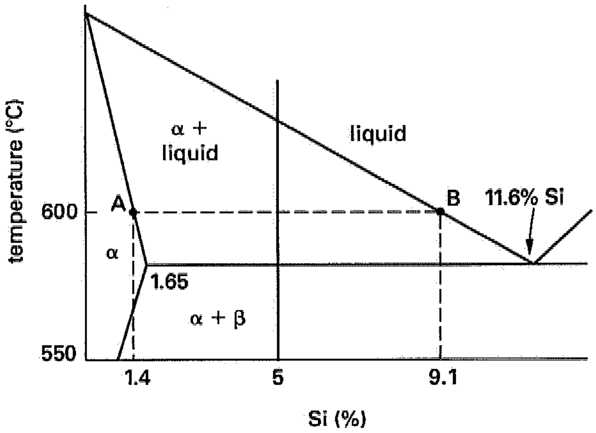
(D) α and γ are present in region III.

β and γ are present in region II. γ is not present in region III, nor is β present in region I.

The answer is (B).

MATERIALS SCIENCE–89

Given the following phase diagram, determine the percentage of liquid remaining at 600°C that results from the equilibrium cooling of an alloy containing 5% silicon and 95% aluminum.



(A) 0.0% (B) 47% (C) 53% (D) 67%

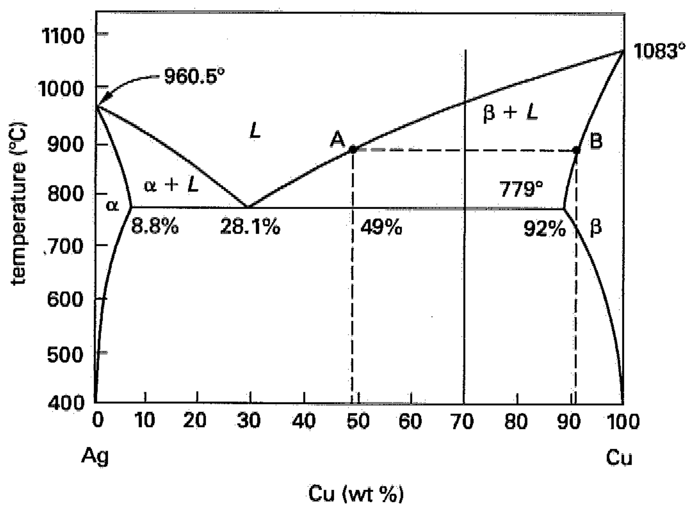
Use the lever rule. At point A there is 1.4% Si and no liquid, while at point B there is 9.1% Si and all liquid. Therefore,

$$\text{percent liquid} = \frac{5\% - 1.4\%}{9.1\% - 1.4\%} \times 100\% = 47\%$$

The answer is (B).

MATERIALS SCIENCE–90

Consider the Ag-Cu phase diagram given. Calculate the equilibrium amount of β in an alloy of 30% Ag, 70% Cu at 850°C.



(A) 0.0% (B) 22% (C) 49% (D) 52%

At 70% Cu, A = 49% Cu and B = 92% Cu.

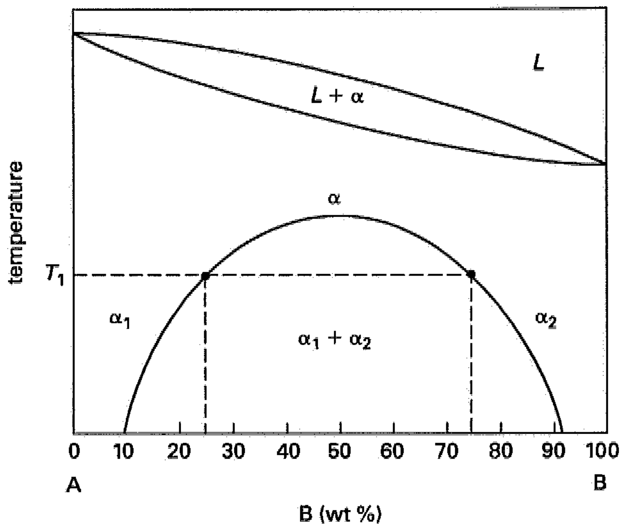
$$\text{percent } \beta = \frac{\%\text{ Cu in alloy} - \%\text{ Cu at point A}}{\%\text{ Cu at point B} - \%\text{ Cu at point A}} \times 100\%$$

$$= \frac{70\% - 49\%}{92\% - 49\%} \times 100\% = 49\%$$

The answer is (C).

MATERIALS SCIENCE–91

Using the given phase diagram, what are the relative weights of phases α_1 and α_2 for an alloy of 70% B at temperature T_1?



(A) 10% α_1, 90% α_2
(B) 30% α_1, 70% α_2
(C) 50% α_1, 50% α_2
(D) 70% α_1, 30% α_2

Let W_{α_1} denote the weight fraction of α_1 and W_{α_2} denote the weight fraction of α_2. From the diagram, $C_{\alpha_1} = 25\%$ and $C_{\alpha_2} = 75\%$. Then,

$$W_{\alpha_1} + W_{\alpha_2} = 1$$
$$W_{\alpha_1}C_{\alpha_1} + W_{\alpha_2}C_{\alpha_2} = C_0$$

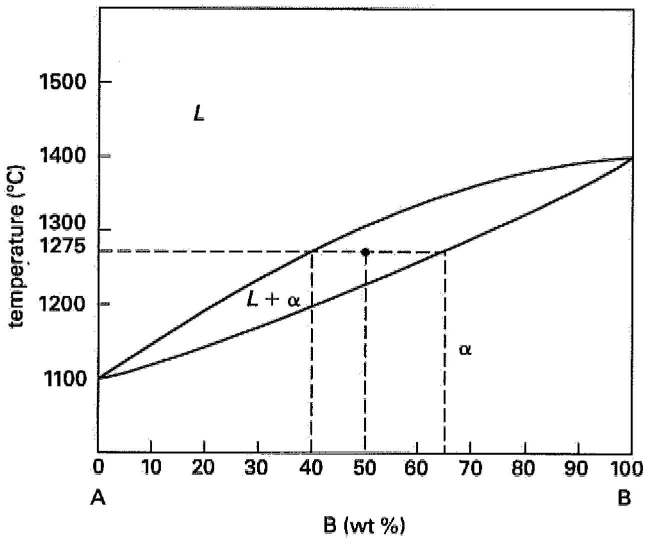
Solving the two equations using $C_0 = 70\%$,

$$W_{\alpha_1} = \frac{C_{\alpha_2} - C_0}{C_{\alpha_2} - C_{\alpha_1}} = \frac{75\% - 70\%}{75\% - 25\%} = 10\%$$

The answer is (A).

MATERIALS SCIENCE–92

For 50% B at 1275°C as shown, what is the relative amount of each phase present?



(A) 40% liquid, 60% solid
(B) 45% liquid, 55% solid
(C) 50% liquid, 50% solid
(D) 60% liquid, 40% solid

From the phase diagram, $C_\alpha = 65\%$ and $C_L = 40\%$. With C_0 given as 50%, and denoting the weight fraction of liquid and solid by W_L and W_α, respectively,

$$W_L + W_\alpha = 1$$
$$W_L C_L + W_\alpha C_\alpha = C_0$$

$$W_L = \frac{C_\alpha - C_0}{C_\alpha - C_L} = \frac{65\% - 50\%}{65\% - 40\%} = 60\%$$

The answer is (D).

MATERIALS SCIENCE–93

Which of the following is NOT a structural class of steels?

(A) carbon
(B) high-strength, low-alloy
(C) low-alloy
(D) tool and die

"Tool and die" steel is an application class, not a structural class.

The answer is (D).

MATERIALS SCIENCE–94

Which of the following phases of steel has a face-centered cubic structure?

(A) ferrite (B) cementite (C) pearlite (D) austenite

Only austenite has a face-centered cubic structure.

The answer is (D).

MATERIALS SCIENCE–95

Low-carbon steels are generally used in the "as rolled" or "as fabricated" state. What is the reason for this?

(A) They come in many different shapes and thicknesses.
(B) Their strength generally cannot be increased by heat treatment.
(C) They degrade severely under heat treatment.
(D) Their chromium content is so low.

Since their strength cannot be increased by heat treatment, low-carbon steels are used as fabricated.

The answer is (B).

MATERIALS SCIENCE–96

The equilibrium cooling of a steel containing 0.8% carbon results in a product with little use because it is extremely brittle. Which of the following is the primary reason for this poor characteristic?

(A) The material has not been cold worked.
(B) The austenite grains are too small, and the carbide grains are too large.
(C) Thick layers of iron carbide surround the coarse ferrite grains.
(D) The carbide forms thin plates that are brittle.

When hypereutectoid steels are slow cooled, brittle carbide plates are formed.

The answer is (D).

MATERIALS SCIENCE–97

Ductile cast iron and gray cast iron both contain 4% carbon. Ductile cast iron, however, has a higher tensile strength and is considerably more ductile. Which of the following is the major difference that accounts for the superior properties of the ductile iron?

(A) The gray cast iron contains iron carbide, whereas the ductile iron contains graphite.
(B) The gray cast iron contains flakes of graphite, whereas the ductile iron contains spheroids of graphite.
(C) The ductile iron is tempered to give better properties.
(D) The ferrite grains in the gray cast iron are excessively large.

Gray cast iron contains flakes of graphite while ductile cast iron contains spheroids. The difference in the shape of the graphite gives the ductile cast iron approximately twice the tensile strength and 20 times the ductility of the gray cast iron.

The answer is (B).

MATERIALS SCIENCE–98

In preparing a metallographic iron specimen, the grain boundaries are made most visible by which of the following steps?

(A) grinding the sample with silicon carbide abrasive
(B) polishing the sample with Al_2O_3
(C) mounting the sample in an epoxy resin mold
(D) etching the sample in a 2% solution of nitric acid in alcohol

Etching the specimen with nitric acid in alcohol dissolves metal from the surface and preferentially attacks the grain boundaries. It is the last step in the sample preparation process.

The answer is (D).

MATERIALS SCIENCE–99

Which of the following statements is FALSE?

(A) Low-alloy steels are a minor group and are rarely used.
(B) Low-alloy steels are used in the heat-treated condition.
(C) Low-alloy steels contain small amounts of nickel and chromium.
(D) The addition of small amounts of molybdenum to low-alloy steels makes it possible to harden and strengthen thick pieces of the metal by heat treatment.

Low-alloy steels are one of the most commonly used classes of structural steels.

The answer is (A).

MATERIALS SCIENCE–100

Which of the following statements is FALSE?

(A) High-strength, low-alloy steels are not as strong as nonalloy, low-carbon steels.
(B) Small amounts of copper increase the tensile strength of steels.
(C) Small amounts of silicon in steels have little influence on toughness or fabricability.
(D) Addition of small amounts of silicon to steel can cause a marked decrease in yield strength of the steel.

Addition of small amounts of silicon to steel increases both the yield strength and the tensile strength.

The answer is (D).

MATERIALS SCIENCE–101

Which of the following statements is FALSE?

(A) Stainless steels contain large amounts of chromium.

(B) There are three basic types of stainless steels: martensitic, austenitic, and ferritic.

(C) The nonmagnetic stainless steels contain large amounts of nickel.

(D) Stabilization of the face-centered cubic crystal structure of stainless steels imparts a nonmagnetic characteristic to the alloy.

There are only two basic types of stainless steels: magnetic (martensitic or ferritic) and nonmagnetic (austenitic).

The answer is (B).

MATERIALS SCIENCE–102

For a completely corrosion-resistant stainless steel, what minimum percentage of chromium in the alloy is required?

(A) 1.1% (B) 3.2% (C) 8.3% (D) 11%

For complete corrosion resistance, the chromium content must be at least 11%.

The answer is (D).

MATERIALS SCIENCE–103

Which of the following would most likely require a steel containing 0.6% carbon that has been spheroidized, cold-drawn, and slightly tempered?

(A) a bridge beam
(B) a water pipe
(C) a cutting tool
(D) a ball bearing

A hypoeutectoid steel that has been worked using the above process has good strength and excellent toughness. A cutting tool undergoes tremendous stress loads due to the relatively small contact area. It requires a stronger material than do the other objects.

The answer is (C).

10 MECHANICS OF MATERIALS

MECHANICS OF MATERIALS–1

Where do stress concentrations occur?

I. near the points of application of concentrated loads
II. along the entire length of high distributed loads
III. at discontinuities

(A) I and II (B) I and III (C) II and III (D) I, II, and III

Stress concentrations occur under concentrated loads and at discontinuities, not under distributed loads.

The answer is (B).

MECHANICS OF MATERIALS–2

What is the definition of normal strain, ϵ? (δ is elongation, and L is the length of the specimen.)

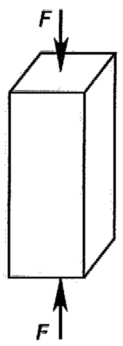
(A) $\epsilon = \frac{L+\delta}{L}$ (B) $\epsilon = \frac{L+\delta}{\delta}$ (C) $\epsilon = \frac{\delta}{L+\delta}$ (D) $\epsilon = \frac{\delta}{L}$

Strain is defined as elongation per unit length.

The answer is (D).

MECHANICS OF MATERIALS–3

The column shown has a cross-sectional area of 13 m^2. What can the approximate maximum load be if the compressive stress cannot exceed 9.6 kPa?



(A) 120 kN (B) 122 kN (C) 125 kN (D) 130 kN

The equation for axial stress is

$$\sigma = \frac{F}{A}$$

$$F = \sigma A$$

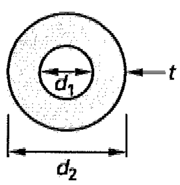
$$= \left(9.6\ \frac{\text{kN}}{\text{m}^2}\right)(13\ \text{m}^2)$$

$$= 124.8\ \text{kN} \quad (125\ \text{kN})$$

The answer is (C).

MECHANICS OF MATERIALS–4

A copper column of annular cross section has an outer diameter, d_2, of 5 m, and is subjected to an axial loading of 200 kN. The allowable compressive stress is 14.4 kPa. The wall thickness, t, should be most nearly



(A) 0.5 m (B) 0.8 m (C) 1 m (D) 2 m

For axial stress,

$$\sigma = \frac{F}{A}$$

Then,

$$A = \frac{F}{\sigma} = \frac{\pi}{4}(d_2^2 - d_1^2)$$

$$d_1 = \sqrt{d_2^2 - \frac{4F}{\pi\sigma}}$$

$$= \sqrt{(5\text{ m})^2 - \frac{(4)(200\text{ kN})}{\pi\left(14.4\ \frac{\text{kN}}{\text{m}^2}\right)}}$$

$$= 2.7\text{ m}$$

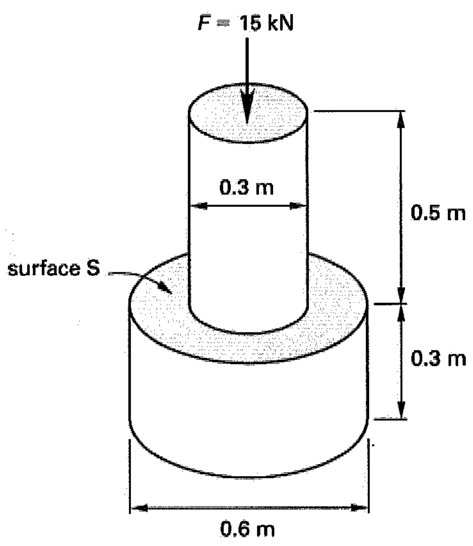
Therefore,

$$t = \frac{d_2 - d_1}{2} = \frac{5\text{ m} - 2.7\text{ m}}{2} = 1.15\text{ m} \quad (1\text{ m})$$

The answer is (C).

MECHANICS OF MATERIALS–5

What is most nearly the stress at surface S of the cylindrical object shown? The specific weight of the material is $\gamma = 76.9\text{ kN/m}^3$.



(A) 100 kPa (B) 150 kPa (C) 200 kPa (D) 250 kPa

The stress at surface S is due to the weight of the material above it in addition to the force F. The total load is

$$F_{\text{total}} = W + F = \gamma V + F$$

$$= \left(76.9\ \frac{\text{kN}}{\text{m}^3}\right)\left(\frac{\pi}{4}\right)(0.3\ \text{m})^2(0.5\ \text{m}) + 15\ \text{kN}$$

$$= 17.72\ \text{kN}$$

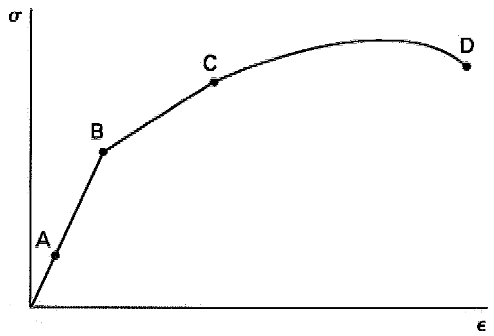
$$\sigma = \frac{F_{\text{total}}}{A} = \frac{17.72\ \text{kN}}{\left(\frac{\pi}{4}\right)(0.3\ \text{m})^2}$$

$$= 250.7\ \text{kN/m}^2 \quad (250\ \text{kPa})$$

The answer is (D).

MECHANICS OF MATERIALS–6

Considering the stress-strain diagram for aluminum, which point is the fracture point?



(A) A (B) B (C) C (D) D

Point D is where fracture occurs.

The answer is (D).

MECHANICS OF MATERIALS–7

In a stress-strain diagram, what is the correct term for the stress level at $\epsilon = 0.2\%$ offset?

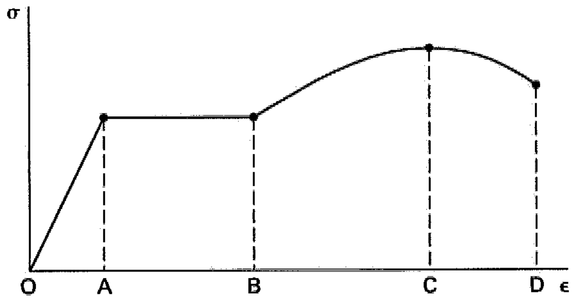
(A) the elastic limit
(B) the plastic limit
(C) the offset rupture stress
(D) the offset yield stress

This is known as the offset yield stress.

The answer is (D).

MECHANICS OF MATERIALS–8

Consider this stress-strain diagram for a carbon steel in tension. Determine the region of perfect plasticity or yielding.



(A) O to A (B) A to B (C) B to C (D) C to D

The plastic region is between points A and B. O to A is known as the linear region, B to C is where strain hardening occurs, and C to D is where reduction in area occurs.

The answer is (B).

MECHANICS OF MATERIALS–9

Under which type of loading does fatigue occur?

(A) static load (B) plane load (C) high load (D) repeated load

Fatigue occurs under repeated loading cycles.

The answer is (D).

MECHANICS OF MATERIALS–10

A specimen is subjected to a load. When the load is removed, the strain disappears. From this information, which of the following can be deduced about this material?

(A) It is elastic.
(B) It is plastic.
(C) It has a high modulus of elasticity.
(D) It does not obey Hooke's law.

By definition, elasticity is the property of a material by which it returns to its original dimensions during unloading.

The answer is (A).

MECHANICS OF MATERIALS–11

Which of the following may be the Poisson ratio of a material?

(A) 0.35 (B) 0.52 (C) 0.55 (D) 0.60

The Poisson ratio must be in the range $0 < \nu < 0.5$. Option (A) is the only answer that satisfies this condition.

The answer is (A).

MECHANICS OF MATERIALS–12

A 2 m long aluminum bar (modulus of elasticity = 70 GPa) is subjected to a tensile stress of 175 MPa. Find the elongation.

(A) 3.5 mm (B) 5.0 mm (C) 7.5 mm (D) 9.0 mm

From Hooke's law,

$$\epsilon = \frac{\sigma}{E} = \frac{\delta}{L}$$

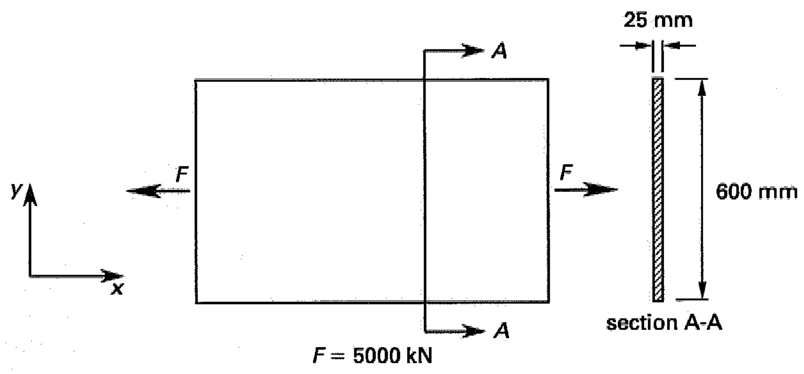
$$\delta = \frac{\sigma L}{E} = \frac{\left(175 \times 10^6 \ \frac{\text{N}}{\text{m}^2}\right)(2 \text{ m})}{70 \times 10^9 \ \frac{\text{N}}{\text{m}^2}}$$

$$= 0.005 \text{ m} \quad (5.0 \text{ mm})$$

The answer is (B).

MECHANICS OF MATERIALS–13

A 600 mm tall thin plate is placed in tension by a 5000 kN force as shown. What is the height (y direction) of the plate while tension is applied? The modulus of elasticity, E, is 200 GPa, and Poisson's ratio, ν, is 0.3. Assume the load is distributed uniformly across the plate and the yield strength is not exceeded.



(A) 599.7 mm (B) 599.9 mm (C) 600.2 mm (D) 600.5 mm

The Poisson ratio is defined as the negative ratio of lateral strain, ϵ_y, to axial strain, ϵ_x. Using this and the equation for axial stress and strain,

$$\nu = -\frac{\epsilon_y}{\epsilon_x}$$

$$\epsilon_y = -\nu\epsilon_x \qquad \text{[I]}$$

$$\epsilon_x = \frac{\sigma}{E} = \frac{F}{EA} \qquad \text{[II]}$$

Combining equations I and II,

$$\begin{aligned}\epsilon_y &= -\frac{\nu F}{EA} = -\frac{(0.3)(5000 \text{ kN})}{\left(200 \times 10^6 \frac{\text{kN}}{\text{m}^2}\right)(0.015 \text{ m}^2)} \\ &= -0.0005\end{aligned}$$

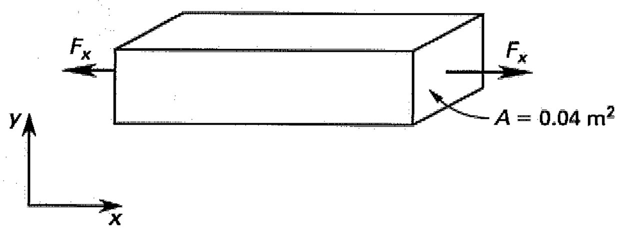
Therefore, the width while the plate is in tension is

$$\begin{aligned}w &= 600 \text{ mm} - \delta_y \\ &= 600 \text{ mm} - (0.0005)(600 \text{ mm}) \\ &= 599.7 \text{ mm}\end{aligned}$$

The answer is (A).

MECHANICS OF MATERIALS-14

What is most nearly the lateral strain, ϵ_y, of the steel specimen shown if $F_x = 3000$ kN, $E = 193$ GPa, and $\nu = 0.29$?



(A) -4×10^{-4} (B) -1×10^{-4} (C) 1×10^{-4} (D) 4×10^{-4}

From Hooke's law and the equation for axial stress,

$$\epsilon_x = \frac{\sigma_x}{E} = \frac{F_x}{EA} = \frac{3000 \text{ kN}}{\left(193 \times 10^6 \ \frac{\text{kN}}{\text{m}^2}\right)(0.04 \text{ m}^2)}$$
$$= 3.89 \times 10^{-4}$$

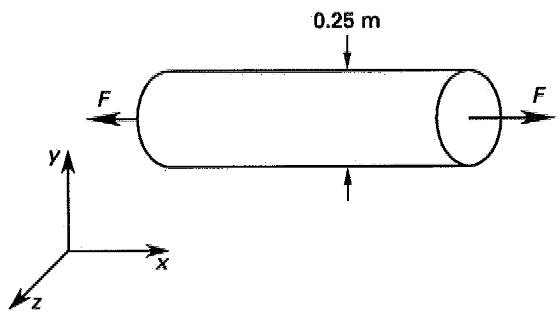
Use Poisson's ratio.

$$\epsilon_y = -\nu\epsilon_x = -(0.29)(3.89 \times 10^{-4})$$
$$= -1.13 \times 10^{-4} \quad (-1 \times 10^{-4})$$

The answer is (B).

MECHANICS OF MATERIALS–15

A steel specimen is subjected to a tensile force, F, of 2000 kN. If Poisson's ratio, ν, is 0.29 and the modulus of elasticity, E, is 193 GPa, the dilatation, e, is most nearly



(A) 6.5×10^{-5} (B) 8.8×10^{-5} (C) 8.8×10^{-4} (D) 6.5×10^{-4}

Dilatation is defined as the sum of the strain in all three coordinate directions. In the axial z direction,

$$\epsilon_z = \frac{F}{EA} = \frac{2000 \text{ kN}}{\left(193 \times 10^6 \ \frac{\text{kN}}{\text{m}^2}\right)(0.049 \text{ m}^2)} = 2.1 \times 10^{-4}$$

From Poisson's ratio,

$$\nu = -\frac{\epsilon_x}{\epsilon_z} = -\frac{\epsilon_y}{\epsilon_z}$$

$$\begin{aligned}\epsilon_x = \epsilon_y &= -\nu\epsilon_z \\ &= -(0.29)(2.1 \times 10^{-4}) \\ &= -6.09 \times 10^{-5}\end{aligned}$$

Therefore,

$$\begin{aligned}e &= \epsilon_x + \epsilon_y + \epsilon_z \\ &= (2.1 \times 10^{-4}) + (2)(-6.09 \times 10^{-5}) \\ &= 8.82 \times 10^{-5} \quad (8.8 \times 10^{-5})\end{aligned}$$

The answer is (B).

MECHANICS OF MATERIALS–16

Given a shear stress of $\tau_{xy} = 35\,000$ kPa and a shear modulus of $G = 75$ GPa, the shear strain is most nearly

(A) 2.5×10^{-5} rad (B) 4.7×10^{-4} rad (C) 5.5×10^{-4} rad (D) 8.3×10^{-4} rad

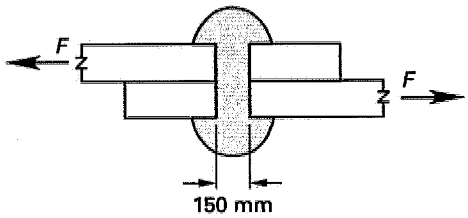
Hooke's law for shear gives

$$\begin{aligned}\gamma = \frac{\tau_{xy}}{G} &= \frac{35\,000\ \frac{\text{kN}}{\text{m}^2}}{75 \times 10^6\ \frac{\text{kN}}{\text{m}^2}} \\ &= 4.67 \times 10^{-4}\ \text{rad} \quad (4.7 \times 10^{-4}\ \text{rad})\end{aligned}$$

The answer is (B).

MECHANICS OF MATERIALS–17

A 150 mm diameter rivet resists a shear force of $V = 8$ kN. Find the average shear stress in the rivet.



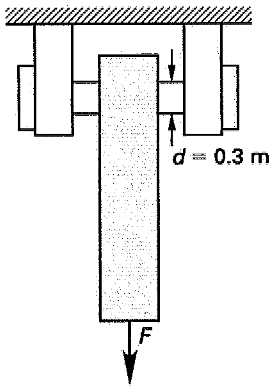
(A) 230 kPa (B) 370 kPa (C) 430 kPa (D) 450 kPa

$$\tau = \frac{V}{A} = \frac{8 \text{ kN}}{\left(\frac{\pi}{4}\right)(0.150 \text{ m})^2} = 452.7 \text{ kN/m}^2 \quad (450 \text{ kPa})$$

The answer is (D).

MECHANICS OF MATERIALS–18

A steel bar carrying a 3000 kN load, F, is attached to a support by a round pin 0.3 m in diameter. What is most nearly the average shear stress in the pin?



(A) 10 MPa (B) 12 MPa (C) 21 MPa (D) 25 MPa

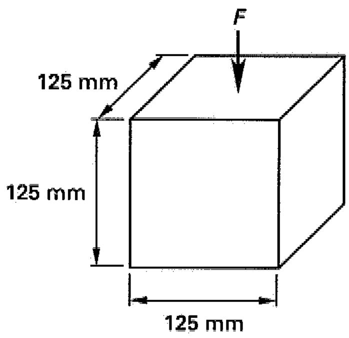
The pin will shear on two cross sections.

$$\tau = \frac{F}{2A} = \frac{3000 \text{ kN}}{(2)\left(\frac{\pi}{4}\right)(0.3 \text{ m})^2} = 21\,221 \text{ kN/m}^2 \quad (21 \text{ MPa})$$

The answer is (C).

MECHANICS OF MATERIALS–19

What is most nearly the maximum allowable load, F, if the factor of safety is 1.5 and the compressive yield stress, σ_{yield}, is 20 670 kPa?



(A) 220 kN (B) 240 kN (C) 300 kN (D) 420 kN

$$\sigma_{\text{allowable}} = \frac{\sigma_{\text{yield}}}{\text{SF}} = \frac{20\,670 \frac{\text{kN}}{\text{m}^2}}{1.5} = 13\,780 \text{ kN/m}^2$$

$$F = \sigma_{\text{allowable}} A = \left(13\,780 \frac{\text{kN}}{\text{m}^2}\right)(0.125 \text{ m})^2 = 215.3 \text{ kN} \quad (220 \text{ kN})$$

The answer is (A).

MECHANICS OF MATERIALS–20

The allowable tensile stress for a 6.25 mm diameter bolt with a thread length of 5.5 mm is 207 MPa. The allowable shear stress of the material is 103 MPa. Where and how will such a bolt be most likely to fail if placed in tension? (Assume threads are perfectly triangular and that the force is carried at the mean thread height.)

(A) at the root diameter due to tension
(B) at the threads due to shear
(C) at the root diameter due to shear
(D) at the threads due to tension

The bolt will most likely fail due to shearing of the threads or due to tensile failure of the bolt diameter.

$$\begin{aligned} F_{\text{allowable,thread}} &= \tau_{\text{allowable}}(\text{average shear area}) \\ &= \tau_{\text{allowable}}\left(\tfrac{1}{2}\pi dh\right) \\ &= \left(103\,000\ \frac{\text{kN}}{\text{m}^2}\right)\left(\tfrac{1}{2}\pi\right)(0.006\,25\ \text{m})(0.0055\ \text{m}) \\ &= 5.56\ \text{kN} \\ F_{\text{allowable,root}} &= \sigma_{\text{allowable}}(\text{root area}) \\ &= \left(207\,000\ \frac{\text{kN}}{\text{m}^2}\right)\left(\frac{\pi}{4}\right)(0.006\,25\ \text{m})^2 \\ &= 6.35\ \text{kN} \end{aligned}$$

The shear stress in the threads will exceed the allowable stress before the tensile load becomes excessive.

The answer is (B).

MECHANICS OF MATERIALS–21

Hexagonal nuts for 6.25 mm diameter bolts have a height of 5.5 mm. If the ultimate strength of the nut material in shear is 103 MPa, what is most nearly the maximum allowable shear force on the nut threads using a safety factor of 5?

(A) 0.72 kN (B) 0.8 kN (C) 1.0 kN (D) 1.1 kN

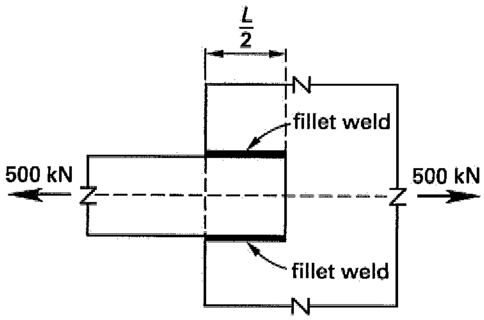
$$\tau_{\text{allowable}} = \frac{\tau}{\text{SF}} = \frac{103\,000\ \frac{\text{kN}}{\text{m}^2}}{5} = 20\,600\ \text{kN/m}^2$$

$$V = \tau_{\text{allowable}} A = \tau_{\text{allowable}} \left(\tfrac{1}{2}\pi dh\right) = \left(20\,600\ \frac{\text{kN}}{\text{m}^2}\right)\left(\tfrac{1}{2}\pi\right)(0.006\,25\ \text{m})(0.0055\ \text{m}) = 1.11\ \text{kN} \quad (1.1\ \text{kN})$$

The answer is (D).

MECHANICS OF MATERIALS–22

Determine the total length, L, of the fillet weld for the lap joint shown. The weld has to resist a tension, F, of 500 kN. The effective throat for the weld, h, is 12 mm, and the allowable stress is 145 MPa.



(A) 247 mm (B) 252 mm (C) 287 mm (D) 312 mm

For a fillet weld, the average normal stress is

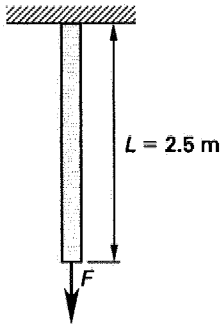
$$\sigma = \frac{F}{hL}$$

$$L = \frac{F}{\sigma h} = \frac{500\ \text{kN}}{\left(145\,000\ \frac{\text{kN}}{\text{m}^2}\right)(0.012\ \text{m})} = 0.287\ \text{m} \quad (287\ \text{mm})$$

The answer is (C).

MECHANICS OF MATERIALS–23

What is most nearly the elongation of the aluminum bar (cross section of 3 cm × 3 cm) shown in the figure when loaded to its yield point? $E = 69$ GPa, and $\sigma_{\text{yield}} = 255$ MPa. Neglect the weight of the bar.



(A) 3.3 mm (B) 9.3 mm (C) 12 mm (D) 15 mm

From Hooke's law, the axial strain is

$$\epsilon = \frac{\sigma}{E} = \frac{255 \times 10^6 \text{ Pa}}{69 \times 10^9 \text{ Pa}} = 0.0037$$

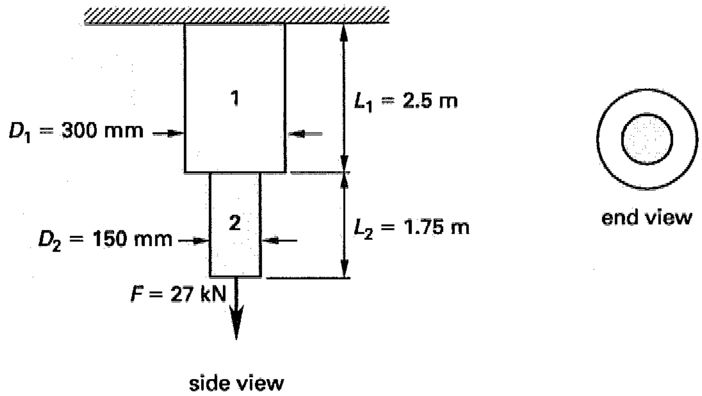
The total elongation is

$$\delta = \epsilon L = (0.0037)(2.5 \text{ m}) = 0.009\,25 \text{ m} \quad (9.3 \text{ mm})$$

The answer is (B).

MECHANICS OF MATERIALS–24

What is most nearly the total elongation of the rod shown if $E = 69$ GPa? Neglect bending.



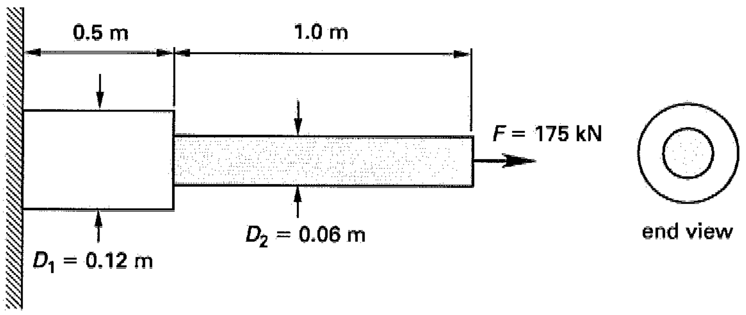
(A) 0.01 mm (B) 0.05 mm (C) 0.2 mm (D) 1.2 mm

$$\delta_{\text{total}} = \frac{FL_1}{EA_1} + \frac{FL_2}{EA_2} = \frac{F}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2}\right)$$
$$= \frac{4F}{\pi E}\left(\frac{L_1}{D_1^2} + \frac{L_2}{D_2^2}\right)$$
$$= \left(\frac{(4)(27 \text{ kN})}{\pi\left(69 \times 10^6 \ \frac{\text{kN}}{\text{m}^2}\right)}\right)\left(\frac{2.5 \text{ m}}{(0.3 \text{ m})^2} + \frac{1.75 \text{ m}}{(0.15 \text{ m})^2}\right)$$
$$= 5.26 \times 10^{-5} \text{ m} \quad (0.05 \text{ mm})$$

The answer is (B).

MECHANICS OF MATERIALS–25

What is most nearly the total elongation of this composite body under a force of 27 kN? $E_1 = 70$ GPa, and $E_2 = 100$ GPa.



(A) 0.075 mm (B) 0.73 mm (C) 1.2 mm (D) 3.0 mm

Total elongation is the elongation of section 1 plus the elongation of section 2.

$$\delta_{\text{total}} = \delta_1 + \delta_2 = \frac{FL_1}{A_1E_1} + \frac{FL_2}{A_2E_2}$$
$$= \frac{(175 \text{ kN})(0.5 \text{ m})}{\left(\frac{\pi}{4}\right)(0.12 \text{ m})^2\left(70 \times 10^6 \ \frac{\text{kN}}{\text{m}^2}\right)} + \frac{(175 \text{ kN})(1.0 \text{ m})}{\left(\frac{\pi}{4}\right)(0.06 \text{ m})^2\left(100 \times 10^6 \ \frac{\text{kN}}{\text{m}^2}\right)}$$
$$= 7.29 \times 10^{-4} \text{ m} \quad (0.73 \text{ mm})$$

The answer is (B).

MECHANICS OF MATERIALS–26

A 200 m cable is suspended vertically. At any point along the cable, the strain is proportional to the length of the cable below that point. If the strain at the top of the cable is 0.001, determine the total elongation of the cable.

(A) 0.050 m (B) 0.10 m (C) 0.15 m (D) 0.20 m

Since the strain is proportional to the cable length, it varies from 0 at the end to the maximum value of 0.001 at the supports. The average strain is

$$\begin{aligned}\epsilon_{\text{ave}} &= \frac{\epsilon_{\text{max}}}{2} = \frac{0.001}{2}\\ &= 0.0005\end{aligned}$$

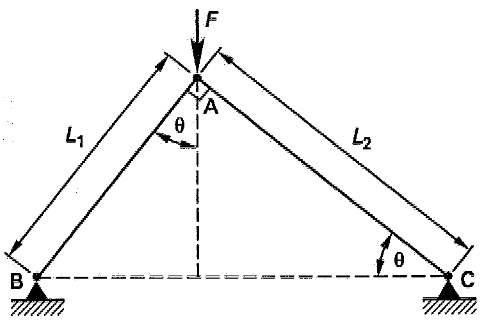
The total elongation is

$$\begin{aligned}\delta &= \epsilon_{\text{ave}}L = (0.0005)(200\ \text{m})\\ &= 0.10\ \text{m}\end{aligned}$$

The answer is (B).

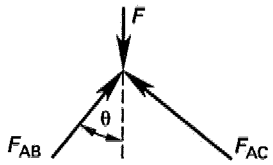
MECHANICS OF MATERIALS–27

The figure shows a two-member truss with a load $F = 50\,000$ kN applied statically. Given that $L_1 = 1.2$ m, $L_2 = 1.5$ m, and each member's cross-sectional area, A, is 4000 mm^2, what is most nearly the elongation of member AB after F is applied? Use $E = 200$ GPa.



(A) −59 mm (B) −48 mm (C) −36 mm (D) −23 mm

A free-body diagram of joint A gives



$$R_{AB} = F\cos\theta = F\frac{L_2}{\sqrt{L_1^2 + L_2^2}}$$

$$= \frac{(50\,000\text{ kN})(1.5\text{ m})}{\sqrt{(1.2\text{ m})^2 + (1.5\text{ m})^2}} = 39\,043\text{ kN} \quad \text{[AB is in compression]}$$

$$F_{AB} = -R_{AB} = -39\,043\text{ kN}$$

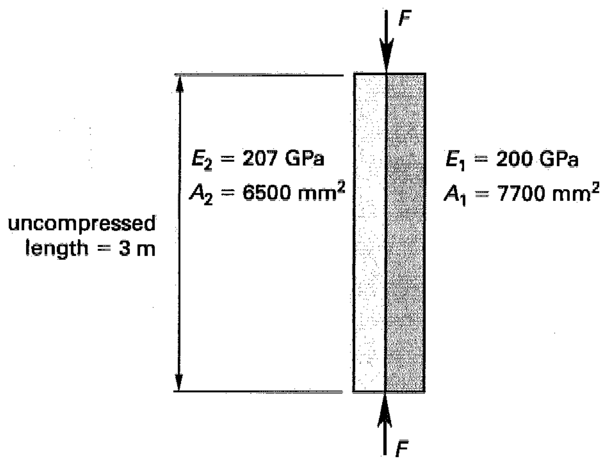
$$\delta_{AB} = \frac{F_{AB}L_1}{EA} = \frac{(-39\,043\text{ kN})(1.2\text{ m})}{\left(200 \times 10^6\ \frac{\text{kN}}{\text{m}^2}\right)(0.004\text{ m}^2)}$$

$$= -0.0586\text{ m} \quad (-59\text{ mm})$$

The answer is (A).

MECHANICS OF MATERIALS–28

The two bars shown are perfectly bonded to a common face to form an assembly. The bars have moduli of elasticity and areas as given. If a force of $F = 1300$ kN compresses the assembly, what is most nearly the reduction in length?



(A) 1.2 mm (B) 1.4 mm (C) 1.5 mm (D) 1.6 mm

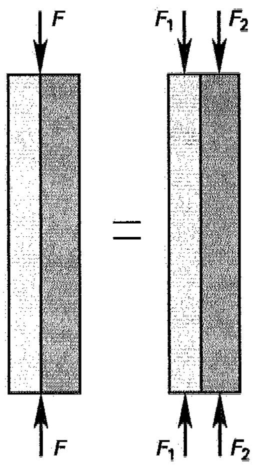
From the principle of compatability, both bars are compressed the same length.

$$\epsilon_1 = \frac{\sigma_1}{E_1}$$

$$= \frac{F_1}{A_1 E_1}$$

$$\epsilon_2 = \frac{\sigma_2}{E_2}$$

$$= \frac{F_2}{A_2 E_2}$$



Since $\epsilon_1 = \epsilon_2$,

$$\frac{F_1}{A_1 E_1} = \frac{F_2}{A_2 E_2}$$

$$F_1 = \left(\frac{A_1 E_1}{A_2 E_2}\right) F_2 \quad \text{[I]}$$

From a force balance,

$$F_1 + F_2 = 1300 \text{ kN} \quad \text{[II]}$$

$$F_1 = 1300 \text{ kN} - F_2$$

Combining equations I and II,

$$1300\ \text{kN} - F_2 = \left(\frac{A_1E_1}{A_2E_2}\right) F_2$$

$$1300\ \text{kN} = \left(1 + \frac{A_1E_1}{A_2E_2}\right) F_2$$

$$F_2 = \frac{1300\ \text{kN}}{1 + \frac{A_1E_1}{A_2E_2}}$$

$$= \frac{1300\ \text{kN}}{1 + \frac{(7700\ \text{mm}^2)(200\ \text{GPa})}{(6500\ \text{mm}^2)(207\ \text{GPa})}}$$

$$= 606.2\ \text{kN}$$

$$\epsilon_2 = \frac{606.2\ \text{kN}}{(0.0065\ \text{m}^2)\left(207 \times 10^6\ \frac{\text{kN}}{\text{m}^2}\right)}$$

$$= 4.51 \times 10^{-4}$$

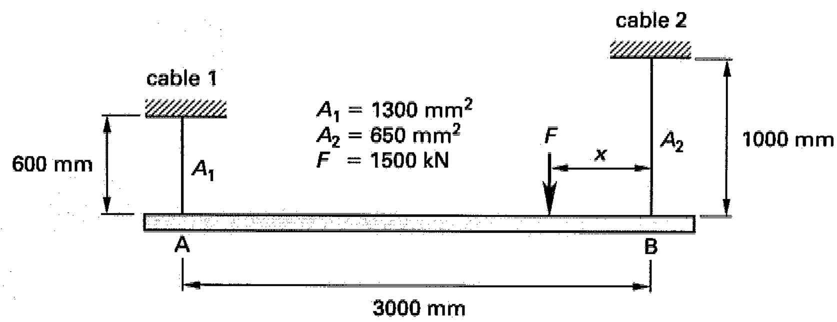
$$\delta = \epsilon L = (4.51 \times 10^{-4})(3\ \text{m})$$

$$= 0.001\,35\ \text{m} \quad (1.4\ \text{mm})$$

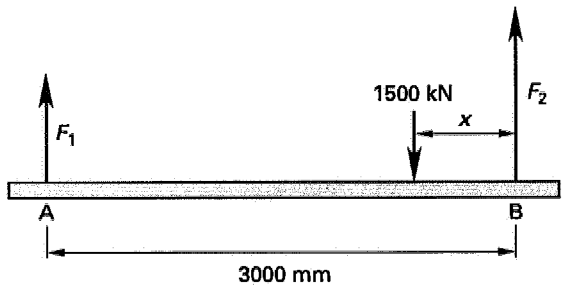
The answer is (B).

MECHANICS OF MATERIALS–29

A rigid weightless bar is suspended horizontally by cables 1 and 2 as shown. The cross-sectional areas of the cables are given in the figure. The modulus of elasticity, E, is the same for both cables. If a concentrated load of $F = 1500$ kN is applied between points A and B, what is most nearly the distance, x, for the bar to remain horizontal?



(A) 1300 mm (B) 1600 mm (C) 1900 mm (D) 2300 mm



From the free-body diagram, taking moments about point B gives

$$\sum M_{\text{B}} = 0 = (1500 \text{ kN})x - (3000 \text{ mm})F_1$$
$$(1500 \text{ kN})x = (3000 \text{ mm})F_1$$
$$x = \left(2 \; \frac{\text{mm}}{\text{kN}}\right) F_1 \qquad \text{[I]}$$

From a vertical force balance,

$$F_1 + F_2 = 1500 \text{ kN} \qquad \text{[II]}$$

For the bar to remain horizontal, the deflection of cable 1 must equal the deflection of cable 2.

$$\delta_1 = \delta_2$$
$$\frac{F_1L_1}{EA_1} = \frac{F_2L_2}{EA_2}$$
$$F_1 = \frac{L_2A_1}{L_1A_2}F_2 = \frac{(1000 \text{ mm})(1300 \text{ mm}^2)}{(600 \text{ mm})(650 \text{ mm}^2)}F_2$$
$$= 3.33F_2 \qquad \text{[III]}$$

Solving equations II and III simultaneously,

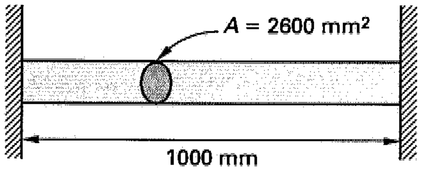
$$3.33F_2 + F_2 = 1500 \text{ kN}$$
$$4.33F_2 = 1500 \text{ kN}$$
$$F_2 = 346.4 \text{ kN}$$
$$F_1 = 1153.6 \text{ kN}$$

$$\begin{aligned} x &= \left(2\ \frac{\text{mm}}{\text{kN}}\right) F \\ &= \left(2\ \frac{\text{mm}}{\text{kN}}\right)(1153.6\ \text{kN}) \\ &= 2307\ \text{mm} \quad (2300\ \text{mm}) \end{aligned}$$

The answer is (D).

MECHANICS OF MATERIALS-30

A prismatic bar at 10°C is constrained in a rigid concrete wall at both ends. The bar is 1000 mm long and has a cross-sectional area of 2600 mm^2. What is most nearly the axial force in the bar if the temperature is raised to 40°C?



E = modulus of elasticity
= 200 GPa
α = coefficient of thermal expansion
= 9.4×10^{-6}/°C

(A) 116 kN (B) 125 kN (C) 134 kN (D) 147 kN

Elongation due to temperature change is given by

$$\begin{aligned} \delta &= \alpha L(T_2 - T_1) \\ &= \left(9.4 \times 10^{-6}\ \frac{1}{^\circ\text{C}}\right)(1000\ \text{mm})(40^\circ\text{C} - 10^\circ\text{C}) \\ &= 0.282\ \text{mm} \end{aligned}$$

Elongation is

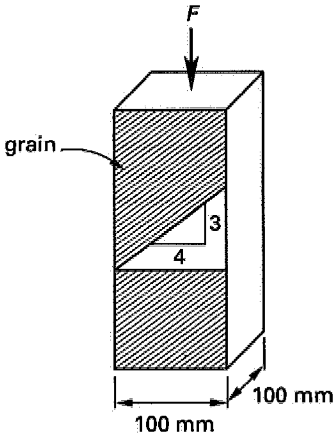
$$\delta = \frac{FL}{EA}$$

$$\begin{aligned} F = \frac{\delta EA}{L} &= \frac{(0.000\,282\ \text{m})\left(200 \times 10^6\ \frac{\text{kN}}{\text{m}^2}\right)(0.0026\ \text{m}^2)}{1\ \text{m}} \\ &= 146.6\ \text{kN} \quad (147\ \text{kN}) \end{aligned}$$

The answer is (D).

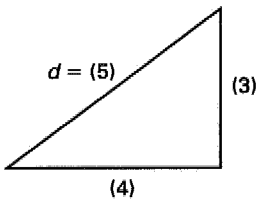
MECHANICS OF MATERIALS–31

What is most nearly the maximum axial load, F, that can be applied to the wood post shown without exceeding a maximum shear stress of 1650 kPa parallel to the grain?



(A) 22 kN (B) 33 kN (C) 44 kN (D) 57 kN

The length of the diagonal parallel to the grain, d, (part of a 3-4-5 triangle) is



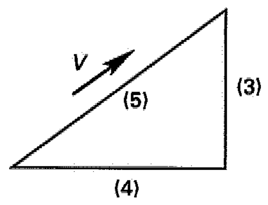
$$d = \left(\frac{5}{4}\right)(100 \text{ mm}) = 125 \text{ mm} \quad (0.125 \text{ m})$$

The area of the inclined plane is

$$A = (0.125 \text{ m})(0.100 \text{ m}) = 0.0125 \text{ m}^2$$

The total shear on the plane is

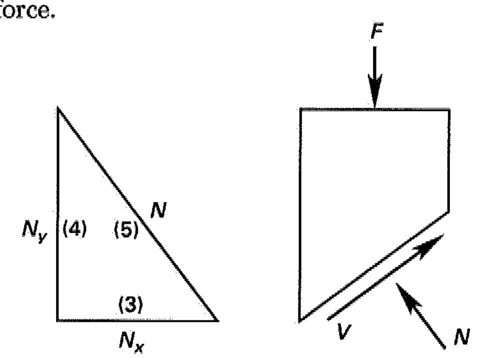
$$V = \tau A = \left(1650 \ \frac{\text{kN}}{\text{m}^2}\right)(0.0125 \text{ m}^2) = 20.63 \text{ kN}$$



The horizontal component of the shear is

$$V_x = \left(\frac{4}{5}\right)(20.63 \text{ kN}) = 16.5 \text{ kN}$$

Draw the free-body diagram of the upper section. Include the normal compressive force.



Balancing the x-components,

$$\sum F_x = 0 = V_x - N_x = 0$$
$$N_x = \tfrac{3}{5}N = V_x$$
$$\tfrac{3}{5}N = 16.5 \text{ kN}$$
$$N = \left(\frac{5}{3}\right)(16.5 \text{ kN}) = 27.5 \text{ kN}$$

Balancing the y-components,

$$\sum F_y = 0 = N_y - F = 0$$
$$F = N_y = \tfrac{4}{5}N = \left(\frac{4}{5}\right)(27.5 \text{ kN})$$
$$= 22 \text{ kN}$$

The answer is (A).

MECHANICS OF MATERIALS–32

The shear strain, ϵ, along a shaft is

$$\epsilon = r\frac{d\phi}{dx}$$

r is the radius from the shaft's centerline, and $d\phi/dx$ is the change of the angle of twist with respect to the axis of the shaft. Which condition is NOT necessary for the above equation to be valid?

(A) The area of interest must be free of connections and other load applications.

(B) The material must be isotropic and homogeneous.

(C) The loading must result in the stress being a torsional couple acting along the axis.

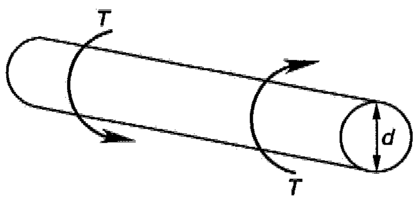
(D) r must be the full radius of the shaft.

The equation may be evaluated for any value of r, giving the stress distribution over the shaft cross section.

The answer is (D).

MECHANICS OF MATERIALS–33

A 3 m diameter bar experiences a torque of 280 N·m. What is most nearly the maximum shear stress in the bar?



(A) 2.2 Pa (B) 31 Pa (C) 42 Pa (D) 53 Pa

Maximum shear stress occurs at the outer surface.

The equation for shear gives

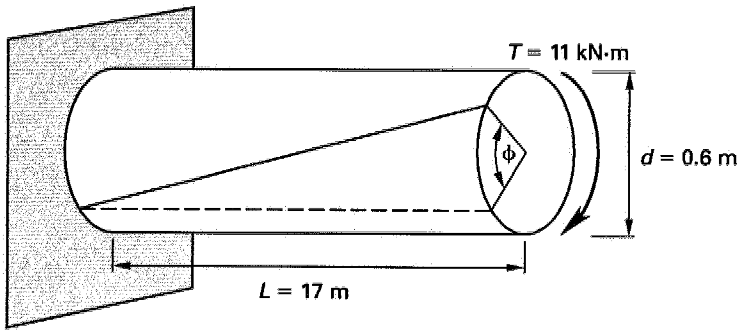
$$\tau = \frac{Tr}{J} = \frac{T\left(\frac{d}{2}\right)}{\frac{\pi}{32}d^4} = \frac{(280\ \text{N·m})\left(\frac{3\ \text{m}}{2}\right)}{\left(\frac{\pi}{32}\right)(3\ \text{m})^4}$$

$$= 52.8\ \text{N/m}^2 \quad (53\ \text{Pa})$$

The answer is (D).

MECHANICS OF MATERIALS–34

What is most nearly the angle of twist, ϕ, for the aluminum bar shown? The shear modulus of elasticity, G, is 26 GPa.



(A) 0.000 55° (B) 0.0055° (C) 0.032° (D) 0.082°

The angle of twist is given by

$$\phi = \frac{TL}{GJ} = \frac{(11\ \text{kN·m})(17\ \text{m})}{\left(26 \times 10^6\ \frac{\text{kN}}{\text{m}^2}\right)\left(\frac{\pi}{32}\right)(0.6\ \text{m})^4}$$

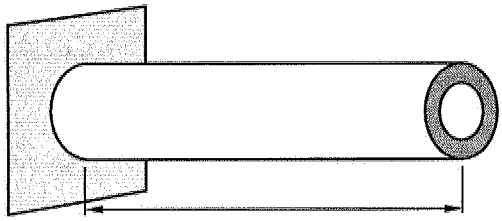
$$= (0.000\,565\ \text{rad})\left(\frac{180^\circ}{\pi\ \text{rad}}\right)$$

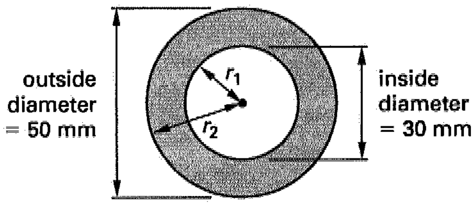
$$= 0.032^\circ$$

The answer is (C).

MECHANICS OF MATERIALS–35

What torque, T, should be applied to the end of the steel shaft shown in order to produce a twist of 1.5°? Use $G = 80$ GPa for the shear modulus.





(A) 420 N·m (B) 560 N·m (C) 830 N·m (D) 1100 N·m

Converting the twist angle to radians and calculating the polar moment of inertia J,

$$\phi = (1.5°)\left(\frac{2\pi \text{ rad}}{360°}\right) = 0.026 \text{ rad}$$

$$r_1 = 0.015 \text{ m}$$

$$r_2 = 0.025 \text{ m}$$

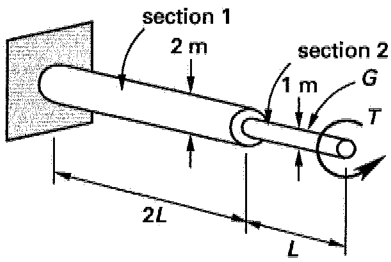
$$J = \frac{\pi}{2}\left(r_2^4 - r_1^4\right) = \left(\frac{\pi}{2}\right)\left((0.025 \text{ m})^4 - (0.015 \text{ m})^4\right)$$
$$= 5.34 \times 10^{-7} \text{ m}^4$$

$$T = \frac{GJ}{L}\phi$$
$$= \left(\frac{\left(80 \times 10^9 \ \frac{\text{N}}{\text{m}^2}\right)(5.34 \times 10^{-7} \text{ m}^4)}{1 \text{ m}}\right)(0.026 \text{ rad})$$
$$= 1110 \text{ N·m} \quad (1100 \text{ N·m})$$

The answer is (D).

MECHANICS OF MATERIALS-36

Determine the maximum torque that can be applied to the shaft, given that the maximum angle of twist is 0.0225 rad. Neglect bending.



(A) $0.000\,625\frac{\pi G}{L}$ (B) $0.0500\frac{\pi G}{L}$

(C) $0.250\frac{\pi G}{L}$ (D) $0.525\frac{\pi G}{L}$

The angle of twist is

$$\phi = \frac{TL}{GJ}$$

J for a circular bar of diameter d is $\frac{1}{2}\pi r^4 = \frac{1}{32}\pi d^4$. The total angle of twist, ϕ_{total}, is equal to the sum of the angles of twist for the two different sections. The torque is the same for both sections.

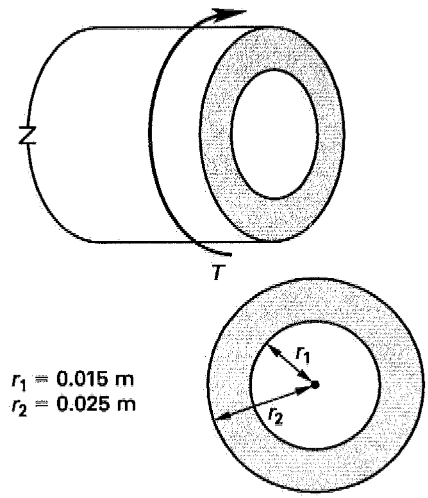
$$\begin{aligned}
\phi_{\text{total}} &= \phi_1 + \phi_2 \\
&= \frac{T(2L)}{GJ_1} + \frac{TL}{GJ_2} \\
&= \left(\frac{32TL}{\pi G}\right)\left(\frac{2}{d_1^4} + \frac{1}{d_2^4}\right) \\
&= \left(\frac{32TL}{\pi G}\right)\left(\frac{2}{(2\text{ m})^4} + \frac{1}{(1\text{ m})^4}\right) \\
&= \frac{36TL}{\pi G}
\end{aligned}$$

$$\begin{aligned}
T &= \frac{\pi G\phi_{\text{total}}}{36L} = \frac{\pi G(0.0225\text{ rad})}{36L} \\
&= 0.000\,625\frac{\pi G}{L}
\end{aligned}$$

The answer is (A).

MECHANICS OF MATERIALS–37

For the given shaft, what is most nearly the largest torque that can be applied if the shear stress is not to exceed 110 MPa?



(A) 1700 N·m (B) 1900 N·m (C) 2400 N·m (D) 3400 N·m

Since the shear stress is largest at the outer diameter, the maximum torque is found using this radius.

$$T_{\text{max}} = \frac{\tau J}{r_2}$$

For an annular region,

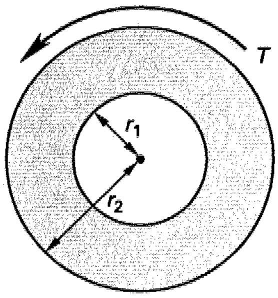
$$\begin{aligned} J &= \frac{\pi}{2}(r_2^4 - r_1^4) = \left(\frac{\pi}{2}\right)\left((0.025\text{ m})^4 - (0.015\text{ m})^4\right) \\ &= 5.34 \times 10^{-7}\text{ m}^4 \end{aligned}$$

$$\begin{aligned} T_{\text{max}} &= \frac{(5.34 \times 10^{-7}\text{ m})^4 \left(110 \times 10^6\ \frac{\text{N}}{\text{m}^2}\right)}{0.025\text{ m}} \\ &= 2350\text{ N·m} \quad (2400\text{ N·m}) \end{aligned}$$

The answer is (C).

MECHANICS OF MATERIALS–38

A hollow circular bar has an inner radius r_1 and an outer radius r_2. If $r_1 = r_2/2$, most nearly what percentage of torque can the shaft carry in comparison with a solid shaft?



(A) 25% (B) 55% (C) 75% (D) 95%

The equation for torsional stress is

$$\tau = \frac{Tr}{J}$$

$$T = \frac{\tau J}{r}$$

For the hollow shaft,

$$T_h = \frac{\tau\left(\frac{\pi}{2}\right)\left(r_2^4 - r_1^4\right)}{r_2}$$

For the solid shaft,

$$T_s = \frac{\tau\left(\frac{\pi}{2}\right)r_2^4}{r_2}$$

Therefore,

$$\frac{T_h}{T_s} = \frac{\tau\left(\frac{\pi}{2}\right)\left(\frac{r_2^4 - r_1^4}{r_2}\right)}{\tau\left(\frac{\pi}{2}\right)\left(\frac{r_2^4}{r_2}\right)}$$

$$= \frac{r_2^4 - r_1^4}{r_2^4}$$

$$= \frac{r_2^4 - \left(\frac{r_2}{2}\right)^4}{r_2^4}$$

$$= \frac{r_2^4 - \frac{r_2^4}{16}}{r_2^4}$$

$$= \frac{\frac{15}{16}r_2^4}{r_2^4}$$

$$= \frac{15}{16}$$

$$= 0.94 \quad (95\%)$$

The answer is (D).

MECHANICS OF MATERIALS–39

What is the minimum solid shaft diameter that can be used for the rotor of a 4.5 kW motor operating at 3500 rpm, if the maximum shear stress for the shaft is 60 MPa?

(A) 1.2 mm (B) 2.1 mm (C) 10 mm (D) 20 mm

The relationship between the power, P, transmitted by a shaft and the torque, T, is

$$P = \frac{\pi n T}{30}$$

n is in rpm, T is in N·m, and P is in W. Rearranging to solve for T,

$$T = \frac{30P}{\pi n} = \frac{(30)(4500\ \text{W})}{\pi\left(3500\ \frac{\text{rev}}{\text{min}}\right)} = 12.28\ \text{N·m}$$

$$\tau_{\max} = \frac{Tr}{J} = \frac{Td}{2J}$$

$$J = \frac{Td}{2\tau_{\max}} = \frac{\pi d^4}{32}$$

Therefore,

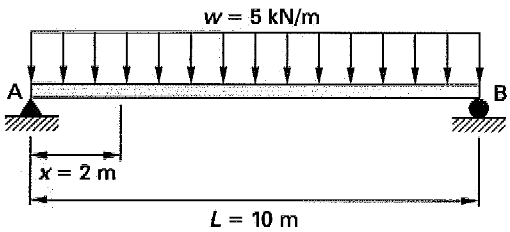
$$d = \left(\frac{32T}{2\pi\tau_{\max}}\right)^{1/3} = \left(\frac{(16)(12.28\ \text{N·m})}{\pi\left(60\times 10^6\ \frac{\text{N}}{\text{m}^2}\right)}\right)^{1/3}$$

$$= 0.0101\ \text{m} \quad (10\ \text{mm})$$

The answer is (C).

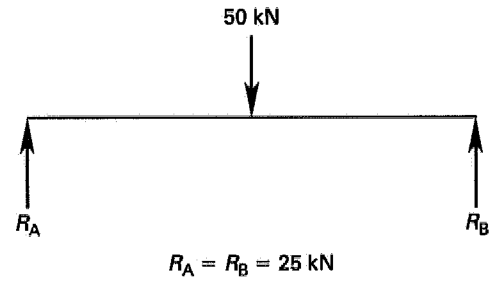
MECHANICS OF MATERIALS–40

A beam supports a distributed load, w, as shown. Find the shear force at $x = 2$ m from the left end.

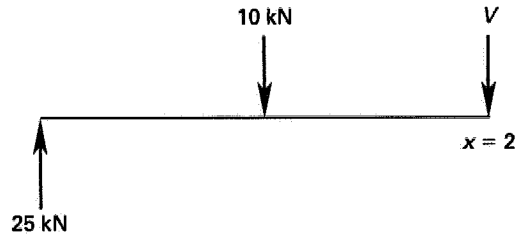


(A) 11 kN (B) 12 kN (C) 13 kN (D) 15 kN

The reactions at A and B are found by observation from symmetry to be $R_A = R_B = 25$ kN.



Sectioning the beam at $x = 2$ m, the free-body diagram with shear force is

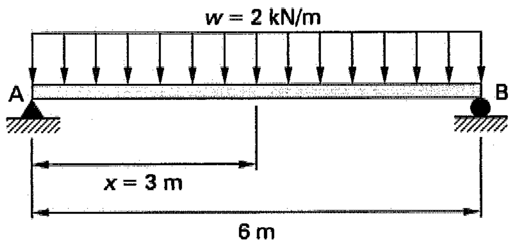


$$\sum F_y = 0$$
$$25 \text{ kN} - 10 \text{ kN} - V = 0 \text{ kN}$$
$$V = 25 \text{ kN} - 10 \text{ kN} = 15 \text{ kN}$$

The answer is (D).

MECHANICS OF MATERIALS–41

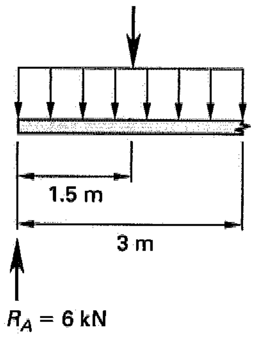
For the beam shown, find the bending moment, M, at $x = 3$ m.



(A) 4.5 kN·m (B) 6.0 kN·m (C) 7.5 kN·m (D) 9.0 kN·m

By inspection from symmetry, $R_A = R_B = 6$ kN. Sectioning the beam at $x = 3$ m gives

$$F = \left(2\ \frac{\text{kN}}{\text{ft}}\right)(3\ \text{ft}) = 6\ \text{kN}$$



$$\sum M_{x=3} = 0$$

$$(6\ \text{kN})(3\ \text{m}) + (6\ \text{kN})(1.5\ \text{m}) + M = 0$$

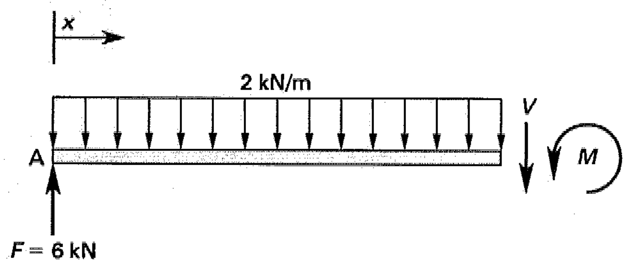
$$M = 18\ \text{kN·m} - 9\ \text{kN·m}$$

$$= 9\ \text{kN·m}$$

The answer is (D).

MECHANICS OF MATERIALS–42

Find the expression for the bending moment as a function of distance from the left end, x, for the following beam.



(A) $M = -x^3 + 2x$ (B) $M = -x^2 + 1$

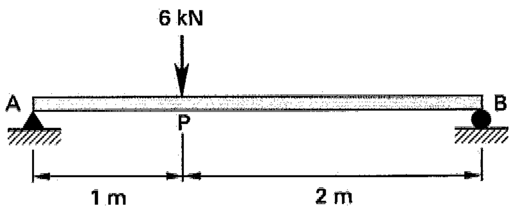
(C) $M = -x^2 + 2x$ (D) $M = x^3 - 2x^2$

$$\sum M = -2x + (2x)\left(\tfrac{1}{2}x\right) + M = 0$$
$$M = -x^2 + 2x$$

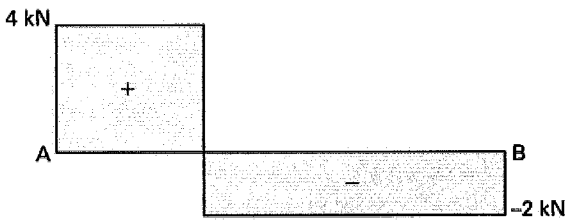
The answer is (C).

MECHANICS OF MATERIALS–43

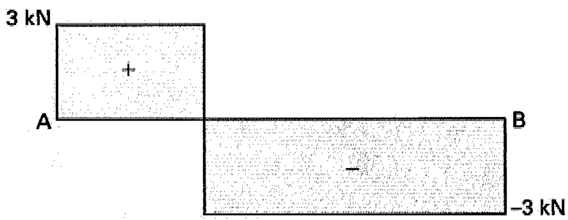
Which of the following is the shear force diagram for this beam?



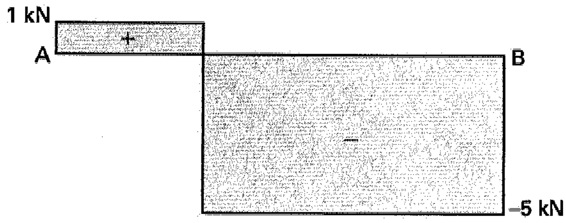
(A)



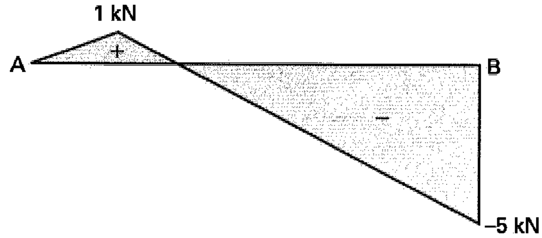
(B)

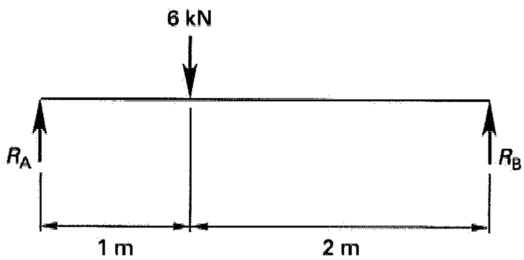


(C)

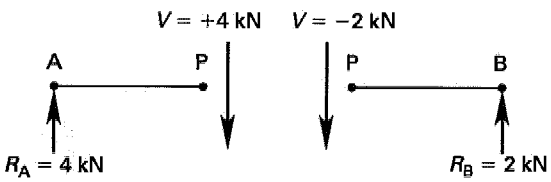


(D)





By observation, the reactions at points A and B are $R_A = 4$ kN and $R_B = 2$ kN. Draw free-body diagrams of the left and right sections of the beam.

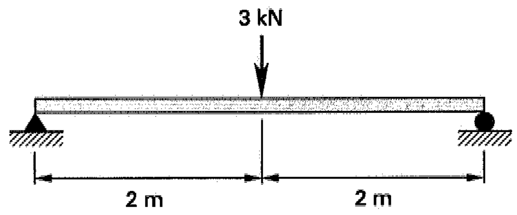


Thus, $V = +4$ kN between points A and P, and $V = -2$ kN between points P and B.

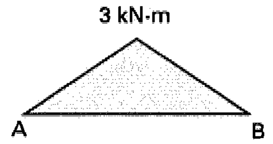
The answer is (A).

MECHANICS OF MATERIALS–44

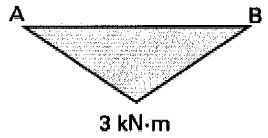
Which of the following is the bending moment diagram for this beam?



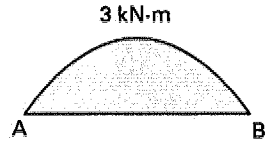
(A)



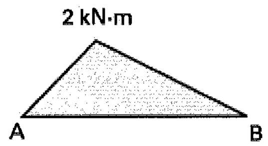
(B)

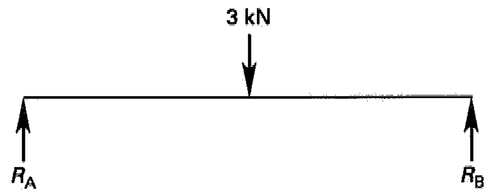


(C)

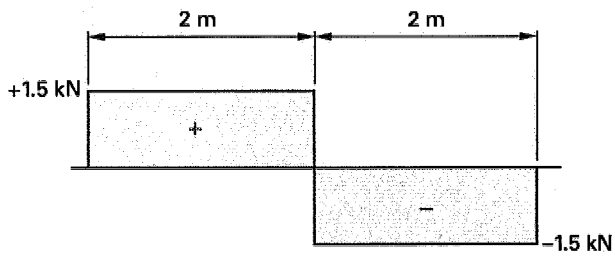


(D)

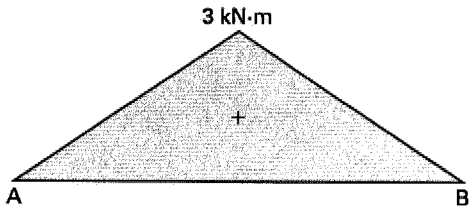




From the free-body diagram, $R_A = R_B = 1.5$ kN. The shear force diagram is therefore,



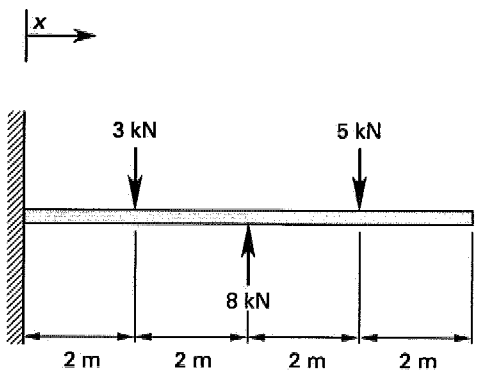
The bending moment increases linearly to (1.5 kN)(2 m) = 3 kN·m, then decreases linearly back to 0 kN·m.



The answer is (A).

MECHANICS OF MATERIALS–45

The cantilever beam shown is loaded by three concentrated forces. What is the maximum shear force in the beam?



(A) 1 kN (B) 2 kN (C) 3 kN (D) 5 kN

Examining the shear force along the beam from left to right, for 0 m $< x < 2$ m,

$$V = 0 \text{ kN}$$

For 2 m $< x <$ 4 m,

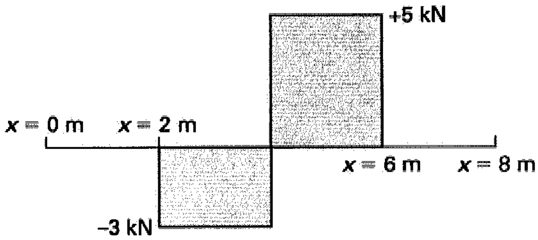
$$V = -3 \text{ kN}$$

For 4 m $< x <$ 6 m,

$$V = 5 \text{ kN}$$

For 6 m $< x <$ 8 m,

$$V = 0 \text{ kN}$$

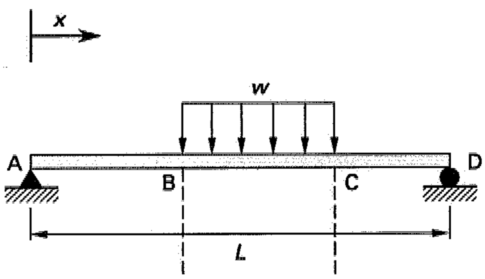


The maximum shear force is, therefore, 5 kN.

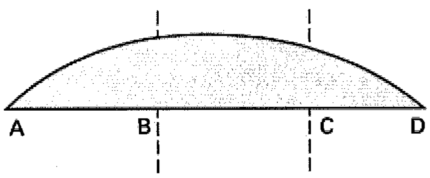
The answer is (D).

MECHANICS OF MATERIALS–46

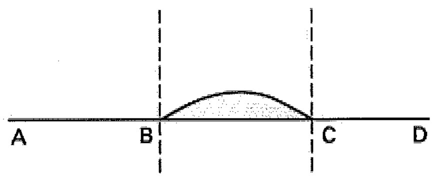
Which of the following bending moment diagrams corresponds to the simply supported beam shown? The beam is subjected to a distributed load, w, between points B and C.



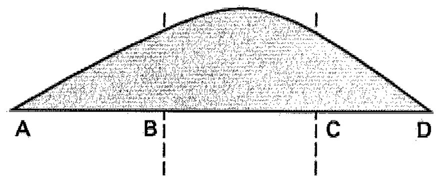
(A)



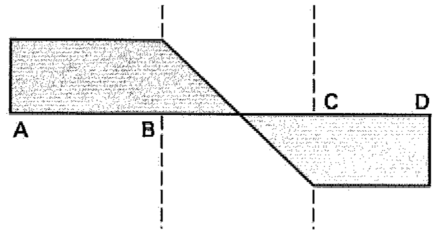
(B)



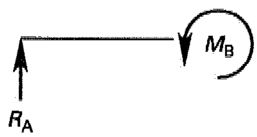
(C)



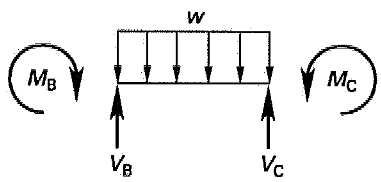
(D)



For sections AB and CD, the beam may be modeled as



$M(x)$ is linear with respect to x. For section BC, the beam is modeled as

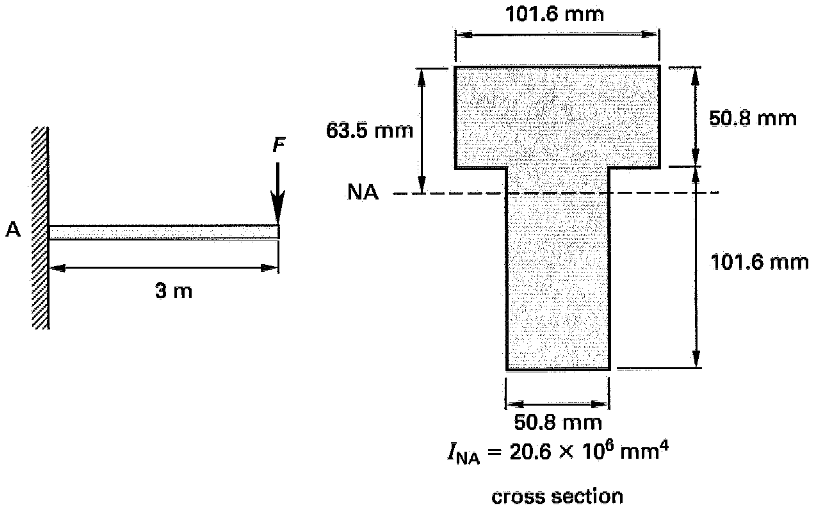


$M(x)$ is parabolic, reaching a maximum near or at the center.

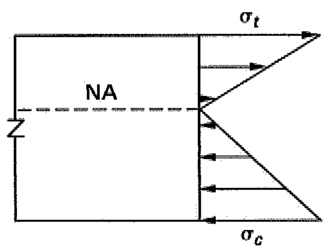
The answer is (C).

MECHANICS OF MATERIALS–47

What is most nearly the maximum allowable load, F, on the cantilever? The maximum compressive stress is 7000 kPa, and the maximum tensile stress is 5500 kPa. The moment of inertia about the centroidal axis, I_{NA}, is 20.6×10^6 mm^4.



(A) 540 N (B) 600 N (C) 610 N (D) 640 N



The maximum bending moment occurs at A, where $M = 3F$.

$$\sigma_{max} = \frac{Mc}{I} = \frac{3Fc}{I}$$

$$F = \frac{\sigma_{max} I}{3c}$$

$$I = (20.6 \times 10^6 \text{ mm}^4)\left(\frac{1 \text{ m}}{1000 \text{ mm}}\right)^4$$

$$= 20.6 \times 10^{-6} \text{ m}^4$$

For compression, $\sigma_{\text{allowable}} = 7000$ kPa.

$$c = 101.6\text{ mm} + 50.8\text{ mm} - 63.5\text{ mm} = 88.9\text{ mm}$$

$$F_{\substack{\text{allowable}\\ \text{compression}}} = \frac{\left(7000\ \frac{\text{kN}}{\text{m}^2}\right)(20.6 \times 10^{-6}\text{ m}^4)}{(3\text{ m})(0.0889\text{ m})}$$

$$= 0.541\text{ kN} \quad (541\text{ N})$$

For tension, $\sigma_{\text{allowable}} = 5500$ kPa, and $c = 63.5$ mm.

$$F_{\substack{\text{allowable}\\ \text{tension}}} = \frac{\left(5500\ \frac{\text{kN}}{\text{m}^2}\right)(20.6 \times 10^{-6}\text{ m}^4)}{(3\text{ m})(0.0635\text{ m})}$$

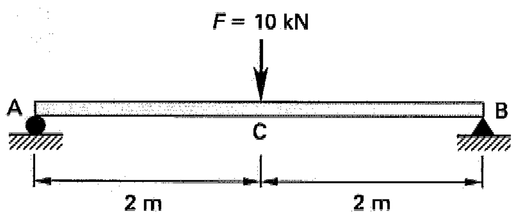
$$= 0.595\text{ kN} \quad (600\text{ N})$$

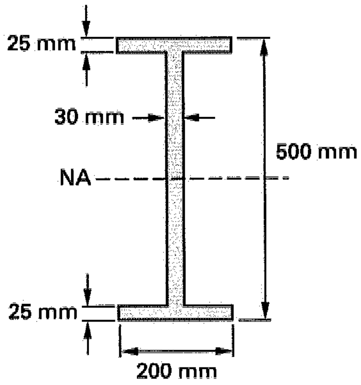
The maximum allowable load is 540 N.

The answer is (A).

MECHANICS OF MATERIALS–48

A simply supported beam with the cross section shown supports a concentrated load, $F = 10$ kN, at its center, C. What is most nearly the maximum bending stress in the beam?





cross section

(A) 2300 kPa (B) 3200 kPa (C) 3800 kPa (D) 4600 kPa

The reactions at A and B are $R_A = R_B = 5$ kN by inspection from symmetry. Since the maximum bending moment occurs at C,

$$M_{max} = (5\ \text{kN})(2\ \text{m}) = 10\ \text{kN·m}$$

The moment of inertia about the neutral axis, NA, is the difference between the moments of inertia of an area measuring 20 mm × 500 mm and two areas measuring 85 mm × 450 mm.

$$I = \frac{bh^3}{12} = \left(\frac{1}{12}\right)(200\ \text{mm})(500\ \text{mm})^3 - (2)\left(\frac{1}{12}\right)(85\ \text{mm})(450\ \text{mm})^3$$

$$= 792 \times 10^6\ \text{mm}^4$$

Since $c = \left(\frac{1}{2}\right)(500\ \text{mm}) = 250\ \text{mm}$,

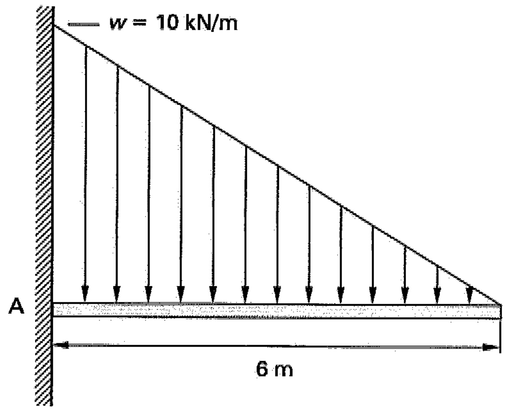
$$\sigma_{max} = \frac{Mc}{I} = \frac{(10\ \text{kN·m})(0.25\ \text{m})}{792 \times 10^{-6}\ \text{m}^4}$$

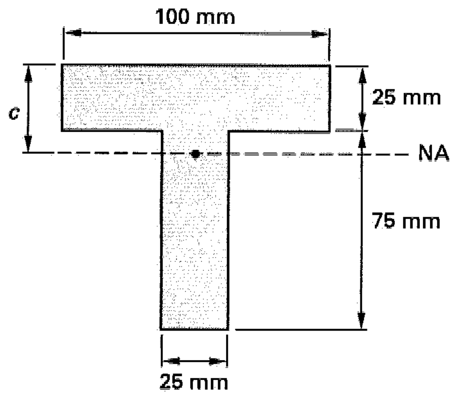
$$= 3157\ \text{kN/m}^2 \quad (3160\ \text{kPa})$$

The answer is (B).

MECHANICS OF MATERIALS–49

For the cantilever beam shown, what is the maximum tensile bending stress?

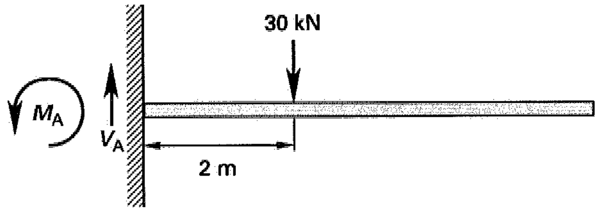




cross section

(A) 230 MPa (B) 320 MPa (C) 480 MPa (D) 550 MPa

The maximum moment occurs at point A and is a result of the distributed load w. w is equivalent to a concentrated load, $W = (1/2)(6\text{ m})(10\text{ kN/m}) = 30\text{ kN}$, acting at a point $(1/3)(6\text{ m}) = 2\text{ m}$ from point A. The equivalent loading diagram for the cantilever is as follows.



$$M_A = (30\text{ kN})(2\text{ m}) = 60\text{ kN·m}$$

The upper part of the beam will be under tension, with c equal to the distance between the neutral axis, NA, and the top edge of the beam.

$$c = \frac{\sum A\bar{y}}{\sum A} = \frac{(25\text{ mm})(100\text{ mm})(12.5\text{ mm}) + (25\text{ mm})(75\text{ mm})(62.5\text{ mm})}{2500\text{ mm}^2 + 1875\text{ mm}^2}$$

$$= 33.9\text{ mm} \quad (0.0339\text{ cm})$$

Use the parallel axis theorem to find I.

$$I_{NA} = \frac{bh^3}{12} = \left(\frac{1}{12}\right)(100\text{ mm})(25\text{ mm})^3 + (100\text{ mm})(25\text{ mm})(34\text{ mm} - 12.5\text{ mm})^2 + \left(\frac{1}{12}\right)(25\text{ mm})(75\text{ mm})^3 + (75\text{ mm})(25\text{ mm})(62.5\text{ mm} - 34\text{ mm})^2$$

$$= 3.7 \times 10^6\text{ mm}^4$$

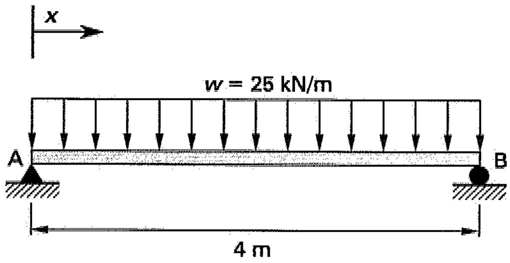
$$\sigma = \frac{Mc}{I} = \frac{(60\text{ kN·m})(0.0339\text{ m})}{(3.7 \times 10^6\text{ mm}^4)\left(\frac{1\text{ m}}{1000\text{ mm}}\right)^4}$$

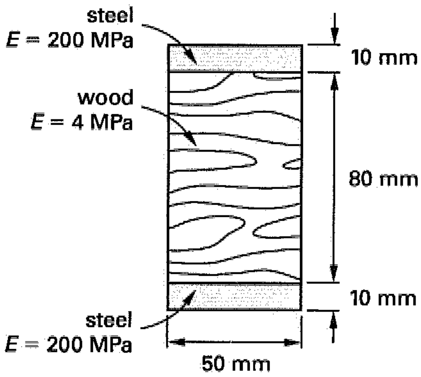
$$= 5.5 \times 10^5\text{ kPa} \quad (550\text{ MPa})$$

The answer is (D).

MECHANICS OF MATERIALS–50

A composite beam made of steel and wood is subjected to a uniform distributed load, w. Determine the maximum compressive stress in the steel.





cross section

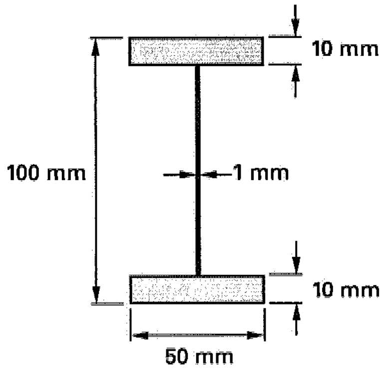
(A) 620 MPa (B) 850 MPa (C) 1100 MPa (D) 1200 MPa

The maximum moment is at the center of the beam, where $x = 2$ m.

$$R_A = R_B = 50 \text{ kN} \quad \text{[by inspection]}$$

$$\begin{aligned} M_{max} &= (50 \text{ kN})(2 \text{ m}) - \left(25 \, \frac{\text{kN}}{\text{m}}\right)(2 \text{ m})(1 \text{ m}) \\ &= 50 \text{ kN·m} \end{aligned}$$

Since $E_{wood}/E_{steel} = 4 \text{ MPa}/200 \text{ MPa} = 1/50$, the wood is equivalent to a steel web 1 mm thick.



cross section

$$I = \frac{bh^3}{12} = \left(\frac{1}{12}\right)(50 \text{ mm})(100 \text{ mm})^3 - (2)\left(\frac{1}{12}\right)(24.5 \text{ mm})(80 \text{ mm})^3$$
$$= 2.076 \times 10^6 \text{ mm}^4$$

$$\sigma = \frac{Mc}{I} = \frac{(50 \text{ kN·m})(0.05 \text{ m})\left(1000 \frac{\text{N}}{\text{kN}}\right)}{(2.076 \times 10^6 \text{ mm}^4)\left(\frac{1 \text{ m}}{1000 \text{ mm}}\right)^4}$$
$$= 1200 \times 10^6 \text{ N/m}^2 \quad (1200 \text{ MPa})$$

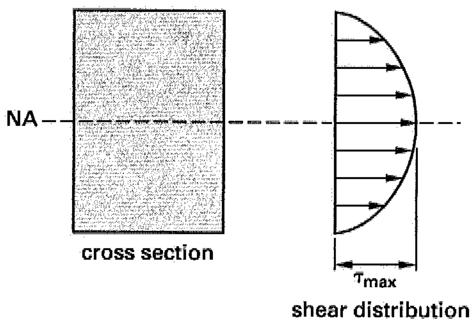
The answer is (D).

MECHANICS OF MATERIALS–51

For a rectangular beam under transverse (bending) loading, where is the location of maximum shear stress?

(A) at the top edge
(B) at the bottom edge
(C) at the neutral axis
(D) at a location between the top edge and the neutral axis

The shear distribution is

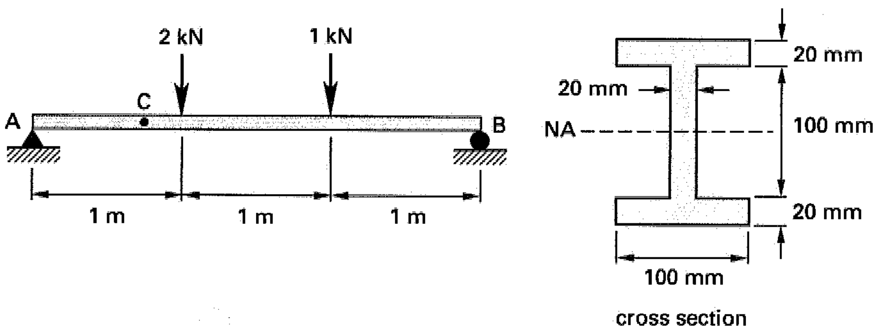


The maximum shear stress is at the neutral axis.

The answer is (C).

MECHANICS OF MATERIALS–52

An I-beam is loaded as shown. What is most nearly the maximum shear stress, τ, in the web at point C along the beam?



(A) 160 kPa (B) 370 kPa (C) 400 kPa (D) 750 kPa

The reaction at point A is found by taking the moment about point B.

$$\begin{aligned} \sum M_B &= 0 \\ &= -R_A(3\text{ m}) + (2\text{ kN})(2\text{ m}) + (1\text{ kN})(1\text{ m}) \\ R_A &= 1.67\text{ kN} \\ V_C = R_A &= 1.67\text{ kN} \end{aligned}$$

The shear stress is given by $\tau = VQ/It$, where Q is the first moment of either the upper half or the lower half of the cross-sectional area with respect to the neutral axis.

$$Q = A'\bar{y} = (50 \text{ mm})(20 \text{ mm})(25 \text{ mm}) + (100 \text{ mm})(20 \text{ mm})(60 \text{ mm})$$
$$= 145\,000 \text{ mm}^3$$
$$I = \frac{bh^3}{12} = \left(\frac{1}{12}\right)(100 \text{ mm})(140 \text{ mm})^3 - (2)\left(\frac{1}{12}\right)(40 \text{ mm})(100 \text{ mm})^3$$
$$= 16.2 \times 10^6 \text{ mm}^4$$

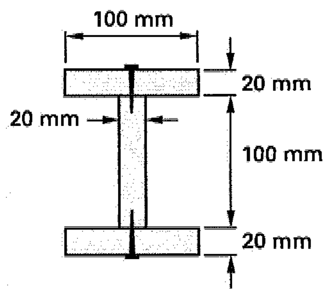
$\tau_{\max}$ occurs at the neutral axis. Thus,

$$\tau_{\max} = \frac{VQ}{It} = \frac{(1670 \text{ N})(145\,000 \text{ mm}^3)}{(16.2 \times 10^6 \text{ mm}^4)(20 \text{ mm})}$$
$$= 0.747 \text{ N/mm}^2 \quad (750 \text{ kPa})$$

The answer is (D).

MECHANICS OF MATERIALS–53

An I-beam is made of three planks, each 20 mm × 100 mm in cross section, nailed together with a single row of nails on top and bottom as shown. If the longitudinal spacing between the nails is 25 mm, and the vertical shear force acting on the cross section is 600 N, what is most nearly the load in shear per nail, F?



(A) 56 N (B) 76 N (C) 110 N (D) 160 N

The shear force per unit distance along the beam's axis is given by

$$f = \frac{VQ}{I}$$

For an I-beam, Q is the first moment of the upper flange area with respect to the z-axis.

$$Q = A\bar{y} = (60\ \text{mm})(100\ \text{mm})(20\ \text{mm}) = 120\,000\ \text{mm}^3$$

$$I = \frac{bh^3}{12} = \left(\frac{1}{12}\right)(100\ \text{mm})(140\ \text{mm})^3 - (2)\left(\frac{1}{12}\right)(40\ \text{mm})(100\ \text{mm})^3$$
$$= 16.2 \times 10^6\ \text{mm}^4$$

$$f = \frac{(600\ \text{N})(120\,000\ \text{mm}^3)}{16.2 \times 10^6\ \text{mm}^4} = 4.44\ \text{N/mm}$$

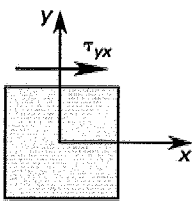
The load capacity of the nails per unit length is F/L. Therefore,

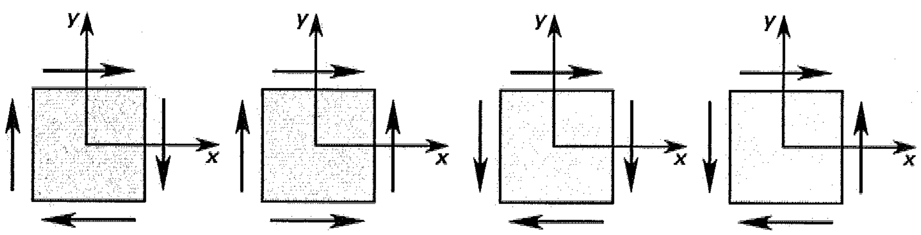
$$F = Lf = (25\ \text{mm})\left(4.44\ \frac{\text{N}}{\text{mm}}\right) = 111\ \text{N} \quad (110\ \text{N})$$

The answer is (C).

MECHANICS OF MATERIALS–54

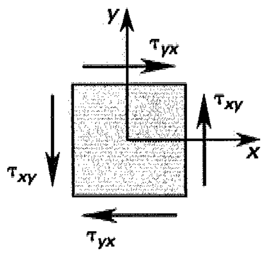
Considering the orientation of shear force τ_{yx} in the illustration, find the direction of the shear stress on the other three sides of the stress element.





(A) (B) (C) (D)

For static equilibrium, the shear stresses on opposite faces of an element must be equal in magnitude and opposite in direction. Also, the shear stresses on adjoining faces must not produce rotation of the element.



The answer is (D).

MECHANICS OF MATERIALS–55

If the principal stresses on a body are $\sigma_1 = 400$ kPa, $\sigma_2 = -700$ kPa, and $\sigma_3 = 600$ kPa, what is the maximum shear stress?

(A) 150 kPa (B) 250 kPa (C) 550 kPa (D) 650 kPa

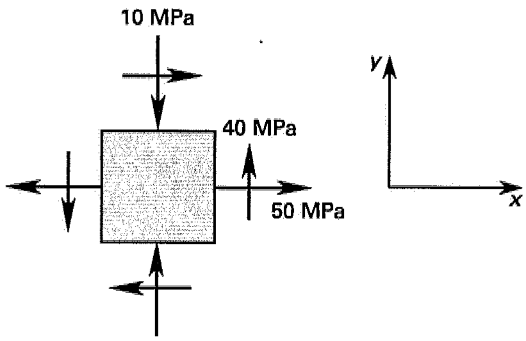
The maximum shear stress is equal to one-half of the difference between the principal stresses. Comparing the three combinations, the maximum shear stress is given by the difference between σ_2 and σ_3.

$$\tau_{\max} = \left|\frac{\sigma_2 - \sigma_3}{2}\right| = \left|\frac{-700 \text{ kPa} - 600 \text{ kPa}}{2}\right| = 650 \text{ kPa}$$

The answer is (D).

MECHANICS OF MATERIALS–56

For the element of plane stress shown, find the principal stresses.



(A) $\sigma_{max} = 35$ MPa, $\sigma_{min} = -25$ MPa
(B) $\sigma_{max} = 45$ MPa, $\sigma_{min} = 55$ MPa
(C) $\sigma_{max} = 70$ MPa, $\sigma_{min} = -30$ MPa
(D) $\sigma_{max} = 85$ MPa, $\sigma_{min} = 15$ MPa

The stresses on the element are

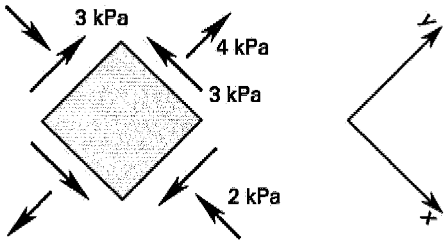
$$\sigma_x = 50 \text{ MPa} \qquad \sigma_y = -10 \text{ MPa} \qquad \tau_{xy} = 40 \text{ MPa}$$

$$\begin{aligned}\sigma_{\text{max,min}} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \frac{50 \text{ MPa} - 10 \text{ MPa}}{2} \pm \sqrt{\left(\frac{50 \text{ MPa} + 10 \text{ MPa}}{2}\right)^2 + (40 \text{ MPa})^2} \\ &= 20 \text{ MPa} \pm 50 \text{ MPa} \\ &= 70 \text{ MPa or } -30 \text{ MPa}\end{aligned}$$

The answer is (C).

MECHANICS OF MATERIALS–57

What are the principal (maximum and minimum) stresses of the stress element shown?



(A) $\sigma_{max} = 1.16$ kPa, $\sigma_{min} = -6.16$ kPa
(B) $\sigma_{max} = 2.00$ kPa, $\sigma_{min} = -4.00$ kPa
(C) $\sigma_{max} = 3.24$ kPa, $\sigma_{min} = -5.24$ kPa
(D) $\sigma_{max} = 5.24$ kPa, $\sigma_{min} = -3.24$ kPa

The stresses on the element are

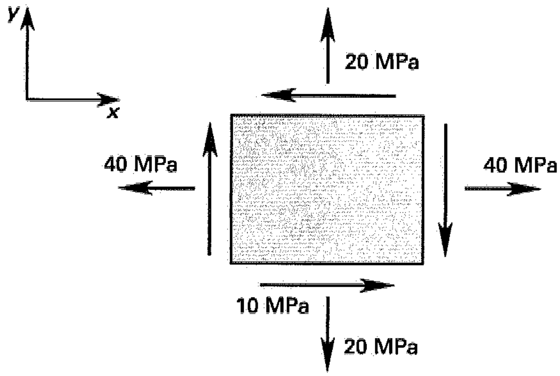
$$\sigma_x = -2 \text{ kPa} \qquad \sigma_y = +4 \text{ kPa} \qquad \tau_{xy} = -3 \text{ kPa}$$

$$\begin{aligned}\sigma_{\text{max,min}} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \frac{-2 \text{ kPa} + 4 \text{ kPa}}{2} \pm \sqrt{\left(\frac{-2 \text{ kPa} - 4 \text{ kPa}}{2}\right)^2 + (-3 \text{ kPa})^2} \\ &= 5.24 \text{ kPa or } -3.24 \text{ kPa}\end{aligned}$$

The answer is (D).

MECHANICS OF MATERIALS–58

What is most nearly the maximum principal stress of the element shown?



(A) 30 MPa (B) 34 MPa (C) 40 MPa (D) 44 MPa

The stresses on the element are

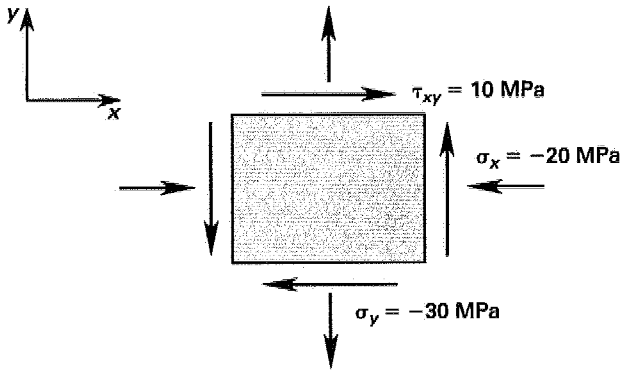
$$\sigma_x = 40 \text{ MPa} \qquad \sigma_y = 20 \text{ MPa} \qquad \tau_{xy} = -10 \text{ MPa}$$

$$\begin{aligned}
\sigma_{\max} &= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\
&= \frac{40 \text{ MPa} + 20 \text{ MPa}}{2} + \sqrt{\left(\frac{40 \text{ MPa} - 20 \text{ MPa}}{2}\right)^2 + (-10 \text{ MPa})^2} \\
&= 44.1 \text{ MPa} \quad (44 \text{ MPa})
\end{aligned}$$

The answer is (D).

MECHANICS OF MATERIALS–59

For the following stress element, what is most nearly the maximum shear stress?



(A) 10 MPa (B) 11 MPa (C) 14 MPa (D) 27 MPa

The maximum shear stress is

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

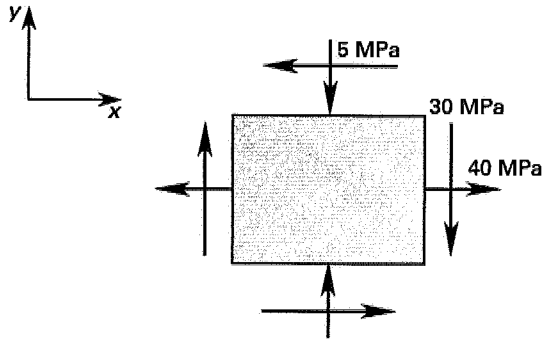
$$= \sqrt{\left(\frac{-20 \text{ MPa} - 30 \text{ MPa}}{2}\right)^2 + (10 \text{ MPa})^2}$$

$$= 26.9 \text{ MPa}$$

The answer is (D).

MECHANICS OF MATERIALS–60

For the state of plane stress shown, what are the inclination angles of the principal planes?



(A) 32.5° and 122°
(B) 25.5° and 115°
(C) −26.5° and −117°
(D) −11.5° and −102°

The stresses on the element are

$$\sigma_x = 40 \text{ MPa} \qquad \sigma_y = -5 \text{ MPa} \qquad \tau_{xy} = -30 \text{ MPa}$$

$$\begin{aligned} \tan 2\theta_p &= \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-30 \text{ MPa})}{40 \text{ MPa} - (-5 \text{ MPa})} \\ &= -1.33 \\ 2\theta_p &= -53.0^\circ \text{ or } -233^\circ \\ \theta_p &= -26.5^\circ \text{ or } -117^\circ \end{aligned}$$

The answer is (C).

MECHANICS OF MATERIALS–61

A steel ($\sigma_{\text{yield}} = 200$ MPa) pressure tank is designed to hold pressures up to 7 MPa. The tank is cylindrical with a diameter of 1 m. If the longitudinal stress must be less than 20% of the yield stress of the steel, what is the necessary wall thickness, t?

(A) 22 mm (B) 44 mm (C) 88 mm (D) 120 mm

For a thin-walled cylinder of diameter d containing a pressure p,

$$\sigma_{\text{long}} = \frac{pd}{4t} = (1 - 0.2)\sigma_{\text{yield}}$$

$$t = \frac{pd}{0.8\sigma_{\text{yield}}} = \frac{\left(7 \times 10^6 \ \frac{\text{N}}{\text{m}^2}\right)(1 \text{ m})}{(0.8)\left(200 \times 10^6 \ \frac{\text{N}}{\text{m}^2}\right)}$$

$$= 0.044 \text{ m} \quad (44 \text{ mm})$$

The answer is (B).

MECHANICS OF MATERIALS–62

In designing a cylindrical pressure tank 1 m in diameter, a factor of safety of 2.5 is used. The cylinder is made of steel ($\sigma_{\text{yield}} = 200$ MPa), and will contain pressures up to 7 MPa. What is the required wall thickness, t, based on circumferential stress considerations?

(A) 22 mm (B) 44 mm (C) 88 mm (D) 120 mm

For a thin-walled cylinder of diameter d containing a pressure p,

$$\sigma_{\text{circumferential}} = \frac{pd}{2t} = \frac{\sigma_{\text{yield}}}{2.5}$$

$$t = \frac{1.25pd}{\sigma_{\text{yield}}} = \frac{(1.25)\left(7 \times 10^6 \ \frac{\text{N}}{\text{m}^2}\right)(1 \text{ m})}{200 \times 10^6 \ \frac{\text{N}}{\text{m}^2}}$$

$$= 0.044 \text{ m} \quad (44 \text{ mm})$$

The answer is (B).

MECHANICS OF MATERIALS–63

What is most nearly the maximum principal strain at a point where $\epsilon_x = 1500$ μm, $\epsilon_y = -750$ μm, and $\epsilon_{xy} = 1000$ μm?

(A) 1160 μm (B) 1490 μm (C) 1610 μm (D) 1830 μm

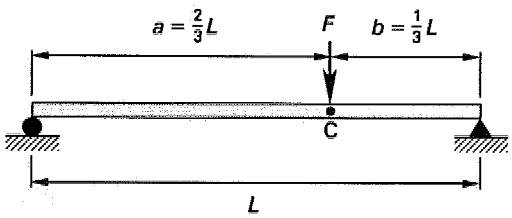
The equation for principal strain gives

$$\begin{aligned}\epsilon_{\text{max,min}} &= \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\epsilon_{xy}}{2}\right)^2} \\ &= \frac{1500\ \mu\text{m} - 750\ \mu\text{m}}{2} \pm \sqrt{\left(\frac{1500\ \mu\text{m} + 750\ \mu\text{m}}{2}\right)^2 + \left(\frac{1000\ \mu\text{m}}{2}\right)^2} \\ &= 375\ \mu\text{m} \pm 1231\ \mu\text{m} \\ \epsilon_{\text{max}} &= 1606\ \mu\text{m} \quad (1610\ \mu\text{m})\end{aligned}$$

The answer is (C).

MECHANICS OF MATERIALS–64

A beam of length L carries a concentrated load, F, at point C. Determine the deflection at point C in terms of F, L, E, and I, where E is the modulus of elasticity, and I is the moment of inertia.



(A) $\frac{2FL^3}{243EI}$ (B) $\frac{4FL^3}{243EI}$ (C) $\frac{FL^3}{27EI}$ (D) $\frac{FL^3}{9EI}$

The equation for bending moment in the beam is

$$EI\frac{d^2\delta}{dx^2} = -M$$

Computing M for the different beam sections,

$$EI\frac{d^2\delta}{dx^2} = -\frac{Fbx}{L} \qquad (0 \le x \le a)$$

$$EI\frac{d^2\delta}{dx^2} = -\frac{Fbx}{L} + F(x-a) \qquad (a \le x \le L)$$

Integrating each equation twice gives

$$EI\delta = -\frac{Fbx^3}{6L} + C_1x + C_3 \qquad (0 \le x \le a)$$

$$EI\delta = -\frac{Fbx^3}{6L} + \frac{F(x-a)^3}{6} + C_2x + C_4 \qquad (a \le x \le L)$$

The constants are determined by the following conditions: (1) at $x = a$, the slopes $d\delta/dx$ and deflections δ are equal; (2) at $x = 0$ and $x = L$, the deflection $\delta = 0$. These conditions give

$$C_1 = C_2 = \frac{Fb(L^2 - b^2)}{6L}$$

$$C_3 = C_4 = 0$$

Evaluating the equation for $(0 \le x \le a)$ at $x = a = {}^2\!/_3L$ and $b = {}^1\!/_3L$,

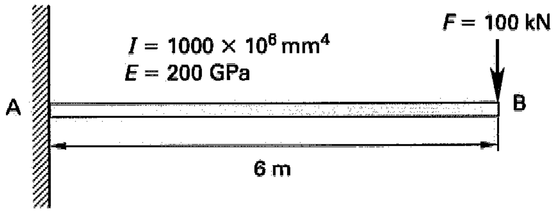
$$EI\delta = \left(\frac{F(\frac{1}{3}L)(\frac{2}{3}L)}{6L}\right)\left(L^2 - \frac{L^2}{9} - \frac{4L^2}{9}\right)$$

$$\delta = \frac{4FL^3}{243EI}$$

The answer is (B).

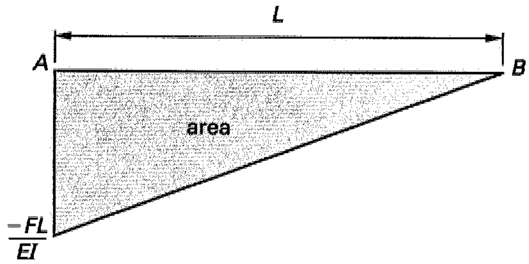
MECHANICS OF MATERIALS–65

What is the deflection at point B for the beam shown?



(A) 17 mm (B) 25 mm (C) 36 mm (D) 48 mm

Using the moment-area method, the M/EI diagram is



$$A = \tfrac{1}{2}L\left(\frac{-FL}{EI}\right) = -\frac{FL^2}{2EI}$$

The first moment with respect to point A is

$$Q = A(\tfrac{2}{3}L) = -\left(\frac{FL^2}{2EI}\right)(\tfrac{2}{3}L) = -\frac{FL^3}{3EI}$$

The deflection is

$$y = -Q = \frac{FL^3}{3EI} = \frac{(100\ \text{kN})(6\ \text{m})^3}{(3)\left(200\times 10^6\ \dfrac{\text{kN}}{\text{m}^2}\right)(1000\times 10^6\ \text{mm}^4)\left(\dfrac{1\ \text{m}}{1000\ \text{mm}}\right)^4}$$

$$= 0.036\ \text{m} \quad (36\ \text{mm})$$

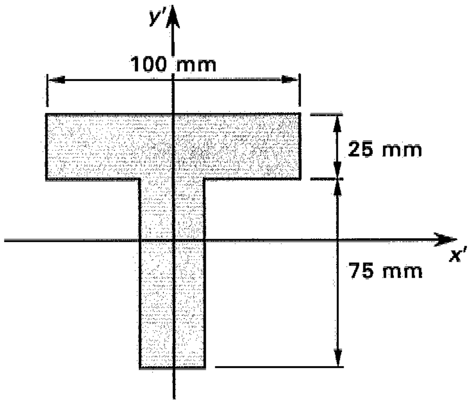
The answer is (C).

MECHANICS OF MATERIALS–66

What is the Euler buckling load for a 10 m long steel column pinned at both ends and with the given properties and cross section?

$$I_{x'x'} = 3.70 \times 10^6\ \text{mm}^4$$

$$E = 200\ \text{GPa}$$



(A) 15 kN (B) 24 kN (C) 43 kN (D) 73 kN

$x'x'$ and $y'y'$ are centroidal axes.

$$I_{x'x'} = 3.70 \times 10^6 \text{ mm}^4$$

$I_{y'y'}$ is computed by applying the equation for I $(bh^3/12)$ about the centroidal axis of a rectangle. For this cross section, $b_1 = 25$ mm, $h_1 = 100$ mm, $b_2 = 75$ mm, and $h_2 = 25$ mm.

$$\begin{aligned} I_{y'y'} &= \left(\left(\frac{1}{12}\right)(25 \text{ mm})(100 \text{ mm})^3 + \left(\frac{1}{12}\right)(75 \text{ mm})(25 \text{ mm})^3\right) \\ &\quad \times \left(\frac{1 \text{ m}}{1000 \text{ mm}}\right)^4 \\ &= 2.18 \times 10^{-6} \text{ m}^4 \end{aligned}$$

The Euler buckling load, P_{cr}, is

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

I is the minimum I value. $I_{y'y'}$ is less that $I_{x'x'}1$.

$$\begin{aligned} P_{cr} &= \frac{\pi^2 \left(200 \times 10^6 \ \frac{\text{kN}}{\text{m}^2}\right)(2.18 \times 10^{-6} \text{ m}^4)}{(10 \text{ m})^2} \\ &= 43 \text{ kN} \end{aligned}$$

The answer is (C).

11 DYNAMICS

DYNAMICS–1

How many degrees of freedom does a coin rolling on the ground have?

(A) one (B) two (C) three (D) five

A coin has two translational degrees of freedom and one rotational degree of freedom.

The answer is (C).

DYNAMICS–2

What is the definition of instantaneous velocity?

(A) $\mathrm{v} = dx\ dt$ (B) $\mathrm{v} = \int x\ dt$ (C) $\mathrm{v} = \frac{dx}{dt}$ (D) $\mathrm{v} = \lim_{\Delta t \to 0} \frac{\Delta t}{\Delta x}$

By definition,

$$\mathrm{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The answer is (C).

DYNAMICS–3

A car travels 100 km to city A in 2 h, then travels 200 km to city B in 3 h. What is the average speed of the car for the trip?

(A) 45 km/h (B) 58 km/h (C) 60 km/h (D) 66 km/h

Average velocity is defined as total distance traveled over total time.

$$\mathrm{v}_{\text{ave}} = \frac{\Delta x}{\Delta t} = \frac{100\ \text{km} + 200\ \text{km}}{2\ \text{h} + 3\ \text{h}}$$
$$= 60\ \text{km/h}$$

The answer is (C).

DYNAMICS–4

The position of a particle moving along the x-axis is given by $x(t) = t^2 - t + 8$, where x is in units of meters, and t is in seconds. Find the velocity of the particle when $t = 5$ s.

(A) 9.0 m/s (B) 10 m/s (C) 11 m/s (D) 12 m/s

The velocity equation is the first derivative of the position equation with respect to time. Therefore,

$$\mathrm{v}(t) = \frac{dx}{dt}$$
$$= \frac{d}{dt}(t^2 - t + 8)$$
$$= 2t - 1$$
$$\mathrm{v}(5) = (2)(5) - 1 = 9.0\ \text{m/s}$$

The answer is (A).

DYNAMICS–5

If a particle's position is given by the expression $x(t) = 3.4t^3 - 5.4t$ m, what is most nearly the acceleration of the particle at $t = 5$ s?

(A) 1.0 m/s^2 (B) 3.4 m/s^2 (C) 18 m/s^2 (D) 100 m/s^2

The acceleration is found from the second derivative of the position equation. Therefore,

$$\begin{aligned} a(t) &= \frac{d^2x}{dt^2} \\ &= \frac{d^2}{dt^2}(3.4t^3 - 5.4t) \\ &= \frac{d}{dt}(10.2t^2 - 5.4) \\ &= 20.4t \end{aligned}$$

$$a(5) = (20.4)(5) = 102 \text{ m/s}^2 \quad (100 \text{ m/s}^2)$$

The answer is (D).

DYNAMICS–6

A car starts from rest and moves with a constant acceleration of 6 m/s^2. What is the speed of the car after 4 s?

(A) 18 m/s (B) 24 m/s (C) 35 m/s (D) 55 m/s

For uniformly accelerated motion,

$$\begin{aligned} v &= v_0 + at = 0 + \left(6 \ \frac{\text{m}}{\text{s}^2}\right)(4 \text{ s}) \\ &= 24 \text{ m/s} \end{aligned}$$

The answer is (B).

DYNAMICS–7

A car starts from rest and has a constant acceleration of 3 m/s^2. What is the average velocity during the first 10 s of motion?

(A) 12 m/s (B) 13 m/s (C) 14 m/s (D) 15 m/s

The distance traveled by the car is

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 = 0 + 0 + \left(\frac{1}{2}\right)\left(3\ \frac{\text{m}}{\text{s}^2}\right)(10\ \text{s})^2$$

$$= 150\ \text{m}$$

$$v_{\text{ave}} = \frac{\Delta x}{\Delta t} = \frac{150\ \text{m}}{10\ \text{s}}$$

$$= 15\ \text{m/s}$$

The answer is (D).

DYNAMICS–8

A truck increases its speed uniformly from 13 km/h to 50 km/h in 25 s. What is most nearly the acceleration of the truck?

(A) 0.22 m/s^2 (B) 0.41 m/s^2 (C) 0.62 m/s^2 (D) 0.92 m/s^2

For uniformly accelerated rectilinear motion,

$$v = v_0 + at$$

$$at = v - v_0$$

$$a = \frac{v - v_0}{t} = \left(\frac{50\ \frac{\text{km}}{\text{h}} - 13\ \frac{\text{km}}{\text{h}}}{25\ \text{s}}\right)\left(\frac{1000\ \frac{\text{m}}{\text{km}}}{3600\ \frac{\text{s}}{\text{h}}}\right)$$

$$= 0.411\ \text{m/s}^2 \quad (0.41\ \text{m/s}^2)$$

The answer is (B).

DYNAMICS–9

A bicycle moves with a constant deceleration of $-2\ \text{m/s}^2$. If the initial velocity of the bike is 4.0 m/s, how far does it travel in 3 s?

(A) 2.0 m (B) 2.5 m (C) 3.0 m (D) 4.0 m

For constant acceleration,

$$\begin{aligned} x &= x_0 + \text{v}_0 t + \tfrac{1}{2}at^2 \\ &= 0 + \left(4.0\ \frac{\text{m}}{\text{s}}\right)(3\ \text{s}) + \left(\frac{1}{2}\right)\left(-2\ \frac{\text{m}}{\text{s}^2}\right)(3\ \text{s})^2 \\ &= 3.0\ \text{m} \end{aligned}$$

The answer is (C).

DYNAMICS–10

A ball is dropped from a height of 60 m above ground. How long does it take to hit the ground?

(A) 1.3 s (B) 2.1 s (C) 3.5 s (D) 5.5 s

The positive y direction is downward, and $y = 0$ at 60 m above ground. For uniformly accelerated motion,

$$\begin{aligned} y &= y_0 + \text{v}_0 t + \tfrac{1}{2}at^2 \\ 60 &= 0 + 0 + \left(\frac{1}{2}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)t^2 \\ t &= 3.5\ \text{s} \end{aligned}$$

The answer is (C).

DYNAMICS–11

A man driving a car at 65 km/h suddenly sees an object in the road 20 m ahead. Assuming an instantaneous reaction on the driver's part, what constant deceleration is required to stop the car in this distance?

(A) 7.1 m/s^2 (B) 7.5 m/s^2 (C) 8.0 m/s^2 (D) 8.1 m/s^2

For uniform deceleration, the velocity equation that is not a function of time is

$$v^2 = v_0^2 + 2a(x - x_0)$$

Using v = 0, v_0 = 65 km/h = 18 m/s, and $(x - x_0)$ = 20 m,

$$0 = \left(18\ \frac{\text{m}}{\text{s}}\right)^2 + 2a(20\ \text{m})$$

$$a = -\frac{\left(18\ \frac{\text{m}}{\text{s}}\right)^2}{(2)(20\ \text{m})} = 8.1\ \text{m/s}^2$$

The answer is (D).

DYNAMICS–12

A ball is thrown vertically upward with an initial speed of 24 m/s. Most nearly how long will it take for the ball to return to the thrower?

(A) 2.3 s (B) 2.6 s (C) 4.1 s (D) 4.9 s

At the apex of its flight, the ball has zero velocity and is at the midpoint of its flight time. If the total flight time is t_{total}, then the time elapsed at this point is $1/2 t_{\text{total}}$.

$$v = v_0 + at$$

Rearranging to solve for t_{total},

$$0 = 24\ \frac{\text{m}}{\text{s}} + \left(-9.81\ \frac{\text{m}}{\text{s}^2}\right)\tfrac{1}{2}t_{\text{total}}$$

$$t_{\text{total}} = (2)\left(\frac{24\ \frac{\text{m}}{\text{s}}}{9.81\ \frac{\text{m}}{\text{s}^2}}\right) = 4.893\ \text{s} \quad (4.9\ \text{s})$$

The answer is (D).

DYNAMICS–13

A projectile is launched upward from level ground at an angle of 60° from the horizontal. It has an initial velocity of 45 m/s. How long will it take before the projectile hits the ground?

(A) 4.1 s (B) 5.8 s (C) 7.9 s (D) 9.5 s

The projectile will experience acceleration only in the y direction due to gravity. The y component of velocity is

$$\mathrm{v}_{0y} = 45 \sin 60° = 39 \text{ m/s}$$

For uniform rectilinear motion with constant acceleration,

$$\begin{aligned} y &= y_0 + \mathrm{v}_{0y}t + \tfrac{1}{2}at^2 \\ &= 0 + \left(39 \ \frac{\text{m}}{\text{s}}\right) t + \left(\frac{1}{2}\right)\left(-9.81 \ \frac{\text{m}}{\text{s}^2}\right) t^2 \\ &= \left(39 \ \frac{\text{m}}{\text{s}}\right) t - \left(4.91 \ \frac{\text{m}}{\text{s}^2}\right) t^2 \end{aligned}$$

When the body is on the ground, $y = 0$.

$$0 = \left(39 \ \frac{\text{m}}{\text{s}}\right) t - \left(4.91 \ \frac{\text{m}}{\text{s}^2}\right) t^2 = t\left(39 \ \frac{\text{m}}{\text{s}} - 4.91 \ \frac{\text{m}}{\text{s}^2} t\right)$$

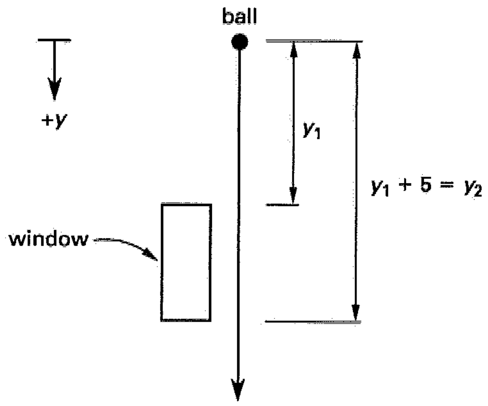
$$t = \frac{3.9 \ \frac{\text{m}}{\text{s}}}{4.9 \ \frac{\text{m}}{\text{s}^2}} = 7.94 \text{ s} \quad (7.9 \text{ s})$$

The answer is (C).

DYNAMICS–14

A man standing at a 5 m tall window watches a falling ball pass by the window in 0.3 s. From approximately how high above the top of the window was the ball released from a stationary position?

(A) 8.2 m (B) 9.6 m (C) 12 m (D) 21 m



The positive y direction is taken as downward, and the initial release point is y_0. Then,

$$y = y_0 + \mathrm{v}_o t + \tfrac{1}{2}at^2 = 0 + 0 + \tfrac{1}{2}at^2$$
$$y_1 = \tfrac{1}{2}at_1^2$$
$$y_2 = \tfrac{1}{2}at_2^2$$

However, $y_2 = y_1 + 5$ m, and $t_2 = t_1 + 0.3$ s. Therefore,

$$y_1 + 5 \text{ m} = \tfrac{1}{2}a(t_1 + 0.3 \text{ s})^2$$
$$\tfrac{1}{2}at_1^2 + 5 \text{ m} = \tfrac{1}{2}a\left(t_1^2 + (2)(0.3 \text{ s})t_1 + (0.09 \text{ s}^2)\right)$$
$$= \tfrac{1}{2}at_1^2 + (0.3 \text{ s})at_1 + (0.045 \text{ s}^2)a$$

Rearrange and solve for t_1.

$$(0.3 \text{ s})at_1 = 5 \text{ m} - (0.045 \text{ s}^2)a$$
$$t_1 = \frac{5 \text{ m} - (0.045 \text{ s}^2)a}{(0.3 \text{ s})a}$$
$$= \frac{5 \text{ m} - (0.045 \text{ s}^2)\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)}{(0.3 \text{ s})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)}$$
$$= 1.55 \text{ s}$$

Solving for y_1,

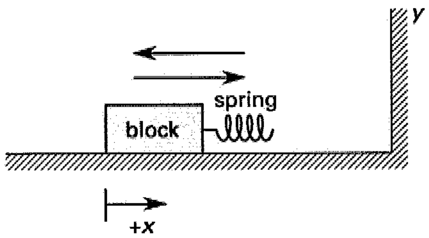
$$y_1 = \tfrac{1}{2}at_1^2 = \left(\frac{1}{2}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1.55\ \text{s})^2$$

$$= 11.8\ \text{m} \quad (12\ \text{m})$$

The answer is (C).

DYNAMICS–15

A block with a spring attached to one end slides along a rough surface with an initial velocity of 7 m/s. After it slides 4 m, it impacts a wall for 0.1 s, and then slides 10 m in the opposite direction before coming to a stop. If the block's deceleration is assumed constant and the contraction of the spring is negligible, what is the average acceleration of the block during impact with the wall?



(A) $-120\ \text{m/s}^2$ (B) $-100\ \text{m/s}^2$ (C) $-99\ \text{m/s}^2$ (D) $-49\ \text{m/s}^2$

$a_{1\text{-}2}$	acceleration before impact	m/s^2
$a_{2\text{-}2'}$	acceleration during impact	m/s^2
$a_{2\text{-}3}$	acceleration after impact	m/s^2
$s_{1\text{-}2}$	distance traveled before impact	m
$s_{2\text{-}3}$	distance traveled after impact	m
v_1	initial velocity	m/s
v_2	velocity just before impact	m/s
v_2'	velocity after impact	m/s
v_3	final velocity	m/s

$$v_1 = 7\ \text{m/s}$$
$$v_3 = 0$$

Because Δx is small and energy is conserved, $v_2 = v_2'$.

$$\begin{aligned} v_2 &= \sqrt{v_1^2 - 2a_{1\text{-}2}s_{1\text{-}2}} \\ &= \sqrt{\left(7\ \frac{\text{m}}{\text{s}}\right)^2 - 2(a_{1\text{-}2})(4\ \text{m})} \\ &= \sqrt{49\ \frac{\text{m}^2}{\text{s}^2} - 8a_{1\text{-}2}} \end{aligned}$$

Alternatively,

$$a_{1\text{-}2} = \frac{49\ \frac{\text{m}^2}{\text{s}^2} - v_2^2}{8\ \text{m}}$$

$$v_3 = \sqrt{v_{2'}^2 - 2a_{2\text{-}3}s_{2\text{-}3}}$$

$$0 = \sqrt{v_{2'}^2 - 2a_{2\text{-}3}(10\ \text{m})}$$

Alternatively,

$$a_{2\text{-}3} = \frac{v_{2'}^2}{20}$$

But $a_{1\text{-}2} = a_{2\text{-}3}$, and $v_2 = v_{2'}$.

$$\frac{49\ \frac{\text{m}^2}{\text{s}^2} - v_2^2}{8\ \text{m}} = \frac{v_2^2}{20\ \text{m}}$$

$$980\ \frac{\text{m}^2}{\text{s}^2} - 20v_2^2 = 8{v_2}^2$$

$$v_2 = 5.9\ \text{m/s}$$

Then, because of the direction change,

$$\begin{aligned} a_{2-2'} &= \frac{v_{2'} - v_2}{\Delta t} = \frac{-5.9\ \frac{\text{m}}{\text{s}} - 5.9\ \frac{\text{m}}{\text{s}}}{0.10\ \text{s}} \\ &= -118\ \text{m/s}^2 \quad (-120\ \text{m/s}^2) \end{aligned}$$

The answer is (A).

DYNAMICS–16

A car starting from rest moves with a constant acceleration of 15 km/h^2 for 1 h, then decelerates at a constant −7.5 km/h^2 until it comes to a stop. Most nearly how far has it traveled?

(A) 15 km (B) 23 km (C) 25 km (D) 35 km

For constant acceleration,

$$x = x_0 + \mathrm{v}_0 t + \tfrac{1}{2}at^2$$

During acceleration, $x_0 = 0$, $\mathrm{v}_0 = 0$, $a = 15$ km/h^2, and $t = 1$ h. Then, the distance over which the car accelerates is

$$x_{\mathrm{acc}} = \left(\frac{1}{2}\right)\left(15\ \frac{\mathrm{km}}{\mathrm{h}^2}\right)(1\ \mathrm{h})^2 = 7.5\ \mathrm{km}$$

At the end of the hour, the car's velocity is

$$\begin{aligned}\mathrm{v} = \mathrm{v}_0 + at &= \left(15\ \frac{\mathrm{km}}{\mathrm{h}^2}\right)(1\ \mathrm{h})\\ &= 15\ \mathrm{km/h}\end{aligned}$$

During deceleration, $x_0 = 0$, $\mathrm{v}_0 = 15$ km/h, and $a = -7.5$ km/h^2. The car has velocity $\mathrm{v} = 0$ when it stops. Therefore,

$$\begin{aligned}\mathrm{v} &= \mathrm{v}_0 + at\\ 0 &= 15\ \frac{\mathrm{km}}{\mathrm{h}} + \left(-7.5\ \frac{\mathrm{km}}{\mathrm{h}^2}\right)t\\ t &= 2\ \mathrm{h}\\ x &= x_0 + \mathrm{v}_0 t + \tfrac{1}{2}at^2\end{aligned}$$

The distance over which the car decelerates is

$$x_{\mathrm{dec}} = \left(15\ \frac{\mathrm{km}}{\mathrm{h}}\right)(2\ \mathrm{h}) + \left(\frac{1}{2}\right)\left(-7.5\ \frac{\mathrm{km}}{\mathrm{h}^2}\right)(2\ \mathrm{h})^2 = 15\ \mathrm{km}$$

$$x_{\mathrm{total}} = x_{\mathrm{acc}} + x_{\mathrm{dec}} = 7.5\ \mathrm{km} + 15\ \mathrm{km} = 22.5\ \mathrm{km} \quad (23\ \mathrm{km})$$

The answer is (B).

DYNAMICS–17

A train with a top speed of 75 km/h cannot accelerate or decelerate faster than 1.2 m/s^2. What is the minimum distance between two train stops in order for the train to be able to reach its top speed?

(A) 300 m (B) 350 m (C) 360 m (D) 365 m

To travel the minimum distance, the train must accelerate from $v_0 = 0$ km/h to $v = 75$ km/h at a constant 1.2 m/s^2 and then decelerate at a constant 1.2 m/s^2 to $v = 0$ km/h. The train travels the same distance during acceleration as during deceleration, since the initial and final speeds are identical, as well as the magnitude of acceleration or deceleration. The following two equations apply for constant acceleration.

$$x = x_0 + v_0 t = \tfrac{1}{2}at^2$$
$$v = v_0 + at$$

During acceleration, $x_0 = 0$ m, $v_0 = 0$ km/h, $v = 75$ km/h $=$ 20.8 m/s, and $a = 1.2$ m/s^2. Then,

$$t = \frac{v - v_0}{a} = \frac{20.8\ \frac{\text{m}}{\text{s}} - 0\ \frac{\text{m}}{\text{s}}}{1.2\ \frac{\text{m}}{\text{s}^2}} = 17.3\ \text{s}$$

$$x = \left(\frac{1}{2}\right)\left(1.2\ \frac{\text{m}}{\text{s}^2}\right)(17.3\ \text{s})^2 = 180\ \text{m}$$

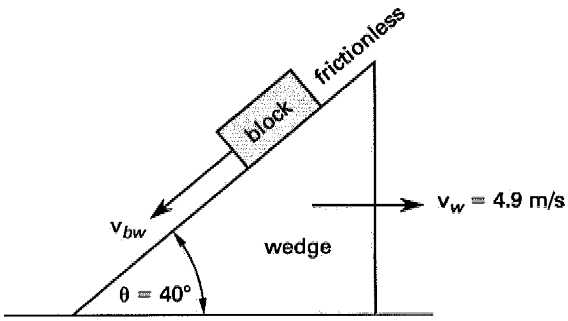
The minimum total distance is

$$x = (2)(180\ \text{m}) = 360\ \text{m}$$

The answer is (C).

DYNAMICS–18

A block with a mass of 150 kg slides down a frictionless wedge with a slope of 40°. The wedge is moving horizontally in the opposite direction at a constant velocity of 4.9 m/s. What is most nearly the absolute speed of the block 2 s after it is released from rest?



(A) 8.9 m/s (B) 9.4 m/s (C) 9.5 m/s (D) 9.8 m/s

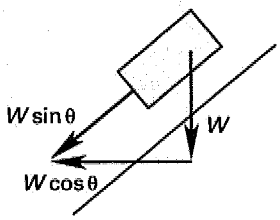
Let v_{bw} equal the velocity of the block relative to the wedge's slope. The component of gravitational force in this direction, F_{slope}, is $W \sin\theta$. Down the slope, relative to the wedge,

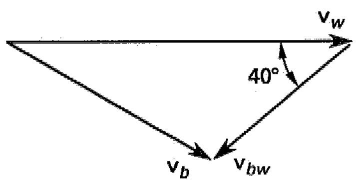
$$F_{slope} = W\sin\theta = ma_{slope}$$

$$a_{slope} = \frac{W\sin\theta}{m} = g\sin\theta$$

$$\begin{aligned} v_{bw} &= v_0 + a_{slope}t = 0 + gt\sin\theta \\ &= \left(9.81\ \frac{m}{s^2}\right)(2\ s)\sin 40° \\ &= 12.6\ m/s \end{aligned}$$

The absolute velocity, v_b, can be found from a velocity triangle.





The law of cosines gives

$$v_b^2 = v_w^2 + v_{bw}^2 - 2v_w v_{bw} \cos\theta$$

$$= \left(4.9\ \frac{m}{s}\right)^2 + \left(12.6\ \frac{m}{s}\right)^2 - (2)\left(4.9\ \frac{m}{s}\right)\left(12.6\ \frac{m}{s}\right)\cos 40°$$

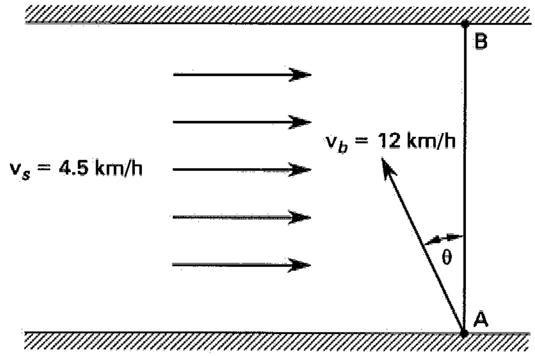
$$= 88.18\ m^2/s^2$$

$$v_b = 9.39\ m/s \quad (9.4\ m/s)$$

The answer is (B).

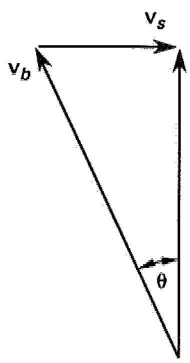
DYNAMICS–19

A stream flows at $v_s = 4.5$ km/h. At what angle, θ, upstream should a boat traveling at $v_b = 12$ km/h be launched in order to reach the shore directly opposite the launch point?



(A) 22° (B) 24° (C) 26° (D) 28°

Draw a velocity triangle.



$$\sin\theta = \frac{\mathrm{v}_s}{\mathrm{v}_b} = \frac{4.5\ \frac{\text{km}}{\text{h}}}{12\ \frac{\text{km}}{\text{h}}}$$

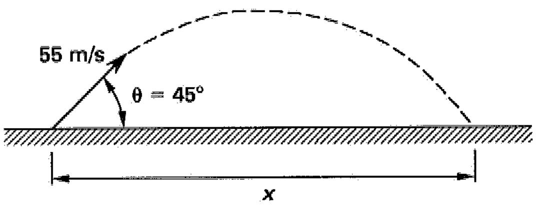
$$\theta = \sin^{-1}\left(\frac{4.5\ \frac{\text{km}}{\text{h}}}{12\ \frac{\text{km}}{\text{h}}}\right)$$

$$= 22°$$

The answer is (A).

DYNAMICS-20

An object is launched at 45° to the horizontal on level ground as shown. What is the range of the projectile if its initial velocity is 55 m/s? Neglect air resistance.



(A) 309 m (B) 617 m (C) 624 m (D) 680 m

Choosing the launch point as the origin of the x and y axes, $x_0 = y_0 = 0$. For uniform acceleration,

$$x = x_0 + \mathrm{v}_{x_0}t + \tfrac{1}{2}a_x t^2$$
$$y = y_0 + \mathrm{v}_{y_0}t + \tfrac{1}{2}a_y t^2$$

However, $a_x = 0\ \text{m/s}^2$, and $a_y = -9.81\ \text{m/s}^2$. Therefore,

$$\begin{aligned} x &= \text{v}_{x_0} t \\ y &= \text{v}_{y_0} t - \tfrac{1}{2} g t^2 \\ \text{v}_{x_0} &= \text{v}_0 \cos\theta \\ &= \left(55\ \frac{\text{m}}{\text{s}}\right) \cos 45^\circ \\ &= 38.9\ \text{m/s} \\ \text{v}_{y_0} &= \text{v}_0 \sin\theta \\ &= \left(55\ \frac{\text{m}}{\text{s}}\right) \sin 45^\circ \\ &= 38.9\ \text{m/s} \\ y &= 38.9\ \frac{\text{m}}{\text{s}} t - 4.9\ \frac{\text{m}}{\text{s}^2} t^2 \end{aligned}$$

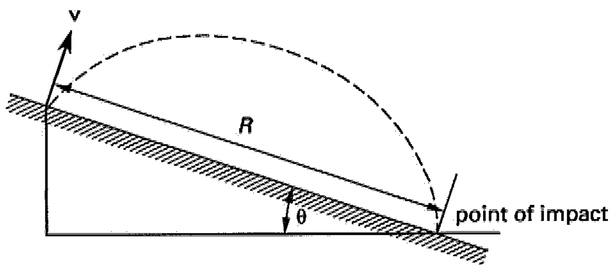
When the projectile is on the ground, $y = 0$ m/s. Thus,

$$\begin{aligned} 0 &= t\left(38.9\ \frac{\text{m}}{\text{s}} - 4.9\ \frac{\text{m}}{\text{s}^2} t\right) \\ t &= 7.94\ \text{s} \\ x &= \left(38.9\ \frac{\text{m}}{\text{s}}\right) t \\ &= \left(38.9\ \frac{\text{m}}{\text{s}}\right)(7.94\ \text{s}) \\ &= 308.9\ \text{m} \quad (309\ \text{m}) \end{aligned}$$

The answer is (A).

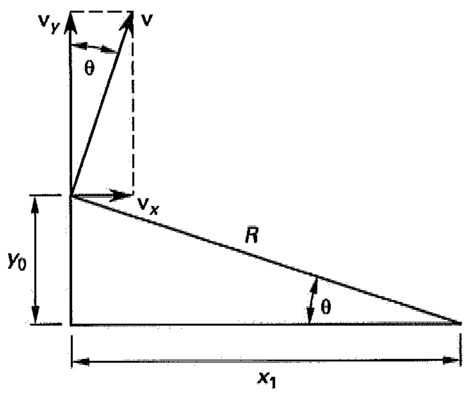
DYNAMICS–21

A projectile is fired with a velocity, v, perpendicular to a surface that is inclined at an angle, θ, with the horizontal. Determine the expression for the distance R to the point of impact.



(A) $R = \frac{2\text{v}^2 \sin\theta}{g\cos^2\theta}$ (B) $R = \frac{2\text{v}^2 \sin\theta}{g\cos\theta}$

(C) $R = \frac{2\text{v}\cos\theta}{g\sin\theta}$ (D) $R = \frac{2\text{v}\sin\theta}{g\cos\theta}$



Using the notation in the figure, $x_0 = 0$, $a_x = 0$, and $y_0 = R\sin\theta$. Therefore,

$$\begin{aligned} x &= x_0 + \text{v}_x t + \tfrac{1}{2}a_x t^2 \\ &= \text{v}_x t \\ &= \text{v}\sin\theta t \\ y &= y_0 + \text{v}_y t + \tfrac{1}{2}a_y t^2 \\ &= R\sin\theta + \text{v}_y t + \tfrac{1}{2}(-g)t^2 \\ &= R\sin\theta + \text{v}\cos\theta t - \tfrac{1}{2}gt^2 \end{aligned}$$

At impact, let $t = t_1$, $x_1 = R\cos\theta$, and $y_1 = 0$. The two equations above give

$$R\cos\theta = \mathrm{v}\sin\theta t_1$$

$$t_1 = \frac{R\cos\theta}{\mathrm{v}\sin\theta} \quad \text{[I]}$$

$$0 = R\sin\theta + \mathrm{v}\cos\theta t_1 - \tfrac{1}{2}gt_1^2 \quad \text{[II]}$$

Equations I and II give

$$0 = R\sin\theta + \mathrm{v}\cos\theta\left(\frac{R\cos\theta}{\mathrm{v}\sin\theta}\right) - \tfrac{1}{2}g\left(\frac{R\cos\theta}{\mathrm{v}\sin\theta}\right)^2$$

$$0 = \sin^2\theta + \cos^2\theta - \tfrac{1}{2}g\left(\frac{R\cos^2\theta}{\mathrm{v}^2\sin\theta}\right)$$

$$1 = \frac{g\cos^2\theta R}{2\mathrm{v}^2\sin\theta}$$

$$R = \frac{2\mathrm{v}^2\sin\theta}{g\cos^2\theta}$$

The answer is (A).

DYNAMICS–22

A cyclist on a circular track of radius $r = 240$ m is traveling at 8 m/s. His speed in the tangential direction (i.e., the direction of his travel) increases at the rate of 1 m/s^2. What is most nearly the cyclist's total acceleration?

(A) -0.9 m/s^2 (B) 0.7 m/s^2 (C) 0.9 m/s^2 (D) 1.0 m/s^2

The total acceleration is made up of tangential and normal components. The tangential component is given as 1 m/s^2. By definition, the normal component is

$$a_n = \frac{\mathrm{v}^2}{r} = \frac{\left(8\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2}{240\ \mathrm{m}} = 0.27\ \mathrm{m/s^2}$$

$$a = \sqrt{a_n^2 + a_t^2}$$

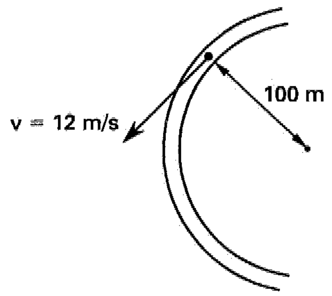
$$= \sqrt{\left(0.27\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)^2 + \left(1\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)^2}$$

$$= 1.04\ \mathrm{m/s^2} \quad (1.0\ \mathrm{m/s^2})$$

The answer is (D).

DYNAMICS–23

A motorcycle moves at a constant speed of v = 12 m/s around a curved road of radius r = 100 m. What is most nearly the magnitude and general direction of the motorcycle's acceleration?



(A) 1.1 m/s^2 away from the center of curvature
(B) 1.1 m/s^2 toward the center of curvature
(C) 1.4 m/s^2 away from the center of curvature
(D) 1.4 m/s^2 toward the center of curvature

The normal acceleration, a_n, is

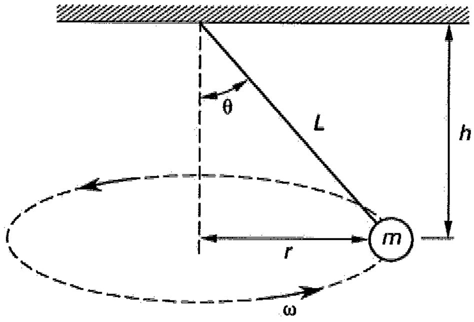
$$a_n = \frac{v^2}{r} = \frac{\left(12\ \frac{m}{s}\right)^2}{100\ m} = 1.44\ m/s^2 \quad (1.4\ m/s^2)$$

Since the velocity in the tangential direction is constant, $a_t = 0$. Thus, only the normal component of acceleration contributes to total acceleration, so $a = 1.44\ m/s^2$. The normal component is always directed toward the center of curvature.

The answer is (D).

DYNAMICS–24

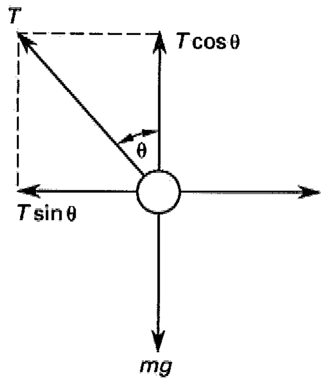
A pendulum of mass m and length L rotates about the vertical axis. If the angular velocity is ω, determine the expression for the height h.



(A) $h = \dfrac{g\cos\theta}{\omega^2}$ (B) $h = \dfrac{g}{\omega^2}$

(C) $h = \dfrac{g}{\omega^2\cos\theta}$ (D) $h = \dfrac{gL\cos\theta}{\omega^2}$

A free-body diagram of the pendulum is



Since the pendulum undergoes uniform circular motion,

$$T\sin\theta = ma_n = mr\omega^2 \qquad \text{[I]}$$

Assuming the pendulum is not accelerating in the vertical direction, a force balance gives

$$T\cos\theta = mg \qquad \text{[II]}$$

Combining equations I and II,

$$\tan\theta = \frac{r\omega^2}{g}$$

However, $\tan\theta = r/h$. Therefore,

$$h = \frac{r}{\tan\theta} = \frac{g}{\omega^2}$$

The answer is (B).

DYNAMICS–25

A 3 kg block is moving at a speed of 5 m/s. The force required to bring the block to a stop in 8×10^{-4} seconds is most nearly

(A) 10 kN (B) 13 kN (C) 15 kN (D) 19 kN

Newton's second law gives

$$\begin{aligned} F &= ma = m\frac{dv}{dt} = m\frac{\Delta v}{\Delta t} \\ &= (3\ \text{kg})\left(\frac{5\ \frac{\text{m}}{\text{s}}}{8\times 10^{-4}\ \text{s}}\right)\left(\frac{1}{1000\ \frac{\text{N}}{\text{kN}}}\right) \\ &= 18.75\ \text{kN} \quad (19\ \text{kN}) \end{aligned}$$

The answer is (D).

DYNAMICS–26

A rope is used to tow an 800 kg car with free-rolling wheels over a smooth, level road. The rope will break if the tension exceeds 2000 N. What is the greatest acceleration that the car can reach without breaking the rope?

(A) 1.2 m/s^2 (B) 2.5 m/s^2 (C) 3.8 m/s^2 (D) 4.5 m/s^2

$$F_{\text{max}} = ma_{\text{max}}$$

$$2000\ \text{N} = (800\ \text{kg})a_{\text{max}}$$

Rearranging to solve for a_{max},

$$a_{max} = \frac{2000 \text{ N}}{800 \text{ kg}} = 2.5 \text{ m/s}^2$$

The answer is (B).

DYNAMICS–27

A force of 15 N acts on a 16 kg body for 2 s. If the body is initially at rest, how far is it displaced by the force?

(A) 1.1 m (B) 1.5 m (C) 1.9 m (D) 2.1 m

The acceleration is found using Newton's second law.

$$a = \frac{F}{m} = \frac{15 \text{ N}}{16 \text{ kg}} = 0.94 \text{ m/s}^2$$

For a body undergoing constant acceleration, with $v_0 = 0 \text{ m/s}^2$ and $t = 2$ s,

$$\Delta x = \tfrac{1}{2}at^2 = \left(\frac{1}{2}\right)\left(0.94 \ \frac{\text{m}}{\text{s}^2}\right)(2 \text{ s})^2$$
$$= 1.88 \text{ m}$$

The answer is (C).

DYNAMICS–28

A car of mass $m = 150$ kg accelerates in 10 s from rest at a constant rate to a speed of v = 6 m/s. What is the resultant force on the car due to this acceleration?

(A) 75 N (B) 90 N (C) 95 N (D) 98 N

For constant acceleration,

$$v = v_0 + at$$

$$a = \frac{v - v_0}{t} = \frac{6\ \frac{\text{m}}{\text{s}} - 0\ \frac{\text{m}}{\text{s}}}{10\ \text{s}}$$

$$= 0.6\ \text{m/s}^2$$

$$F = ma = (150\ \text{kg})\left(0.6\ \frac{\text{m}}{\text{s}^2}\right) = 90\ \text{N}$$

The answer is (B).

DYNAMICS–29

A man weighs himself twice in an elevator. When the elevator is at rest, he weighs 824 N; when the elevator starts moving upward, he weighs 932 N. Most nearly how fast is the elevator accelerating, assuming constant acceleration?

(A) 0.64 m/s^2 (B) 1.1 m/s^2 (C) 1.3 m/s^2 (D) 9.8 m/s^2

The mass of the man can be determined from his weight at rest.

$$W = mg$$

$$m = \frac{W}{g} = \frac{824\ \text{N}}{9.81\ \frac{\text{m}}{\text{s}^2}}$$

$$= 84.0\ \text{kg}$$

At constant acceleration,

$$F = ma$$

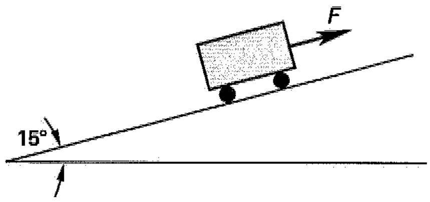
$$a = \frac{F}{m} = \frac{932\ \text{N} - 824\ \text{N}}{84.0\ \text{kg}}$$

$$= 1.29\ \text{m/s}^2 \quad (1.3\ \text{m/s}^2)$$

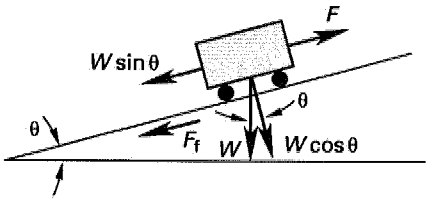
The answer is (C).

DYNAMICS–30

A truck weighing 1.4 kN moves up a slope of 15°. What is the force generated by the engine if the truck is accelerating at a rate of 3 m/s^2? Assume the coefficient of friction is $\mu = 0.1$.



(A) 876 N (B) 926 N (C) 930 N (D) 958 N



In the direction parallel to the slope, a force balance gives

$$\sum F_x = ma = F - (W \sin\theta + F_f)$$

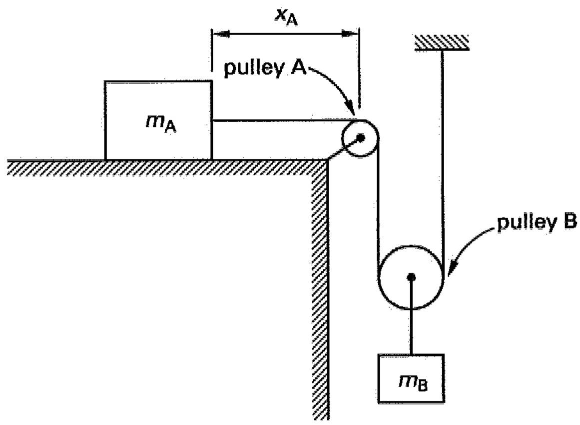
F_f is the friction force, which is equal to $\mu N = \mu W \cos\theta$.

$$\begin{aligned} F &= W(\sin\theta + \mu\cos\theta) + ma \\ &= (1400\ \text{N})(\sin 15° + 0.1\cos 15°) + \left(\frac{1400\ \text{N}}{9.81\ \frac{\text{m}}{\text{s}^2}}\right)\left(3\ \frac{\text{m}}{\text{s}^2}\right) \\ &= 925.7\ \text{N} \quad (926\ \text{N}) \end{aligned}$$

The answer is (B).

DYNAMICS–31

In the illustration, the two pulleys and the horizontal surface are frictionless. The cord connecting the masses m_A and m_B is weightless. What is the ratio of the acceleration of mass A to the acceleration of mass B? Assume the system is released from rest.

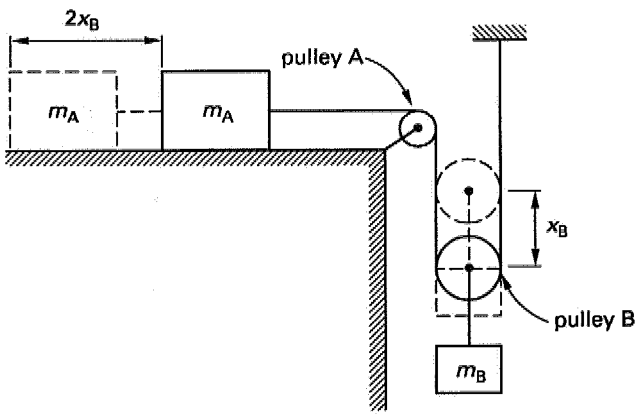


(A) 1/2 (B) 1 (C) 2 (D) m_A/m_B

Assuming the accelerations of both masses are constant, their respective displacement equations are

$$x_A = \tfrac{1}{2}a_A t^2$$

$$x_B = \tfrac{1}{2}a_B t^2$$



Taking the ratio of x_A to x_B,

$$\frac{x_A}{x_B} = \frac{a_A}{a_B}$$

Since mass B is supported by a two-segment rope section and mass A is pulled by only one rope, the displacement of mass B is half the displacement of mass A. Therefore,

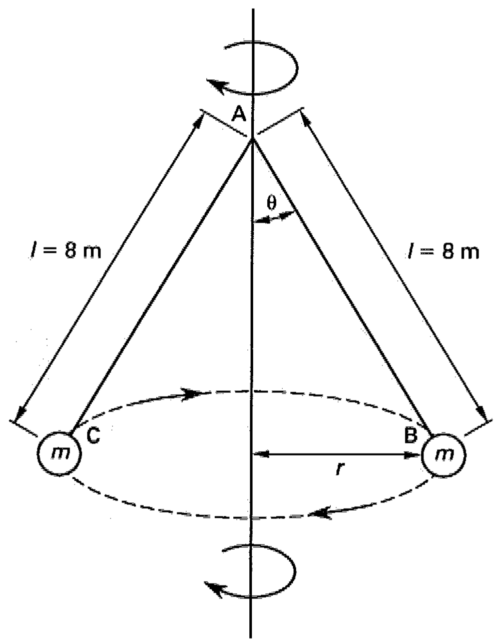
$$x_A = 2x_B$$

$$\frac{a_A}{a_B} = \frac{2x_B}{x_B} = 2$$

The answer is (C).

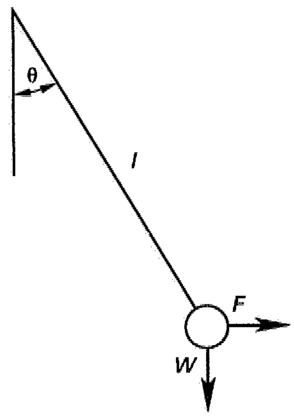
DYNAMICS–32

A simplified model of a carousel is illustrated. The 8 m long arms AB and AC attach the seats B and C, each with a mass of 200 kg, to a vertical rotating shaft. What is the maximum angle of tilt, θ, for the seats, if the carousel operates at 12 rpm?



(A) 39° (B) 40° (C) 45° (D) 51°

The free-body diagram is



The angular velocity, ω, of the carousel is

$$\omega = \frac{\left(12\ \frac{\text{rev}}{\text{min}}\right)\left(2\pi\ \frac{\text{rad}}{\text{rev}}\right)}{60\ \frac{\text{s}}{\text{min}}}$$
$$= 1.257\ \text{rad/s}$$

The rotational force, F, expressed in terms of θ is

$$\begin{aligned} F &= ma = mr\omega^2 \\ &= ml\sin\theta\omega^2 \\ &= (200\ \text{kg})(8\ \text{m})\sin\theta\left(1.257\ \frac{\text{rad}}{\text{s}}\right)^2 \\ &= (2528\ \text{N})\sin\theta \end{aligned}$$

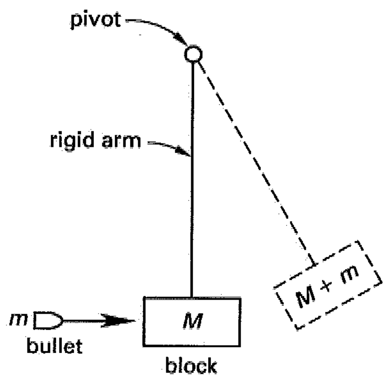
From the free-body diagram,

$$\tan\theta = \frac{F}{W} = \frac{(2528\ \text{N})\sin\theta}{(200\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$
$$= 1.288\sin\theta$$
$$\cos\theta = \frac{1}{1.288}$$
$$\theta = \cos^{-1}\left(\frac{1}{1.288}\right)$$
$$= 39.07^\circ \quad (39^\circ)$$

The answer is (A).

DYNAMICS–33

In the ballistic pendulum shown, a bullet of mass m is fired into a block of mass M that can swing freely. Which of the following is true for the system during the swing motion after impact?



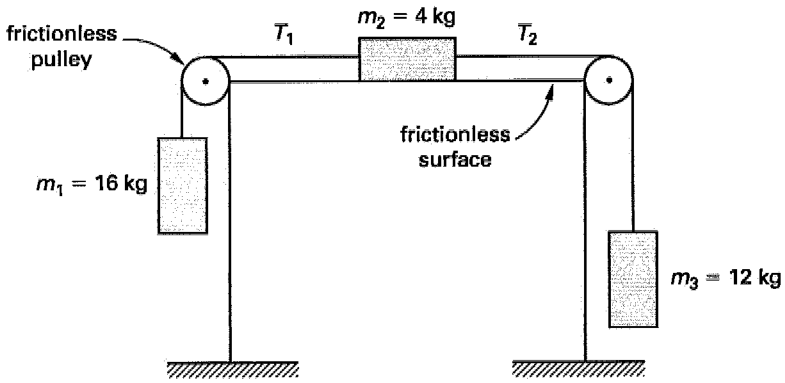
(A) Both mechanical energy and momentum are conserved.
(B) Mechanical energy is conserved; momentum is not conserved.
(C) Momentum is conserved; mechanical energy is not conserved.
(D) Neither mechanical energy nor momentum is conserved.

Momentum is not conserved since an external force, gravity, acts on the bullet-block mass. Only mechanical energy is conserved.

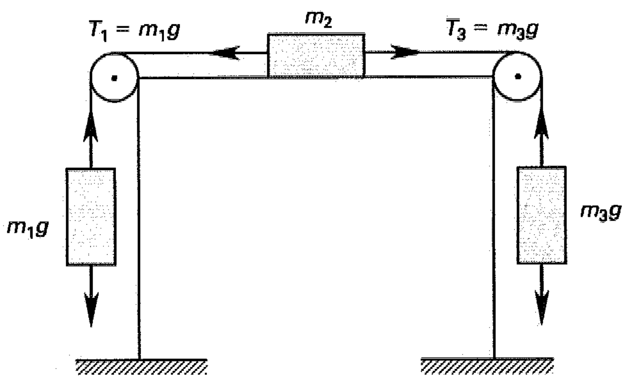
The answer is (B).

DYNAMICS–34

Three masses are attached by a weightless cord as shown. If mass m_2 is exactly halfway between the other masses and is located at the center of the flat surface when the masses are released, what is most nearly its initial acceleration? Assume there is no friction in the system and that the pulleys have no mass.



(A) 1.0 m/s² (B) 1.2 m/s² (C) 9.8 m/s² (D) 12 m/s²



Since $m_1 > m_3$, m_1 will move downward and m_2 will be displaced to the left. All masses contribute to the inertia of the system.

$$T_1 - T_3 = a_2 m_{\text{total}} = a_2(m_1 + m_2 + m_3)$$

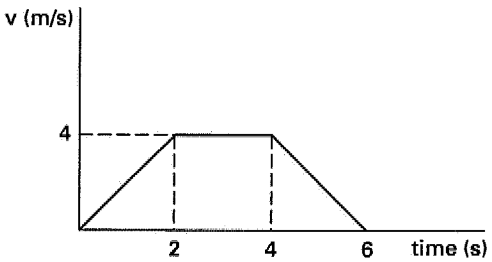
T_1 and T_3 are the tensions in the cord due to the masses m_1 and m_3.

$$T_1 = m_1 g$$
$$T_3 = m_3 g$$
$$a_2 = g\left(\frac{m_1 - m_3}{m_1 + m_2 + m_3}\right)$$
$$= \left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{16\ \text{kg} - 12\ \text{kg}}{16\ \text{kg} + 4\ \text{kg} + 12\ \text{kg}}\right)$$
$$= 1.23\ \text{m/s}^2 \quad (1.2\ \text{m/s}^2)$$

The answer is (B).

DYNAMICS–35

The maximum capacity (occupant load) of an elevator is 1000 N. The elevator starts from rest, and its velocity varies with time as shown in the graph. What is most nearly the maximum additional tension in the elevator cable due to the occupants at full capacity? Neglect the mass of the elevator.



(A) 960 N (B) 1000 N (C) 1200 N (D) 1400 N

The maximum tension occurs during the period of maximum acceleration. This occurs for $0\ \text{s} < t < 2\ \text{s}$, with acceleration, a, equal to $\text{v}/t = 4\ \text{m/s}/2\ \text{s} = 2\ \text{m/s}^2$. The mass of the occupants is $m = 1000\ \text{N}/9.81\ \text{m/s}^2$. During this time,

$$\sum F = ma = T - W$$
$$T = W + ma = 1000\ \text{N} + \left(\frac{1000\ \text{N}}{9.81\ \frac{\text{m}}{\text{s}^2}}\right)\left(2\ \frac{\text{m}}{\text{s}^2}\right)$$
$$= 1204\ \text{N} \quad (1200\ \text{N})$$

The answer is (C).

DYNAMICS–36

What is most nearly the kinetic energy of a 3924 N motorcycle traveling at 40 km/h?

(A) 11 100 J (B) 12 300 J (C) 23 600 J (D) 24 600 J

$$m = \frac{W}{g} = \frac{3924 \text{ N}}{9.81 \frac{\text{m}}{\text{s}^2}} = 400 \text{ kg}$$

$$\text{v} = \left(40 \ \frac{\text{km}}{\text{h}}\right)\left(\frac{1000 \ \frac{\text{m}}{\text{km}}}{3600 \ \frac{\text{s}}{\text{h}}}\right) = 11.1 \text{ m/s}$$

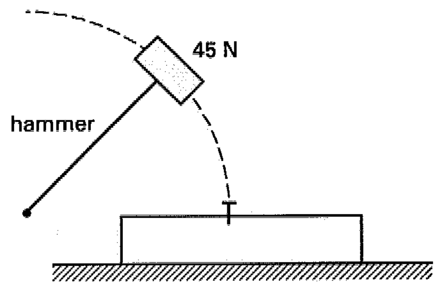
The kinetic energy is

$$E_k = \tfrac{1}{2}m\text{v}^2 = \left(\frac{1}{2}\right)(400 \text{ kg})\left(11.1 \ \frac{\text{m}}{\text{s}}\right)^2$$
$$= 24\,640 \text{ J} \quad (24\,600 \text{ J})$$

The answer is (D).

DYNAMICS–37

A lead hammer weighs 45 N. In one swing of the hammer, a nail is driven 1.5 cm into a wood block. The velocity of the hammer's head at impact is 4.5 m/s. What is most nearly the average resistance of the wood block?



(A) 3090 N (B) 3100 N (C) 3920 N (D) 4090 N

Because energy is conserved, the kinetic energy of the hammer before impact is equal to the work done by the resistance force of the wood block. $m = 45\ \text{N}/9.81\ \text{m/s}^2 = 4.59\ \text{kg}$, $\text{v} = 4.5\ \text{m/s}$, and $x_{\text{nail}} = 0.015\ \text{m}$.

$$\tfrac{1}{2}m\text{v}^2 = Fx$$

$$F = \frac{\frac{1}{2}m\text{v}^2}{x} = \frac{\left(\frac{1}{2}\right)(4.59\ \text{kg})\left(4.5\ \frac{\text{m}}{\text{s}}\right)^2}{0.015\ \text{m}}$$

$$= 3098\ \text{N} \quad (3100\ \text{N})$$

The answer is (B).

DYNAMICS-38

An automobile uses 74.6 kW to maintain a uniform speed of 96 km/h. What is the thrust force provided by the engine?

(A) 0.87 kN (B) 2.8 kN (C) 3.2 kN (D) 5.6 kN

Power is defined as work done per unit time, which, for a linear system, is equivalent to force times velocity. Therefore,

$$P = F\text{v}$$

$$F = \frac{P}{\text{v}}$$

$$= \frac{(74.6\ \text{kW})\left(1000\ \frac{\text{W}}{\text{kW}}\right)}{\left(96\ \frac{\text{km}}{\text{h}}\right)\left(1000\ \frac{\text{m}}{\text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)}$$

$$= 2797.5\ \text{N} \quad (2.8\ \text{kN})$$

The answer is (B).

DYNAMICS–39

A 580 N man is standing on the top of a building 40 m above the ground. What is his potential energy relative to the ground?

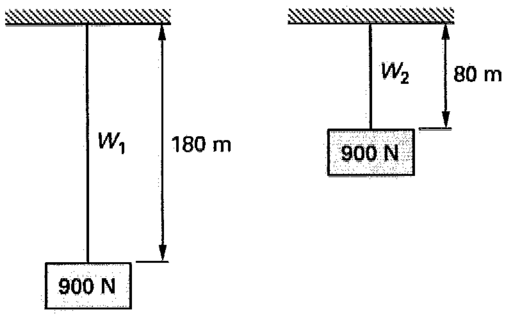
(A) 10 kJ (B) 12 kJ (C) 20 kJ (D) 23 kJ

$$E_p = Wy = (580\ \text{N})(40\ \text{m}) = 23\,200\ \text{J}$$

The answer is (D).

DYNAMICS–40

A 900 N object is initially suspended on a 180 m long cable. The object is then raised 100 m. If the cable weighs 16 N/m, how much work is done?



(A) 100 000 J (B) 298 000 J (C) 320 000 J (D) 398 000 J

The weight of the extended cable for the two situations is

$$W_1 = (180\ \text{m})\left(16\ \frac{\text{N}}{\text{m}}\right) = 2880\ \text{N}$$

$$W_2 = (80\ \text{m})\left(16\ \frac{\text{N}}{\text{m}}\right) = 1280\ \text{N}$$

These weights may be considered to be concentrated at the midpoints of the extended cables. Choosing the datum to be at the top of the cable, and using the work-energy principle, the work done is equal to the difference in potential energies of the two situations.

$$E_p = (\text{weight})(\text{distance}) + W_1 L$$

$$E_{p1} = (900\ \text{N})(-180\ \text{m}) + (2880\ \text{N})\left(\frac{-180\ \text{m}}{2}\right) = -421\,200\ \text{J}$$

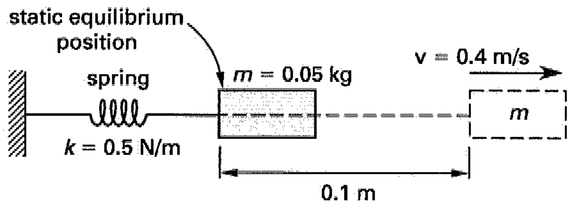
$$E_{p2} = (900\ \text{N})(-80\ \text{m}) + (1280\ \text{N})\left(\frac{-80\ \text{m}}{2}\right) = -123\,200\ \text{J}$$

$$W = E_{p2} - E_{p1} = -123\,200\ \text{J} - (-421\,200\ \text{J}) = 298\,000\ \text{J}$$

The answer is (B).

DYNAMICS–41

A 0.05 kg mass attached to a spring (spring constant, $k = 0.5$ N/m) is accelerated to a velocity of 0.4 m/s. What is the total energy for the body in the following diagram? Neglect the spring mass.



(A) 0.0025 J (B) 0.0040 J (C) 0.0065 J (D) 0.0092 J

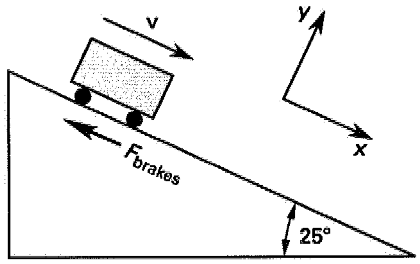
The total energy is the sum of the kinetic and potential energies.

$$\begin{aligned} E &= E_k + E_p \\ &= \tfrac{1}{2}m\mathrm{v}^2 + \tfrac{1}{2}kx^2 \\ &= \left(\frac{1}{2}\right)(0.05\ \text{kg})\left(0.4\ \frac{\text{m}}{\text{s}}\right)^2 + \left(\frac{1}{2}\right)\left(0.5\ \frac{\text{N}}{\text{m}}\right)(0.1\ \text{m})^2 \\ &= 0.0065\ \text{J} \end{aligned}$$

The answer is (C).

DYNAMICS–42

A 1000 kg car is traveling down a 25° slope. At the instant that the speed is 13 m/s, the driver applies the brakes. What constant force parallel to the road must be generated by the brakes if the car is to stop in 90 m?



(A) 1290 N (B) 2900 N (C) 5080 N (D) 8630 N

The change in energy is equal to the work done by the brakes. The change in velocity squared is

$$\mathrm{v}^2 - \mathrm{v}_0^2 = 0 - \left(13\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2 = -169\ \mathrm{m}^2/\mathrm{s}^2$$

The change in elevation of the car is

$$h - h_0 = 0 - (90\ \mathrm{m})\sin 25° = -38\ \mathrm{m}$$

$$\Delta E_k + \Delta E_p = Fx$$

$$\tfrac{1}{2}m(\mathrm{v}^2 - \mathrm{v}_0^2) + mg(h - h_0) = Fx$$

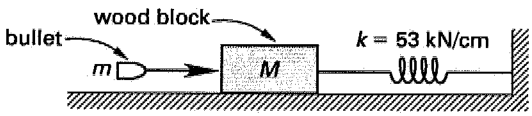
$$\left(\frac{1}{2}\right)(1000\ \mathrm{kg})\left(-169\ \frac{\mathrm{m}^2}{\mathrm{s}^2}\right) + (1000\ \mathrm{kg})\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)(-38\ \mathrm{m}) = F(90\ \mathrm{m})$$

$$F = -5080\ \mathrm{N}$$

The answer is (C).

DYNAMICS–43

A bullet of mass 100 g is fired at a wooden block resting on a horizontal surface. A spring with stiffness $k = 53$ kN/cm resists the motion of the block. If the maximum displacement of the block produced by the impact of the bullet is 3.4 cm, what is most nearly the velocity of the bullet at impact? Assume there are no losses at impact, and the spring has no mass.



(A) 250 km/h (B) 450 km/s (C) 630 km/h (D) 890 km/h

Due to the conservation of energy, the kinetic energy of the bullet before impact is equal to the potential energy of the spring-mass-bullet system at maximum compression.

$$E_{k,\text{bullet}} = E_{p,\text{system}}$$

$$\tfrac{1}{2}m_{\text{bullet}}\text{v}^2 = \tfrac{1}{2}kx^2$$

$$\text{v} = \sqrt{\frac{kx^2}{m_{\text{bullet}}}}$$

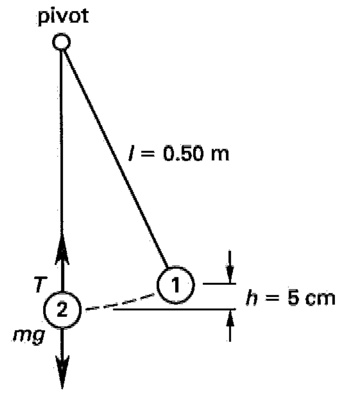
$$= \frac{\sqrt{\dfrac{\left(53\,000\ \dfrac{\text{N}}{\text{cm}}\right)(3.4\ \text{cm})^2\left(\dfrac{1\ \text{m}}{100\ \text{cm}}\right)}{0.1\ \text{kg}}}\left(60\ \dfrac{\text{s}}{\text{min}}\right)\left(60\ \dfrac{\text{min}}{\text{h}}\right)}{1000\ \dfrac{\text{m}}{\text{km}}}$$

$= 891$ km/h (890 km/h)

The answer is (D).

DYNAMICS–44

A simple pendulum consists of a 100 g mass attached to a weightless cord. If the mass is moved laterally such that $h = 5$ cm and then released, what is the maximum tension in the cord, T?



(A) 1.08 N (B) 1.12 N (C) 1.18 N (D) 1.25 N

The maximum tension will occur when the pendulum is at its lowest point, position 2 in the figure. The force balance in the vertical y direction gives

$$ma_y = T - mg$$
$$T = ma_y + mg$$
$$= \frac{mv^2}{l} + mg \qquad [I]$$

From the conservation of energy,

$$E_{p1} = E_{k2}$$
$$mgh = \tfrac{1}{2}mv^2$$
$$v = \sqrt{2gh} \qquad [II]$$

Equations I and II give

$$T_{max} = \frac{m\left(\sqrt{2gh}\right)^2}{l} + mg$$
$$= mg\left(\frac{2h + l}{l}\right)$$
$$= (100 \text{ g})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{(2)(5 \text{ cm}) + 50 \text{ cm}}{50 \text{ cm}}\right)\left(\frac{1 \text{ kg}}{1000 \text{ g}}\right)$$
$$= 1.177 \text{ kg·m/s}^2 \quad (1.18 \text{ N})$$

The answer is (C).

DYNAMICS–45

A stationary passenger car of a train is set into motion by the impact of a moving locomotive. What is the impulse delivered to the car if it has a velocity of 11 m/s immediately after the collision? The weight of the car is 56.8 kN.

(A) 45.5 kN·s (B) 57.5 kN·s (C) 63.7 kN·s (D) 64.1 kN·s

From the impulse-momentum principle,

$$\text{Imp} = \Delta m\text{v}$$

$$m\text{v}_1 + \text{Imp} = m\text{v}_2$$

$$\text{Imp} = m(\text{v}_2 - \text{v}_1)$$

$$= \left(\frac{56.8 \text{ kN}}{9.81 \ \frac{\text{m}}{\text{s}^2}}\right)\left(11 \ \frac{\text{m}}{\text{s}} - 0 \ \frac{\text{m}}{\text{s}}\right)$$

$$= 63.7 \text{ kN·s}$$

The answer is (C).

DYNAMICS–46

Which of the following statements is FALSE?

(A) The time rate of change of the angular momentum about a fixed point is equal to the total moment of the external forces acting on the system about the point.

(B) The coefficient of restitution can be less than zero.

(C) The frictional force always acts to resist motion.

(D) Momentum is conserved during elastic collisions.

The coefficient of restitution is defined as the ratio of the impulses corresponding to the period of restitution and to the period of deformation of a body, respectively. Its value is always between 0 and 1.

The answer is (B).

DYNAMICS–47

Two identical balls hit head-on in a perfectly elastic collision. Given that the initial velocity of one ball is 0.85 m/s and the initial velocity of the other is −0.53 m/s, what is the relative velocity of each ball after the collision?

(A) 0.85 m/s and −0.53 m/s
(B) 1.2 m/s and −0.72 m/s
(C) 1.2 m/s and −5.1 m/s
(D) 1.8 m/s and −0.98 m/s

Let v_1 and v_2 be the velocities of balls 1 and 2, respectively, after the collision. The conservation of momentum equation is

$$\begin{aligned} m v_{01} + m v_{02} &= m v_1 + m v_2 \\ 0.85\ \frac{\text{m}}{\text{s}} + \left(-0.53\ \frac{\text{m}}{\text{s}}\right) &= v_1 + v_2 \\ v_1 + v_2 &= 0.32\ \text{m/s} \end{aligned} \qquad \text{[I]}$$

Since kinetic energy is conserved,

$$\begin{aligned} \tfrac{1}{2} m v_{01}^2 + \tfrac{1}{2} m v_{02}^2 &= \tfrac{1}{2} m v_1^2 + \tfrac{1}{2} m v_2^2 \\ \left(0.85\ \frac{\text{m}}{\text{s}}\right)^2 + \left(-0.53\ \frac{\text{m}}{\text{s}}\right)^2 &= v_1^2 + v_2^2 \\ v_1^2 + v_2^2 &= 1\ \text{m}^2/\text{s}^2 \end{aligned} \qquad \text{[II]}$$

Combining Eqs. I and II,

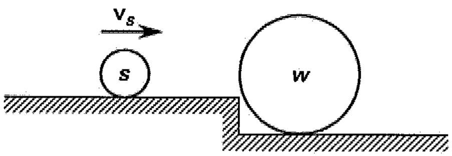
$$v_2^2 - 0.32 v_2 - 0.4488 = 0$$

$$\begin{aligned} v_2 &= \frac{0.32\ \frac{\text{m}}{\text{s}} \pm \sqrt{\left(-0.32\ \frac{\text{m}}{\text{s}}\right)^2 - (4)(1)\left(-0.4488\ \frac{\text{m}^2}{\text{s}^2}\right)}}{2} \\ &= 0.85\ \text{m/s or } -0.53\ \text{m/s} \quad \text{[negative value not used]} \\ v_1 &= 0.32 - v_2 = 0.32\ \frac{\text{m}}{\text{s}} - 0.85\ \frac{\text{m}}{\text{s}} \\ &= -0.53\ \text{m/s} \end{aligned}$$

The answer is (A).

DYNAMICS–48

A steel ball weighing 490 N strikes a stationary wooden ball weighing 490 N. If the steel ball has a velocity of 5.1 m/s at impact, what is its velocity immediately after impact? Assume the collision is central and perfectly elastic.



(A) −5 m/s (B) −2 m/s (C) 0 m/s (D) 5 m/s

Since the balls have the same weight, they have equal mass. Denoting the instances before and after the collision by the subscripts 1 and 2, respectively, $v_{s1} = 5.1$ m/s and $v_{w1} = 0$. Conservation of momentum gives

$$m_s v_{s1} + m_w v_{w1} = m_s v_{s2} + m_w v_{w2}$$
$$v_{s2} + v_{w2} = v_{s1} = 5.1 \text{ m/s} \qquad \text{[I]}$$

Conservation of energy gives

$$\tfrac{1}{2} m_s v_{s1}^2 + \tfrac{1}{2} m_w v_{w1}^2 = \tfrac{1}{2} m_s v_{s2}^2 + \tfrac{1}{2} m_w v_{w2}^2$$
$$v_{s2}^2 + v_{w2}^2 = v_{s1}^2 = \left(5.1 \ \frac{\text{m}}{\text{s}}\right)^2$$
$$= 26.01 \text{ m}^2/\text{s}^2 \qquad \text{[II]}$$

Solving Eqs. I and II simultaneously,

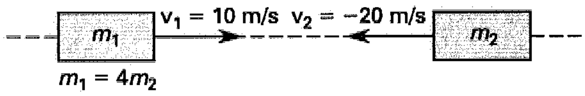
$$v_{s2}^2 + \left(26.01 \ \frac{\text{m}^2}{\text{s}^2} - 10.2 v_{s2} + v_{s2}^2\right) = 26.01 \text{ m}^2/\text{s}^2$$
$$2v_{s2}^2 - 10.2 v_{s2} = 0$$
$$v_{s2}^2 - 5.1 \ \frac{\text{m}}{\text{s}} v_{s2} = 0 \ \frac{\text{m}}{\text{s}^2}$$
$$v_{s2} = 0 \text{ m/s}, \ 5.1 \text{ m/s}$$

If $v_{s2} = 5.1$ m/s, then $v_{w2} = 0$ m/s, and no change has occurred during the collision. This is physically impossible, so $v_{s2} = 0$ m/s.

The answer is (C).

DYNAMICS–49

Two masses collide in a perfectly inelastic collision. Given the data in the illustration, find the velocity and direction of motion of the resulting combined mass.



(A) The mass is stationary.
(B) 4 m/s to the right
(C) 5 m/s to the left
(D) 10 m/s to the right

Let the positive direction of motion be to the right. Let m_3 be the resultant combined mass moving at velocity v_3 after the collision. Since momentum is conserved,

$$m_1 v_1 + m_2 v_2 = m_3 v_3$$

However, $m_3 = m_1 + m_2 = 4m_2 + m_2 = 5m_2$. Therefore,

$$4m_2\left(10\ \frac{\text{m}}{\text{s}}\right) + m_2\left(-20\ \frac{\text{m}}{\text{s}}\right) = 5m_2 v_3$$

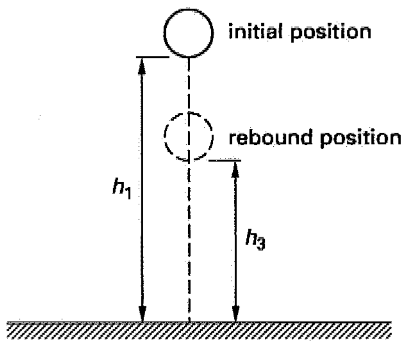
$$40m_2 - 20m_2 = 5m_2 v_3$$

$$v_3 = 4\ \text{m/s to the right}$$

The answer is (B).

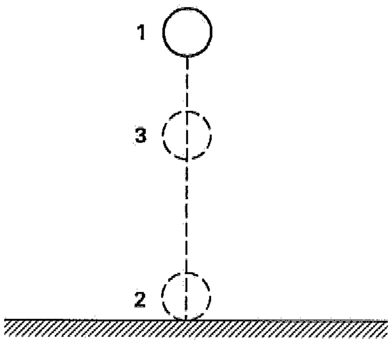
DYNAMICS–50

A ball is dropped onto a solid floor from an initial height, h_0. If the coefficient of restitution, e, is 0.90, how high will the ball rebound?



(A) $0.45h_1$ (B) $0.81h_1$ (C) $0.85h_1$ (D) $0.90h_1$

The subscripts 1, 2, and 3 denote the positions shown.



Conservation of energy gives, before impact,

$$E_{1,\text{total}} = E_{2,\text{total}}$$

Since the kinetic energy at position 1 and the potential energy at position 2 are zero,

$$mgh_1 = \tfrac{1}{2}m\text{v}_2^2$$
$$\text{v}_2 = \sqrt{2gh_1}$$

After impact, the kinetic energy at position 3 is zero.

$$\tfrac{1}{2}m\text{v}_2^2 = mgh_3$$
$$\text{v}_2 = \sqrt{2gh_3}$$

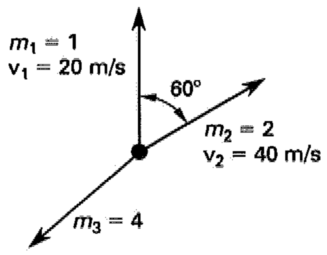
By definition, the coefficient of restitution is

$$e = \frac{\text{v}_{\text{ball}} - \text{v}_{\text{floor}}}{\text{v}_{1,\text{floor}} - \text{v}_{1,\text{ball}}} = -\frac{\text{v}}{\text{v}_1}$$
$$= -\frac{\sqrt{2gh_3}}{\sqrt{2gh_1}} = -\sqrt{\frac{h_3}{h_1}}$$
$$h_3 = e^2h_1 = (0.9)^2h_1$$
$$= 0.81h_1$$

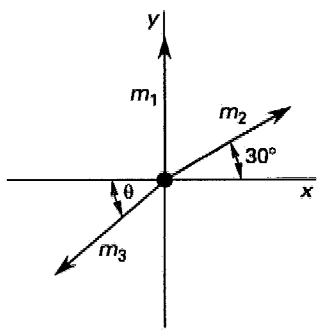
The answer is (B).

DYNAMICS–51

A mass suspended in space explodes into three pieces whose masses, initial velocities, and directions are given in the illustration. All motion is within a single plane. Find the velocity of m_3.



(A) 20 m/s (B) 23 m/s (C) 35 m/s (D) 40 m/s



Defining the x and y axes as shown, conservation of momentum for the x direction gives

$$m_2 v_2 \cos 30° + m_3 v_3 \cos\theta = 0$$

$$2 v_2 \cos 30° + 4 v_3 \cos\theta = 0$$

$$(2)\left(40\ \frac{\text{m}}{\text{s}}\right)\cos 30° = -4 v_3 \cos\theta$$

$$20\ \frac{\text{m}}{\text{s}}\cos 30° = -v_3 \cos\theta$$

$$v_3 = -\frac{17.32\ \frac{\text{m}}{\text{s}}}{\cos\theta} \qquad \text{[I]}$$

For the y direction,

$$m_1 v_1 + m_2 v_2 \sin 30° + m_3 v_3 \sin\theta = 0$$

$$m_1\left(20\ \frac{\text{m}}{\text{s}}\right) + 2m_1\left(40\ \frac{\text{m}}{\text{s}}\right)\sin 30° = -4m_1 v_3 \sin\theta$$

$$-4v_3\sin\theta = 60\ \frac{\text{m}}{\text{s}}$$

$$v_3 = -\frac{60\ \frac{\text{m}}{\text{s}}}{4\sin\theta} \qquad \text{[II]}$$

Equations I and II give

$$\tan\theta = \frac{60\ \frac{\text{m}}{\text{s}}}{\left(17.32\ \frac{\text{m}}{\text{s}}\right)(4)}$$

$$\theta = 40.9°$$

$$v_3 = \frac{-17.32\ \frac{\text{m}}{\text{s}}}{-\cos 40.9°}$$

$$= 22.9\ \text{m/s}$$

The answer is (B).

DYNAMICS–52

Which of the following statements is FALSE?

(A) Kinematics is the study of the effects of motion, while kinetics is the study of the causes of motion.

(B) The radius of gyration for a mass of uniform thickness is identical to the radius of gyration for a planar area of the same shape.

(C) Angular momentum for rigid bodies may be regarded as the product of angular velocity and inertia.

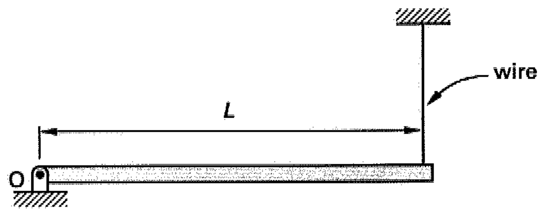
(D) The acceleration of any point within a homogenous body rotating with a constant angular velocity is proportional to the distance of that point to the center of mass.

A body rotating at a constant angular velocity has no angular acceleration.

The answer is (D).

DYNAMICS–53

A uniform beam of weight W is supported by a pin joint and a wire. What will be the angular acceleration, α, at the instant that the wire is cut?



(A) $\frac{g}{L}$ (B) $\frac{3g}{2L}$ (C) $\frac{2g}{L}$ (D) $\frac{Wg}{L}$

The only force on the beam is its weight acting at a distance of $L/2$ from point O. Taking the moment about O,

$$W\left(\frac{L}{2}\right) = I_O\alpha$$

For a slender beam rotating about its end,

$$I_O = \frac{1}{3}mL^2$$

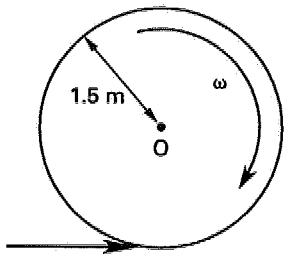
$$\alpha = \frac{WL}{2I_O} = \frac{WL}{2\left(\frac{1}{3}mL^2\right)}$$

$$= \frac{3g}{2L}$$

The answer is (B).

DYNAMICS–54

A thin circular disk of mass 25 kg and radius 1.5 m is spinning about its axis with an angular velocity of $\omega = 1800$ rpm. It takes 2.5 min to stop the motion by applying a constant force, F, to the edge of the disk. The force required is most nearly



(A) 7.2 N (B) 16 N (C) 24 N (D) 32 N

The relationship between the retarding moment, Fr, and the deceleration is

$$Fr = -I_O\alpha$$

Designating the positive rotational direction as counterclockwise, $\omega = -1800$ rpm. Therefore,

$$\begin{aligned} F &= -\frac{I_O\alpha}{r} = -\frac{\frac{1}{2}mr^2\alpha}{r} \\ &= -\tfrac{1}{2}mr\frac{\Delta\omega}{\Delta t} \\ &= \left(-\frac{1}{2}\right)(25\text{ kg})(1.5\text{ m})\left(\left(\frac{-1800\ \frac{\text{rev}}{\text{min}}}{2.5\text{ min}}\right)(2\pi)\left(\frac{1\text{ min}^2}{3600\text{ s}^2}\right)\right) \\ &= 23.6\text{ N}\quad(24\text{ N}) \end{aligned}$$

The answer is (C).

DYNAMICS–55

A mass, m, of 0.025 kg is hanging from a spring whose spring constant, k, is 0.44 N/m. If the mass is pulled down and released, what is the period of oscillation?

(A) 0.50 s (B) 1.2 s (C) 1.5 s (D) 2.1 s

By definition, the period T is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.025\ \text{kg}}{0.44\ \frac{\text{N}}{\text{m}}}}$$
$$= 1.5\ \text{s}$$

The answer is (C).

DYNAMICS–56

A body hangs from an ideal spring. What is the frequency of oscillation of the body if its mass, m, is 0.015 kg, and k is 0.5 N/m?

(A) 0.51 Hz (B) 0.66 Hz (C) 0.78 Hz (D) 0.92 Hz

By definition, the frequency, f, is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{0.5\ \frac{\text{N}}{\text{m}}}{0.015\ \text{kg}}}$$
$$= 0.92\ \text{Hz}$$

The answer is (D).

DYNAMICS–57

What is the natural frequency, ω, of an oscillating body whose period of oscillation is 1.8 s?

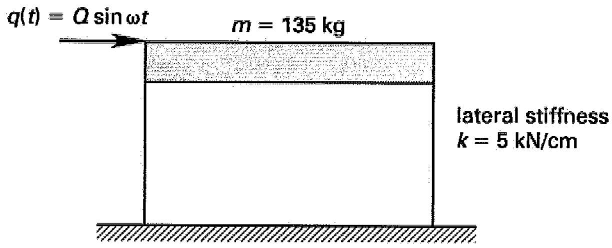
(A) 1.8 rad/s (B) 2.7 rad/s (C) 3.5 rad/s (D) 4.2 rad/s

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1.8\ \text{s}}$$
$$= 3.5\ \text{rad/s}$$

The answer is (C).

DYNAMICS–58

A one-story frame is subjected to a sinusoidal forcing function $q(t) = Q \sin \omega t$ at the transom. What is most nearly the frequency of $q(t)$, in hertz, if the frame is in resonance with the force?



(A) 2.6 Hz (B) 2.9 Hz (C) 3.6 Hz (D) 9.7 Hz

Resonance occurs when the forced frequency, ω, equals the natural frequency, ω_n.

$$m = 135 \text{ kg}$$

$$k = \left(5000 \ \frac{\text{N}}{\text{cm}}\right)\left(100 \ \frac{\text{cm}}{\text{m}}\right) = 500\,000 \text{ N/m}$$

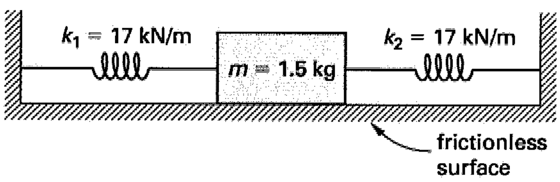
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{500\,000 \ \frac{\text{N}}{\text{m}}}{135 \text{ kg}}}$$

$$= 9.69 \text{ Hz} \quad (9.7 \text{ Hz})$$

The answer is (D).

DYNAMICS–59

In the mass-spring system shown, the mass, m, is displaced 0.09 m to the right of the equilibrium position and then released. Find the maximum velocity of m.



(A) 0.3 m/s (B) 5 m/s (C) 8 m/s (D) 14 m/s

The kinetic energy before the mass is released is zero. The maximum velocity will occur when the mass returns to the point of static equilibrium, where the deflection is zero and, hence, the potential energy equals zero. Therefore, since the total energy of the system is constant,

$$E_{p,1} = E_{k,2}$$

$$\tfrac{1}{2}k_1x_1^2 + \tfrac{1}{2}k_2x_2^2 = \tfrac{1}{2}m\text{v}^2$$

The displacement of each spring is

$$x = 0.09 \text{ m}$$

$$\text{v} = \sqrt{\frac{k_1x_1^2 + k_2x_2^2}{m}}$$

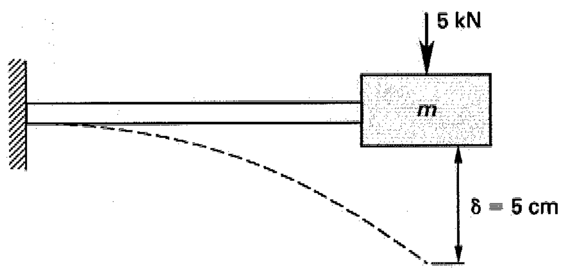
$$= \sqrt{\frac{\left(\left(17\ \frac{\text{kN}}{\text{m}}\right)(0.09 \text{ m})^2 + \left(17\ \frac{\text{kN}}{\text{m}}\right)(0.09 \text{ m})^2\right)\left(1000\ \frac{\text{N}}{\text{kN}}\right)}{1.5 \text{ kg}}}$$

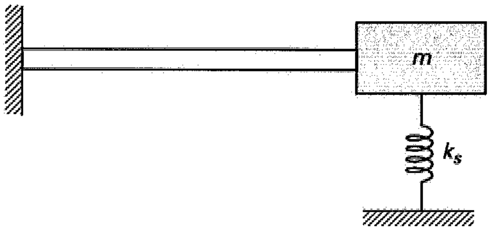
$$= 13.5 \text{ m/s} \quad (14 \text{ m/s})$$

The answer is (D).

DYNAMICS–60

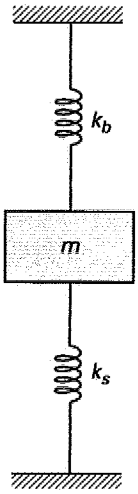
A cantilever beam with an end mass, $m = 7000$ kg, deflects 5 cm when a force of 5 kN is applied at the end. The beam is subsequently mounted on a spring of stiffness, $k_s = 1.5$ kN/cm. What is most nearly the natural frequency of the mass-beam-spring system?





(A) 1.5 rad/s (B) 3.1 rad/s (C) 6.0 rad/s (D) 6.3 rad/s

A cantilever with an end mass m can be modeled as follows.



$$k_b = \frac{5000 \text{ N}}{5 \text{ cm}} = 1000 \text{ N/cm}$$

For this model, both springs undergo the same deflection. Hence,

$$\begin{aligned} k &= k_b + k_s = 1000 \frac{\text{N}}{\text{cm}} + 1500 \frac{\text{N}}{\text{cm}} \\ &= 2500 \text{ N/cm} \end{aligned}$$

The natural frequency is, therefore,

$$\begin{aligned} \omega &= \sqrt{\frac{k}{m}} = \sqrt{\frac{\left(2500 \frac{\text{N}}{\text{cm}}\right)\left(100 \frac{\text{cm}}{\text{m}}\right)}{7000 \text{ kg}}} \\ &= 5.98 \text{ rad/s} \quad (6.0 \text{ rad/s}) \end{aligned}$$

The answer is (C).

12 DC ELECTRICITY

DC ELECTRICITY-1

Which statement about a charge placed on a dielectric material is true?

(A) The charge diffuses across the material's surface.
(B) The charge diffuses through the interior of the material.
(C) The charge is confined to the region in which the charge was placed.
(D) The charge increases the conductivity of the material.

In a dielectric, all charges are attached to specific atoms or molecules.

The answer is (C).

DC ELECTRICITY-2

The coulomb force, F, acts on two charges a distance, r, apart. What is F proportional to?

(A) r (B) r^2 (C) $\frac{1}{r^2}$ (D) $\frac{1}{r^3}$

The coulomb force is

$$F = \frac{q_1 q_2}{4\pi\epsilon r^2}$$

q_1 and q_2 are the charges, and ϵ is the permittivity of the surrounding medium. Hence, F is proportional to the inverse of r^2.

The answer is (C).

DC ELECTRICITY–3

The force between two electrons in a vacuum is 1×10^{-15} N. Approximately how far apart are the electrons?

(A) 1.4×10^{-12} m (B) 5.1×10^{-12} m (C) 4.8×10^{-7} m (D) 1.7×10^{-6} m

Coulomb's law is

$$F = \frac{q_1 q_2}{4\pi\epsilon_o r^2}$$

$\epsilon_o = 8.85 \times 10^{-12}$ $C^2/N{\cdot}m^2$. Also, for an electron, $q = 1.6 \times 10^{-19}$ C. Solving for r,

$$\begin{aligned} r &= q\sqrt{\frac{1}{4\pi\epsilon_o F}} \\ &= (1.6 \times 10^{-19}\ \text{C})\sqrt{\frac{1}{4\pi\left(8.85 \times 10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(1 \times 10^{-15}\ \text{N})}} \\ &= 4.8 \times 10^{-7}\text{m} \end{aligned}$$

The answer is (C).

DC ELECTRICITY–4

Two solid spheres have charges of 1 C and −8 C, respectively. The permittivity, ϵ_o, is 8.85×10^{-12} $C^2/N{\cdot}m^2$, and the distance between the sphere centers, r, is 0.3 m. Determine the force on the spheres.

(A) -1×10^{13} N (B) -8×10^{11} N (C) 0 N (D) 8×10^{11} N

Because of their symmetry, charged spheres may be treated as point charges. Use Coulomb's law.

$$\begin{aligned} F &= \frac{q_1 q_2}{4\pi\epsilon_o r^2} = \frac{(1\ \text{C})(-8\ \text{C})}{4\pi\left(8.85 \times 10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.3\ \text{m})^2} \\ &= -8 \times 10^{11}\ \text{N} \end{aligned}$$

The answer is (B).

DC ELECTRICITY–5

A parallel plate capacitor with plates of area A that are separated a distance d by air is initially charged with charge q_c. The energy stored in the capacitor initially is E. The plates are then separated by $2d$. What is the new energy stored in the capacitor?

(A) 0 (B) $0.5E$ (C) E (D) $2E$

The energy initially stored in the capacitor is

$$E = \frac{q_c^2}{2C}$$

C is the initial capacitance. After the increase in plate separation, the capacitance, C', is

$$C' = \frac{\varepsilon_0 A}{2d} = \tfrac{1}{2}C$$

Therefore the energy stored, E', after the plate distance is increased is

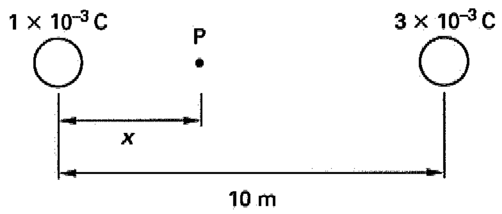
$$E' = \frac{q_c^2}{2C'} = \frac{q_c^2}{(2)\left(\frac{1}{2}C\right)} = 2E$$

The increased energy is added into the system when force is used to separate the plates against the electrostatic force between them.

The answer is (D).

DC ELECTRICITY–6

A 0.001 C charge is separated from a 0.003 C charge by 10 m. If P denotes the point of zero electric field between the charges, determine the distance, x, between the 0.001 C charge and point P.



(A) 2.2 m (B) 3.7 m (C) 6.3 m (D) 14 m

Electric field intensity E at point 2 due to a point charge, Q, at point 1 is

$$E = \frac{Q_1}{4\pi\epsilon r^2}$$

r is the distance between points 1 and 2.

At the point where E is zero, the electric field due to the 0.001 C charge equals the field due to the 0.003 C charge in magnitude.

$$\frac{0.001\ \text{C}}{4\pi\epsilon_o x^2} = \frac{0.003\ \text{C}}{4\pi\epsilon_o(10\ \text{m} - x)^2}$$

$$(10\ \text{m} - x)^2 = 3x^2$$

$$x^2 + 10x - 50 = 0$$

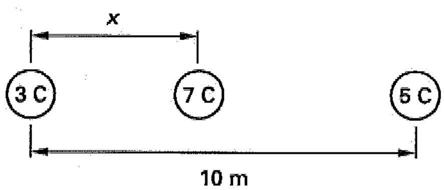
Solving for the positive x value,

$$\begin{aligned} x &= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-10\ \text{m} + \sqrt{(10\ \text{m})^2 - (4)(1\ \text{m})(-50\ \text{m})}}{2} \\ &= 3.66\ \text{m} \quad (3.7\ \text{m}) \end{aligned}$$

The answer is (B).

DC ELECTRICITY–7

A 3 C charge and a 5 C charge are 10 m apart. A 7 C charge is placed on a line connecting the two charges, x meters away from the 3 C charge. If the 7 C charge is in equilibrium, find the value of x.



(A) 3.9 m (B) 4.4 m (C) 5.0 m (D) 5.7 m

At equilibrium, $F_{37} = F_{75}$. Using Coulomb's law,

$$\frac{(3\ \text{C})(7\ \text{C})}{4\pi\epsilon_o x^2} = \frac{(7\ \text{C})(5\ \text{C})}{4\pi\epsilon_o (10\ \text{m} - x)^2}$$
$$(21\ \text{C}^2)(10\ \text{m} - x)^2 = 35x^2$$
$$x^2 + 30x - 150 = 0$$

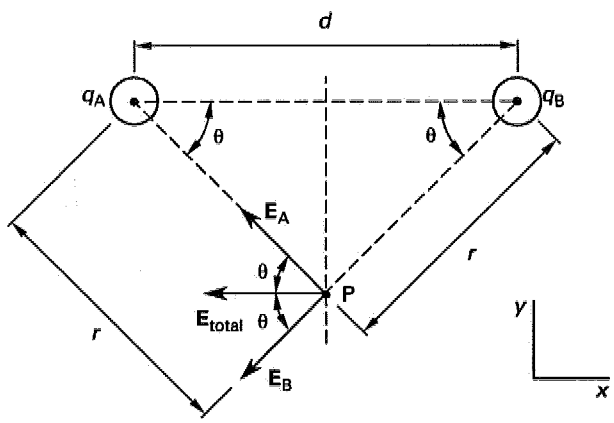
Solving for a positive value of x,

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-30\ \text{m} + \sqrt{(30\ \text{m})^2 - (4)(1\ \text{m})(-150\ \text{m})}}{2}$$
$$= 4.36\ \text{m} \quad (4.4\ \text{m})$$

The answer is (B).

DC ELECTRICITY-8

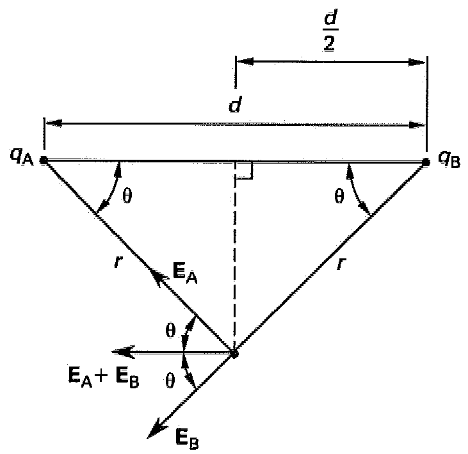
Two charges, A and B, of equal and opposite value are separated by a distance, d. r is the distance from a charge to any point, P, lying on the normal plane that bisects the length d. What is the electric field at point P if K is a constant equal to $1/4\pi\epsilon$?



(A) $\frac{Kqd}{r^3}$ (B) $\frac{Kq^2d}{r^3}$ (C) $\frac{Kq}{r^2}$ (D) $\frac{2Kq}{r^2}$

The total electric field will be in the x direction only, since the y components of the charges cancel each other out. By definition, with $\mathbf{a_r}$ denoting the unit radial vector,

$$\mathbf{E}_{\text{total}} = \left(\frac{Kq}{r^2}\right)\mathbf{a_r}$$



Therefore,

$$E_{\text{total}} = E_A + E_B = \left(\frac{Kq_A}{r^2}\right)\cos\theta + \left(\frac{Kq_B}{r^2}\right)\cos\theta$$

$$= \left(\frac{2Kq}{r^2}\right)\left(\frac{\frac{1}{2}d}{r}\right)$$

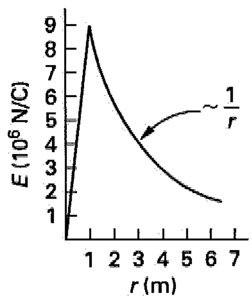
$$= \frac{Kqd}{r^3}$$

The answer is (A).

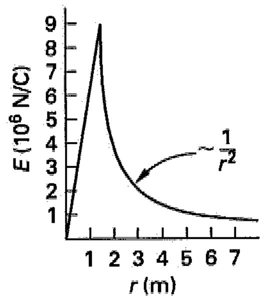
DC ELECTRICITY-9

A hollow metallic spherical shell has a charge of 0.001 C. The shell is 2 m in diameter. Which of the following correctly shows the variation of electric field with respect to the distance, r, from the center of the sphere?

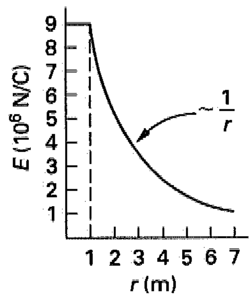
(A)



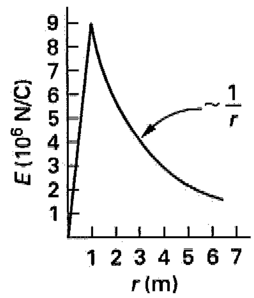
(B)



(C)



(D)



Outside the sphere, Coulomb's law can be used to find the electric field. Thus, the electric field varies as $1/r^2$ for $r > 1$ m. On the surface of the sphere, $r = 1$ m.

$$E = \frac{q}{4\pi\epsilon_o r^2} = \frac{0.001 \text{ C}}{4\pi\left(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(1 \text{ m})^2} = 9 \times 10^6 \text{ N/C}$$

Gauss' law states that the electric flux passing through a given closed surface is proportional to the charge enclosed by the surface. There is no charge within the sphere. Therefore, the electric field is zero for $r < 1$ m. Only (D) is correct.

The answer is (D).

DC ELECTRICITY–10

Approximately how far away must an isolated positive point charge of 1×10^{-8} C be in order for it to produce an electric potential of 100 V? The charge is in free space with $\epsilon_o = 8.85 \times 10^{-12}\ \mathrm{C^2/N{\cdot}m^2}$.

(A) 0.90 m (B) 1.2 m (C) 5.3 m (D) 8.6 m

At a distance, r, from a point charge, q,

$$V = -\int E\,dr\,\frac{q}{4\pi\epsilon r}$$

$$r = \frac{q}{4\pi\epsilon V}$$

$$= \frac{1 \times 10^{-8}\ \mathrm{C}}{4\pi\left(8.85 \times 10^{-12}\ \frac{\mathrm{C^2}}{\mathrm{N{\cdot}m^2}}\right)(100\ \mathrm{V})}$$

$$= 0.90\ \mathrm{m}$$

The answer is (A).

DC ELECTRICITY–11

A point charge, q, in a vacuum creates a potential, V, at a distance, r. A reference voltage of zero is arbitrarily selected when $r = a$. If $K = 1/4\pi\epsilon_o$, which of the following is the correct expression for V?

(A) $Kq\left(\frac{1}{r^2} - \frac{1}{a^2}\right)$ (B) $Kq\frac{1-a}{r^2}$

(C) $Kq\left(\frac{1}{r} - \frac{1}{a}\right)$ (D) $Kq\left(\frac{1}{r^3} - \frac{1}{a^3}\right)$

From Coulomb's law for a point charge,

$$E = \frac{Kq}{r^2}$$

The total voltage is measured between the reference voltage, a, and r.

$$V = -\int E\,dr = -\int_a^r \frac{Kq}{r^2}dr$$

$$= Kq\left(\frac{1}{r} - \frac{1}{a}\right)$$

The answer is (C).

DC ELECTRICITY–12

What accelerating voltage is required to accelerate an electron to a kinetic energy of 5×10^{-15} J? The charge of an electron is 1.6×10^{-19} C.

(A) 8 kV (B) 13 kV (C) 19 kV (D) 31 kV

For an electron, after the potential energy has been converted to kinetic energy, kinetic energy is

$$E_k = qV$$

$$V = \frac{E_k}{q} = \frac{5 \times 10^{-15}\ \text{J}}{1.6 \times 10^{-19}\ \text{C}}$$

$$= 31\,250\ \text{V} \quad (31\ \text{kV})$$

The answer is (D).

DC ELECTRICITY–13

A certain potential variation in the xy plane is given by the expression

$$\nabla V = \left(\frac{1}{\sqrt{x^2 + 4y^2}}\right)(\mathbf{i} + \mathbf{j})$$

Which of the following gives the magnitude and direction (angle made with the x-axis) of the electric field intensity at the point (2,1)?

(A) $-\sqrt{2}/4,\ \pi$ (B) $1/2,\ -\pi/4$ (C) $\sqrt{2}/2,\ \pi/4$ (D) $1/2,\ \pi/4$

$$V = \int E\,dr$$

$$\mathbf{E} = \nabla V = \frac{\partial V}{\partial x}\mathbf{i} + \frac{\partial V}{\partial y}\mathbf{j}$$

Since this is equivalent to the expression given,

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{\left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2}$$

$$= \sqrt{\left(\frac{1}{\sqrt{x^2 + 4y^2}}\right)^2 + \left(\frac{1}{\sqrt{x^2 + 4y^2}}\right)^2}$$

$$= \sqrt{\frac{2}{x^2 + 4y^2}}$$

Evaluating at the point (2,1),

$$E = \sqrt{\frac{2}{(2)^2 + (4)(1)^2}}$$
$$= 1/2$$

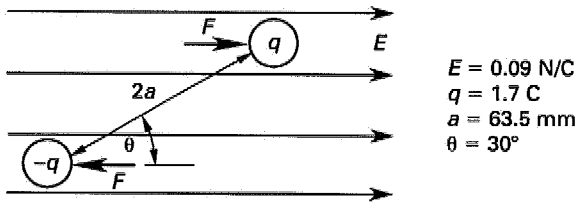
The angle from horizontal that the **E** field is directed is

$$\tan\theta = \frac{E_x}{E_y}$$
$$= \frac{\frac{1}{\sqrt{x^2+4y^2}}}{\frac{1}{\sqrt{x^2+4y^2}}} = 1$$
$$\theta = \pi/4$$

The answer is (D).

DC ELECTRICITY–14

An electric dipole is placed in a uniform electric field of intensity, E. Given the information in the figure, what is most nearly the torque acting on the dipole?



(A) 1.7×10^{-3} N·m (B) 3.3×10^{-3} N·m (C) 4.8×10^{-3} N·m (D) 9.5×10^{-2} N·m

The torque is

$$T = F(2a)\sin\theta$$

F is the force from the electric field.

$$F = Eq = 0.09\ \frac{\text{N}}{\text{C}} \times 1.7\ \text{C}$$
$$= 0.15\ \text{N}$$

Solving for torque,

$$T = (0.15 \text{ N})(2)\left(\frac{63.5 \text{ mm}}{1000 \frac{\text{mm}}{\text{m}}}\right)\sin 30^\circ$$
$$= 9.5 \times 10^{-2} \text{ N·m}$$

The answer is (D).

DC ELECTRICITY-15

Current i is applied to a long N-turn solenoid with cross-section area A and length d. The magnetic field intensity inside the solenoid is $H = Ni/d$ when d is very large. What is the inductance of this long solenoid in air?

(A) $\frac{\mu_0 NA}{d}$ (B) $\frac{\mu_0 N^2 A}{d}$ (C) $\frac{\mu_0 NA}{i}$ (D) $\frac{\mu_0 NA}{id}$

The magnetic flux passing through one turn of the solenoid is

$$\Phi_1 = \int \mathbf{B} \cdot d\mathbf{S} = \mu_0 HA$$

The total flux enclosed by the N turns is obtained by summing the contribution of all the turns.

$$\Phi = \sum_{\text{turns}} \Phi_1 = \mu_0 NHA$$

The inductance is

$$L = \frac{\Phi}{i} = \frac{\mu_0 NHA}{i} = \frac{\mu_0 N\left(\frac{Ni}{d}\right)A}{i} = \frac{\mu_0 N^2 A}{d}$$

The answer is (B).

DC ELECTRICITY–16

Which of the following is NOT a property of magnetic field lines?

(A) The field is stronger where the lines are closer together.
(B) The lines intersect surfaces of equal intensity at right angles.
(C) Magnetic field lines have no beginnings and no ends.
(D) The lines cross themselves only at right angles.

Magnetic field lines do not cross. Their direction at any given point is unique.

The answer is (D).

DC ELECTRICITY–17

The tesla is a unit of

(A) permittivity
(B) capacitance
(C) inductance
(D) magnetic flux density

The tesla is a unit of magnetic flux density.

The answer is (D).

DC ELECTRICITY–18

The south poles of two bar magnets are 7.5 cm apart in air. The magnets are of equal strength and repel each other with a force of 4.9×10^{-4} N. What is most nearly the strength of each magnet?

(A) 6.6×10^{-6} Wb (B) 0.86 Wb (C) 11 Wb (D) 53 Wb

The force between two magnets of strength M_1 and M_2 is

$$F = \frac{M_1 M_2}{4\pi\mu r^2}$$

$M_1 = M_2$. Therefore,

$$M = \sqrt{4\pi\mu r^2 F}$$
$$= \sqrt{4\pi\left(4\pi \times 10^{-7}\ \frac{\text{H}}{\text{m}}\right)(0.075\ \text{m})^2(4.9 \times 10^{-4}\ \text{N})}$$
$$= 6.60 \times 10^{-6}\ \text{Wb} \quad \text{[unit poles]}$$

The answer is (A).

DC ELECTRICITY–19

A conductor has length of 1 m, electrical resistivity of 0.1 Ω·m, and area of 0.01 m^2. A uniform direct current having a density of 100 A/m^2 flows through this conductor. What is the power loss in the conductor?

(A) 0 W (B) 1 W (C) 10 W (D) 100 W

The resistance of the conductor is

$$R = \frac{\rho L}{A} = \frac{(0.1\ \Omega\cdot\text{m})(1\ \text{m})}{0.01\ \text{m}^2} = 10\ \Omega$$

The current flows through the conductor is

$$I = JA = \left(100\ \frac{\text{A}}{\text{m}^2}\right)(0.01\ \text{m}^2) = 1\ \text{A}$$

Therefore, the power consumed in the conductor is

$$P = I^2 R = (1\ \text{A})^2(10\ \Omega) = 10\ \text{W}$$

The answer is (C).

DC ELECTRICITY–20

For a field given by $B = \mu H$ (Wb/m^2), what is the energy storage per unit volume?

(A) $U = \frac{B^2}{2\mu}$ (B) $U = \frac{H^2}{2}$ (C) $U = \frac{H^2}{2\mu}$ (D) $U = \frac{H^2}{2\mu^2}$

The energy stored in a magnetic field H per unit volume is $U = {}^1/_2 BH$. Since $B = \mu H$, $H = B/\mu$. Therefore,

$$U = \tfrac{1}{2}B\frac{B}{\mu} = \frac{B^2}{2\mu}$$

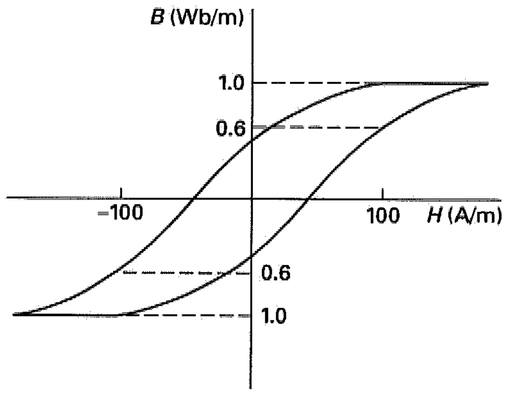
The answer is (A).

DC ELECTRICITY–21

The magnetic flux density, B, and the magnetic field intensity, H, have the following relationship.

$$B = \mu_o(H + M)$$

μ_o is the permeability of free space (in H/m), and M is the magnetic polarization of the material (in A/m). If B is increasing, which of the following may be true about the state of metal X at a value of $H = 100$ A/m? The B-H curve of the metal is as shown.



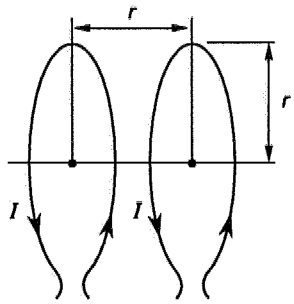
(A) $B = 0.6$ Wb/m; metal X is nonferrous
(B) $B = 0.6$ Wb/m; metal X is ferrous
(C) $B = 1.0$ Wb/m; metal X is nonferrous
(D) $B = 1.0$ Wb/m; metal X is ferrous

Nonferrous metals do not exhibit hysteresis; hence, metal X is ferrous. The hysteresis curve follows a counterclockwise path. Therefore, for B to be increasing at an H value of 100 A/m, $B = 0.6$ Wb/m.

The answer is (B).

DC ELECTRICITY–22

Two identical coils of radius r are placed at a distance r apart as shown. Such a configuration is called a Helmholtz coil. Which of the following describes the magnetic field created by passing a uniform current through the assembly?



(A) The magnetic field is negligible regardless of the magnitude of I.
(B) The magnetic field is zero midway between the two coils.
(C) The magnetic field is fairly uniform between the two coils.
(D) The magnetic field is zero at the centers of the coils.

The magnetic field between the two coils is the superposition of the field created by each coil. Since the currents in both coils are in the same direction, the direction of each individual magnetic field is also the same, using the right-hand rule. Therefore, the fields will not cancel each other.

Since the two coils are circular with their centers aligned, the field between them will be fairly uniform.

The answer is (C).

DC ELECTRICITY–23

Which statement is true?

(A) Magnetic flux lines have sources only.
(B) Magnetic flux lines have sinks only.
(C) Magnetic flux lines have both sources and sinks.
(D) Magnetic flux lines do not have sources or sinks.

Magnetic flux lines are closed loops with no sources or sinks. No known particle produces lines of magnetism.

The answer is (D).

DC ELECTRICITY–24

A charge of 0.75 C passes through a wire every 15 s. What is most nearly the current in the wire?

(A) 5.0 mA (B) 9.4 mA (C) 20 mA (D) 50 mA

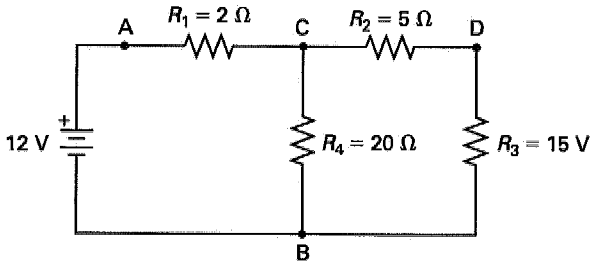
Current is the charge per unit time passing through the wire.

$$I = \frac{q}{t} = \frac{(0.75\ \text{C})\left(1000\ \frac{\text{mA}}{\text{A}}\right)}{15\ \text{s}}$$
$$= 50\ \text{mA}$$

The answer is (D).

DC ELECTRICITY–25

What is most nearly the total resistance between points A and B?



(A) 0 Ω (B) 12 Ω (C) 16 Ω (D) 22 Ω

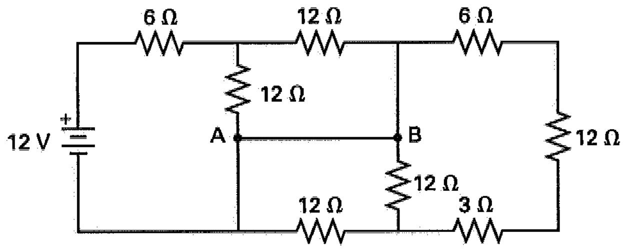
The total resistance is the sum of the resistance between points A and C, plus the equivalent resistance of the resistors in parallel between points C and B.

$$R_{\text{total}} = R_1 + R_4 \parallel (R_2 + R_3)$$
$$= R_1 + \frac{1}{\frac{1}{R_4} + \frac{1}{R_2 + R_3}}$$
$$= 2\ \Omega + \frac{1}{\frac{1}{20\ \Omega} + \frac{1}{5\ \Omega + 15\ \Omega}}$$
$$= 2\ \Omega + 10\ \Omega$$
$$= 12\ \Omega$$

The answer is (B).

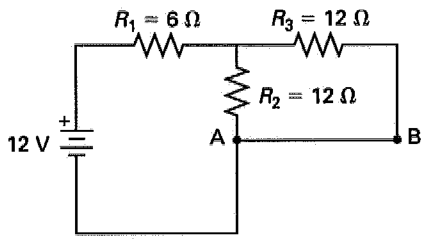
DC ELECTRICITY–26

What is the total resistance (as seen by the battery) of the following network?



(A) 6.0 Ω (B) 12 Ω (C) 15 Ω (D) 24 Ω

AB is a short circuit. Therefore, the rest of the circuit does not contribute to the resistance. The effective circuit is

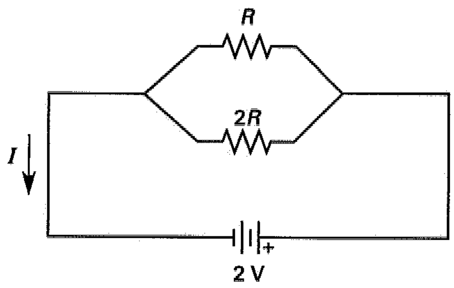


$$
\begin{aligned}
R_{\text{total}} &= R_1 + R_2 \parallel R_3 \\
&= R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} \\
&= 6\ \Omega + \frac{1}{\frac{1}{12\ \Omega} + \frac{1}{12\ \Omega}} \\
&= 6\ \Omega + 6\ \Omega \\
&= 12\ \Omega
\end{aligned}
$$

The answer is (B).

DC ELECTRICITY–27

In the circuit shown, $R = 10\ \Omega$, and the electromagnetic force, V, is 2 V. What is most nearly the current, I?



(A) 0.10 A (B) 0.30 A (C) 0.67 A (D) 3.3 A

$$R_{\text{total}} = R \parallel 2R = \frac{1}{\frac{1}{R} + \frac{1}{2R}}$$
$$= \frac{2R}{3}$$
$$= \frac{(2)(10\ \Omega)}{3}$$
$$= 6.67\ \Omega$$

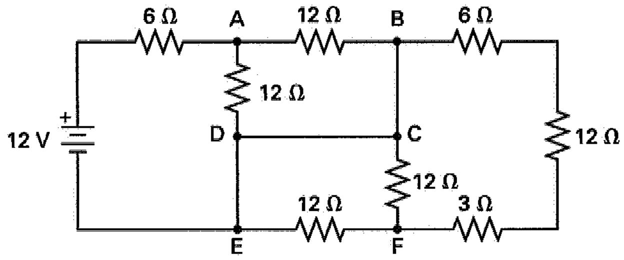
Use Ohm's law. Current is calculated as voltage divided by total resistance.

$$I = \frac{V}{R_{\text{total}}}$$
$$= \frac{2\ \text{V}}{6.67\ \Omega}$$
$$= 0.30\ \text{A}$$

The answer is (B).

DC ELECTRICITY–28

Find the current passing through the 3 Ω resistor.



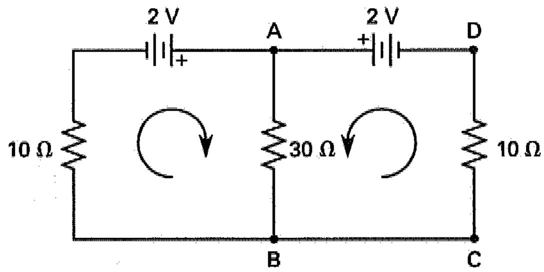
(A) 0 A (B) 0.3 A (C) 1 A (D) 4 A

Current from a battery will always follow a path of zero resistance in a circuit. Instead of flowing through the 3 Ω resistor and its neighboring resistors, the current will follow the path BCDE, a short circuit. There will be no current in the resistor.

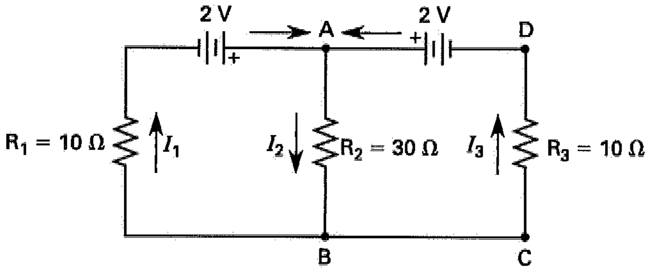
The answer is (A).

DC ELECTRICITY–29

What is most nearly the current passing through the 30 Ω resistor?



(A) 0.0 A (B) 29 mA (C) 50 mA (D) 57 mA



The circuit is symmetrical. Therefore, a current, I_1, flows through the resistors, R_1, and R_3. Another current, I_2, flows through resistor R_2. From Kirchhoff's current law at point A,

$$\sum I = 0$$
$$I_2 = 2I_1$$

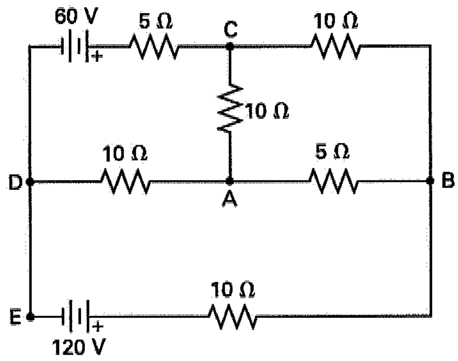
Using Kirchhoff's voltage law around the loop ABCDA.

$$V = R_2I_2 + R_3I_1$$
$$2V = (30\ \Omega)I_2 + (10\ \Omega)I_1$$
$$2V = (30\ \Omega)(2I_1) + (10\ \Omega)I_1$$
$$2V = (70\ \Omega)I_1$$
$$I_1 = 0.0286\ \text{A}$$
$$I_2 = 2I_1 = (2)(0.0286\ \text{A})\left(1000\ \frac{\text{mA}}{\text{A}}\right)$$
$$= 57\ \text{mA}$$

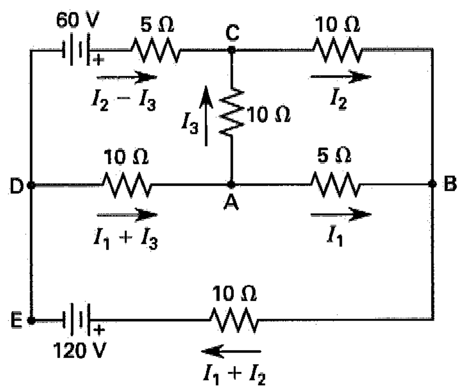
The answer is (D).

DC ELECTRICITY–30

What is most nearly the current through AB?



(A) 0.5 A (B) 1 A (C) 3 A (D) 4 A



By redrawing the circuit and designating the currents as shown in loop ACBA, the currents through the remaining loops can be expressed in terms of I_1, I_2, and I_3. Since voltage equals resistance multiplied by current, for loop CDAC,

$$60 \text{ V} = (5\ \Omega)(I_2 - I_3) - (10\ \Omega)(I_1 + I_3) - (10\ \Omega)I_3$$

$$2I_1 - I_2 + 5I_3 = -12 \text{ A} \qquad \text{[I]}$$

For loop BEDAB,

$$120 \text{ V} = (10\ \Omega)(I_1 + I_2) + (10\ \Omega)(I_1 + I_3) + (5\ \Omega)I_1$$

$$0 = 25I_1 + 10I_2 + 10I_3 - 120$$

$$5I_1 + 2I_2 + 2I_3 = 24 \text{ A} \qquad \text{[II]}$$

Around loop ACBA,

$$0\text{ V} = -(5\ \Omega)I_1 + (10\ \Omega)I_2 + (10\ \Omega)I_3$$

$$I_1 - 2I_2 - 2I_3 = 0\text{ A} \qquad \text{[III]}$$

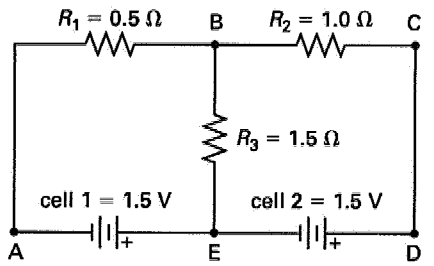
Observe that adding Eqs. II and III can directly solve I_1.

$$6I_1 = 24\text{ A}$$
$$I_1 = 4\text{ A}$$

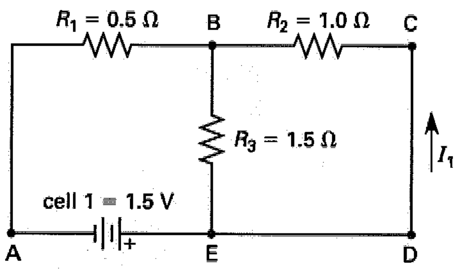
The answer is (D).

DC ELECTRICITY–31

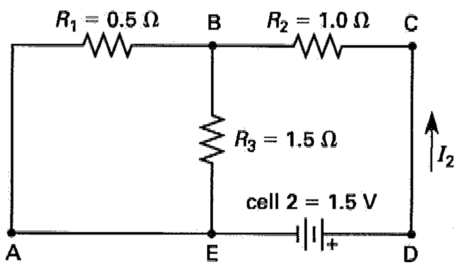
In the circuit shown, what is the current through CD?



(A) 0.20 A (B) 0.60 A (C) 1.0 A (D) 1.9 A



(a)



(b)

The method of superposition is used to find the current, I. Let I_1 be the current from cell 1, and let I_2 be the current from cell 2. Then, $I = I_1 + I_2$. Short circuiting cell 2 to find I_1 as shown in illustration (a), the equivalent total resistance is

$$\begin{aligned} R_{\text{total},1} &= R_1 + R_2 \parallel R_3 \\ &= R_1 + \frac{R_2 R_3}{R_2 + R_3} \\ &= 0.5\ \Omega + \frac{(1.5\ \Omega)(1\ \Omega)}{2.5\ \Omega} \\ &= 1.1\ \Omega \end{aligned}$$

$$\begin{aligned} I_1 &= \left(\frac{1.5\ \Omega}{2.5\ \Omega}\right)\left(\frac{1.5\ \text{V}}{1.1\ \Omega}\right) \\ &= 0.82\ \text{A} \end{aligned}$$

Short circuiting cell 1 to find I_2, as shown in illustration (b), the equivalent total resistance is

$$\begin{aligned} R_{\text{total},2} &= R_1 \parallel R_3 + R_2 \\ &= \frac{R_1 R_3}{R_1 + R_3} + R_2 \\ &= 1\ \Omega + \frac{(0.5\ \Omega)(1.5\ \Omega)}{2\ \Omega} \\ &= 1.375\ \Omega \end{aligned}$$

$$\begin{aligned} I_2 &= \frac{1.5\ \text{V}}{1.375\ \Omega} \\ &= 1.1\ \text{A} \end{aligned}$$

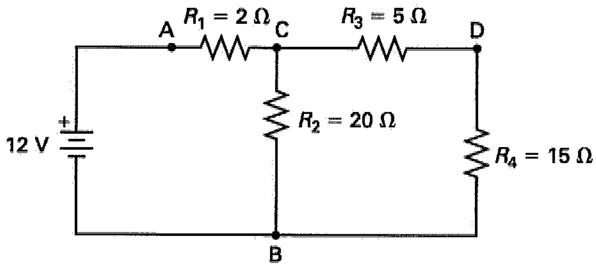
The total current is

$$\begin{aligned} I &= I_1 + I_2 = 0.82\ \text{A} + 1.1\ \text{A} \\ &= 1.92\ \text{A} \quad (1.9\ \text{A}) \end{aligned}$$

The answer is (D).

DC ELECTRICITY–32

For the network shown, find the voltage drop from C to D.



(A) 2.0 V (B) 2.5 V (C) 3.0 V (D) 8.0 V

The total resistance is

$$\begin{aligned} R_{\text{total}} &= R_1 + R_2 \parallel (R_3 + R_4) \\ &= R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3 + R_4}} \\ &= 2\ \Omega + \frac{1}{\frac{1}{20\ \Omega} + \frac{1}{15\ \Omega + 5\ \Omega}} \\ &= 12\ \Omega \end{aligned}$$

$$I_{\text{total}} = \frac{V}{R_{\text{total}}} = \frac{12\ \text{V}}{12\ \Omega} = 1\ \text{A}$$

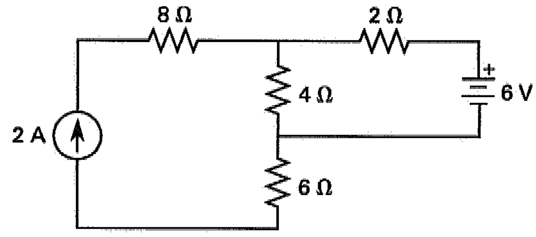
Use a current divider to find the current in section CDB.

$$\begin{aligned} I_{\text{CDB}} &= I_{\text{total}}\left(\frac{R_2}{R_2 + R_3 + R_4}\right) \\ &= (1\ \text{A})\left(\frac{20\ \Omega}{40\ \Omega}\right) \\ &= 0.5\ \text{A} \\ V_{\text{CD}} &= I_{\text{CDB}} R_3 \\ &= (0.5\ \text{A})(5\ \Omega) \\ &= 2.5\ \text{V} \end{aligned}$$

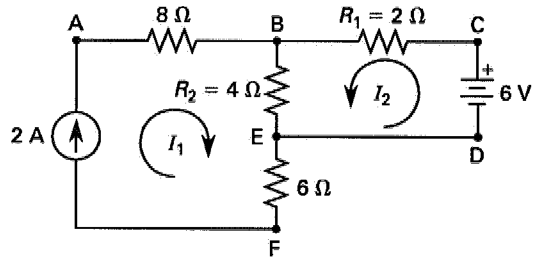
The answer is (B).

DC ELECTRICITY–33

Determine the voltage drop across the 4 Ω resistor in the network shown.



(A) 4.3 V (B) 6.7 V (C) 12 V (D) 24 V



The network is redrawn with the currents and circuit points labeled as shown. The current through BE is equal to the sum of currents from AB and CB.

$$I_{BE} = I_1 + I_2 = 2\ \text{A} + I_2$$

Kirchhoff's voltage law around loop DCBE gives

$$V_{CD} = R_1 I_2 + R_2 I_{BC}$$

$$6V = (2\ \Omega)I_2 + (4\ \Omega)I_{BE} = (2\ \Omega)I_2 + (4\ \Omega)(2\ \text{A} + I_2)$$

$$I_2 = -0.333\ \text{A} \quad \text{[opposite to the direction that it was defined]}$$

$$I_{BE} = 2\ \text{A} - 0.333\ \text{A} = 1.67\ \text{A}$$

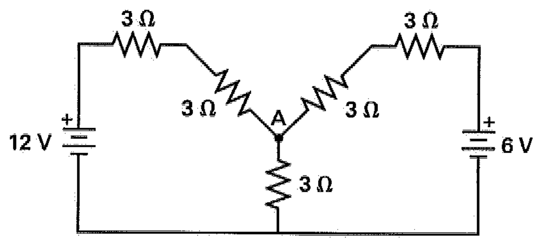
$$V_{BE} = (1.67\ \text{A})(4\ \Omega)$$

$$= 6.68\ \text{V} \quad (6.7\ \text{V})$$

The answer is (B).

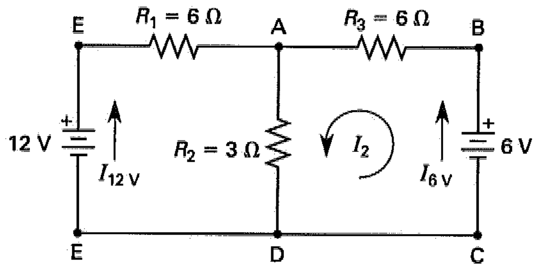
DC ELECTRICITY–34

The voltage at point A in the network shown is most nearly



(A) 1.0 V (B) 2.3 V (C) 3.0 V (D) 4.5 V

The circuit is redrawn.



Superposition is used to find I_2.

$$I_2 = I_{6\,\text{V}} - I_{12\,\text{V}}$$

$I_{6\,\text{V}}$ is the current through BA from the 6 V source, and $I_{12\,\text{V}}$ is the current through BA from the 12 V source. The equivalent resistances are calculated by short circuit for each voltage source.

$$\begin{aligned} R_{6\,\text{V}} &= R_3 + R_1 \parallel R_2 \\ &= R_3 + \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \\ &= 6\ \Omega + \frac{1}{\frac{1}{6\,\Omega} + \frac{1}{3\,\Omega}} \\ &= 8\ \Omega \end{aligned}$$

$$I_{6\,\text{V}} = \frac{V_{6\,\text{V}}}{R_{6\,\text{V}}} = \frac{6\ \text{V}}{8\ \Omega} = 0.75\ \text{A}$$

$$R_{12\,V} = R_1 + R_2 \parallel R_3$$
$$= R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}}$$
$$= 6\ \Omega + \frac{1}{\frac{1}{6\,\Omega} + \frac{1}{3\,\Omega}}$$
$$= 8\ \Omega$$

$$I_{12\,V} = \left(\frac{V_{12\,V}}{R_{12\,V}}\right)\left(\frac{R_2}{R_2 + R_3}\right) = \left(\frac{3\ \Omega}{9\ \Omega}\right)\left(\frac{12\ V}{8\ \Omega}\right) = 0.5\ A$$

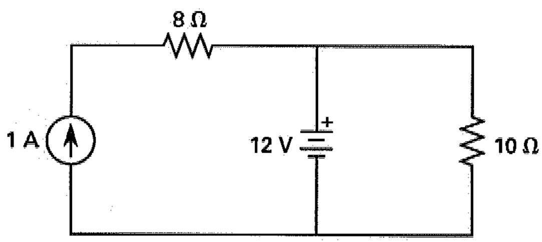
$$I_2 = I_{6\,V} - I_{12\,V}$$
$$= 0.75\ A - 0.5\ A$$
$$= 0.25\ A$$

$$V_A = V_{6\,V} - I_2R_3$$
$$= 6\ V - (0.25\ A)(6\ \Omega)$$
$$= 4.5\ V$$

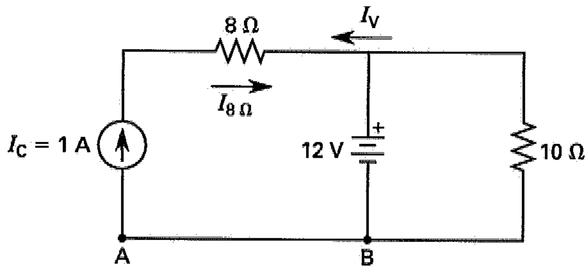
The answer is (D).

DC ELECTRICITY–35

What is the voltage drop across the 8 Ω resistor in the following circuit?



(A) 8 V (B) 12 V (C) 20 V (D) 22 V



Redrawing the circuit as shown, with I_C equal to the component of the current through the 8 Ω resistor due to the current source, and I_V equal to the component of the current through the resistor due to the voltage source,

$$I_{8\,\Omega} = I_C - I_V$$

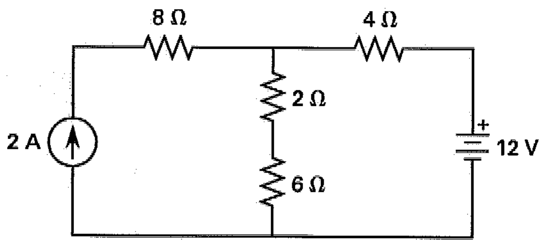
But, $I_C = 1$ A, and $I_V = 0$ A. Therefore,

$$\begin{aligned} I_{8\,\Omega} &= 1 \text{ A} \\ V_{8\,\Omega} &= IR = (1 \text{ A})(8\ \Omega) \\ &= 8 \text{ V} \end{aligned}$$

The answer is (A).

DC ELECTRICITY–36

Determine the voltage drop across the 6 Ω resistor.



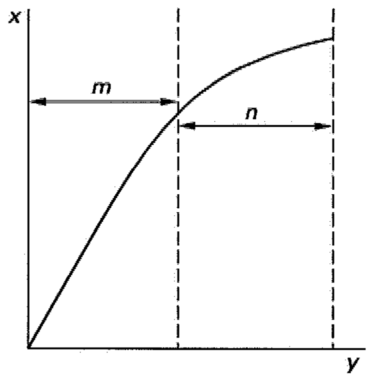
(A) 6.0 V (B) 9.0 V (C) 10 V (D) 18 V

From the graphs, the modulus of elasticity of material B is greater than that of material A. This means that material A is more ductile, that is, it can undergo more strain before fracturing. However, material B can withstand higher loads than material A. Only option (D) is correct.

The answer is (D).

MATERIALS SCIENCE–49

If the diagram below represents deformation of rigid bodies, what do x, y, m, and n refer to?



(A) x = stress, y = strain, m = plastic deformation, n = elastic deformation
(B) x = strain, y = stress, m = plastic deformation, n = elastic deformation
(C) x = stress, y = strain, m = elastic deformation, n = plastic deformation
(D) x = strain, y = stress, m = elastic deformation, n = plastic deformation

Option (C) is the only choice that fits the graph.

The answer is (C).

MATERIALS SCIENCE–50

Which of the following best describes the 0.2% offset yield stress?

(A) It is the elastic limit after which a measurable plastic strain has occurred.
(B) It is the stress at which the material plastically strains 0.2%.
(C) It is the stress at which the material elastically strains 0.2%.
(D) It is 0.2% below the fracture point of the material.

By definition, the offset yield stress is where the material undergoes a 0.2% plastic strain.

The answer is (B).

MATERIALS SCIENCE–51

Which of the following is true regarding the ductile-to-brittle transition temperature?

I. It is important for structures used in cold environments.
II. It is the point at which the size of the shear lip or tearing rim goes to zero.
III. It is the temperature at which 20 J of energy causes failure in a Charpy v-notch specimen of standard dimensions.

(A) I only (B) I and II (C) I and III (D) II and III

II is the only choice that is false. A test piece broken at 20 J of energy usually has a small shear lip.

The answer is (C).

MATERIALS SCIENCE–52

Which of the following are true regarding creep?

I. It is caused by the diffusion of vacancies to edge dislocations, permitting dislocation climb.
II. It involves the plastic deformation of materials at loads below the yield stress.
III. It may involve whole grain sliding.

(A) I only (B) II only (C) I and III (D) I, II, and III

All are true.

The answer is (D).

MATERIALS SCIENCE–53

Under conditions of very slow deformation and high temperature, it is possible to have plastic flow in a crystal at shear stresses lower than the critical shear stress. What is this phenomenon called?

(A) slip (B) twinning (C) creep (D) bending

Creep involves the flow of material.

The answer is (C).

MATERIALS SCIENCE–54

What does the Charpy impact test measure?

I. the energy required to break a test sample
II. the strength of a test sample
III. the ductile to brittle transition temperature of metals

(A) I only (B) II only (C) III only (D) I and III

The Charpy test measures toughness, the energy required to break a sample. By conducting the test at different temperatures, the brittle transition temperature can be determined.

The answer is (D).

MATERIALS SCIENCE–55

A shaft made of good quality steel breaks in half due to fatigue. What would the surface of the fracture site look like?

(A) like a cup and cone
(B) quite smooth to the unaided eye, with ripples apparent under low-power magnification
(C) smooth over most of the surface, with tearing at the location of fracture
(D) very jagged and rough

Typically, the surface is mostly smooth. Where final fracture took place however, the surface is torn.

The answer is (C).

MATERIALS SCIENCE–56

To which of the following can the large discrepancy between the actual and theoretical strengths of metals mainly be attributed?

(A) heat (B) dislocations (C) low density (D) stress direction

Although point defects do contribute to the discrepancy in strengths, the major reason for the difference is the presence of dislocations.

The answer is (B).

MATERIALS SCIENCE–57

The ease with which dislocations are able to move through a crystal under stress accounts for which of the following?

I. ductility
II. lower yield strength
III. hardness

(A) I only (B) II only (C) III only (D) I and II

The ease with which dislocations move through a crystal accounts for its ductility and lower yield strength.

The answer is (D).

MATERIALS SCIENCE–58

As the amount of slip increases, additional deformation becomes more difficult and decreases until the plastic flow finally stops. Slip may begin again only if a larger stress is applied. What is this phenomenon known as?

(A) cooling (B) crowding (C) strain hardening (D) twinning

This is known as strain hardening.

The answer is (C).

MATERIALS SCIENCE–59

Which word combination best completes the following sentence?

"Plastic deformation of a single crystal occurs either by ______ or by ______, but ______ is the more common method."

(A) bending; compression; bending
(B) shearing; compression; compression
(C) slip; twinning; slip
(D) twinning; slip; twinning

Bending, compression, and shear are elastic phenomena. Slip is a more common method of plastic deformation than twinning.

The answer is (C).

MATERIALS SCIENCE–60

Which one of these statements is true for twinning?

(A) It occurs at lower shear stresses than slip.
(B) It is the most significant form of plastic deformation.
(C) It cannot be caused by impact or thermal treatment.
(D) It frequently occurs in hexagonal close-packed structures.

Options (A), (B), and (C) are false. Twinning requires a relatively high shear stress, is much less common than slip, and can be caused by impact or thermal treatment. It occurs in hexagonal close-packed crystal structures.

The answer is (D).

MATERIALS SCIENCE–61

Which of the following does NOT produce vacancies, interstitial defects, or impurity defects in a material?

(A) plastic deformation
(B) slow equilibrium cooling
(C) quenching
(D) increasing the temperature (which increases atomic energy)

Slow equilibrium cooling is used to reduce variations in the material.

The answer is (B).

MATERIALS SCIENCE–62

Which of the following are true statements about the modulus of elasticity, E?

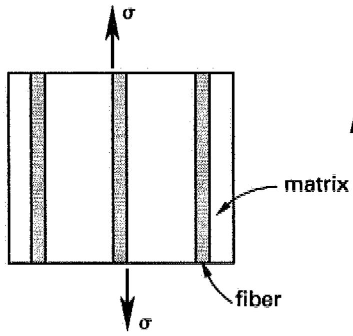
(A) It is the same as the rupture modulus.
(B) It is the slope of the stress-strain diagram in the linearly elastic region.
(C) It is the ratio of stress to volumetric strain.
(D) Its value depends only on the temperature of the material.

The modulus of elasticity is equal to the ratio of stress to strain for a particular material. It is the slope of the stress-strain diagram in the linearly elastic region.

The answer is (B).

MATERIALS SCIENCE–63

What is the modulus of elasticity, E, for a composite material in which the fibers take up 20% of the total volume and the load is applied parallel to the fibers as shown?



(A) 2.76×10^{10} Pa (B) 2.95×10^{10} Pa (C) 1.38×10^{11} Pa (D) 3.45×10^{11} Pa

The matrix and fibers experience the same strain, ϵ. The total stress, σ, is the sum of the stresses carried by the fibers and the matrix.

$$\sigma = E_f \epsilon V_f + E_m \epsilon (1 - V_f)$$

V_f is the fraction of the total volume taken up by the fibers. Thus,

$$\begin{aligned} E &= \frac{\sigma}{\epsilon} = E_f V_f + E_m (1 - V_f) \\ &= (1.38 \times 10^{11}\ \text{Pa})(0.2) + (3.45 \times 10^{6}\ \text{Pa})(1 - 0.2) \\ &= 2.76 \times 10^{10}\ \text{Pa} \end{aligned}$$

The answer is (A).

MATERIALS SCIENCE–64

What is the proper relationship between the modulus of elasticity, E, the Poisson ratio, ν, and the bulk modulus of elasticity, K?

(A) $E = K(1 - 2\nu)$ (B) $E = K(1 - \nu)$

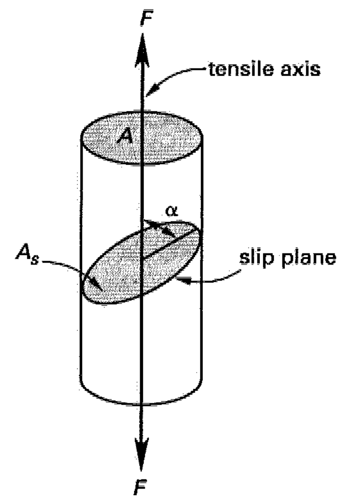
(C) $E = \dfrac{3K}{1 - 2\nu}$ (D) $E = 3K(1 - 2\nu)$

For an element in triaxial stress, the unit volume change can be obtained from Hooke's law. The resultant equation is given by option (D).

The answer is (D).

MATERIALS SCIENCE–65

A crystal is subjected to a tensile load acting along its axis. α is the angle between the tensile axis and the slip plane as shown. At what value of α will the shear stress in the slip plane be a maximum?



(A) 0° (B) 30° (C) 45° (D) 60°

The component of force along the shear surface is equal to $F\cos\alpha$. The area of the shear surface, A_s, is related to the cross-sectional area, A, by $A_s = A/\sin\alpha$.

$$\tau = \frac{F\cos\alpha}{\frac{A}{\sin\alpha}} = \left(\frac{F}{A}\right)\sin\alpha\cos\alpha$$

Taking the first derivative and setting it equal to zero,

$$\frac{\partial \tau}{\partial \alpha} = \left(\frac{F}{A}\right)(\cos^2\alpha - \sin^2\alpha) = 0$$

$$\cos^2\alpha - \sin^2\alpha = 0$$

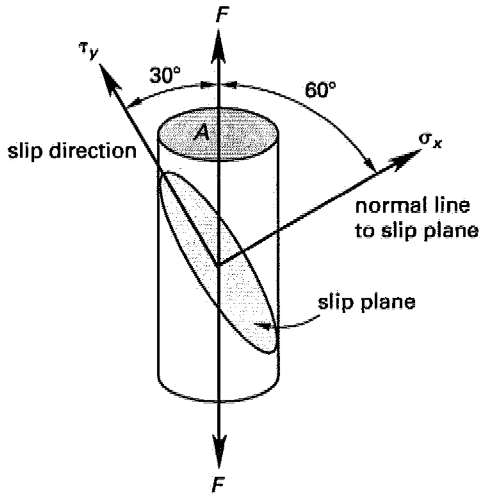
$$\cos\alpha = \sin\alpha$$

$$\alpha = 45^\circ$$

The answer is (C).

MATERIALS SCIENCE–66

An axial stress $\sigma_x = F/A$ is applied as shown. Calculate the resolved shear stress, τ_y, along the slip plane.



(A) $\tau_y = \frac{1}{4}\sigma_x$ (B) $\tau_y = \frac{1}{2}\sigma_x$ (C) $\tau_y = \frac{\sqrt{2}}{3}\sigma_x$ (D) $\tau_y = \frac{3}{4}\sigma_x$

$$\tau_y = \left(\frac{F}{A}\right)\sin 60^\circ \cos 30^\circ$$

$$= \sigma_x\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \tfrac{3}{4}\sigma_x$$

The answer is (D).

MATERIALS SCIENCE–67

If G is the shear modulus, b is the magnitude of the Burgers vector, and r is half the distance between particles, what is the local stress, τ, required to bend dislocations around a particle?

(A) $\frac{Gb}{r}$ (B) Gbr (C) $\frac{br}{G}$ (D) $\frac{Gr}{b}$

Line tension is given by $\tau = 2T/bl$. $T = Gb^2$ and $l = 2r$. Therefore, $\tau = Gb/r$.

The answer is (A).

MATERIALS SCIENCE–68

Given that d is the distance between dislocations and b is the magnitude of the Burgers vector, what is the expression for the misorientation angle θ of a tilt boundary?

(A) $\sin\theta = \frac{d}{b}$ (B) $\tan\theta = \frac{b}{d}$ (C) $\theta = \frac{b}{d}$ (D) $\theta = \frac{d}{b}$

By definition, $\tan\theta = b/d$.

The answer is (B).

MATERIALS SCIENCE–69

In general, what are the effects of cold working a metal?

(A) increased strength and ductility
(B) increased strength, decreased ductility
(C) decreased strength and ductility
(D) decreased strength, increased ductility

The strength of the metal will increase at the expense of a loss in ductility.

The answer is (B).

MATERIALS SCIENCE–70

Which of the following does cold working a metal cause?

(A) elongation of grains in the flow direction, an increase in dislocation density, and an overall increase in energy of the metal

(B) elongation of grains in the flow direction, a decrease in dislocation density, and an overall decrease in energy of the metal

(C) elongation of grains in the flow direction, a decrease in dislocation density, and an overall increase in energy of the metal

(D) shortening of grains in the flow direction, a decrease in dislocation density, and an overall decrease in the energy of the metal

Cold working a metal produces elongations of grains coupled with increases in both dislocation density and energy.

The answer is (A).

MATERIALS SCIENCE–71

Which of the following statements is FALSE?

(A) The amount or percentage of cold work cannot be obtained from information about change in the area or thickness of a metal.

(B) The process of applying force to a metal at temperatures below the temperature of crystallization in order to plastically deform the metal is called cold working.

(C) Annealing eliminates most of the defects caused by the cold working of a metal.

(D) Annealing reduces the hardness of the metal.

The percentage of cold work can be calculated directly from the reduction in thickness or area of the metal.

The answer is (A).

MATERIALS SCIENCE–72

Which of the following statements is FALSE?

(A) There is a considerable increase in the hardness and the strength of a cold-worked metal.

(B) Cold working a metal significantly reduces its ductility.

(C) Cold working causes a slight decrease in the density and electrical conductivity of a metal.

(D) Cold work decreases the yield point of metal.

Cold working increases the yield point as well as the strength and hardness of metal.

The answer is (D).

MATERIALS SCIENCE–73

Which of the following statements is FALSE?

(A) Hot working can be regarded as the simultaneous combination of cold working and annealing.

(B) Hot working increases the density of the metal.

(C) One of the primary goals of hot working is to produce a fine-grained product.

(D) Hot working causes much strain hardening of the metal.

In hot working, the high temperature immediately releases any strain hardening that could occur in the deformation of the metal.

The answer is (D).

MATERIALS SCIENCE–74

Which of the following is FALSE?

(A) Grain size is of minor importance in considering the properties of polycrystalline materials.

(B) Fine-grained materials usually exhibit greater yield stresses than coarse-grained materials at low temperatures.

(C) At high temperatures, grain boundaries become weak, and sliding occurs.

(D) Grain boundary sliding is the relative movement of two grains by a shear movement parallel to the grain boundary between them.

Grain size is an important factor to consider in understanding the properties of polycrystalline materials because it affects the area and length of the grain boundaries.

The answer is (A).

MATERIALS SCIENCE–75

Which of the following correctly describes atoms located at grain boundaries?

(A) They are subjected to the same type of interatomic forces that are present in the interior atoms of the crystal.

(B) They are located primarily in highly strained and distorted positions.

(C) They have a higher free energy than atoms in the undisturbed part of the crystal lattice.

(D) All of the above are correct.

All are correct statements regarding atoms at the grain boundary.

The answer is (D).

MATERIALS SCIENCE–76

What causes the vinyl interiors of automobiles to crack when subjected to prolonged direct sunlight?

(A) the volatilization (evaporation) of plasticizers
(B) repetitive expansion and contraction of the plastic
(C) oxidation of the plastic by sunlight and oxygen
(D) all of the above

All of the statements are true.

The answer is (D).

MATERIALS SCIENCE–77

Low-density polyethylene undergoes extensive (over 100%) elongation prior to rupture, while polystyrene undergoes only 1–2% elongation. What is the main reason for this difference?

(A) The polyethylene is less dense.
(B) The large styrene groups in the polystyrene prevent slippage.
(C) More cross-linking occurs in the polystyrene.
(D) Polyethylene is less crystalline.

Polystyrene has large styrene groups on the side of its carbon chain. These prevent slippage, making the polystyrene brittle.

The answer is (B).

MATERIALS SCIENCE–78

Which of the following describe(s) the modulus of elasticity of an elastomer?

I. It is directly proportional to the number of cross links in the elastomer.
II. Its value increases with temperature.
III. It is directly proportional to the number of double bonds in the chemical structure.

(A) I only (B) II only (C) III only (D) I and II

Choice III is false, since a double bond prevents rotation along the bond, inhibiting elasticity.

The answer is (D).

MATERIALS SCIENCE–79

Which statement(s) describe(s) the glass transition temperature?

I. It is the temperature at which the rate of volume contraction decreases abruptly.
II. It is the temperature at which residual stresses in the glass can be relieved.
III. It is the point where the material behaves more like a solid than a viscous liquid.

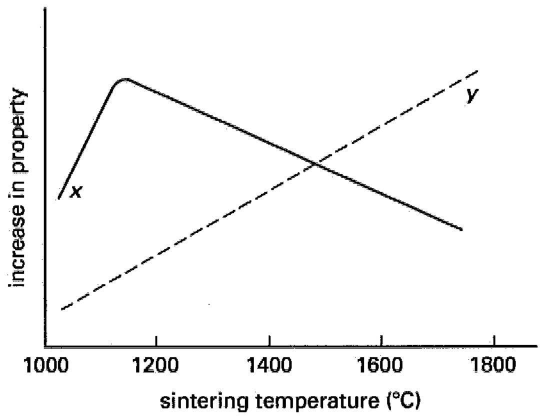
(A) I only (B) I and II (C) I and III (D) II and III

The glass transition temperature is the point at which the free movement of the glass molecules past each other becomes difficult. The glass begins to act like a solid, increasing in specific volume.

The answer is (C).

MATERIALS SCIENCE–80

If the following diagram represents the sintering of the ceramic MgO, what could the curves x and y refer to?



(A) x = grain size; y = porosity
(B) x = grain size; y = strength
(C) x = porosity; y = grain size
(D) x = strength; y = grain size

As the sintering temperature increases, the strength of a ceramic will increase first and then drop abruptly. The grain size will increase linearly with rising temperature.

The answer is (D).

MATERIALS SCIENCE–81

Of the following inorganic glasses, which have tetrahedral lattice structures?

$$SiO_2,\ B_2O_3,\ BeF_2,\ GeO_2$$

(A) SiO_2 and B_2O_3
(B) SiO_2 and BeF_2
(C) SiO_2, B_2O_3, and BeF_2
(D) SiO_2, BeF_2, and GeO_2

SiO_2, BeF_2, and GeO_2 have tetrahedral structures. B_2O_3 has an almost triangular structure.

The answer is (D).

MATERIALS SCIENCE–82

Which of the following is NOT an important criterion for forming a complete binary solid solution?

(A) The difference in radii should be less than 15%.
(B) The constituent elements must have the same crystal structure.
(C) The atoms should be close to one another in the periodic table.
(D) The difference in atomic numbers should be small.

All choices except option (D) are criteria for a binary solid solution.

The answer is (D).

MATERIALS SCIENCE–83

How can an ordered solid solution be distinguished from a compound?

(A) In an ordered solid solution, the solute atoms occupy interstitial positions within the lattice.
(B) The solute atoms in an ordered solid solution substitute for atoms in the parent lattice.
(C) The atoms in an ordered solid solution form layers in the lattice structure.
(D) When heated, an ordered solid solution becomes disordered before melting.

Unlike a compound, an ordered solid solution becomes disordered when heated.

The answer is (D).

MATERIALS SCIENCE–84

What is transformed in a eutectoid reaction?

(A) One liquid is transformed into two solids of different composition.
(B) A solid becomes a liquid at the eutectic temperature.
(C) A liquid becomes a solid at the solidus temperature.
(D) A solid becomes a liquid at the liquidus temperature.

In a eutectoid reaction, one liquid is transformed into two different solids.

The answer is (A).

MATERIALS SCIENCE–85

Which of the following is the correct representation of a eutectic cooling reaction? (The subscripts denote different compositions.)

(A) $(\text{liquid}) \longrightarrow (\text{solid})_1 + (\text{solid})_2$
(B) $(\text{solid})_1 + (\text{liquid}) \longrightarrow (\text{solid})_2$
(C) $(\text{solid})_1 \longrightarrow (\text{solid})_2 + (\text{solid})_3$
(D) $(\text{solid})_1 + (\text{solid})_2 \longrightarrow (\text{solid})_3$

A eutectic reaction is the transformation from one liquid phase to two solid phases.

The answer is (A).

MATERIALS SCIENCE–86

Two pieces of copper are brazed together using a eutectic alloy of copper and silver. The braze material melts at 780°C. If a second braze is attempted in order to attach another piece of copper, which of the following is true?

(A) The first braze will melt if the braze temperature is again 780°C.
(B) The braze temperature must be lowered below 780°C.
(C) The first braze will partially melt, causing the parts to slide.
(D) The first braze will not melt at 780°C, but the second braze will.

All compositions of copper and silver other than the eutectic will have a melting point higher than the eutectic temperature. The alloy of the first braze will dissolve somewhat into the copper pieces, changing its composition. It will not melt again at the second braze temperature of 780°C.

The answer is (D).

MATERIALS SCIENCE–87

On an alloy phase diagram, what is the solidus temperature?

(A) The point at which all solids completely reach the liquid stage.

(B) The temperature of the liquid phase at which the first solid forms for a given overall composition.

(C) The temperature of the solid phase at which the first liquid forms for a given overall composition.

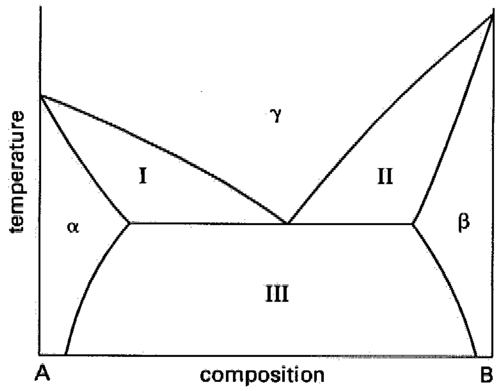
(D) The temperature at which the solid is at equilibrium.

The solidus temperature is the temperature at which liquid first forms.

The answer is (C).

MATERIALS SCIENCE–88

In this phase diagram, what can be said about the phases present in regions I, II, and III?



(A) α, β, and γ are present in region I.

(B) β and γ are present in region II.

(C) α, β, and γ are present in region III.

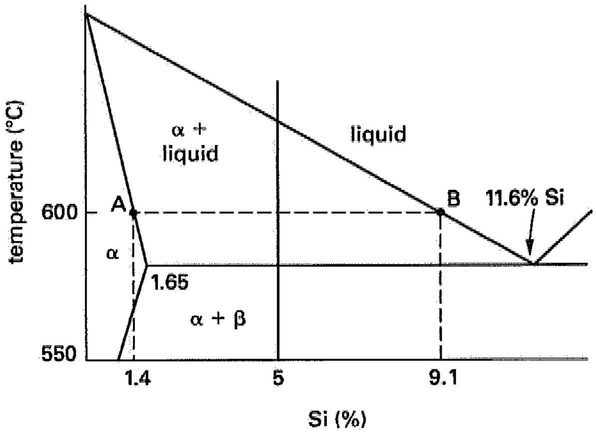
(D) α and γ are present in region III.

β and γ are present in region II. γ is not present in region III, nor is β present in region I.

The answer is (B).

MATERIALS SCIENCE–89

Given the following phase diagram, determine the percentage of liquid remaining at 600°C that results from the equilibrium cooling of an alloy containing 5% silicon and 95% aluminum.



(A) 0.0% (B) 47% (C) 53% (D) 67%

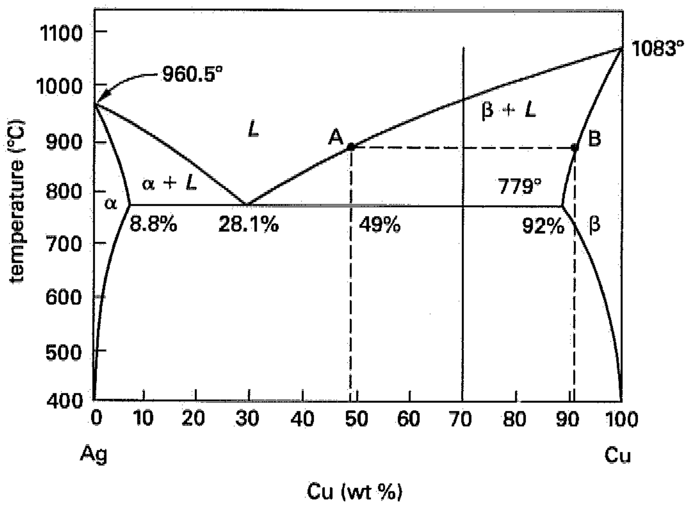
Use the lever rule. At point A there is 1.4% Si and no liquid, while at point B there is 9.1% Si and all liquid. Therefore,

$$\text{percent liquid} = \frac{5\% - 1.4\%}{9.1\% - 1.4\%} \times 100\% = 47\%$$

The answer is (B).

MATERIALS SCIENCE–90

Consider the Ag-Cu phase diagram given. Calculate the equilibrium amount of β in an alloy of 30% Ag, 70% Cu at 850°C.



(A) 0.0% (B) 22% (C) 49% (D) 52%

At 70% Cu, A = 49% Cu and B = 92% Cu.

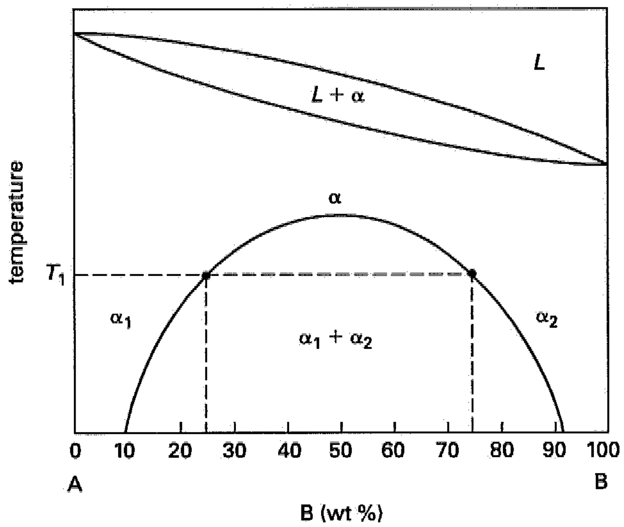
$$\text{percent } \beta = \frac{\text{\% Cu in alloy} - \text{\% Cu at point A}}{\text{\% Cu at point B} - \text{\% Cu at point A}} \times 100\%$$

$$= \frac{70\% - 49\%}{92\% - 49\%} \times 100\% = 49\%$$

The answer is (C).

MATERIALS SCIENCE–91

Using the given phase diagram, what are the relative weights of phases α_1 and α_2 for an alloy of 70% B at temperature T_1?



(A) 10% α_1, 90% α_2
(B) 30% α_1, 70% α_2
(C) 50% α_1, 50% α_2
(D) 70% α_1, 30% α_2

Let W_{α_1} denote the weight fraction of α_1 and W_{α_2} denote the weight fraction of α_2. From the diagram, $C_{\alpha_1} = 25\%$ and $C_{\alpha_2} = 75\%$. Then,

$$W_{\alpha_1} + W_{\alpha_2} = 1$$
$$W_{\alpha_1}C_{\alpha_1} + W_{\alpha_2}C_{\alpha_2} = C_0$$

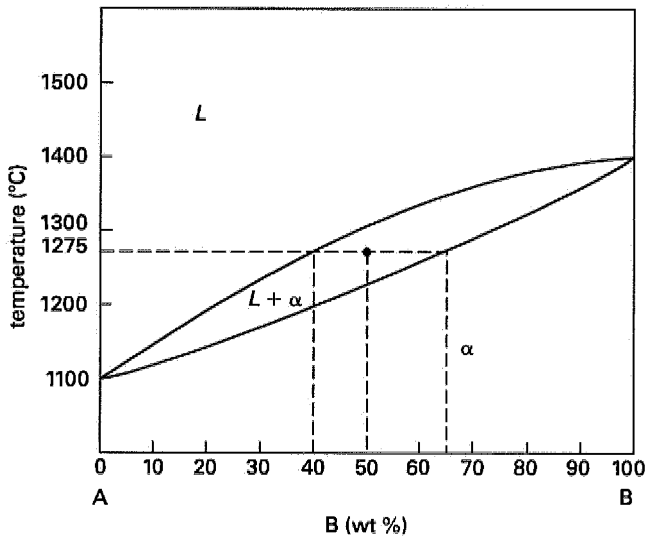
Solving the two equations using $C_0 = 70\%$,

$$W_{\alpha_1} = \frac{C_{\alpha_2} - C_0}{C_{\alpha_2} - C_{\alpha_1}} = \frac{75\% - 70\%}{75\% - 25\%} = 10\%$$

The answer is (A).

MATERIALS SCIENCE–92

For 50% B at 1275°C as shown, what is the relative amount of each phase present?



(A) 40% liquid, 60% solid
(B) 45% liquid, 55% solid
(C) 50% liquid, 50% solid
(D) 60% liquid, 40% solid

From the phase diagram, $C_\alpha = 65\%$ and $C_L = 40\%$. With C_0 given as 50%, and denoting the weight fraction of liquid and solid by W_L and W_α, respectively,

$$W_L + W_\alpha = 1$$
$$W_L C_L + W_\alpha C_\alpha = C_0$$

$$W_L = \frac{C_\alpha - C_0}{C_\alpha - C_L} = \frac{65\% - 50\%}{65\% - 40\%} = 60\%$$

The answer is (D).

MATERIALS SCIENCE–93

Which of the following is NOT a structural class of steels?

(A) carbon
(B) high-strength, low-alloy
(C) low-alloy
(D) tool and die

"Tool and die" steel is an application class, not a structural class.

The answer is (D).

MATERIALS SCIENCE–94

Which of the following phases of steel has a face-centered cubic structure?

(A) ferrite (B) cementite (C) pearlite (D) austenite

Only austenite has a face-centered cubic structure.

The answer is (D).

MATERIALS SCIENCE–95

Low-carbon steels are generally used in the "as rolled" or "as fabricated" state. What is the reason for this?

(A) They come in many different shapes and thicknesses.
(B) Their strength generally cannot be increased by heat treatment.
(C) They degrade severely under heat treatment.
(D) Their chromium content is so low.

Since their strength cannot be increased by heat treatment, low-carbon steels are used as fabricated.

The answer is (B).

MATERIALS SCIENCE–96

The equilibrium cooling of a steel containing 0.8% carbon results in a product with little use because it is extremely brittle. Which of the following is the primary reason for this poor characteristic?

(A) The material has not been cold worked.
(B) The austenite grains are too small, and the carbide grains are too large.
(C) Thick layers of iron carbide surround the coarse ferrite grains.
(D) The carbide forms thin plates that are brittle.

When hypereutectoid steels are slow cooled, brittle carbide plates are formed.

The answer is (D).

MATERIALS SCIENCE–97

Ductile cast iron and gray cast iron both contain 4% carbon. Ductile cast iron, however, has a higher tensile strength and is considerably more ductile. Which of the following is the major difference that accounts for the superior properties of the ductile iron?

(A) The gray cast iron contains iron carbide, whereas the ductile iron contains graphite.
(B) The gray cast iron contains flakes of graphite, whereas the ductile iron contains spheroids of graphite.
(C) The ductile iron is tempered to give better properties.
(D) The ferrite grains in the gray cast iron are excessively large.

Gray cast iron contains flakes of graphite while ductile cast iron contains spheroids. The difference in the shape of the graphite gives the ductile cast iron approximately twice the tensile strength and 20 times the ductility of the gray cast iron.

The answer is (B).

MATERIALS SCIENCE–98

In preparing a metallographic iron specimen, the grain boundaries are made most visible by which of the following steps?

(A) grinding the sample with silicon carbide abrasive
(B) polishing the sample with Al_2O_3
(C) mounting the sample in an epoxy resin mold
(D) etching the sample in a 2% solution of nitric acid in alcohol

Etching the specimen with nitric acid in alcohol dissolves metal from the surface and preferentially attacks the grain boundaries. It is the last step in the sample preparation process.

The answer is (D).

MATERIALS SCIENCE–99

Which of the following statements is FALSE?

(A) Low-alloy steels are a minor group and are rarely used.
(B) Low-alloy steels are used in the heat-treated condition.
(C) Low-alloy steels contain small amounts of nickel and chromium.
(D) The addition of small amounts of molybdenum to low-alloy steels makes it possible to harden and strengthen thick pieces of the metal by heat treatment.

Low-alloy steels are one of the most commonly used classes of structural steels.

The answer is (A).

MATERIALS SCIENCE–100

Which of the following statements is FALSE?

(A) High-strength, low-alloy steels are not as strong as nonalloy, low-carbon steels.
(B) Small amounts of copper increase the tensile strength of steels.
(C) Small amounts of silicon in steels have little influence on toughness or fabricability.
(D) Addition of small amounts of silicon to steel can cause a marked decrease in yield strength of the steel.

Addition of small amounts of silicon to steel increases both the yield strength and the tensile strength.

The answer is (D).

MATERIALS SCIENCE–101

Which of the following statements is FALSE?

(A) Stainless steels contain large amounts of chromium.

(B) There are three basic types of stainless steels: martensitic, austenitic, and ferritic.

(C) The nonmagnetic stainless steels contain large amounts of nickel.

(D) Stabilization of the face-centered cubic crystal structure of stainless steels imparts a nonmagnetic characteristic to the alloy.

There are only two basic types of stainless steels: magnetic (martensitic or ferritic) and nonmagnetic (austenitic).

The answer is (B).

MATERIALS SCIENCE–102

For a completely corrosion-resistant stainless steel, what minimum percentage of chromium in the alloy is required?

(A) 1.1% (B) 3.2% (C) 8.3% (D) 11%

For complete corrosion resistance, the chromium content must be at least 11%.

The answer is (D).

MATERIALS SCIENCE–103

Which of the following would most likely require a steel containing 0.6% carbon that has been spheroidized, cold-drawn, and slightly tempered?

(A) a bridge beam
(B) a water pipe
(C) a cutting tool
(D) a ball bearing

A hypoeutectoid steel that has been worked using the above process has good strength and excellent toughness. A cutting tool undergoes tremendous stress loads due to the relatively small contact area. It requires a stronger material than do the other objects.

The answer is (C).

10 MECHANICS OF MATERIALS

MECHANICS OF MATERIALS–1

Where do stress concentrations occur?

I. near the points of application of concentrated loads
II. along the entire length of high distributed loads
III. at discontinuities

(A) I and II (B) I and III (C) II and III (D) I, II, and III

Stress concentrations occur under concentrated loads and at discontinuities, not under distributed loads.

The answer is (B).

MECHANICS OF MATERIALS–2

What is the definition of normal strain, ϵ? (δ is elongation, and L is the length of the specimen.)

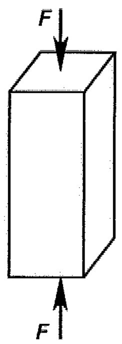
(A) $\epsilon = \frac{L+\delta}{L}$ (B) $\epsilon = \frac{L+\delta}{\delta}$ (C) $\epsilon = \frac{\delta}{L+\delta}$ (D) $\epsilon = \frac{\delta}{L}$

Strain is defined as elongation per unit length.

The answer is (D).

MECHANICS OF MATERIALS–3

The column shown has a cross-sectional area of 13 m^2. What can the approximate maximum load be if the compressive stress cannot exceed 9.6 kPa?



(A) 120 kN (B) 122 kN (C) 125 kN (D) 130 kN

The equation for axial stress is

$$\sigma = \frac{F}{A}$$

$$F = \sigma A$$

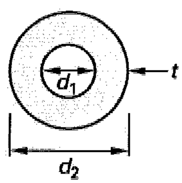
$$= \left(9.6\ \frac{\text{kN}}{\text{m}^2}\right)(13\ \text{m}^2)$$

$$= 124.8\ \text{kN} \quad (125\ \text{kN})$$

The answer is (C).

MECHANICS OF MATERIALS–4

A copper column of annular cross section has an outer diameter, d_2, of 5 m, and is subjected to an axial loading of 200 kN. The allowable compressive stress is 14.4 kPa. The wall thickness, t, should be most nearly



(A) 0.5 m (B) 0.8 m (C) 1 m (D) 2 m

For axial stress,

$$\sigma = \frac{F}{A}$$

Then,

$$A = \frac{F}{\sigma} = \frac{\pi}{4}(d_2^2 - d_1^2)$$

$$d_1 = \sqrt{d_2^2 - \frac{4F}{\pi\sigma}}$$

$$= \sqrt{(5\ \text{m})^2 - \frac{(4)(200\ \text{kN})}{\pi\left(14.4\ \frac{\text{kN}}{\text{m}^2}\right)}}$$

$$= 2.7\ \text{m}$$

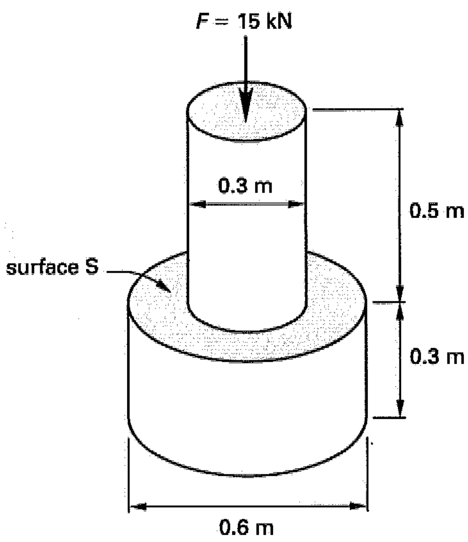
Therefore,

$$t = \frac{d_2 - d_1}{2} = \frac{5\ \text{m} - 2.7\ \text{m}}{2} = 1.15\ \text{m} \quad (1\ \text{m})$$

The answer is (C).

MECHANICS OF MATERIALS–5

What is most nearly the stress at surface S of the cylindrical object shown? The specific weight of the material is $\gamma = 76.9\ \text{kN/m}^3$.



(A) 100 kPa (B) 150 kPa (C) 200 kPa (D) 250 kPa

The stress at surface S is due to the weight of the material above it in addition to the force F. The total load is

$$F_{\text{total}} = W + F = \gamma V + F$$

$$= \left(76.9\ \frac{\text{kN}}{\text{m}^3}\right)\left(\frac{\pi}{4}\right)(0.3\ \text{m})^2(0.5\ \text{m}) + 15\ \text{kN}$$

$$= 17.72\ \text{kN}$$

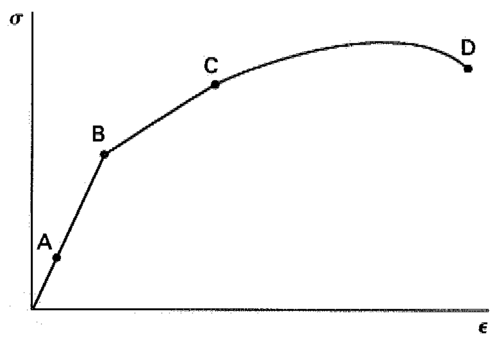
$$\sigma = \frac{F_{\text{total}}}{A} = \frac{17.72\ \text{kN}}{\left(\frac{\pi}{4}\right)(0.3\ \text{m})^2}$$

$$= 250.7\ \text{kN/m}^2 \quad (250\ \text{kPa})$$

The answer is (D).

MECHANICS OF MATERIALS–6

Considering the stress-strain diagram for aluminum, which point is the fracture point?



(A) A (B) B (C) C (D) D

Point D is where fracture occurs.

The answer is (D).

MECHANICS OF MATERIALS–7

In a stress-strain diagram, what is the correct term for the stress level at $\epsilon = 0.2\%$ offset?

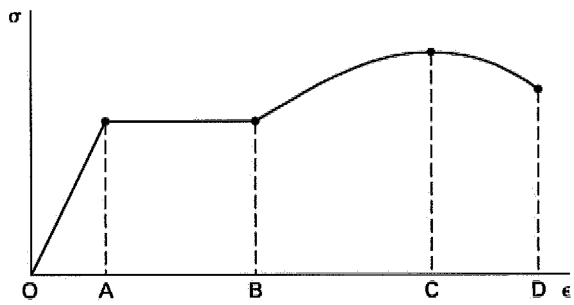
(A) the elastic limit
(B) the plastic limit
(C) the offset rupture stress
(D) the offset yield stress

This is known as the offset yield stress.

The answer is (D).

MECHANICS OF MATERIALS–8

Consider this stress-strain diagram for a carbon steel in tension. Determine the region of perfect plasticity or yielding.



(A) O to A (B) A to B (C) B to C (D) C to D

The plastic region is between points A and B. O to A is known as the linear region, B to C is where strain hardening occurs, and C to D is where reduction in area occurs.

The answer is (B).

MECHANICS OF MATERIALS–9

Under which type of loading does fatigue occur?

(A) static load (B) plane load (C) high load (D) repeated load

Fatigue occurs under repeated loading cycles.

The answer is (D).

MECHANICS OF MATERIALS–10

A specimen is subjected to a load. When the load is removed, the strain disappears. From this information, which of the following can be deduced about this material?

(A) It is elastic.
(B) It is plastic.
(C) It has a high modulus of elasticity.
(D) It does not obey Hooke's law.

By definition, elasticity is the property of a material by which it returns to its original dimensions during unloading.

The answer is (A).

MECHANICS OF MATERIALS–11

Which of the following may be the Poisson ratio of a material?

(A) 0.35 (B) 0.52 (C) 0.55 (D) 0.60

The Poisson ratio must be in the range $0 < \nu < 0.5$. Option (A) is the only answer that satisfies this condition.

The answer is (A).

MECHANICS OF MATERIALS–12

A 2 m long aluminum bar (modulus of elasticity = 70 GPa) is subjected to a tensile stress of 175 MPa. Find the elongation.

(A) 3.5 mm (B) 5.0 mm (C) 7.5 mm (D) 9.0 mm

From Hooke's law,

$$\epsilon = \frac{\sigma}{E} = \frac{\delta}{L}$$

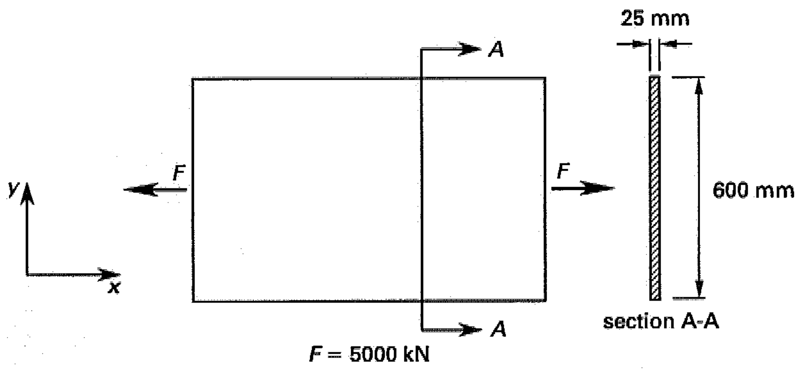
$$\delta = \frac{\sigma L}{E} = \frac{\left(175 \times 10^6 \ \frac{\text{N}}{\text{m}^2}\right)(2 \text{ m})}{70 \times 10^9 \ \frac{\text{N}}{\text{m}^2}}$$

$$= 0.005 \text{ m} \quad (5.0 \text{ mm})$$

The answer is (B).

MECHANICS OF MATERIALS–13

A 600 mm tall thin plate is placed in tension by a 5000 kN force as shown. What is the height (y direction) of the plate while tension is applied? The modulus of elasticity, E, is 200 GPa, and Poisson's ratio, ν, is 0.3. Assume the load is distributed uniformly across the plate and the yield strength is not exceeded.



(A) 599.7 mm (B) 599.9 mm (C) 600.2 mm (D) 600.5 mm

The Poisson ratio is defined as the negative ratio of lateral strain, ϵ_y, to axial strain, ϵ_x. Using this and the equation for axial stress and strain,

$$\nu = -\frac{\epsilon_y}{\epsilon_x}$$

$$\epsilon_y = -\nu\epsilon_x \quad \text{[I]}$$

$$\epsilon_x = \frac{\sigma}{E} = \frac{F}{EA} \quad \text{[II]}$$

Combining equations I and II,

$$\epsilon_y = -\frac{\nu F}{EA} = -\frac{(0.3)(5000 \text{ kN})}{\left(200 \times 10^6 \ \frac{\text{kN}}{\text{m}^2}\right)(0.015 \text{ m}^2)}$$
$$= -0.0005$$

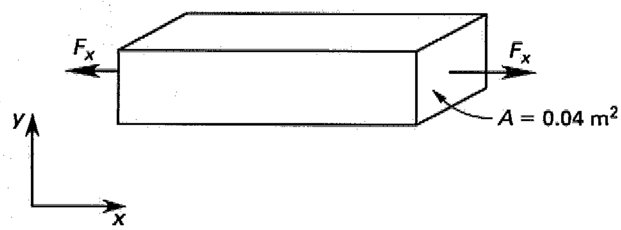
Therefore, the width while the plate is in tension is

$$\begin{aligned} w &= 600 \text{ mm} - \delta_y \\ &= 600 \text{ mm} - (0.0005)(600 \text{ mm}) \\ &= 599.7 \text{ mm} \end{aligned}$$

The answer is (A).

MECHANICS OF MATERIALS–14

What is most nearly the lateral strain, ϵ_y, of the steel specimen shown if F_x = 3000 kN, E = 193 GPa, and ν = 0.29?



(A) -4×10^{-4} (B) -1×10^{-4} (C) 1×10^{-4} (D) 4×10^{-4}

From Hooke's law and the equation for axial stress,

$$\epsilon_x = \frac{\sigma_x}{E} = \frac{F_x}{EA} = \frac{3000 \text{ kN}}{\left(193 \times 10^6 \ \frac{\text{kN}}{\text{m}^2}\right)(0.04 \text{ m}^2)}$$
$$= 3.89 \times 10^{-4}$$

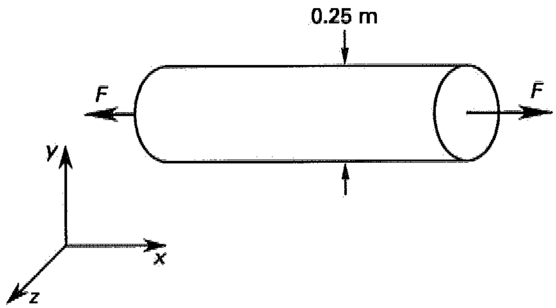
Use Poisson's ratio.

$$\epsilon_y = -\nu\epsilon_x = -(0.29)(3.89 \times 10^{-4})$$
$$= -1.13 \times 10^{-4} \quad (-1 \times 10^{-4})$$

The answer is (B).

MECHANICS OF MATERIALS–15

A steel specimen is subjected to a tensile force, F, of 2000 kN. If Poisson's ratio, ν, is 0.29 and the modulus of elasticity, E, is 193 GPa, the dilatation, e, is most nearly



(A) 6.5×10^{-5} (B) 8.8×10^{-5} (C) 8.8×10^{-4} (D) 6.5×10^{-4}

Dilatation is defined as the sum of the strain in all three coordinate directions. In the axial z direction,

$$\epsilon_z = \frac{F}{EA} = \frac{2000 \text{ kN}}{\left(193 \times 10^6 \ \frac{\text{kN}}{\text{m}^2}\right)(0.049 \text{ m}^2)} = 2.1 \times 10^{-4}$$

From Poisson's ratio,

$$\nu = -\frac{\epsilon_x}{\epsilon_z} = -\frac{\epsilon_y}{\epsilon_z}$$

$$\begin{aligned} \epsilon_x = \epsilon_y &= -\nu\epsilon_z \\ &= -(0.29)(2.1 \times 10^{-4}) \\ &= -6.09 \times 10^{-5} \end{aligned}$$

Therefore,

$$\begin{aligned} e &= \epsilon_x + \epsilon_y + \epsilon_z \\ &= (2.1 \times 10^{-4}) + (2)(-6.09 \times 10^{-5}) \\ &= 8.82 \times 10^{-5} \quad (8.8 \times 10^{-5}) \end{aligned}$$

The answer is (B).

MECHANICS OF MATERIALS–16

Given a shear stress of $\tau_{xy} = 35\,000$ kPa and a shear modulus of $G = 75$ GPa, the shear strain is most nearly

(A) 2.5×10^{-5} rad (B) 4.7×10^{-4} rad (C) 5.5×10^{-4} rad (D) 8.3×10^{-4} rad

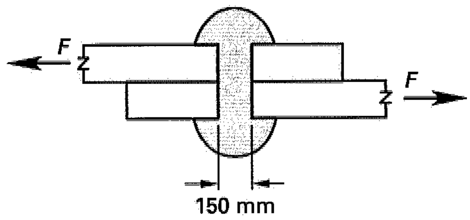
Hooke's law for shear gives

$$\begin{aligned} \gamma = \frac{\tau_{xy}}{G} &= \frac{35\,000\ \frac{\text{kN}}{\text{m}^2}}{75 \times 10^6\ \frac{\text{kN}}{\text{m}^2}} \\ &= 4.67 \times 10^{-4}\ \text{rad} \quad (4.7 \times 10^{-4}\ \text{rad}) \end{aligned}$$

The answer is (B).

MECHANICS OF MATERIALS–17

A 150 mm diameter rivet resists a shear force of $V = 8$ kN. Find the average shear stress in the rivet.



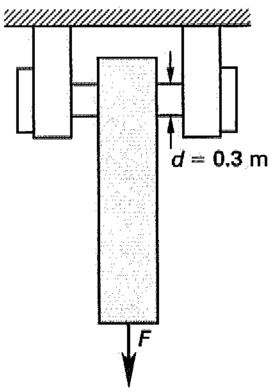
(A) 230 kPa (B) 370 kPa (C) 430 kPa (D) 450 kPa

$$\tau = \frac{V}{A} = \frac{8 \text{ kN}}{\left(\frac{\pi}{4}\right)(0.150 \text{ m})^2} = 452.7 \text{ kN/m}^2 \quad (450 \text{ kPa})$$

The answer is (D).

MECHANICS OF MATERIALS–18

A steel bar carrying a 3000 kN load, F, is attached to a support by a round pin 0.3 m in diameter. What is most nearly the average shear stress in the pin?



(A) 10 MPa (B) 12 MPa (C) 21 MPa (D) 25 MPa

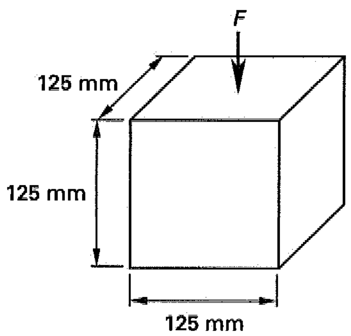
The pin will shear on two cross sections.

$$\tau = \frac{F}{2A} = \frac{3000 \text{ kN}}{(2)\left(\frac{\pi}{4}\right)(0.3 \text{ m})^2} = 21\,221 \text{ kN/m}^2 \quad (21 \text{ MPa})$$

The answer is (C).

MECHANICS OF MATERIALS–19

What is most nearly the maximum allowable load, F, if the factor of safety is 1.5 and the compressive yield stress, σ_{yield}, is 20 670 kPa?



(A) 220 kN (B) 240 kN (C) 300 kN (D) 420 kN

$$\sigma_{\text{allowable}} = \frac{\sigma_{\text{yield}}}{\text{SF}}$$

$$= \frac{20\,670 \frac{\text{kN}}{\text{m}^2}}{1.5} = 13\,780 \text{ kN/m}^2$$

$$F = \sigma_{\text{allowable}} A$$

$$= \left(13\,780 \frac{\text{kN}}{\text{m}^2}\right)(0.125 \text{ m})^2$$

$$= 215.3 \text{ kN} \quad (220 \text{ kN})$$

The answer is (A).

MECHANICS OF MATERIALS–20

The allowable tensile stress for a 6.25 mm diameter bolt with a thread length of 5.5 mm is 207 MPa. The allowable shear stress of the material is 103 MPa. Where and how will such a bolt be most likely to fail if placed in tension? (Assume threads are perfectly triangular and that the force is carried at the mean thread height.)

(A) at the root diameter due to tension
(B) at the threads due to shear
(C) at the root diameter due to shear
(D) at the threads due to tension

The bolt will most likely fail due to shearing of the threads or due to tensile failure of the bolt diameter.

$$\begin{aligned} F_{\text{allowable,thread}} &= \tau_{\text{allowable}}(\text{average shear area}) \\ &= \tau_{\text{allowable}}\left(\tfrac{1}{2}\pi dh\right) \\ &= \left(103\,000\ \frac{\text{kN}}{\text{m}^2}\right)\left(\tfrac{1}{2}\pi\right)(0.006\,25\ \text{m})(0.0055\ \text{m}) \\ &= 5.56\ \text{kN} \end{aligned}$$

$$\begin{aligned} F_{\text{allowable,root}} &= \sigma_{\text{allowable}}(\text{root area}) \\ &= \left(207\,000\ \frac{\text{kN}}{\text{m}^2}\right)\left(\frac{\pi}{4}\right)(0.006\,25\ \text{m})^2 \\ &= 6.35\ \text{kN} \end{aligned}$$

The shear stress in the threads will exceed the allowable stress before the tensile load becomes excessive.

The answer is (B).

MECHANICS OF MATERIALS–21

Hexagonal nuts for 6.25 mm diameter bolts have a height of 5.5 mm. If the ultimate strength of the nut material in shear is 103 MPa, what is most nearly the maximum allowable shear force on the nut threads using a safety factor of 5?

(A) 0.72 kN (B) 0.8 kN (C) 1.0 kN (D) 1.1 kN

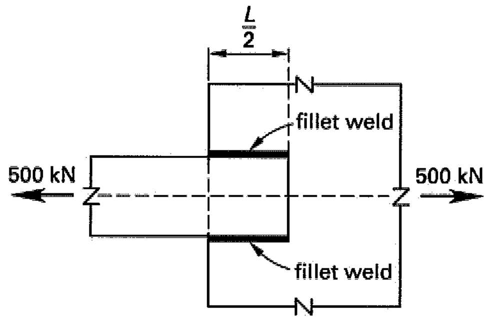
$$\begin{aligned}
\tau_{\text{allowable}} &= \frac{\tau}{\text{SF}} \\
&= \frac{103\,000 \ \frac{\text{kN}}{\text{m}^2}}{5} \\
&= 20\,600 \ \text{kN/m}^2
\end{aligned}$$

$$\begin{aligned}
V &= \tau_{\text{allowable}} A = \tau_{\text{allowable}} \left(\tfrac{1}{2}\pi dh\right) \\
&= \left(20\,600 \ \frac{\text{kN}}{\text{m}^2}\right) \left(\tfrac{1}{2}\pi\right) (0.006\,25 \ \text{m})(0.0055 \ \text{m}) \\
&= 1.11 \ \text{kN} \quad (1.1 \ \text{kN})
\end{aligned}$$

The answer is (D).

MECHANICS OF MATERIALS–22

Determine the total length, L, of the fillet weld for the lap joint shown. The weld has to resist a tension, F, of 500 kN. The effective throat for the weld, h, is 12 mm, and the allowable stress is 145 MPa.



(A) 247 mm (B) 252 mm (C) 287 mm (D) 312 mm

For a fillet weld, the average normal stress is

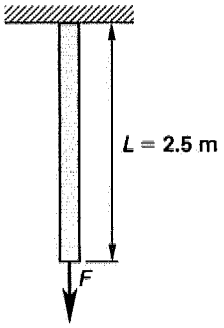
$$\sigma = \frac{F}{hL}$$

$$\begin{aligned}
L &= \frac{F}{\sigma h} = \frac{500 \ \text{kN}}{\left(145\,000 \ \frac{\text{kN}}{\text{m}^2}\right)(0.012 \ \text{m})} \\
&= 0.287 \ \text{m} \quad (287 \ \text{mm})
\end{aligned}$$

The answer is (C).

MECHANICS OF MATERIALS–23

What is most nearly the elongation of the aluminum bar (cross section of 3 cm × 3 cm) shown in the figure when loaded to its yield point? $E = 69$ GPa, and $\sigma_{\text{yield}} = 255$ MPa. Neglect the weight of the bar.



(A) 3.3 mm (B) 9.3 mm (C) 12 mm (D) 15 mm

From Hooke's law, the axial strain is

$$\epsilon = \frac{\sigma}{E} = \frac{255 \times 10^6 \text{ Pa}}{69 \times 10^9 \text{ Pa}} = 0.0037$$

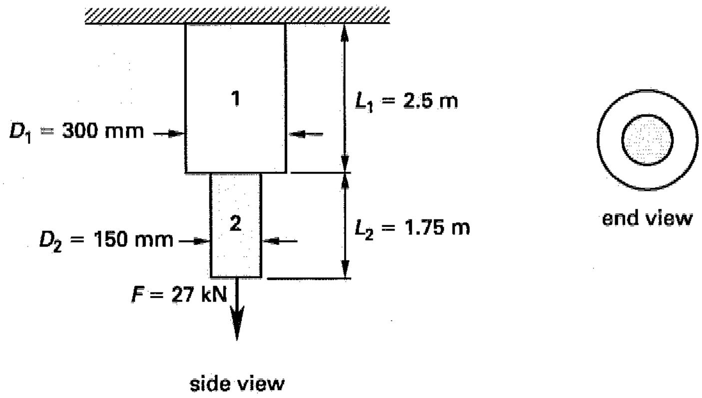
The total elongation is

$$\delta = \epsilon L = (0.0037)(2.5 \text{ m}) = 0.00925 \text{ m} \quad (9.3 \text{ mm})$$

The answer is (B).

MECHANICS OF MATERIALS–24

What is most nearly the total elongation of the rod shown if $E = 69$ GPa? Neglect bending.



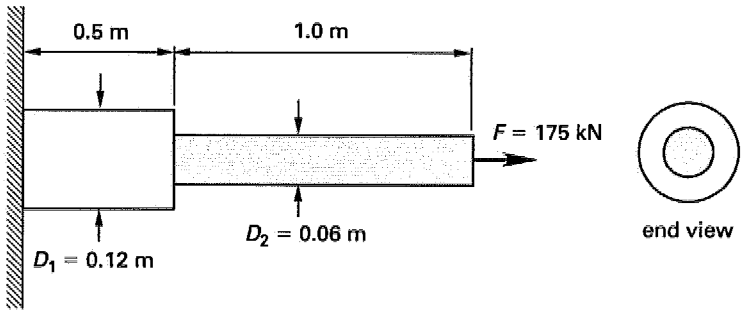
(A) 0.01 mm (B) 0.05 mm (C) 0.2 mm (D) 1.2 mm

$$\delta_{\text{total}} = \frac{FL_1}{EA_1} + \frac{FL_2}{EA_2} = \frac{F}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2}\right)$$
$$= \frac{4F}{\pi E}\left(\frac{L_1}{D_1^2} + \frac{L_2}{D_2^2}\right)$$
$$= \left(\frac{(4)(27\text{ kN})}{\pi\left(69 \times 10^6\ \frac{\text{kN}}{\text{m}^2}\right)}\right)\left(\frac{2.5\text{ m}}{(0.3\text{ m})^2} + \frac{1.75\text{ m}}{(0.15\text{ m})^2}\right)$$
$$= 5.26 \times 10^{-5}\text{ m} \quad (0.05\text{ mm})$$

The answer is (B).

MECHANICS OF MATERIALS–25

What is most nearly the total elongation of this composite body under a force of 27 kN? E_1 = 70 GPa, and E_2 = 100 GPa.



(A) 0.075 mm (B) 0.73 mm (C) 1.2 mm (D) 3.0 mm

Total elongation is the elongation of section 1 plus the elongation of section 2.

$$\delta_{\text{total}} = \delta_1 + \delta_2 = \frac{FL_1}{A_1E_1} + \frac{FL_2}{A_2E_2}$$
$$= \frac{(175\text{ kN})(0.5\text{ m})}{\left(\frac{\pi}{4}\right)(0.12\text{ m})^2\left(70 \times 10^6\ \frac{\text{kN}}{\text{m}^2}\right)} + \frac{(175\text{ kN})(1.0\text{ m})}{\left(\frac{\pi}{4}\right)(0.06\text{ m})^2\left(100 \times 10^6\ \frac{\text{kN}}{\text{m}^2}\right)}$$
$$= 7.29 \times 10^{-4}\text{ m} \quad (0.73\text{ mm})$$

The answer is (B).

MECHANICS OF MATERIALS–26

A 200 m cable is suspended vertically. At any point along the cable, the strain is proportional to the length of the cable below that point. If the strain at the top of the cable is 0.001, determine the total elongation of the cable.

(A) 0.050 m (B) 0.10 m (C) 0.15 m (D) 0.20 m

Since the strain is proportional to the cable length, it varies from 0 at the end to the maximum value of 0.001 at the supports. The average strain is

$$\begin{aligned}\epsilon_{\text{ave}} &= \frac{\epsilon_{\text{max}}}{2} = \frac{0.001}{2} \\ &= 0.0005\end{aligned}$$

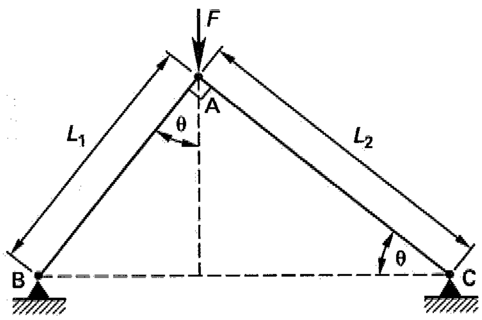
The total elongation is

$$\begin{aligned}\delta &= \epsilon_{\text{ave}}L = (0.0005)(200 \text{ m}) \\ &= 0.10 \text{ m}\end{aligned}$$

The answer is (B).

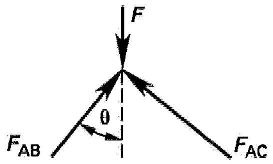
MECHANICS OF MATERIALS–27

The figure shows a two-member truss with a load $F = 50\,000$ kN applied statically. Given that $L_1 = 1.2$ m, $L_2 = 1.5$ m, and each member's cross-sectional area, A, is 4000 mm^2, what is most nearly the elongation of member AB after F is applied? Use $E = 200$ GPa.



(A) −59 mm (B) −48 mm (C) −36 mm (D) −23 mm

A free-body diagram of joint A gives



$$R_{AB} = F\cos\theta = F\frac{L_2}{\sqrt{L_1^2 + L_2^2}}$$

$$= \frac{(50\,000\text{ kN})(1.5\text{ m})}{\sqrt{(1.2\text{ m})^2 + (1.5\text{ m})^2}} = 39\,043\text{ kN} \quad \text{[AB is in compression]}$$

$$F_{AB} = -R_{AB} = -39\,043\text{ kN}$$

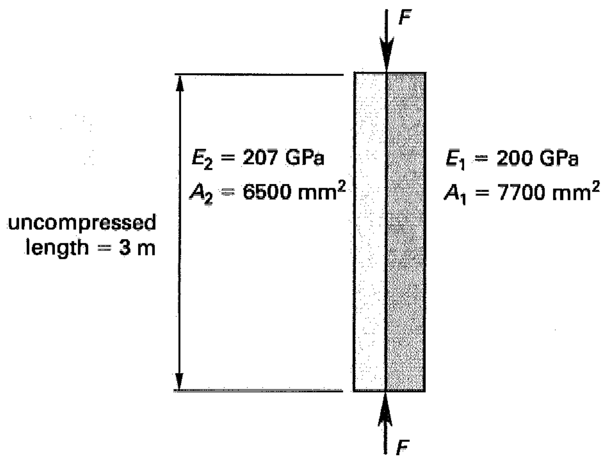
$$\delta_{AB} = \frac{F_{AB}L_1}{EA} = \frac{(-39\,043\text{ kN})(1.2\text{ m})}{\left(200 \times 10^6\ \frac{\text{kN}}{\text{m}^2}\right)(0.004\text{ m}^2)}$$

$$= -0.0586\text{ m} \quad (-59\text{ mm})$$

The answer is (A).

MECHANICS OF MATERIALS–28

The two bars shown are perfectly bonded to a common face to form an assembly. The bars have moduli of elasticity and areas as given. If a force of $F = 1300$ kN compresses the assembly, what is most nearly the reduction in length?

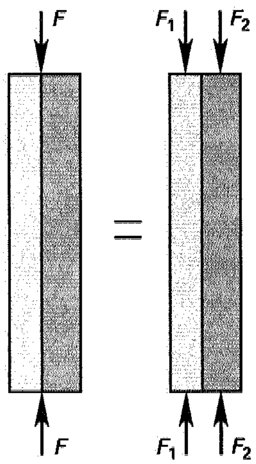


(A) 1.2 mm (B) 1.4 mm (C) 1.5 mm (D) 1.6 mm

From the principle of compatability, both bars are compressed the same length.

$$\epsilon_1 = \frac{\sigma_1}{E_1} = \frac{F_1}{A_1 E_1}$$

$$\epsilon_2 = \frac{\sigma_2}{E_2} = \frac{F_2}{A_2 E_2}$$



Since $\epsilon_1 = \epsilon_2$,

$$\frac{F_1}{A_1 E_1} = \frac{F_2}{A_2 E_2}$$

$$F_1 = \left(\frac{A_1 E_1}{A_2 E_2}\right) F_2 \quad \text{[I]}$$

From a force balance,

$$F_1 + F_2 = 1300 \text{ kN} \quad \text{[II]}$$

$$F_1 = 1300 \text{ kN} - F_2$$

Combining equations I and II,

$$1300\ \text{kN} - F_2 = \left(\frac{A_1E_1}{A_2E_2}\right) F_2$$

$$1300\ \text{kN} = \left(1 + \frac{A_1E_1}{A_2E_2}\right) F_2$$

$$F_2 = \frac{1300\ \text{kN}}{1 + \dfrac{A_1E_1}{A_2E_2}}$$

$$= \frac{1300\ \text{kN}}{1 + \dfrac{(7700\ \text{mm}^2)(200\ \text{GPa})}{(6500\ \text{mm}^2)(207\ \text{GPa})}}$$

$$= 606.2\ \text{kN}$$

$$\epsilon_2 = \frac{606.2\ \text{kN}}{(0.0065\ \text{m}^2)\left(207 \times 10^6\ \dfrac{\text{kN}}{\text{m}^2}\right)}$$

$$= 4.51 \times 10^{-4}$$

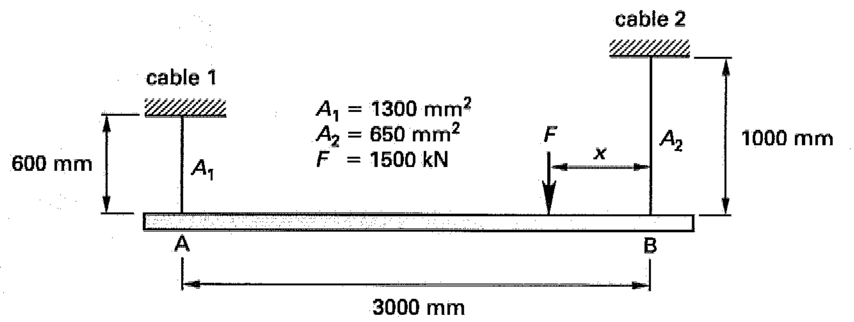
$$\delta = \epsilon L = (4.51 \times 10^{-4})(3\ \text{m})$$

$$= 0.001\,35\ \text{m} \quad (1.4\ \text{mm})$$

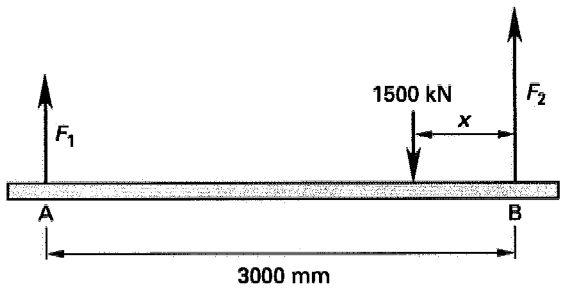
The answer is (B).

MECHANICS OF MATERIALS–29

A rigid weightless bar is suspended horizontally by cables 1 and 2 as shown. The cross-sectional areas of the cables are given in the figure. The modulus of elasticity, E, is the same for both cables. If a concentrated load of $F = 1500$ kN is applied between points A and B, what is most nearly the distance, x, for the bar to remain horizontal?



(A) 1300 mm (B) 1600 mm (C) 1900 mm (D) 2300 mm



From the free-body diagram, taking moments about point B gives

$$\sum M_B = 0 = (1500\ \text{kN})x - (3000\ \text{mm})F_1$$

$$(1500\ \text{kN})x = (3000\ \text{mm})F_1$$

$$x = \left(2\ \frac{\text{mm}}{\text{kN}}\right)F_1 \qquad \text{[I]}$$

From a vertical force balance,

$$F_1 + F_2 = 1500\ \text{kN} \qquad \text{[II]}$$

For the bar to remain horizontal, the deflection of cable 1 must equal the deflection of cable 2.

$$\delta_1 = \delta_2$$

$$\frac{F_1 L_1}{EA_1} = \frac{F_2 L_2}{EA_2}$$

$$F_1 = \frac{L_2 A_1}{L_1 A_2} F_2 = \frac{(1000\ \text{mm})(1300\ \text{mm}^2)}{(600\ \text{mm})(650\ \text{mm}^2)} F_2$$

$$= 3.33 F_2 \qquad \text{[III]}$$

Solving equations II and III simultaneously,

$$3.33F_2 + F_2 = 1500\ \text{kN}$$

$$4.33F_2 = 1500\ \text{kN}$$

$$F_2 = 346.4\ \text{kN}$$

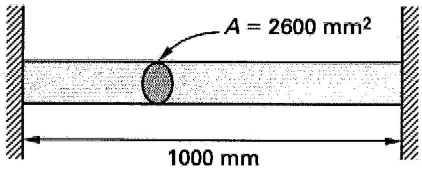
$$F_1 = 1153.6\ \text{kN}$$

$$\begin{aligned} x &= \left(2\ \frac{\text{mm}}{\text{kN}}\right) F \\ &= \left(2\ \frac{\text{mm}}{\text{kN}}\right)(1153.6\ \text{kN}) \\ &= 2307\ \text{mm} \quad (2300\ \text{mm}) \end{aligned}$$

The answer is (D).

MECHANICS OF MATERIALS–30

A prismatic bar at 10°C is constrained in a rigid concrete wall at both ends. The bar is 1000 mm long and has a cross-sectional area of 2600 mm^2. What is most nearly the axial force in the bar if the temperature is raised to 40°C?



E = modulus of elasticity
= 200 GPa
α = coefficient of thermal expansion
= 9.4×10^{-6}/°C

(A) 116 kN (B) 125 kN (C) 134 kN (D) 147 kN

Elongation due to temperature change is given by

$$\begin{aligned} \delta &= \alpha L (T_2 - T_1) \\ &= \left(9.4 \times 10^{-6}\ \frac{1}{°\text{C}}\right)(1000\ \text{mm})(40°\text{C} - 10°\text{C}) \\ &= 0.282\ \text{mm} \end{aligned}$$

Elongation is

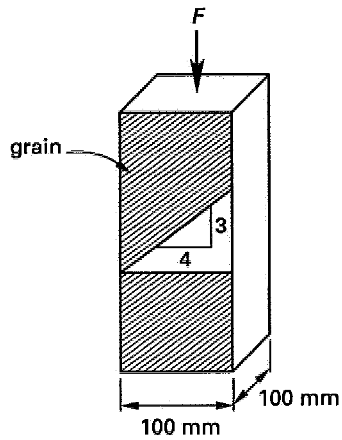
$$\delta = \frac{FL}{EA}$$

$$\begin{aligned} F = \frac{\delta E A}{L} &= \frac{(0.000\,282\ \text{m})\left(200 \times 10^6\ \frac{\text{kN}}{\text{m}^2}\right)(0.0026\ \text{m}^2)}{1\ \text{m}} \\ &= 146.6\ \text{kN} \quad (147\ \text{kN}) \end{aligned}$$

The answer is (D).

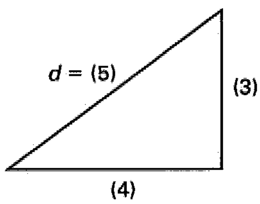
MECHANICS OF MATERIALS–31

What is most nearly the maximum axial load, F, that can be applied to the wood post shown without exceeding a maximum shear stress of 1650 kPa parallel to the grain?



(A) 22 kN (B) 33 kN (C) 44 kN (D) 57 kN

The length of the diagonal parallel to the grain, d, (part of a 3-4-5 triangle) is



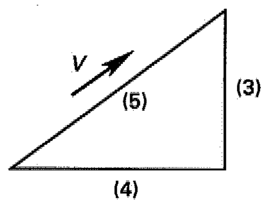
$$d = \left(\frac{5}{4}\right)(100 \text{ mm}) = 125 \text{ mm} \quad (0.125 \text{ m})$$

The area of the inclined plane is

$$A = (0.125 \text{ m})(0.100 \text{ m}) = 0.0125 \text{ m}^2$$

The total shear on the plane is

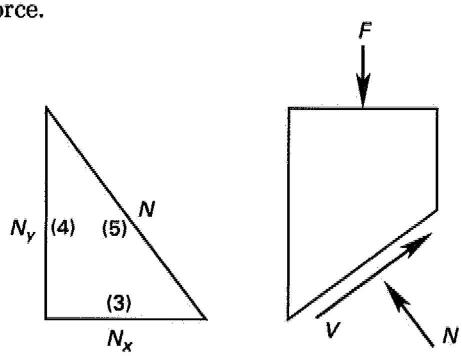
$$V = \tau A = \left(1650 \frac{\text{kN}}{\text{m}^2}\right)(0.0125 \text{ m}^2) = 20.63 \text{ kN}$$



The horizontal component of the shear is

$$V_x = \left(\frac{4}{5}\right)(20.63 \text{ kN}) = 16.5 \text{ kN}$$

Draw the free-body diagram of the upper section. Include the normal compressive force.



Balancing the x-components,

$$\sum F_x = 0 = V_x - N_x = 0$$
$$N_x = \tfrac{3}{5}N = V_x$$
$$\tfrac{3}{5}N = 16.5 \text{ kN}$$
$$N = \left(\frac{5}{3}\right)(16.5 \text{ kN}) = 27.5 \text{ kN}$$

Balancing the y-components,

$$\sum F_y = 0 = N_y - F = 0$$
$$F = N_y = \tfrac{4}{5}N = \left(\frac{4}{5}\right)(27.5 \text{ kN})$$
$$= 22 \text{ kN}$$

The answer is (A).

MECHANICS OF MATERIALS–32

The shear strain, ϵ, along a shaft is

$$\epsilon = r\frac{d\phi}{dx}$$

r is the radius from the shaft's centerline, and $d\phi/dx$ is the change of the angle of twist with respect to the axis of the shaft. Which condition is NOT necessary for the above equation to be valid?

(A) The area of interest must be free of connections and other load applications.

(B) The material must be isotropic and homogeneous.

(C) The loading must result in the stress being a torsional couple acting along the axis.

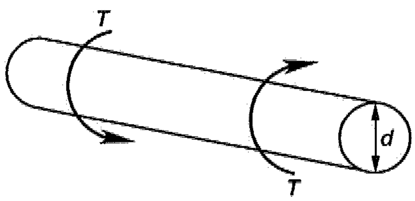
(D) r must be the full radius of the shaft.

The equation may be evaluated for any value of r, giving the stress distribution over the shaft cross section.

The answer is (D).

MECHANICS OF MATERIALS–33

A 3 m diameter bar experiences a torque of 280 N·m. What is most nearly the maximum shear stress in the bar?



(A) 2.2 Pa (B) 31 Pa (C) 42 Pa (D) 53 Pa

Maximum shear stress occurs at the outer surface.

The equation for shear gives

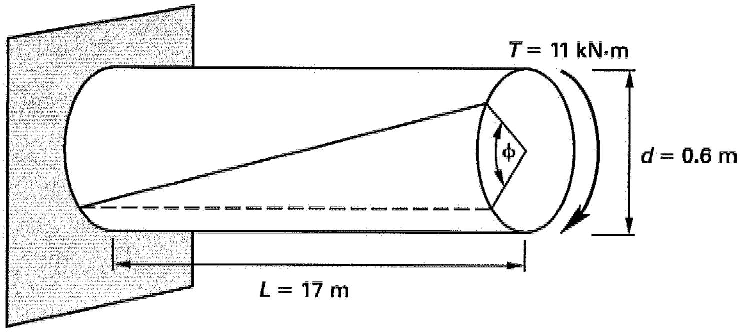
$$\tau = \frac{Tr}{J} = \frac{T\left(\frac{d}{2}\right)}{\frac{\pi}{32}d^4} = \frac{(280\ \text{N}\cdot\text{m})\left(\frac{3\ \text{m}}{2}\right)}{\left(\frac{\pi}{32}\right)(3\ \text{m})^4}$$

$$= 52.8\ \text{N/m}^2 \quad (53\ \text{Pa})$$

The answer is (D).

MECHANICS OF MATERIALS–34

What is most nearly the angle of twist, ϕ, for the aluminum bar shown? The shear modulus of elasticity, G, is 26 GPa.



(A) 0.000 55° (B) 0.0055° (C) 0.032° (D) 0.082°

The angle of twist is given by

$$\phi = \frac{TL}{GJ} = \frac{(11\ \text{kN·m})(17\ \text{m})}{\left(26 \times 10^6\ \frac{\text{kN}}{\text{m}^2}\right)\left(\frac{\pi}{32}\right)(0.6\ \text{m})^4}$$

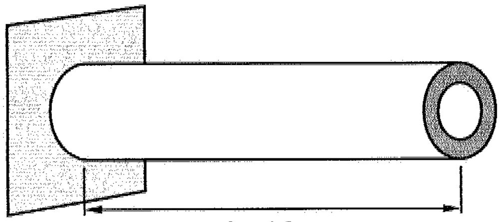
$$= (0.000\,565\ \text{rad})\left(\frac{180°}{\pi\ \text{rad}}\right)$$

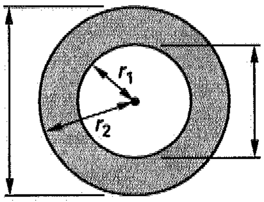
$$= 0.032°$$

The answer is (C).

MECHANICS OF MATERIALS–35

What torque, T, should be applied to the end of the steel shaft shown in order to produce a twist of 1.5°? Use $G = 80$ GPa for the shear modulus.





(A) 420 N·m (B) 560 N·m (C) 830 N·m (D) 1100 N·m

Converting the twist angle to radians and calculating the polar moment of inertia J,

$$\phi = (1.5°)\left(\frac{2\pi \text{ rad}}{360°}\right) = 0.026 \text{ rad}$$

$$r_1 = 0.015 \text{ m}$$

$$r_2 = 0.025 \text{ m}$$

$$J = \frac{\pi}{2}\left(r_2^4 - r_1^4\right) = \left(\frac{\pi}{2}\right)\left((0.025 \text{ m})^4 - (0.015 \text{ m})^4\right)$$

$$= 5.34 \times 10^{-7} \text{ m}^4$$

$$T = \frac{GJ}{L}\phi$$

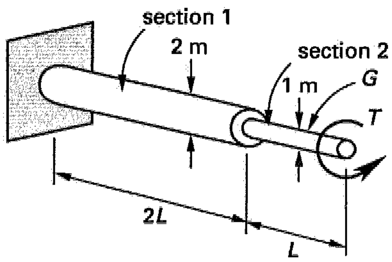
$$= \left(\frac{\left(80 \times 10^9 \ \frac{\text{N}}{\text{m}^2}\right)(5.34 \times 10^{-7} \text{ m}^4)}{1 \text{ m}}\right)(0.026 \text{ rad})$$

$$= 1110 \text{ N·m} \quad (1100 \text{ N·m})$$

The answer is (D).

MECHANICS OF MATERIALS–36

Determine the maximum torque that can be applied to the shaft, given that the maximum angle of twist is 0.0225 rad. Neglect bending.



(A) $0.000\,625\dfrac{\pi G}{L}$ (B) $0.0500\dfrac{\pi G}{L}$

(C) $0.250\dfrac{\pi G}{L}$ (D) $0.525\dfrac{\pi G}{L}$

The angle of twist is

$$\phi = \frac{TL}{GJ}$$

J for a circular bar of diameter d is $\frac{1}{2}\pi r^4 = \frac{1}{32}\pi d^4$. The total angle of twist, ϕ_{total}, is equal to the sum of the angles of twist for the two different sections. The torque is the same for both sections.

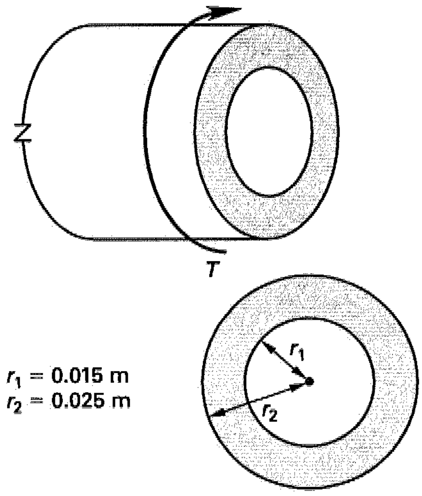
$$\begin{aligned}
\phi_{\text{total}} &= \phi_1 + \phi_2 \\
&= \frac{T(2L)}{GJ_1} + \frac{TL}{GJ_2} \\
&= \left(\frac{32TL}{\pi G}\right)\left(\frac{2}{d_1^4} + \frac{1}{d_2^4}\right) \\
&= \left(\frac{32TL}{\pi G}\right)\left(\frac{2}{(2\text{ m})^4} + \frac{1}{(1\text{ m})^4}\right) \\
&= \frac{36TL}{\pi G}
\end{aligned}$$

$$\begin{aligned}
T &= \frac{\pi G\phi_{\text{total}}}{36L} = \frac{\pi G(0.0225\text{ rad})}{36L} \\
&= 0.000\,625\frac{\pi G}{L}
\end{aligned}$$

The answer is (A).

MECHANICS OF MATERIALS-37

For the given shaft, what is most nearly the largest torque that can be applied if the shear stress is not to exceed 110 MPa?



(A) 1700 N·m (B) 1900 N·m (C) 2400 N·m (D) 3400 N·m

Since the shear stress is largest at the outer diameter, the maximum torque is found using this radius.

$$T_{\text{max}} = \frac{\tau J}{r_2}$$

For an annular region,

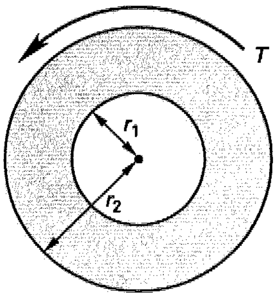
$$\begin{aligned} J &= \frac{\pi}{2}(r_2^4 - r_1^4) = \left(\frac{\pi}{2}\right)\left((0.025 \text{ m})^4 - (0.015 \text{ m})^4\right) \\ &= 5.34 \times 10^{-7} \text{ m}^4 \end{aligned}$$

$$\begin{aligned} T_{\text{max}} &= \frac{(5.34 \times 10^{-7} \text{ m})^4 \left(110 \times 10^6 \dfrac{\text{N}}{\text{m}^2}\right)}{0.025 \text{ m}} \\ &= 2350 \text{ N·m} \quad (2400 \text{ N·m}) \end{aligned}$$

The answer is (C).

MECHANICS OF MATERIALS–38

A hollow circular bar has an inner radius r_1 and an outer radius r_2. If $r_1 = r_2/2$, most nearly what percentage of torque can the shaft carry in comparison with a solid shaft?



(A) 25% (B) 55% (C) 75% (D) 95%

The equation for torsional stress is

$$\tau = \frac{Tr}{J}$$

$$T = \frac{\tau J}{r}$$

For the hollow shaft,

$$T_h = \frac{\tau\left(\frac{\pi}{2}\right)\left(r_2^4 - r_1^4\right)}{r_2}$$

For the solid shaft,

$$T_s = \frac{\tau\left(\frac{\pi}{2}\right)r_2^4}{r_2}$$

Therefore,

$$\frac{T_h}{T_s} = \frac{\tau\left(\frac{\pi}{2}\right)\left(\frac{r_2^4 - r_1^4}{r_2}\right)}{\tau\left(\frac{\pi}{2}\right)\left(\frac{r_2^4}{r_2}\right)}$$

$$= \frac{r_2^4 - r_1^4}{r_2^4}$$

$$= \frac{r_2^4 - \left(\frac{r_2}{2}\right)^4}{r_2^4}$$

$$= \frac{r_2^4 - \frac{r_2^4}{16}}{r_2^4}$$

$$= \frac{\frac{15}{16}r_2^4}{r_2^4}$$

$$= \frac{15}{16}$$

$$= 0.94 \quad (95\%)$$

The answer is (D).

MECHANICS OF MATERIALS–39

What is the minimum solid shaft diameter that can be used for the rotor of a 4.5 kW motor operating at 3500 rpm, if the maximum shear stress for the shaft is 60 MPa?

(A) 1.2 mm (B) 2.1 mm (C) 10 mm (D) 20 mm

The relationship between the power, P, transmitted by a shaft and the torque, T, is

$$P = \frac{\pi n T}{30}$$

n is in rpm, T is in N·m, and P is in W. Rearranging to solve for T,

$$T = \frac{30P}{\pi n} = \frac{(30)(4500 \text{ W})}{\pi \left(3500 \frac{\text{rev}}{\text{min}}\right)} = 12.28 \text{ N·m}$$

$$\tau_{\max} = \frac{Tr}{J} = \frac{Td}{2J}$$

$$J = \frac{Td}{2\tau_{\max}} = \frac{\pi d^4}{32}$$

Therefore,

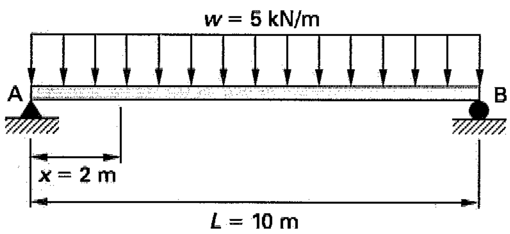
$$d = \left(\frac{32T}{2\pi\tau_{\max}}\right)^{1/3} = \left(\frac{(16)(12.28 \text{ N·m})}{\pi\left(60 \times 10^6 \frac{\text{N}}{\text{m}^2}\right)}\right)^{1/3}$$

$$= 0.0101 \text{ m} \quad (10 \text{ mm})$$

The answer is (C).

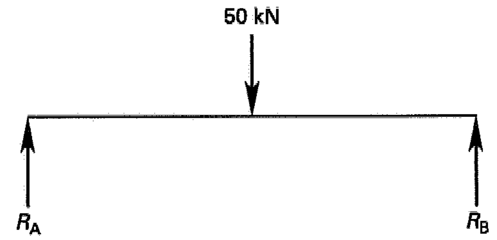
MECHANICS OF MATERIALS–40

A beam supports a distributed load, w, as shown. Find the shear force at $x = 2$ m from the left end.



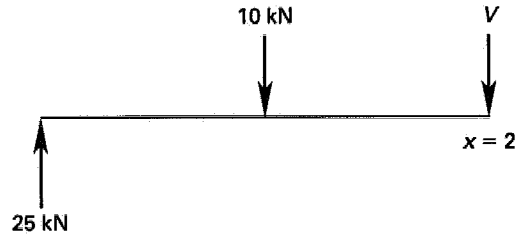
(A) 11 kN (B) 12 kN (C) 13 kN (D) 15 kN

The reactions at A and B are found by observation from symmetry to be $R_A = R_B = 25$ kN.



$R_A = R_B = 25$ kN

Sectioning the beam at $x = 2$ m, the free-body diagram with shear force is



$$\sum F_y = 0$$

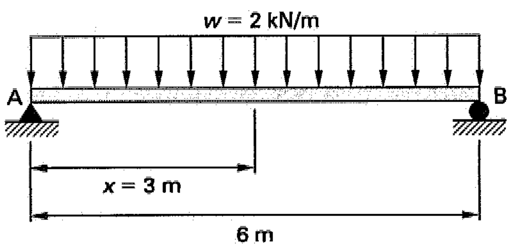
$$25\text{ kN} - 10\text{ kN} - V = 0\text{ kN}$$

$$V = 25\text{ kN} - 10\text{ kN} = 15\text{ kN}$$

The answer is (D).

MECHANICS OF MATERIALS–41

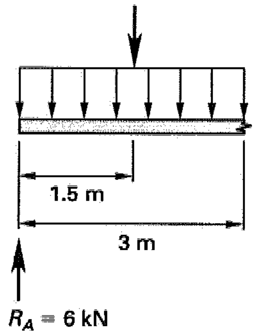
For the beam shown, find the bending moment, M, at $x = 3$ m.



(A) 4.5 kN·m (B) 6.0 kN·m (C) 7.5 kN·m (D) 9.0 kN·m

By inspection from symmetry, $R_A = R_B = 6$ kN. Sectioning the beam at $x = 3$ m gives

$$F = \left(2\ \frac{\text{kN}}{\text{ft}}\right)(3\ \text{ft}) = 6\ \text{kN}$$



$$\sum M_{x=3} = 0$$

$$(6\ \text{kN})(3\ \text{m}) + (6\ \text{kN})(1.5\ \text{m}) + M = 0$$

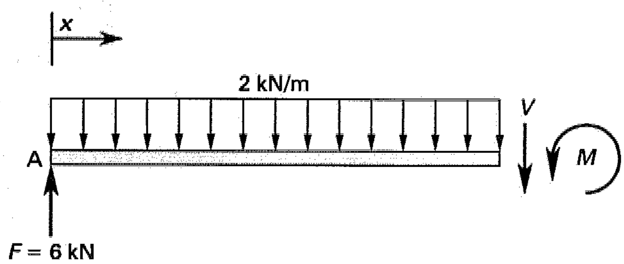
$$M = 18\ \text{kN·m} - 9\ \text{kN·m}$$

$$= 9\ \text{kN·m}$$

The answer is (D).

MECHANICS OF MATERIALS–42

Find the expression for the bending moment as a function of distance from the left end, x, for the following beam.



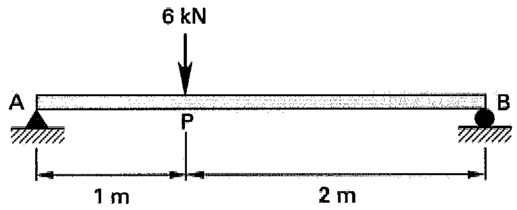
(A) $M = -x^3 + 2x$ (B) $M = -x^2 + 1$
(C) $M = -x^2 + 2x$ (D) $M = x^3 - 2x^2$

$$\sum M = -2x + (2x)\left(\tfrac{1}{2}x\right) + M = 0$$
$$M = -x^2 + 2x$$

The answer is (C).

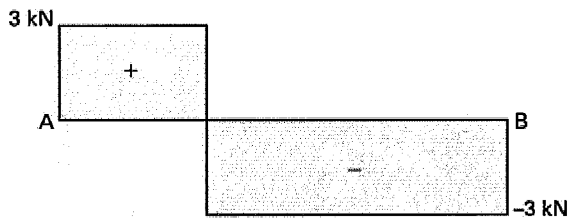
MECHANICS OF MATERIALS–43

Which of the following is the shear force diagram for this beam?

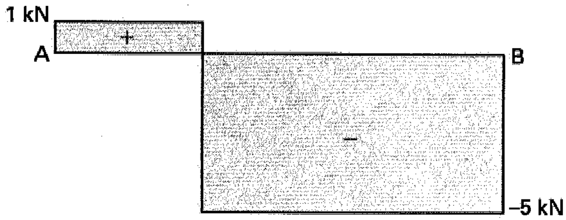


(A)

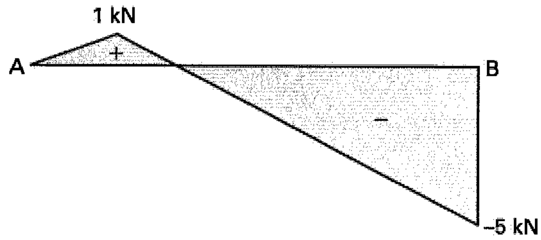
(B)

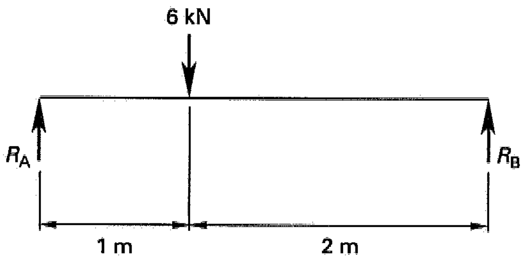


(C)

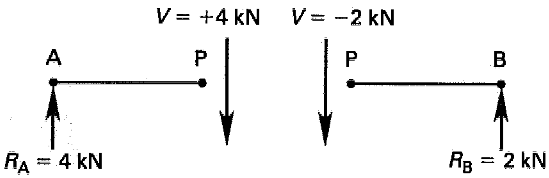


(D)





By observation, the reactions at points A and B are $R_A = 4$ kN and $R_B = 2$ kN. Draw free-body diagrams of the left and right sections of the beam.

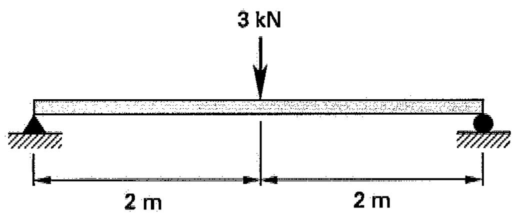


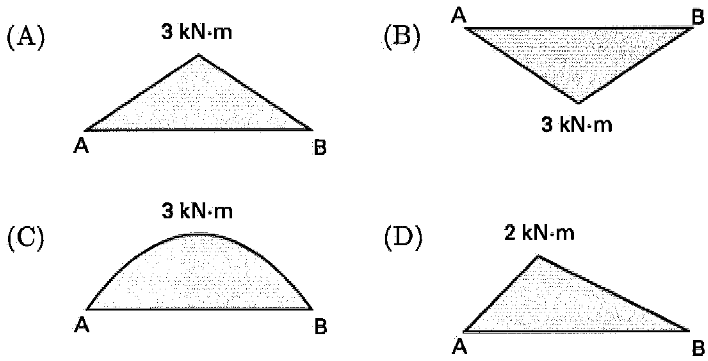
Thus, $V = +4$ kN between points A and P, and $V = -2$ kN between points P and B.

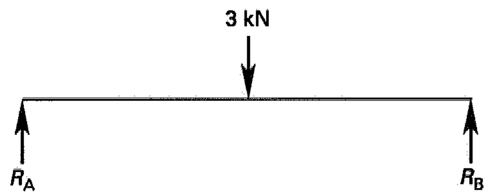
The answer is (A).

MECHANICS OF MATERIALS—44

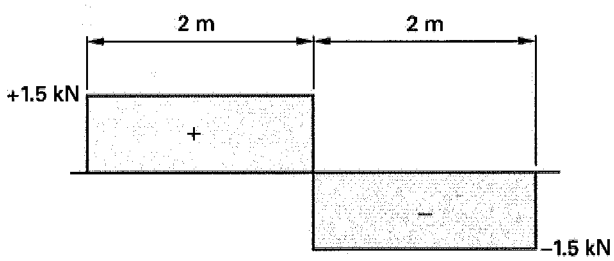
Which of the following is the bending moment diagram for this beam?



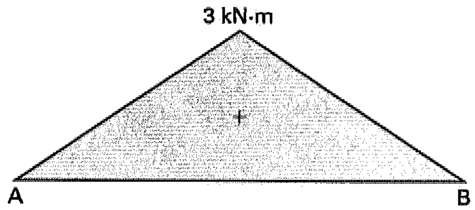




From the free-body diagram, $R_A = R_B = 1.5$ kN. The shear force diagram is therefore,



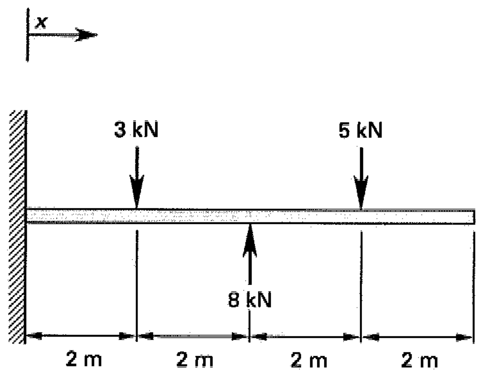
The bending moment increases linearly to (1.5 kN)(2 m) = 3 kN·m, then decreases linearly back to 0 kN·m.



The answer is (A).

MECHANICS OF MATERIALS–45

The cantilever beam shown is loaded by three concentrated forces. What is the maximum shear force in the beam?



(A) 1 kN (B) 2 kN (C) 3 kN (D) 5 kN

Examining the shear force along the beam from left to right, for 0 m $< x < 2$ m,

$$V = 0 \text{ kN}$$

For 2 m $< x <$ 4 m,

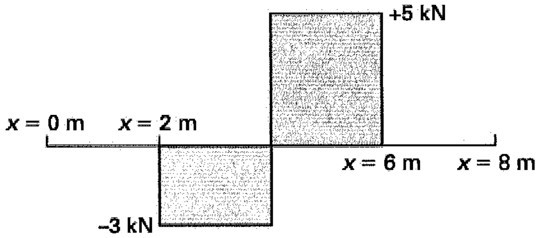
$$V = -3 \text{ kN}$$

For 4 m $< x <$ 6 m,

$$V = 5 \text{ kN}$$

For 6 m $< x <$ 8 m,

$$V = 0 \text{ kN}$$

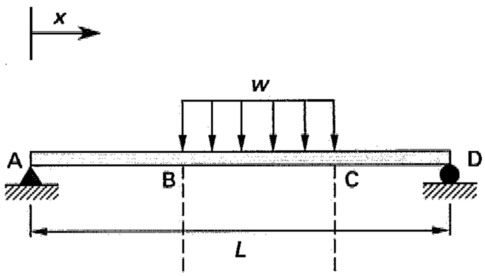


The maximum shear force is, therefore, 5 kN.

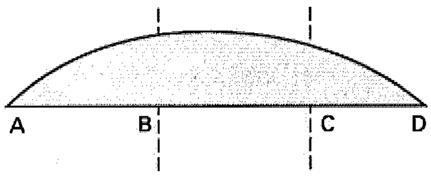
The answer is (D).

MECHANICS OF MATERIALS–46

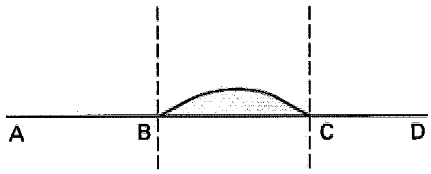
Which of the following bending moment diagrams corresponds to the simply supported beam shown? The beam is subjected to a distributed load, w, between points B and C.



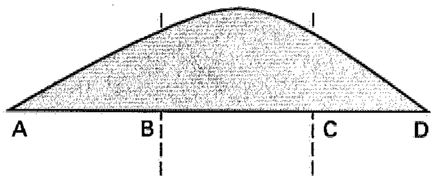
(A)



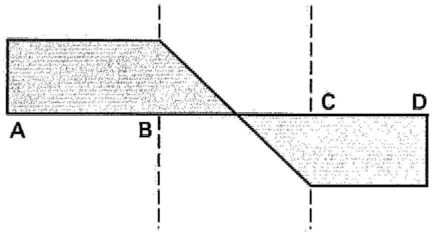
(B)



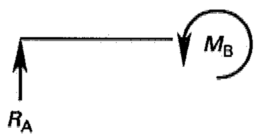
(C)



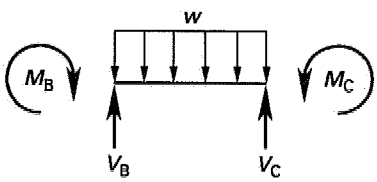
(D)



For sections AB and CD, the beam may be modeled as



$M(x)$ is linear with respect to x. For section BC, the beam is modeled as

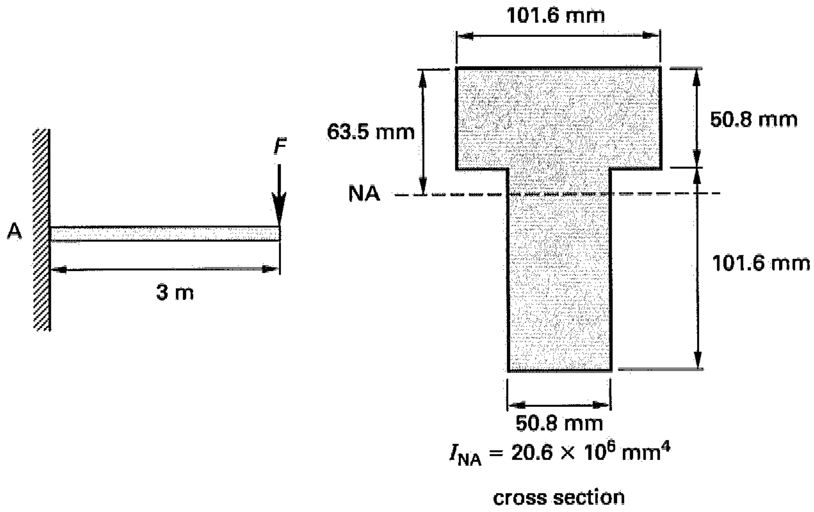


$M(x)$ is parabolic, reaching a maximum near or at the center.

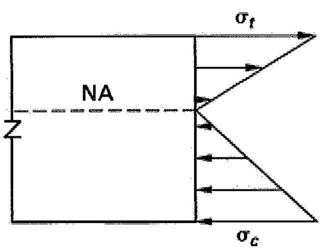
The answer is (C).

MECHANICS OF MATERIALS–47

What is most nearly the maximum allowable load, F, on the cantilever? The maximum compressive stress is 7000 kPa, and the maximum tensile stress is 5500 kPa. The moment of inertia about the centroidal axis, I_{NA}, is 20.6×10^6 mm^4.



(A) 540 N (B) 600 N (C) 610 N (D) 640 N



The maximum bending moment occurs at A, where $M = 3F$.

$$\sigma_{max} = \frac{Mc}{I} = \frac{3Fc}{I}$$

$$F = \frac{\sigma_{max} I}{3c}$$

$$I = (20.6 \times 10^6 \text{ mm}^4)\left(\frac{1 \text{ m}}{1000 \text{ mm}}\right)^4$$

$$= 20.6 \times 10^{-6} \text{ m}^4$$

For compression, $\sigma_{\text{allowable}} = 7000$ kPa.

$$c = 101.6 \text{ mm} + 50.8 \text{ mm} - 63.5 \text{ mm} = 88.9 \text{ mm}$$

$$F_{\substack{\text{allowable} \\ \text{compression}}} = \frac{\left(7000 \ \frac{\text{kN}}{\text{m}^2}\right)(20.6 \times 10^{-6} \text{ m}^4)}{(3 \text{ m})(0.0889 \text{ m})}$$

$$= 0.541 \text{ kN} \quad (541 \text{ N})$$

For tension, $\sigma_{\text{allowable}} = 5500$ kPa, and $c = 63.5$ mm.

$$F_{\substack{\text{allowable} \\ \text{tension}}} = \frac{\left(5500 \ \frac{\text{kN}}{\text{m}^2}\right)(20.6 \times 10^{-6} \text{ m}^4)}{(3 \text{ m})(0.0635 \text{ m})}$$

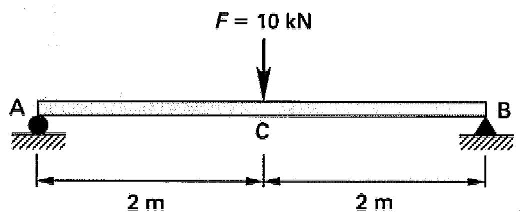
$$= 0.595 \text{ kN} \quad (600 \text{ N})$$

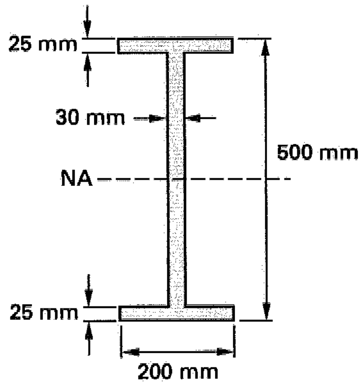
The maximum allowable load is 540 N.

The answer is (A).

MECHANICS OF MATERIALS–48

A simply supported beam with the cross section shown supports a concentrated load, $F = 10$ kN, at its center, C. What is most nearly the maximum bending stress in the beam?





cross section

(A) 2300 kPa (B) 3200 kPa (C) 3800 kPa (D) 4600 kPa

The reactions at A and B are $R_A = R_B = 5$ kN by inspection from symmetry. Since the maximum bending moment occurs at C,

$$M_{max} = (5\ \text{kN})(2\ \text{m}) = 10\ \text{kN·m}$$

The moment of inertia about the neutral axis, NA, is the difference between the moments of inertia of an area measuring 20 mm × 500 mm and two areas measuring 85 mm × 450 mm.

$$I = \frac{bh^3}{12} = \left(\frac{1}{12}\right)(200\ \text{mm})(500\ \text{mm})^3 - (2)\left(\frac{1}{12}\right)(85\ \text{mm})(450\ \text{mm})^3$$

$$= 792 \times 10^6\ \text{mm}^4$$

Since $c = \left(\frac{1}{2}\right)(500\ \text{mm}) = 250\ \text{mm}$,

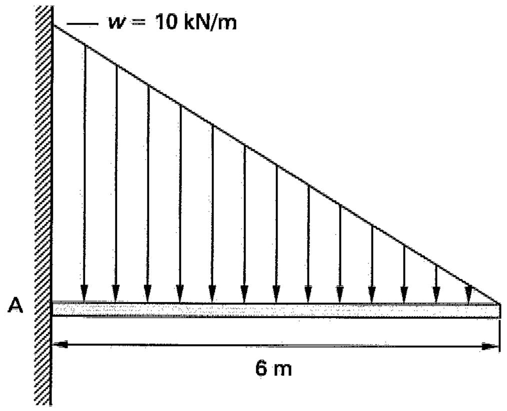
$$\sigma_{max} = \frac{Mc}{I} = \frac{(10\ \text{kN·m})(0.25\ \text{m})}{792 \times 10^{-6}\ \text{m}^4}$$

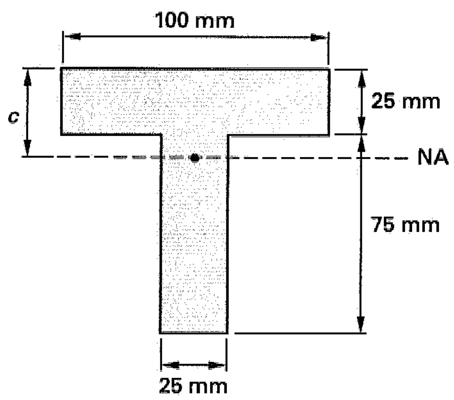
$$= 3157\ \text{kN/m}^2 \quad (3160\ \text{kPa})$$

The answer is (B).

MECHANICS OF MATERIALS–49

For the cantilever beam shown, what is the maximum tensile bending stress?

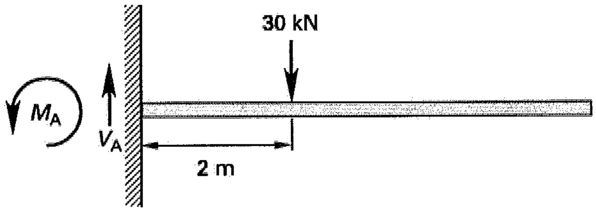




cross section

(A) 230 MPa (B) 320 MPa (C) 480 MPa (D) 550 MPa

The maximum moment occurs at point A and is a result of the distributed load w. w is equivalent to a concentrated load, $W = (1/2)(6\text{ m})(10\text{ kN/m}) = 30\text{ kN}$, acting at a point $(1/3)(6\text{ m}) = 2\text{ m}$ from point A. The equivalent loading diagram for the cantilever is as follows.



$$M_A = (30\text{ kN})(2\text{ m}) = 60\text{ kN·m}$$

The upper part of the beam will be under tension, with c equal to the distance between the neutral axis, NA, and the top edge of the beam.

$$c = \frac{\sum A\bar{y}}{\sum A} = \frac{(25\text{ mm})(100\text{ mm})(12.5\text{ mm}) + (25\text{ mm})(75\text{ mm})(62.5\text{ mm})}{2500\text{ mm}^2 + 1875\text{ mm}^2}$$

$$= 33.9\text{ mm} \quad (0.0339\text{ cm})$$

Use the parallel axis theorem to find I.

$$\begin{aligned} I_{NA} = \frac{bh^3}{12} &= \left(\frac{1}{12}\right)(100\text{ mm})(25\text{ mm})^3 \\ &\quad + (100\text{ mm})(25\text{ mm})(34\text{ mm} - 12.5\text{ mm})^2 \\ &\quad + \left(\frac{1}{12}\right)(25\text{ mm})(75\text{ mm})^3 + (75\text{ mm})(25\text{ mm})(62.5\text{ mm} - 34\text{ mm})^2 \\ &= 3.7 \times 10^6\text{ mm}^4 \end{aligned}$$

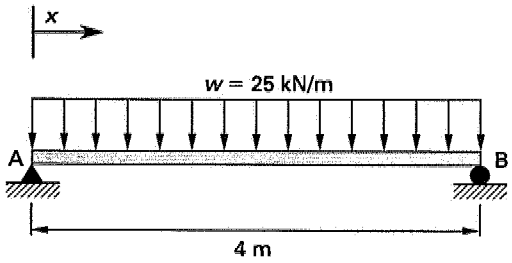
$$\sigma = \frac{Mc}{I} = \frac{(60\text{ kN·m})(0.0339\text{ m})}{(3.7 \times 10^6\text{ mm}^4)\left(\frac{1\text{ m}}{1000\text{ mm}}\right)^4}$$

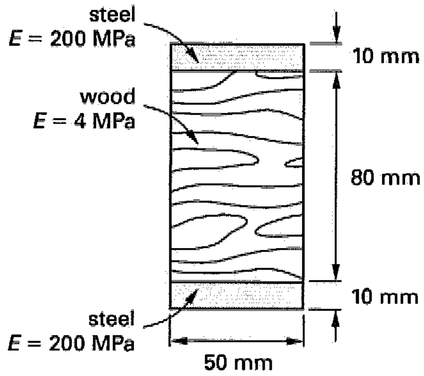
$$= 5.5 \times 10^5\text{ kPa} \quad (550\text{ MPa})$$

The answer is (D).

MECHANICS OF MATERIALS-50

A composite beam made of steel and wood is subjected to a uniform distributed load, w. Determine the maximum compressive stress in the steel.





cross section

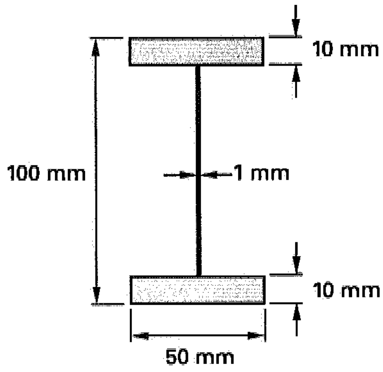
(A) 620 MPa (B) 850 MPa (C) 1100 MPa (D) 1200 MPa

The maximum moment is at the center of the beam, where $x = 2$ m.

$$R_A = R_B = 50 \text{ kN} \quad \text{[by inspection]}$$

$$M_{max} = (50 \text{ kN})(2 \text{ m}) - \left(25 \frac{\text{kN}}{\text{m}}\right)(2 \text{ m})(1 \text{ m})$$
$$= 50 \text{ kN·m}$$

Since $E_{wood}/E_{steel} = 4$ MPa/200 MPa $= 1/50$, the wood is equivalent to a steel web 1 mm thick.



cross section

$$I = \frac{bh^3}{12} = \left(\frac{1}{12}\right)(50\text{ mm})(100\text{ mm})^3 - (2)\left(\frac{1}{12}\right)(24.5\text{ mm})(80\text{ mm})^3$$
$$= 2.076 \times 10^6\text{ mm}^4$$

$$\sigma = \frac{Mc}{I} = \frac{(50\text{ kN}\cdot\text{m})(0.05\text{ m})\left(1000\ \frac{\text{N}}{\text{kN}}\right)}{(2.076 \times 10^6\text{ mm}^4)\left(\frac{1\text{ m}}{1000\text{ mm}}\right)^4}$$
$$= 1200 \times 10^6\text{ N/m}^2 \quad (1200\text{ MPa})$$

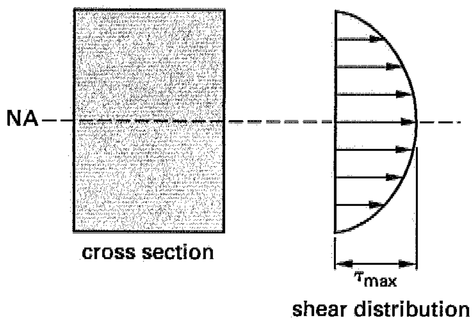
The answer is (D).

MECHANICS OF MATERIALS–51

For a rectangular beam under transverse (bending) loading, where is the location of maximum shear stress?

(A) at the top edge
(B) at the bottom edge
(C) at the neutral axis
(D) at a location between the top edge and the neutral axis

The shear distribution is

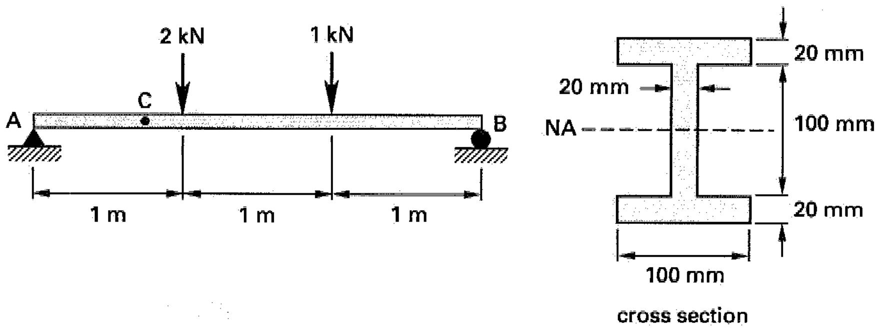


The maximum shear stress is at the neutral axis.

The answer is (C).

MECHANICS OF MATERIALS–52

An I-beam is loaded as shown. What is most nearly the maximum shear stress, τ, in the web at point C along the beam?



(A) 160 kPa (B) 370 kPa (C) 400 kPa (D) 750 kPa

The reaction at point A is found by taking the moment about point B.

$$\begin{aligned} \sum M_B &= 0 \\ &= -R_A(3\text{ m}) + (2\text{ kN})(2\text{ m}) + (1\text{ kN})(1\text{ m}) \\ R_A &= 1.67\text{ kN} \\ V_C &= R_A = 1.67\text{ kN} \end{aligned}$$

The shear stress is given by $\tau = VQ/It$, where Q is the first moment of either the upper half or the lower half of the cross-sectional area with respect to the neutral axis.

$$Q = A'\bar{y} = (50 \text{ mm})(20 \text{ mm})(25 \text{ mm}) + (100 \text{ mm})(20 \text{ mm})(60 \text{ mm})$$
$$= 145\,000 \text{ mm}^3$$
$$I = \frac{bh^3}{12} = \left(\frac{1}{12}\right)(100 \text{ mm})(140 \text{ mm})^3 - (2)\left(\frac{1}{12}\right)(40 \text{ mm})(100 \text{ mm})^3$$
$$= 16.2 \times 10^6 \text{ mm}^4$$

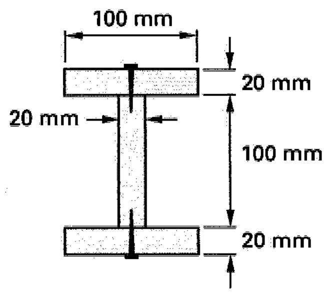
τ_{max} occurs at the neutral axis. Thus,

$$\tau_{\text{max}} = \frac{VQ}{It} = \frac{(1670 \text{ N})(145\,000 \text{ mm}^3)}{(16.2 \times 10^6 \text{ mm}^4)(20 \text{ mm})}$$
$$= 0.747 \text{ N/mm}^2 \quad (750 \text{ kPa})$$

The answer is (D).

MECHANICS OF MATERIALS–53

An I-beam is made of three planks, each 20 mm × 100 mm in cross section, nailed together with a single row of nails on top and bottom as shown. If the longitudinal spacing between the nails is 25 mm, and the vertical shear force acting on the cross section is 600 N, what is most nearly the load in shear per nail, F?



(A) 56 N (B) 76 N (C) 110 N (D) 160 N

The shear force per unit distance along the beam's axis is given by

$$f = \frac{VQ}{I}$$

For an I-beam, Q is the first moment of the upper flange area with respect to the z-axis.

$$Q = A\bar{y} = (60 \text{ mm})(100 \text{ mm})(20 \text{ mm}) = 120\,000 \text{ mm}^3$$

$$I = \frac{bh^3}{12} = \left(\frac{1}{12}\right)(100 \text{ mm})(140 \text{ mm})^3 - (2)\left(\frac{1}{12}\right)(40 \text{ mm})(100 \text{ mm})^3$$
$$= 16.2 \times 10^6 \text{ mm}^4$$

$$f = \frac{(600 \text{ N})(120\,000 \text{ mm}^3)}{16.2 \times 10^6 \text{ mm}^4} = 4.44 \text{ N/mm}$$

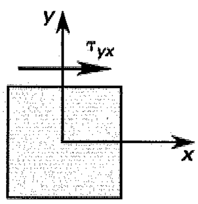
The load capacity of the nails per unit length is F/L. Therefore,

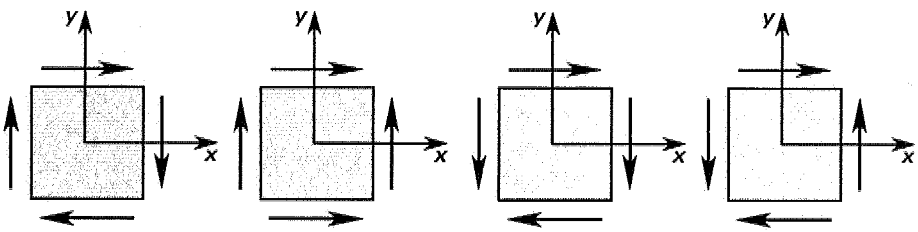
$$F = Lf = (25 \text{ mm})\left(4.44 \frac{\text{N}}{\text{mm}}\right) = 111 \text{ N} \quad (110 \text{ N})$$

The answer is (C).

MECHANICS OF MATERIALS–54

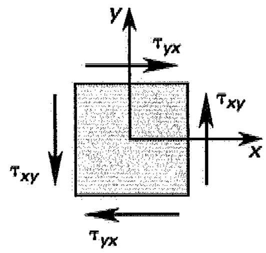
Considering the orientation of shear force τ_{yx} in the illustration, find the direction of the shear stress on the other three sides of the stress element.





(A) (B) (C) (D)

For static equilibrium, the shear stresses on opposite faces of an element must be equal in magnitude and opposite in direction. Also, the shear stresses on adjoining faces must not produce rotation of the element.



The answer is (D).

MECHANICS OF MATERIALS–55

If the principal stresses on a body are $\sigma_1 = 400$ kPa, $\sigma_2 = -700$ kPa, and $\sigma_3 = 600$ kPa, what is the maximum shear stress?

(A) 150 kPa (B) 250 kPa (C) 550 kPa (D) 650 kPa

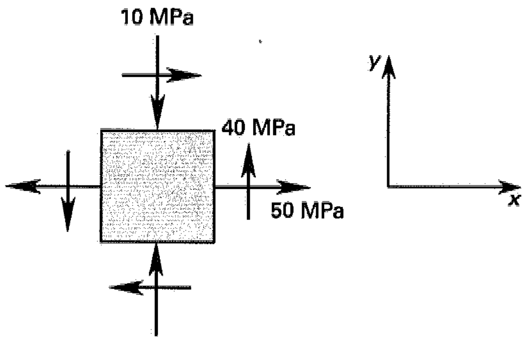
The maximum shear stress is equal to one-half of the difference between the principal stresses. Comparing the three combinations, the maximum shear stress is given by the difference between σ_2 and σ_3.

$$\tau_{\text{max}} = \left|\frac{\sigma_2 - \sigma_3}{2}\right| = \left|\frac{-700 \text{ kPa} - 600 \text{ kPa}}{2}\right| = 650 \text{ kPa}$$

The answer is (D).

MECHANICS OF MATERIALS–56

For the element of plane stress shown, find the principal stresses.



(A) $\sigma_{\max} = 35$ MPa, $\sigma_{\min} = -25$ MPa
(B) $\sigma_{\max} = 45$ MPa, $\sigma_{\min} = 55$ MPa
(C) $\sigma_{\max} = 70$ MPa, $\sigma_{\min} = -30$ MPa
(D) $\sigma_{\max} = 85$ MPa, $\sigma_{\min} = 15$ MPa

The stresses on the element are

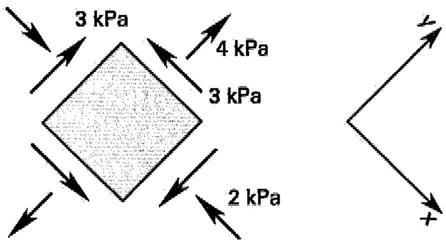
$$\sigma_x = 50 \text{ MPa} \qquad \sigma_y = -10 \text{ MPa} \qquad \tau_{xy} = 40 \text{ MPa}$$

$$\begin{aligned}\sigma_{\max,\min} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \frac{50 \text{ MPa} - 10 \text{ MPa}}{2} \pm \sqrt{\left(\frac{50 \text{ MPa} + 10 \text{ MPa}}{2}\right)^2 + (40 \text{ MPa})^2} \\ &= 20 \text{ MPa} \pm 50 \text{ MPa} \\ &= 70 \text{ MPa or } -30 \text{ MPa}\end{aligned}$$

The answer is (C).

MECHANICS OF MATERIALS–57

What are the principal (maximum and minimum) stresses of the stress element shown?



(A) $\sigma_{\text{max}} = 1.16$ kPa, $\sigma_{\text{min}} = -6.16$ kPa
(B) $\sigma_{\text{max}} = 2.00$ kPa, $\sigma_{\text{min}} = -4.00$ kPa
(C) $\sigma_{\text{max}} = 3.24$ kPa, $\sigma_{\text{min}} = -5.24$ kPa
(D) $\sigma_{\text{max}} = 5.24$ kPa, $\sigma_{\text{min}} = -3.24$ kPa

The stresses on the element are

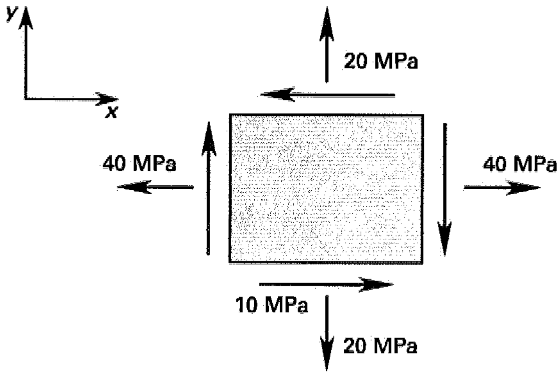
$$\sigma_x = -2 \text{ kPa} \qquad \sigma_y = +4 \text{ kPa} \qquad \tau_{xy} = -3 \text{ kPa}$$

$$\begin{aligned} \sigma_{\text{max,min}} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \frac{-2 \text{ kPa} + 4 \text{ kPa}}{2} \pm \sqrt{\left(\frac{-2 \text{ kPa} - 4 \text{ kPa}}{2}\right)^2 + (-3 \text{ kPa})^2} \\ &= 5.24 \text{ kPa or } -3.24 \text{ kPa} \end{aligned}$$

The answer is (D).

MECHANICS OF MATERIALS-58

What is most nearly the maximum principal stress of the element shown?



(A) 30 MPa (B) 34 MPa (C) 40 MPa (D) 44 MPa

The stresses on the element are

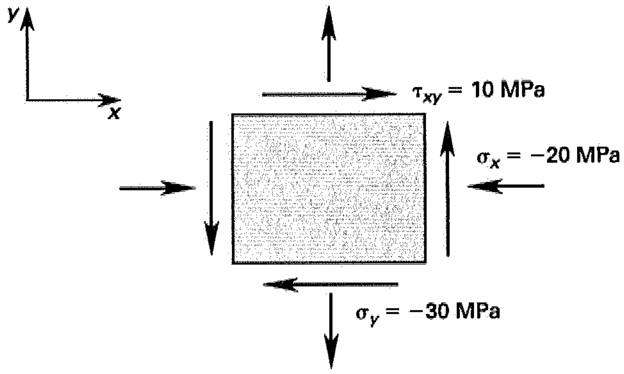
$$\sigma_x = 40 \text{ MPa} \qquad \sigma_y = 20 \text{ MPa} \qquad \tau_{xy} = -10 \text{ MPa}$$

$$\begin{aligned}\sigma_{\max} &= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \frac{40 \text{ MPa} + 20 \text{ MPa}}{2} + \sqrt{\left(\frac{40 \text{ MPa} - 20 \text{ MPa}}{2}\right)^2 + (-10 \text{ MPa})^2} \\ &= 44.1 \text{ MPa} \quad (44 \text{ MPa})\end{aligned}$$

The answer is (D).

MECHANICS OF MATERIALS–59

For the following stress element, what is most nearly the maximum shear stress?



(A) 10 MPa (B) 11 MPa (C) 14 MPa (D) 27 MPa

The maximum shear stress is

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

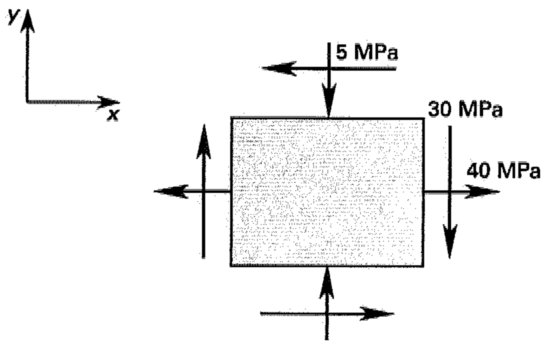
$$= \sqrt{\left(\frac{-20\text{ MPa} - 30\text{ MPa}}{2}\right)^2 + (10\text{ MPa})^2}$$

$$= 26.9\text{ MPa}$$

The answer is (D).

MECHANICS OF MATERIALS-60

For the state of plane stress shown, what are the inclination angles of the principal planes?



(A) 32.5° and 122°
(B) 25.5° and 115°
(C) −26.5° and −117°
(D) −11.5° and −102°

The stresses on the element are

$$\sigma_x = 40 \text{ MPa} \qquad \sigma_y = -5 \text{ MPa} \qquad \tau_{xy} = -30 \text{ MPa}$$

$$\begin{aligned} \tan 2\theta_p &= \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-30 \text{ MPa})}{40 \text{ MPa} - (-5 \text{ MPa})} \\ &= -1.33 \\ 2\theta_p &= -53.0^\circ \text{ or } -233^\circ \\ \theta_p &= -26.5^\circ \text{ or } -117^\circ \end{aligned}$$

The answer is (C).

MECHANICS OF MATERIALS–61

A steel ($\sigma_{yield} = 200$ MPa) pressure tank is designed to hold pressures up to 7 MPa. The tank is cylindrical with a diameter of 1 m. If the longitudinal stress must be less than 20% of the yield stress of the steel, what is the necessary wall thickness, t?

(A) 22 mm (B) 44 mm (C) 88 mm (D) 120 mm

For a thin-walled cylinder of diameter d containing a pressure p,

$$\sigma_{long} = \frac{pd}{4t} = (1 - 0.2)\sigma_{yield}$$

$$t = \frac{pd}{0.8\sigma_{yield}} = \frac{\left(7 \times 10^6 \ \frac{\text{N}}{\text{m}^2}\right)(1 \text{ m})}{(0.8)\left(200 \times 10^6 \ \frac{\text{N}}{\text{m}^2}\right)}$$

$$= 0.044 \text{ m} \quad (44 \text{ mm})$$

The answer is (B).

MECHANICS OF MATERIALS–62

In designing a cylindrical pressure tank 1 m in diameter, a factor of safety of 2.5 is used. The cylinder is made of steel ($\sigma_{yield} = 200$ MPa), and will contain pressures up to 7 MPa. What is the required wall thickness, t, based on circumferential stress considerations?

(A) 22 mm (B) 44 mm (C) 88 mm (D) 120 mm

For a thin-walled cylinder of diameter d containing a pressure p,

$$\sigma_{circumferential} = \frac{pd}{2t} = \frac{\sigma_{yield}}{2.5}$$

$$t = \frac{1.25pd}{\sigma_{yield}} = \frac{(1.25)\left(7 \times 10^6 \ \frac{\text{N}}{\text{m}^2}\right)(1 \text{ m})}{200 \times 10^6 \ \frac{\text{N}}{\text{m}^2}}$$

$$= 0.044 \text{ m} \quad (44 \text{ mm})$$

The answer is (B).

MECHANICS OF MATERIALS–63

What is most nearly the maximum principal strain at a point where $\epsilon_x = 1500$ μm, $\epsilon_y = -750$ μm, and $\epsilon_{xy} = 1000$ μm?

(A) 1160 μm (B) 1490 μm (C) 1610 μm (D) 1830 μm

The equation for principal strain gives

$$\epsilon_{\text{max,min}} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\epsilon_{xy}}{2}\right)^2}$$

$$= \frac{1500\ \mu\text{m} - 750\ \mu\text{m}}{2} \pm \sqrt{\left(\frac{1500\ \mu\text{m} + 750\ \mu\text{m}}{2}\right)^2 + \left(\frac{1000\ \mu\text{m}}{2}\right)^2}$$

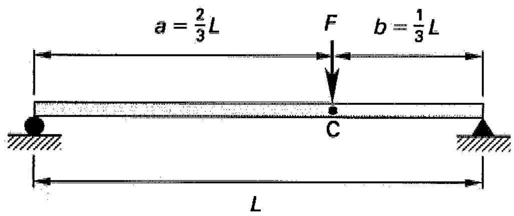
$$= 375\ \mu\text{m} \pm 1231\ \mu\text{m}$$

$$\epsilon_{\text{max}} = 1606\ \mu\text{m} \quad (1610\ \mu\text{m})$$

The answer is (C).

MECHANICS OF MATERIALS–64

A beam of length L carries a concentrated load, F, at point C. Determine the deflection at point C in terms of F, L, E, and I, where E is the modulus of elasticity, and I is the moment of inertia.



(A) $\dfrac{2FL^3}{243EI}$ (B) $\dfrac{4FL^3}{243EI}$ (C) $\dfrac{FL^3}{27EI}$ (D) $\dfrac{FL^3}{9EI}$

The equation for bending moment in the beam is

$$EI\frac{d^2\delta}{dx^2} = -M$$

Computing M for the different beam sections,

$$EI\frac{d^2\delta}{dx^2} = -\frac{Fbx}{L} \qquad (0 \le x \le a)$$

$$EI\frac{d^2\delta}{dx^2} = -\frac{Fbx}{L} + F(x-a) \qquad (a \le x \le L)$$

Integrating each equation twice gives

$$EI\delta = -\frac{Fbx^3}{6L} + C_1x + C_3 \qquad (0 \le x \le a)$$

$$EI\delta = -\frac{Fbx^3}{6L} + \frac{F(x-a)^3}{6} + C_2x + C_4 \qquad (a \le x \le L)$$

The constants are determined by the following conditions: (1) at $x = a$, the slopes $d\delta/dx$ and deflections δ are equal; (2) at $x = 0$ and $x = L$, the deflection $\delta = 0$. These conditions give

$$C_1 = C_2 = \frac{Fb(L^2 - b^2)}{6L}$$

$$C_3 = C_4 = 0$$

Evaluating the equation for $(0 \le x \le a)$ at $x = a = {}^2/_3L$ and $b = {}^1/_3L$,

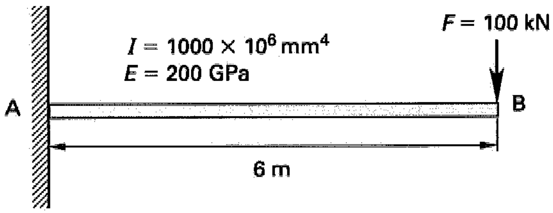
$$EI\delta = \left(\frac{F(\frac{1}{3}L)(\frac{2}{3}L)}{6L}\right)\left(L^2 - \frac{L^2}{9} - \frac{4L^2}{9}\right)$$

$$\delta = \frac{4FL^3}{243EI}$$

The answer is (B).

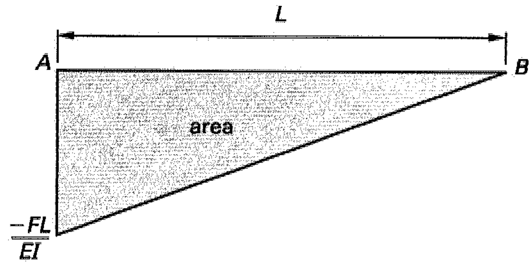
MECHANICS OF MATERIALS–65

What is the deflection at point B for the beam shown?



(A) 17 mm (B) 25 mm (C) 36 mm (D) 48 mm

Using the moment-area method, the M/EI diagram is



$$A = \tfrac{1}{2}L\left(\frac{-FL}{EI}\right) = -\frac{FL^2}{2EI}$$

The first moment with respect to point A is

$$Q = A(\tfrac{2}{3}L) = -\left(\frac{FL^2}{2EI}\right)(\tfrac{2}{3}L) = -\frac{FL^3}{3EI}$$

The deflection is

$$y = -Q = \frac{FL^3}{3EI} = \frac{(100\ \text{kN})(6\ \text{m})^3}{(3)\left(200 \times 10^6\ \frac{\text{kN}}{\text{m}^2}\right)(1000 \times 10^6\ \text{mm}^4)\left(\frac{1\ \text{m}}{1000\ \text{mm}}\right)^4}$$

$$= 0.036\ \text{m} \quad (36\ \text{mm})$$

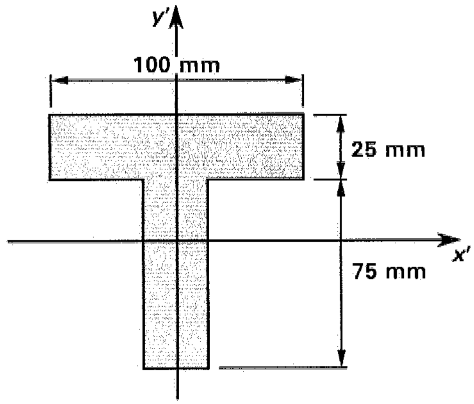
The answer is (C).

MECHANICS OF MATERIALS–66

What is the Euler buckling load for a 10 m long steel column pinned at both ends and with the given properties and cross section?

$$I_{x'x'} = 3.70 \times 10^6\ \text{mm}^4$$

$$E = 200\ \text{GPa}$$



(A) 15 kN (B) 24 kN (C) 43 kN (D) 73 kN

$x'x'$ and $y'y'$ are centroidal axes.

$$I_{x'x'} = 3.70 \times 10^6 \text{ mm}^4$$

$I_{y'y'}$ is computed by applying the equation for I $(bh^3/12)$ about the centroidal axis of a rectangle. For this cross section, $b_1 = 25$ mm, $h_1 = 100$ mm, $b_2 = 75$ mm, and $h_2 = 25$ mm.

$$\begin{aligned} I_{y'y'} &= \left(\left(\frac{1}{12}\right)(25 \text{ mm})(100 \text{ mm})^3 + \left(\frac{1}{12}\right)(75 \text{ mm})(25 \text{ mm})^3\right) \\ &\quad \times \left(\frac{1 \text{ m}}{1000 \text{ mm}}\right)^4 \\ &= 2.18 \times 10^{-6} \text{ m}^4 \end{aligned}$$

The Euler buckling load, P_{cr}, is

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

I is the minimum I value. $I_{y'y'}$ is less that $I_{x'x'}1$.

$$\begin{aligned} P_{cr} &= \frac{\pi^2 \left(200 \times 10^6 \frac{\text{kN}}{\text{m}^2}\right)(2.18 \times 10^{-6} \text{ m}^4)}{(10 \text{ m})^2} \\ &= 43 \text{ kN} \end{aligned}$$

The answer is (C).

11 DYNAMICS

DYNAMICS–1

How many degrees of freedom does a coin rolling on the ground have?

(A) one (B) two (C) three (D) five

A coin has two translational degrees of freedom and one rotational degree of freedom.

The answer is (C).

DYNAMICS–2

What is the definition of instantaneous velocity?

(A) $\mathrm{v} = dx\ dt$ (B) $\mathrm{v} = \int x\ dt$

(C) $\mathrm{v} = \frac{dx}{dt}$ (D) $\mathrm{v} = \lim_{\Delta t \to 0} \frac{\Delta t}{\Delta x}$

By definition,

$$\mathrm{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The answer is (C).

DYNAMICS–3

A car travels 100 km to city A in 2 h, then travels 200 km to city B in 3 h. What is the average speed of the car for the trip?

(A) 45 km/h (B) 58 km/h (C) 60 km/h (D) 66 km/h

Average velocity is defined as total distance traveled over total time.

$$\mathrm{v}_{\text{ave}} = \frac{\Delta x}{\Delta t} = \frac{100\ \text{km} + 200\ \text{km}}{2\ \text{h} + 3\ \text{h}}$$
$$= 60\ \text{km/h}$$

The answer is (C).

DYNAMICS–4

The position of a particle moving along the x-axis is given by $x(t) = t^2 - t + 8$, where x is in units of meters, and t is in seconds. Find the velocity of the particle when $t = 5$ s.

(A) 9.0 m/s (B) 10 m/s (C) 11 m/s (D) 12 m/s

The velocity equation is the first derivative of the position equation with respect to time. Therefore,

$$\mathrm{v}(t) = \frac{dx}{dt}$$
$$= \frac{d}{dt}(t^2 - t + 8)$$
$$= 2t - 1$$
$$\mathrm{v}(5) = (2)(5) - 1 = 9.0\ \text{m/s}$$

The answer is (A).

DYNAMICS–5

If a particle's position is given by the expression $x(t) = 3.4t^3 - 5.4t$ m, what is most nearly the acceleration of the particle at $t = 5$ s?

(A) 1.0 m/s^2 (B) 3.4 m/s^2 (C) 18 m/s^2 (D) 100 m/s^2

The acceleration is found from the second derivative of the position equation. Therefore,

$$\begin{aligned} a(t) &= \frac{d^2x}{dt^2} \\ &= \frac{d^2}{dt^2}(3.4t^3 - 5.4t) \\ &= \frac{d}{dt}(10.2t^2 - 5.4) \\ &= 20.4t \end{aligned}$$

$$a(5) = (20.4)(5) = 102 \text{ m/s}^2 \quad (100 \text{ m/s}^2)$$

The answer is (D).

DYNAMICS–6

A car starts from rest and moves with a constant acceleration of 6 m/s^2. What is the speed of the car after 4 s?

(A) 18 m/s (B) 24 m/s (C) 35 m/s (D) 55 m/s

For uniformly accelerated motion,

$$\begin{aligned} \text{v} &= \text{v}_0 + at = 0 + \left(6 \ \frac{\text{m}}{\text{s}^2}\right)(4 \text{ s}) \\ &= 24 \text{ m/s} \end{aligned}$$

The answer is (B).

DYNAMICS–7

A car starts from rest and has a constant acceleration of 3 m/s^2. What is the average velocity during the first 10 s of motion?

(A) 12 m/s (B) 13 m/s (C) 14 m/s (D) 15 m/s

The distance traveled by the car is

$$x = x_0 + \mathrm{v}_0 t + \tfrac{1}{2}at^2 = 0 + 0 + \left(\frac{1}{2}\right)\left(3\ \frac{\text{m}}{\text{s}^2}\right)(10\ \text{s})^2$$
$$= 150\ \text{m}$$
$$\mathrm{v}_{\text{ave}} = \frac{\Delta x}{\Delta t} = \frac{150\ \text{m}}{10\ \text{s}}$$
$$= 15\ \text{m/s}$$

The answer is (D).

DYNAMICS–8

A truck increases its speed uniformly from 13 km/h to 50 km/h in 25 s. What is most nearly the acceleration of the truck?

(A) 0.22 m/s^2 (B) 0.41 m/s^2 (C) 0.62 m/s^2 (D) 0.92 m/s^2

For uniformly accelerated rectilinear motion,

$$\mathrm{v} = \mathrm{v}_0 + at$$
$$at = \mathrm{v} - \mathrm{v}_0$$
$$a = \frac{\mathrm{v} - \mathrm{v}_0}{t} = \left(\frac{50\ \frac{\text{km}}{\text{h}} - 13\ \frac{\text{km}}{\text{h}}}{25\ \text{s}}\right)\left(\frac{1000\ \frac{\text{m}}{\text{km}}}{3600\ \frac{\text{s}}{\text{h}}}\right)$$
$$= 0.411\ \text{m/s}^2 \quad (0.41\ \text{m/s}^2)$$

The answer is (B).

DYNAMICS–9

A bicycle moves with a constant deceleration of $-2\ \text{m/s}^2$. If the initial velocity of the bike is 4.0 m/s, how far does it travel in 3 s?

(A) 2.0 m (B) 2.5 m (C) 3.0 m (D) 4.0 m

For constant acceleration,

$$\begin{aligned} x &= x_0 + \text{v}_0 t + \tfrac{1}{2}at^2 \\ &= 0 + \left(4.0\ \frac{\text{m}}{\text{s}}\right)(3\ \text{s}) + \left(\frac{1}{2}\right)\left(-2\ \frac{\text{m}}{\text{s}^2}\right)(3\ \text{s})^2 \\ &= 3.0\ \text{m} \end{aligned}$$

The answer is (C).

DYNAMICS–10

A ball is dropped from a height of 60 m above ground. How long does it take to hit the ground?

(A) 1.3 s (B) 2.1 s (C) 3.5 s (D) 5.5 s

The positive y direction is downward, and $y = 0$ at 60 m above ground. For uniformly accelerated motion,

$$\begin{aligned} y &= y_0 + \text{v}_0 t + \tfrac{1}{2}at^2 \\ 60 &= 0 + 0 + \left(\frac{1}{2}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)t^2 \\ t &= 3.5\ \text{s} \end{aligned}$$

The answer is (C).

DYNAMICS–11

A man driving a car at 65 km/h suddenly sees an object in the road 20 m ahead. Assuming an instantaneous reaction on the driver's part, what constant deceleration is required to stop the car in this distance?

(A) 7.1 m/s^2 (B) 7.5 m/s^2 (C) 8.0 m/s^2 (D) 8.1 m/s^2

For uniform deceleration, the velocity equation that is not a function of time is

$$v^2 = v_0^2 + 2a(x - x_0)$$

Using $v = 0$, $v_0 = 65$ km/h $= 18$ m/s, and $(x - x_0) = 20$ m,

$$0 = \left(18\ \frac{m}{s}\right)^2 + 2a(20\ m)$$

$$a = -\frac{\left(18\ \frac{m}{s}\right)^2}{(2)(20\ m)} = 8.1\ m/s^2$$

The answer is (D).

DYNAMICS–12

A ball is thrown vertically upward with an initial speed of 24 m/s. Most nearly how long will it take for the ball to return to the thrower?

(A) 2.3 s (B) 2.6 s (C) 4.1 s (D) 4.9 s

At the apex of its flight, the ball has zero velocity and is at the midpoint of its flight time. If the total flight time is t_{total}, then the time elapsed at this point is $^1/_2 t_{total}$.

$$v = v_0 + at$$

Rearranging to solve for t_{total},

$$0 = 24\ \frac{m}{s} + \left(-9.81\ \frac{m}{s^2}\right) \tfrac{1}{2} t_{total}$$

$$t_{total} = (2)\left(\frac{24\ \frac{m}{s}}{9.81\ \frac{m}{s^2}}\right) = 4.893\ s \quad (4.9\ s)$$

The answer is (D).

DYNAMICS–13

A projectile is launched upward from level ground at an angle of 60° from the horizontal. It has an initial velocity of 45 m/s. How long will it take before the projectile hits the ground?

(A) 4.1 s (B) 5.8 s (C) 7.9 s (D) 9.5 s

The projectile will experience acceleration only in the y direction due to gravity. The y component of velocity is

$$\mathrm{v}_{0y} = 45 \sin 60^\circ = 39 \text{ m/s}$$

For uniform rectilinear motion with constant acceleration,

$$\begin{aligned} y &= y_0 + \mathrm{v}_{0y}t + \tfrac{1}{2}at^2 \\ &= 0 + \left(39\ \frac{\text{m}}{\text{s}}\right)t + \left(\frac{1}{2}\right)\left(-9.81\ \frac{\text{m}}{\text{s}^2}\right)t^2 \\ &= \left(39\ \frac{\text{m}}{\text{s}}\right)t - \left(4.91\ \frac{\text{m}}{\text{s}^2}\right)t^2 \end{aligned}$$

When the body is on the ground, $y = 0$.

$$0 = \left(39\ \frac{\text{m}}{\text{s}}\right)t - \left(4.91\ \frac{\text{m}}{\text{s}^2}\right)t^2 = t\left(39\ \frac{\text{m}}{\text{s}} - 4.91\ \frac{\text{m}}{\text{s}^2}t\right)$$

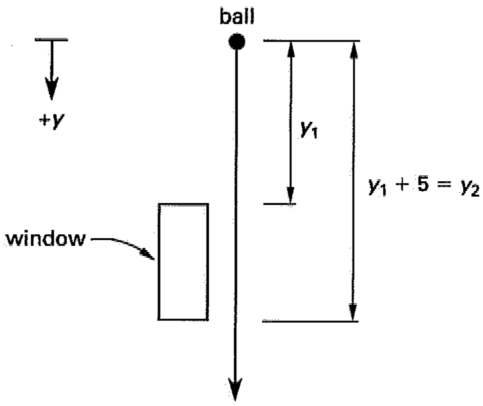
$$t = \frac{3.9\ \frac{\text{m}}{\text{s}}}{4.9\ \frac{\text{m}}{\text{s}^2}} = 7.94 \text{ s} \quad (7.9 \text{ s})$$

The answer is (C).

DYNAMICS–14

A man standing at a 5 m tall window watches a falling ball pass by the window in 0.3 s. From approximately how high above the top of the window was the ball released from a stationary position?

(A) 8.2 m (B) 9.6 m (C) 12 m (D) 21 m



The positive y direction is taken as downward, and the initial release point is y_0. Then,

$$y = y_0 + v_o t + \tfrac{1}{2}at^2 = 0 + 0 + \tfrac{1}{2}at^2$$
$$y_1 = \tfrac{1}{2}at_1^2$$
$$y_2 = \tfrac{1}{2}at_2^2$$

However, $y_2 = y_1 + 5$ m, and $t_2 = t_1 + 0.3$ s. Therefore,

$$\begin{aligned} y_1 + 5 \text{ m} &= \tfrac{1}{2}a(t_1 + 0.3 \text{ s})^2 \\ \tfrac{1}{2}at_1^2 + 5 \text{ m} &= \tfrac{1}{2}a\big(t_1^2 + (2)(0.3 \text{ s})t_1 + (0.09 \text{ s}^2)\big) \\ &= \tfrac{1}{2}at_1^2 + (0.3 \text{ s})at_1 + (0.045 \text{ s}^2)a \end{aligned}$$

Rearrange and solve for t_1.

$$\begin{aligned} (0.3 \text{ s})at_1 &= 5 \text{ m} - (0.045 \text{ s}^2)a \\ t_1 &= \frac{5 \text{ m} - (0.045 \text{ s}^2)a}{(0.3 \text{ s})a} \\ &= \frac{5 \text{ m} - (0.045 \text{ s}^2)\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)}{(0.3 \text{ s})\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)} \\ &= 1.55 \text{ s} \end{aligned}$$

Solving for y_1,

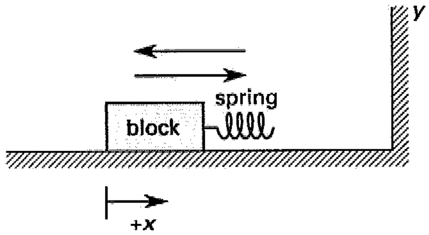
$$y_1 = \tfrac{1}{2}at_1^2 = \left(\frac{1}{2}\right)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(1.55\ \text{s})^2$$

$$= 11.8\ \text{m} \quad (12\ \text{m})$$

The answer is (C).

DYNAMICS–15

A block with a spring attached to one end slides along a rough surface with an initial velocity of 7 m/s. After it slides 4 m, it impacts a wall for 0.1 s, and then slides 10 m in the opposite direction before coming to a stop. If the block's deceleration is assumed constant and the contraction of the spring is negligible, what is the average acceleration of the block during impact with the wall?



(A) $-120\ \text{m/s}^2$ (B) $-100\ \text{m/s}^2$ (C) $-99\ \text{m/s}^2$ (D) $-49\ \text{m/s}^2$

$a_{1\text{-}2}$	acceleration before impact	m/s^2
$a_{2\text{-}2'}$	acceleration during impact	m/s^2
$a_{2\text{-}3}$	acceleration after impact	m/s^2
$s_{1\text{-}2}$	distance traveled before impact	m
$s_{2\text{-}3}$	distance traveled after impact	m
v_1	initial velocity	m/s
v_2	velocity just before impact	m/s
v_2'	velocity after impact	m/s
v_3	final velocity	m/s

$$\text{v}_1 = 7\ \text{m/s}$$

$$\text{v}_3 = 0$$

Because Δx is small and energy is conserved, $v_2 = v_2'$.

$$\begin{aligned} v_2 &= \sqrt{v_1^2 - 2a_{1\text{-}2}s_{1\text{-}2}} \\ &= \sqrt{\left(7\ \frac{\text{m}}{\text{s}}\right)^2 - 2(a_{1\text{-}2})(4\ \text{m})} \\ &= \sqrt{49\ \frac{\text{m}^2}{\text{s}^2} - 8a_{1\text{-}2}} \end{aligned}$$

Alternatively,

$$\begin{aligned} a_{1\text{-}2} &= \frac{49\ \frac{\text{m}^2}{\text{s}^2} - v_2^2}{8\ \text{m}} \\ v_3 &= \sqrt{v_{2'}^2 - 2a_{2\text{-}3}s_{2\text{-}3}} \\ 0 &= \sqrt{v_{2'}^2 - 2a_{2\text{-}3}(10\ \text{m})} \end{aligned}$$

Alternatively,

$$a_{2\text{-}3} = \frac{v_{2'}^2}{20}$$

But $a_{1\text{-}2} = a_{2\text{-}3}$, and $v_2 = v_{2'}$.

$$\begin{aligned} \frac{49\ \frac{\text{m}^2}{\text{s}^2} - v_2^2}{8\ \text{m}} &= \frac{v_2^2}{20\ \text{m}} \\ 980\ \frac{\text{m}^2}{\text{s}^2} - 20v_2^2 &= 8{v_2}^2 \\ v_2 &= 5.9\ \text{m/s} \end{aligned}$$

Then, because of the direction change,

$$\begin{aligned} a_{2-2'} &= \frac{v_{2'} - v_2}{\Delta t} = \frac{-5.9\ \frac{\text{m}}{\text{s}} - 5.9\ \frac{\text{m}}{\text{s}}}{0.10\ \text{s}} \\ &= -118\ \text{m/s}^2 \quad (-120\ \text{m/s}^2) \end{aligned}$$

The answer is (A).

DYNAMICS–16

A car starting from rest moves with a constant acceleration of 15 km/h^2 for 1 h, then decelerates at a constant $-7.5\ \text{km/h}^2$ until it comes to a stop. Most nearly how far has it traveled?

(A) 15 km (B) 23 km (C) 25 km (D) 35 km

For constant acceleration,

$$x = x_0 + \text{v}_0 t + \tfrac{1}{2}at^2$$

During acceleration, $x_0 = 0$, $\text{v}_0 = 0$, $a = 15\ \text{km/h}^2$, and $t = 1$ h. Then, the distance over which the car accelerates is

$$x_{\text{acc}} = \left(\frac{1}{2}\right)\left(15\ \frac{\text{km}}{\text{h}^2}\right)(1\ \text{h})^2 = 7.5\ \text{km}$$

At the end of the hour, the car's velocity is

$$\begin{aligned} \text{v} &= \text{v}_0 + at = \left(15\ \frac{\text{km}}{\text{h}^2}\right)(1\ \text{h}) \\ &= 15\ \text{km/h} \end{aligned}$$

During deceleration, $x_0 = 0$, $\text{v}_0 = 15\ \text{km/h}$, and $a = -7.5\ \text{km/h}^2$. The car has velocity $\text{v} = 0$ when it stops. Therefore,

$$\begin{aligned} \text{v} &= \text{v}_0 + at \\ 0 &= 15\ \frac{\text{km}}{\text{h}} + \left(-7.5\ \frac{\text{km}}{\text{h}^2}\right)t \\ t &= 2\ \text{h} \\ x &= x_0 + \text{v}_0 t + \tfrac{1}{2}at^2 \end{aligned}$$

The distance over which the car decelerates is

$$x_{\text{dec}} = \left(15\ \frac{\text{km}}{\text{h}}\right)(2\ \text{h}) + \left(\frac{1}{2}\right)\left(-7.5\ \frac{\text{km}}{\text{h}^2}\right)(2\ \text{h})^2 = 15\ \text{km}$$

$$x_{\text{total}} = x_{\text{acc}} + x_{\text{dec}} = 7.5\ \text{km} + 15\ \text{km} = 22.5\ \text{km} \quad (23\ \text{km})$$

The answer is (B).

DYNAMICS–17

A train with a top speed of 75 km/h cannot accelerate or decelerate faster than 1.2 m/s^2. What is the minimum distance between two train stops in order for the train to be able to reach its top speed?

(A) 300 m (B) 350 m (C) 360 m (D) 365 m

To travel the minimum distance, the train must accelerate from $v_0 = 0$ km/h to $v = 75$ km/h at a constant 1.2 m/s^2 and then decelerate at a constant 1.2 m/s^2 to $v = 0$ km/h. The train travels the same distance during acceleration as during deceleration, since the initial and final speeds are identical, as well as the magnitude of acceleration or deceleration. The following two equations apply for constant acceleration.

$$x = x_0 + v_0 t = \tfrac{1}{2}at^2$$
$$v = v_0 + at$$

During acceleration, $x_0 = 0$ m, $v_0 = 0$ km/h, $v = 75$ km/h $=$ 20.8 m/s, and $a = 1.2$ m/s^2. Then,

$$t = \frac{v - v_0}{a} = \frac{20.8\ \frac{\text{m}}{\text{s}} - 0\ \frac{\text{m}}{\text{s}}}{1.2\ \frac{\text{m}}{\text{s}^2}} = 17.3\ \text{s}$$

$$x = \left(\frac{1}{2}\right)\left(1.2\ \frac{\text{m}}{\text{s}^2}\right)(17.3\ \text{s})^2 = 180\ \text{m}$$

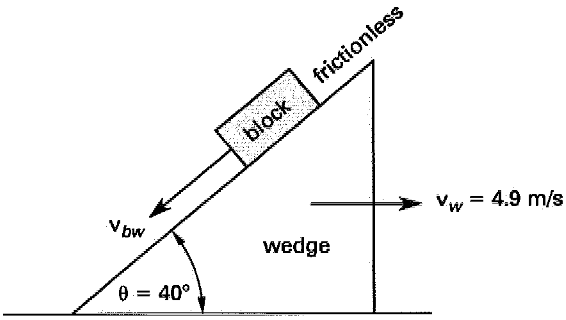
The minimum total distance is

$$x = (2)(180\ \text{m}) = 360\ \text{m}$$

The answer is (C).

DYNAMICS–18

A block with a mass of 150 kg slides down a frictionless wedge with a slope of 40°. The wedge is moving horizontally in the opposite direction at a constant velocity of 4.9 m/s. What is most nearly the absolute speed of the block 2 s after it is released from rest?



(A) 8.9 m/s (B) 9.4 m/s (C) 9.5 m/s (D) 9.8 m/s

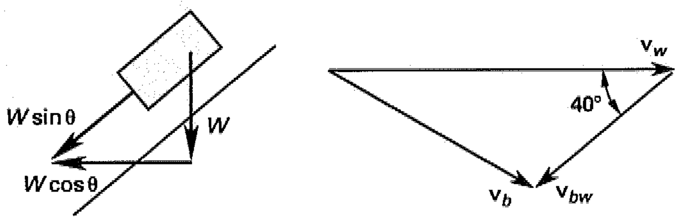
Let v_{bw} equal the velocity of the block relative to the wedge's slope. The component of gravitational force in this direction, F_{slope}, is $W\sin\theta$. Down the slope, relative to the wedge,

$$F_{slope} = W\sin\theta = ma_{slope}$$

$$a_{slope} = \frac{W\sin\theta}{m} = g\sin\theta$$

$$\begin{aligned} v_{bw} &= v_0 + a_{slope}t = 0 + gt\sin\theta \\ &= \left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(2\ \text{s})\sin 40^\circ \\ &= 12.6\ \text{m/s} \end{aligned}$$

The absolute velocity, v_b, can be found from a velocity triangle.



The law of cosines gives

$$v_b^2 = v_w^2 + v_{bw}^2 - 2v_w v_{bw}\cos\theta$$

$$= \left(4.9\ \frac{m}{s}\right)^2 + \left(12.6\ \frac{m}{s}\right)^2 - (2)\left(4.9\ \frac{m}{s}\right)\left(12.6\ \frac{m}{s}\right)\cos 40°$$

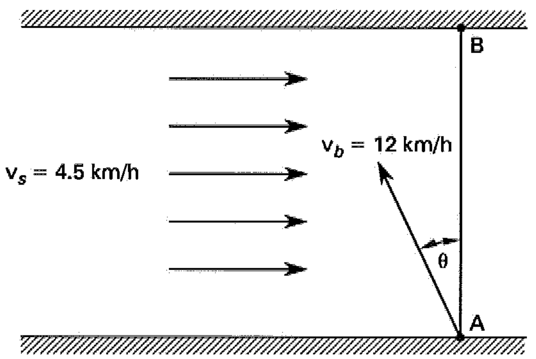
$$= 88.18\ m^2/s^2$$

$$v_b = 9.39\ m/s \quad (9.4\ m/s)$$

The answer is (B).

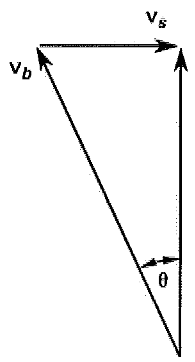
DYNAMICS-19

A stream flows at $v_s = 4.5$ km/h. At what angle, θ, upstream should a boat traveling at $v_b = 12$ km/h be launched in order to reach the shore directly opposite the launch point?



(A) 22° (B) 24° (C) 26° (D) 28°

Draw a velocity triangle.



$$\sin\theta = \frac{v_s}{v_b} = \frac{4.5\ \frac{\text{km}}{\text{h}}}{12\ \frac{\text{km}}{\text{h}}}$$

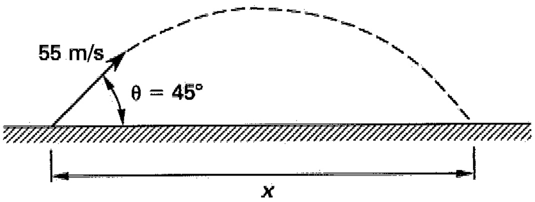
$$\theta = \sin^{-1}\left(\frac{4.5\ \frac{\text{km}}{\text{h}}}{12\ \frac{\text{km}}{\text{h}}}\right)$$

$$= 22°$$

The answer is (A).

DYNAMICS–20

An object is launched at 45° to the horizontal on level ground as shown. What is the range of the projectile if its initial velocity is 55 m/s? Neglect air resistance.



(A) 309 m (B) 617 m (C) 624 m (D) 680 m

Choosing the launch point as the origin of the x and y axes, $x_0 = y_0 = 0$. For uniform acceleration,

$$x = x_0 + v_{x_0}t + \tfrac{1}{2}a_x t^2$$
$$y = y_0 + v_{y_0}t + \tfrac{1}{2}a_y t^2$$

However, $a_x = 0\ \text{m/s}^2$, and $a_y = -9.81\ \text{m/s}^2$. Therefore,

$$x = \text{v}_{x_0} t$$

$$y = \text{v}_{y_0} t - \tfrac{1}{2} g t^2$$

$$\begin{aligned}\text{v}_{x_0} &= \text{v}_0 \cos\theta \\ &= \left(55\ \frac{\text{m}}{\text{s}}\right)\cos 45^\circ \\ &= 38.9\ \text{m/s}\end{aligned}$$

$$\begin{aligned}\text{v}_{y_0} &= \text{v}_0 \sin\theta \\ &= \left(55\ \frac{\text{m}}{\text{s}}\right)\sin 45^\circ \\ &= 38.9\ \text{m/s}\end{aligned}$$

$$y = 38.9\ \frac{\text{m}}{\text{s}} t - 4.9\ \frac{\text{m}}{\text{s}^2} t^2$$

When the projectile is on the ground, $y = 0$ m/s. Thus,

$$0 = t\left(38.9\ \frac{\text{m}}{\text{s}} - 4.9\ \frac{\text{m}}{\text{s}^2} t\right)$$

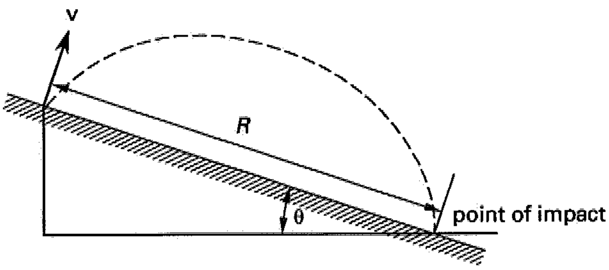
$$t = 7.94\ \text{s}$$

$$\begin{aligned}x &= \left(38.9\ \frac{\text{m}}{\text{s}}\right) t \\ &= \left(38.9\ \frac{\text{m}}{\text{s}}\right)(7.94\ \text{s}) \\ &= 308.9\ \text{m} \quad (309\ \text{m})\end{aligned}$$

The answer is (A).

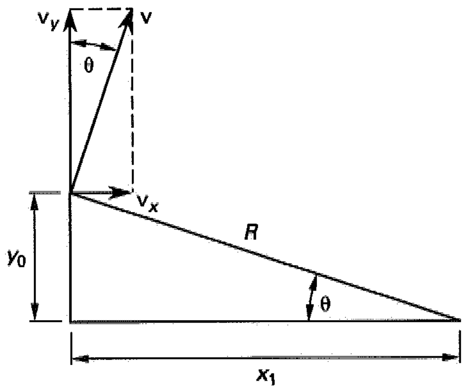
DYNAMICS–21

A projectile is fired with a velocity, v, perpendicular to a surface that is inclined at an angle, θ, with the horizontal. Determine the expression for the distance R to the point of impact.



(A) $R = \dfrac{2\text{v}^2 \sin\theta}{g\cos^2\theta}$ (B) $R = \dfrac{2\text{v}^2 \sin\theta}{g\cos\theta}$

(C) $R = \dfrac{2\text{v}\cos\theta}{g\sin\theta}$ (D) $R = \dfrac{2\text{v}\sin\theta}{g\cos\theta}$



Using the notation in the figure, $x_0 = 0$, $a_x = 0$, and $y_0 = R\sin\theta$. Therefore,

$$\begin{aligned} x &= x_0 + \text{v}_x t + \tfrac{1}{2}a_x t^2 \\ &= \text{v}_x t \\ &= \text{v}\sin\theta t \\ y &= y_0 + \text{v}_y t + \tfrac{1}{2}a_y t^2 \\ &= R\sin\theta + \text{v}_y t + \tfrac{1}{2}(-g)t^2 \\ &= R\sin\theta + \text{v}\cos\theta t - \tfrac{1}{2}gt^2 \end{aligned}$$

At impact, let $t = t_1$, $x_1 = R\cos\theta$, and $y_1 = 0$. The two equations above give

$$R\cos\theta = \mathrm{v}\sin\theta t_1$$

$$t_1 = \frac{R\cos\theta}{\mathrm{v}\sin\theta} \qquad \text{[I]}$$

$$0 = R\sin\theta + \mathrm{v}\cos\theta t_1 - \tfrac{1}{2}gt_1^2 \qquad \text{[II]}$$

Equations I and II give

$$0 = R\sin\theta + \mathrm{v}\cos\theta\left(\frac{R\cos\theta}{\mathrm{v}\sin\theta}\right) - \tfrac{1}{2}g\left(\frac{R\cos\theta}{\mathrm{v}\sin\theta}\right)^2$$

$$0 = \sin^2\theta + \cos^2\theta - \tfrac{1}{2}g\left(\frac{R\cos^2\theta}{\mathrm{v}^2\sin\theta}\right)$$

$$1 = \frac{g\cos^2\theta R}{2\mathrm{v}^2\sin\theta}$$

$$R = \frac{2\mathrm{v}^2\sin\theta}{g\cos^2\theta}$$

The answer is (A).

DYNAMICS–22

A cyclist on a circular track of radius $r = 240$ m is traveling at 8 m/s. His speed in the tangential direction (i.e., the direction of his travel) increases at the rate of 1 m/s^2. What is most nearly the cyclist's total acceleration?

(A) -0.9 m/s^2 (B) 0.7 m/s^2 (C) 0.9 m/s^2 (D) 1.0 m/s^2

The total acceleration is made up of tangential and normal components. The tangential component is given as 1 m/s^2. By definition, the normal component is

$$a_n = \frac{\mathrm{v}^2}{r} = \frac{\left(8\ \frac{\mathrm{m}}{\mathrm{s}}\right)^2}{240\ \mathrm{m}} = 0.27\ \mathrm{m/s^2}$$

$$a = \sqrt{a_n^2 + a_t^2}$$

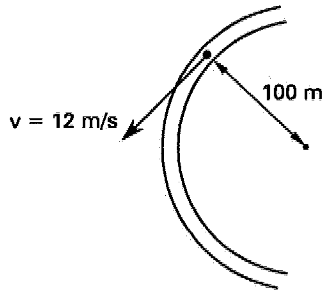
$$= \sqrt{\left(0.27\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)^2 + \left(1\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)^2}$$

$$= 1.04\ \mathrm{m/s^2} \quad (1.0\ \mathrm{m/s^2})$$

The answer is (D).

DYNAMICS–23

A motorcycle moves at a constant speed of v = 12 m/s around a curved road of radius $r = 100$ m. What is most nearly the magnitude and general direction of the motorcycle's acceleration?



(A) 1.1 m/s^2 away from the center of curvature
(B) 1.1 m/s^2 toward the center of curvature
(C) 1.4 m/s^2 away from the center of curvature
(D) 1.4 m/s^2 toward the center of curvature

The normal acceleration, a_n, is

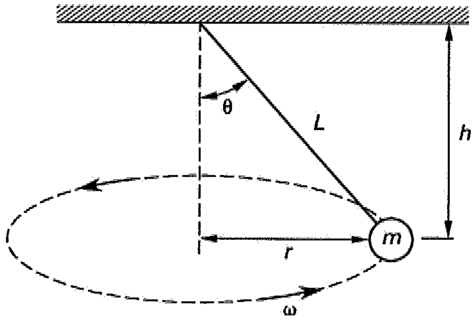
$$a_n = \frac{v^2}{r} = \frac{\left(12\ \frac{m}{s}\right)^2}{100\ m} = 1.44\ m/s^2 \quad (1.4\ m/s^2)$$

Since the velocity in the tangential direction is constant, $a_t = 0$. Thus, only the normal component of acceleration contributes to total acceleration, so $a = 1.44\ m/s^2$. The normal component is always directed toward the center of curvature.

The answer is (D).

DYNAMICS–24

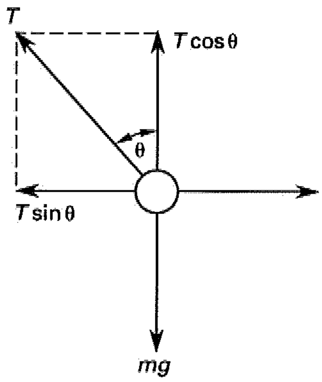
A pendulum of mass m and length L rotates about the vertical axis. If the angular velocity is ω, determine the expression for the height h.



(A) $h = \dfrac{g\cos\theta}{\omega^2}$ (B) $h = \dfrac{g}{\omega^2}$

(C) $h = \dfrac{g}{\omega^2\cos\theta}$ (D) $h = \dfrac{gL\cos\theta}{\omega^2}$

A free-body diagram of the pendulum is



Since the pendulum undergoes uniform circular motion,

$$T\sin\theta = ma_n = mr\omega^2 \quad \text{[I]}$$

Assuming the pendulum is not accelerating in the vertical direction, a force balance gives

$$T\cos\theta = mg \quad \text{[II]}$$

Combining equations I and II,

$$\tan\theta = \frac{r\omega^2}{g}$$

However, $\tan\theta = r/h$. Therefore,

$$h = \frac{r}{\tan\theta} = \frac{g}{\omega^2}$$

The answer is (B).

DYNAMICS–25

A 3 kg block is moving at a speed of 5 m/s. The force required to bring the block to a stop in 8×10^{-4} seconds is most nearly

(A) 10 kN (B) 13 kN (C) 15 kN (D) 19 kN

Newton's second law gives

$$F = ma = m\frac{d\mathrm{v}}{dt} = m\frac{\Delta \mathrm{v}}{\Delta t}$$

$$= (3\ \mathrm{kg})\left(\frac{5\ \frac{\mathrm{m}}{\mathrm{s}}}{8 \times 10^{-4}\ \mathrm{s}}\right)\left(\frac{1}{1000\ \frac{\mathrm{N}}{\mathrm{kN}}}\right)$$

$$= 18.75\ \mathrm{kN} \quad (19\ \mathrm{kN})$$

The answer is (D).

DYNAMICS–26

A rope is used to tow an 800 kg car with free-rolling wheels over a smooth, level road. The rope will break if the tension exceeds 2000 N. What is the greatest acceleration that the car can reach without breaking the rope?

(A) 1.2 m/s^2 (B) 2.5 m/s^2 (C) 3.8 m/s^2 (D) 4.5 m/s^2

$$F_{\mathrm{max}} = ma_{\mathrm{max}}$$

$$2000\ \mathrm{N} = (800\ \mathrm{kg})a_{\mathrm{max}}$$

Rearranging to solve for a_{max},

$$a_{max} = \frac{2000 \text{ N}}{800 \text{ kg}} = 2.5 \text{ m/s}^2$$

The answer is (B).

DYNAMICS–27

A force of 15 N acts on a 16 kg body for 2 s. If the body is initially at rest, how far is it displaced by the force?

(A) 1.1 m (B) 1.5 m (C) 1.9 m (D) 2.1 m

The acceleration is found using Newton's second law.

$$a = \frac{F}{m} = \frac{15 \text{ N}}{16 \text{ kg}} = 0.94 \text{ m/s}^2$$

For a body undergoing constant acceleration, with $v_0 = 0 \text{ m/s}^2$ and $t = 2$ s,

$$\Delta x = \tfrac{1}{2}at^2 = \left(\frac{1}{2}\right)\left(0.94 \ \frac{\text{m}}{\text{s}^2}\right)(2 \text{ s})^2$$
$$= 1.88 \text{ m}$$

The answer is (C).

DYNAMICS–28

A car of mass $m = 150$ kg accelerates in 10 s from rest at a constant rate to a speed of v = 6 m/s. What is the resultant force on the car due to this acceleration?

(A) 75 N (B) 90 N (C) 95 N (D) 98 N

For constant acceleration,

$$v = v_0 + at$$
$$a = \frac{v - v_0}{t} = \frac{6\ \frac{m}{s} - 0\ \frac{m}{s}}{10\ s}$$
$$= 0.6\ m/s^2$$
$$F = ma = (150\ kg)\left(0.6\ \frac{m}{s^2}\right) = 90\ N$$

The answer is (B).

DYNAMICS–29

A man weighs himself twice in an elevator. When the elevator is at rest, he weighs 824 N; when the elevator starts moving upward, he weighs 932 N. Most nearly how fast is the elevator accelerating, assuming constant acceleration?

(A) 0.64 m/s^2 (B) 1.1 m/s^2 (C) 1.3 m/s^2 (D) 9.8 m/s^2

The mass of the man can be determined from his weight at rest.

$$W = mg$$
$$m = \frac{W}{g} = \frac{824\ N}{9.81\ \frac{m}{s^2}}$$
$$= 84.0\ kg$$

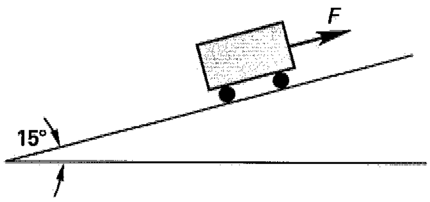
At constant acceleration,

$$F = ma$$
$$a = \frac{F}{m} = \frac{932\ N - 824\ N}{84.0\ kg}$$
$$= 1.29\ m/s^2 \quad (1.3\ m/s^2)$$

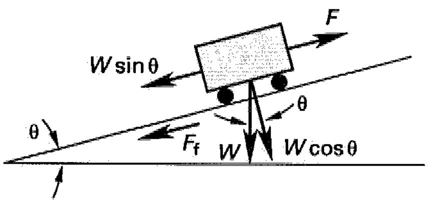
The answer is (C).

DYNAMICS–30

A truck weighing 1.4 kN moves up a slope of 15°. What is the force generated by the engine if the truck is accelerating at a rate of 3 $\mathrm{m/s^2}$? Assume the coefficient of friction is $\mu = 0.1$.



(A) 876 N (B) 926 N (C) 930 N (D) 958 N



In the direction parallel to the slope, a force balance gives

$$\sum F_x = ma = F - (W \sin\theta + F_f)$$

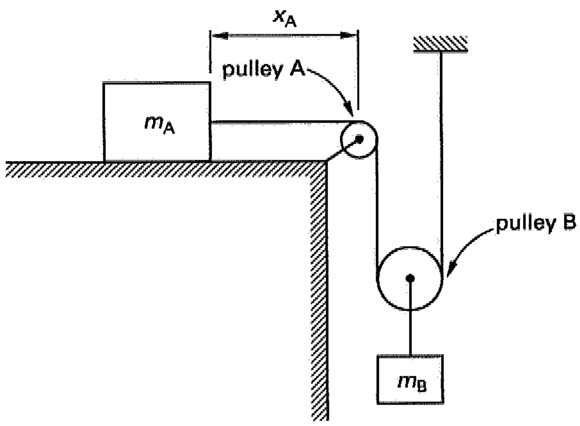
F_f is the friction force, which is equal to $\mu N = \mu W \cos\theta$.

$$\begin{aligned} F &= W(\sin\theta + \mu\cos\theta) + ma \\ &= (1400\ \mathrm{N})(\sin 15° + 0.1\cos 15°) + \left(\frac{1400\ \mathrm{N}}{9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}}\right)\left(3\ \frac{\mathrm{m}}{\mathrm{s}^2}\right) \\ &= 925.7\ \mathrm{N} \quad (926\ \mathrm{N}) \end{aligned}$$

The answer is (B).

DYNAMICS–31

In the illustration, the two pulleys and the horizontal surface are frictionless. The cord connecting the masses m_A and m_B is weightless. What is the ratio of the acceleration of mass A to the acceleration of mass B? Assume the system is released from rest.

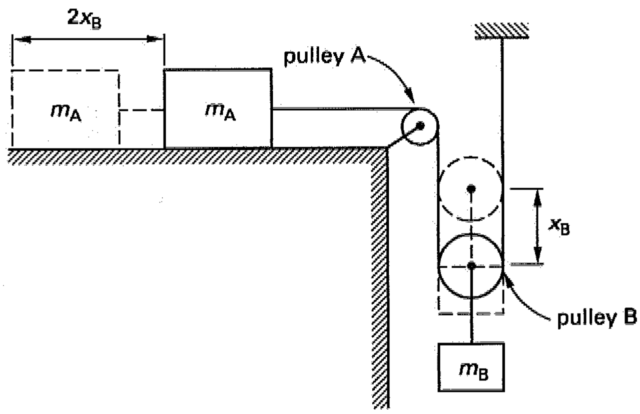


(A) 1/2 (B) 1 (C) 2 (D) m_A/m_B

Assuming the accelerations of both masses are constant, their respective displacement equations are

$$x_A = \tfrac{1}{2}a_A t^2$$

$$x_B = \tfrac{1}{2}a_B t^2$$



Taking the ratio of x_A to x_B,

$$\frac{x_A}{x_B} = \frac{a_A}{a_B}$$

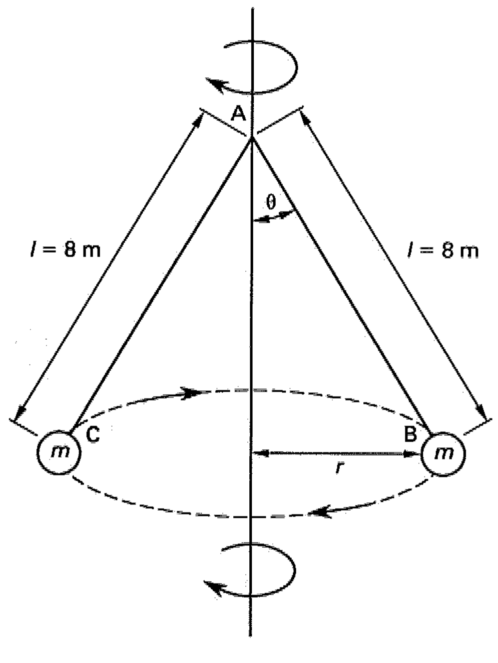
Since mass B is supported by a two-segment rope section and mass A is pulled by only one rope, the displacement of mass B is half the displacement of mass A. Therefore,

$$x_A = 2x_B$$
$$\frac{a_A}{a_B} = \frac{2x_B}{x_B} = 2$$

The answer is (C).

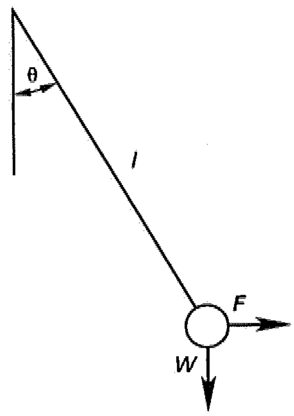
DYNAMICS–32

A simplified model of a carousel is illustrated. The 8 m long arms AB and AC attach the seats B and C, each with a mass of 200 kg, to a vertical rotating shaft. What is the maximum angle of tilt, θ, for the seats, if the carousel operates at 12 rpm?



(A) 39°
(B) 40°
(C) 45°
(D) 51°

The free-body diagram is



The angular velocity, ω, of the carousel is

$$\omega = \frac{\left(12\ \frac{\text{rev}}{\text{min}}\right)\left(2\pi\ \frac{\text{rad}}{\text{rev}}\right)}{60\ \frac{\text{s}}{\text{min}}}$$
$$= 1.257\ \text{rad/s}$$

The rotational force, F, expressed in terms of θ is

$$F = ma = mr\omega^2$$
$$= ml\sin\theta\omega^2$$
$$= (200\ \text{kg})(8\ \text{m})\sin\theta\left(1.257\ \frac{\text{rad}}{\text{s}}\right)^2$$
$$= (2528\ \text{N})\sin\theta$$

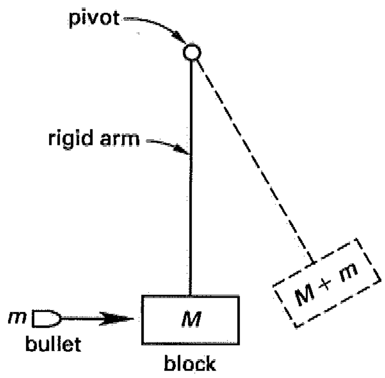
From the free-body diagram,

$$\tan\theta = \frac{F}{W} = \frac{(2528\ \text{N})\sin\theta}{(200\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$
$$= 1.288\sin\theta$$
$$\cos\theta = \frac{1}{1.288}$$
$$\theta = \cos^{-1}\left(\frac{1}{1.288}\right)$$
$$= 39.07^\circ \quad (39^\circ)$$

The answer is (A).

DYNAMICS–33

In the ballistic pendulum shown, a bullet of mass m is fired into a block of mass M that can swing freely. Which of the following is true for the system during the swing motion after impact?



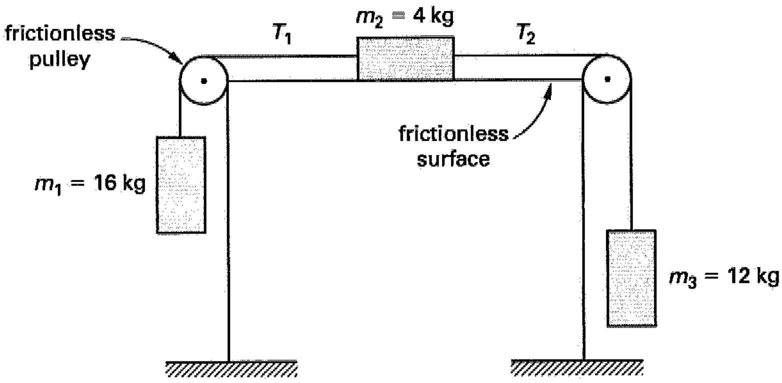
(A) Both mechanical energy and momentum are conserved.
(B) Mechanical energy is conserved; momentum is not conserved.
(C) Momentum is conserved; mechanical energy is not conserved.
(D) Neither mechanical energy nor momentum is conserved.

Momentum is not conserved since an external force, gravity, acts on the bullet-block mass. Only mechanical energy is conserved.

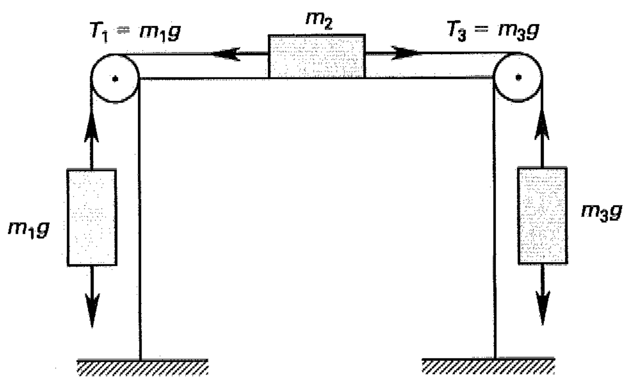
The answer is (B).

DYNAMICS–34

Three masses are attached by a weightless cord as shown. If mass m_2 is exactly halfway between the other masses and is located at the center of the flat surface when the masses are released, what is most nearly its initial acceleration? Assume there is no friction in the system and that the pulleys have no mass.



(A) 1.0 m/s^2 (B) 1.2 m/s^2 (C) 9.8 m/s^2 (D) 12 m/s^2



Since $m_1 > m_3$, m_1 will move downward and m_2 will be displaced to the left. All masses contribute to the inertia of the system.

$$T_1 - T_3 = a_2 m_{\text{total}} = a_2(m_1 + m_2 + m_3)$$

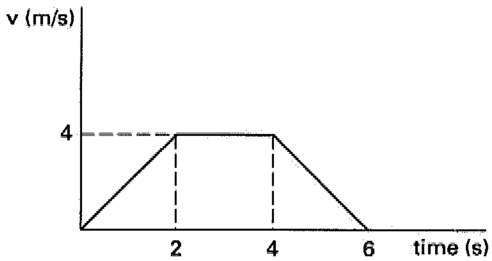
T_1 and T_3 are the tensions in the cord due to the masses m_1 and m_3.

$$T_1 = m_1 g$$
$$T_3 = m_3 g$$
$$a_2 = g\left(\frac{m_1 - m_3}{m_1 + m_2 + m_3}\right)$$
$$= \left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{16\ \text{kg} - 12\ \text{kg}}{16\ \text{kg} + 4\ \text{kg} + 12\ \text{kg}}\right)$$
$$= 1.23\ \text{m/s}^2 \quad (1.2\ \text{m/s}^2)$$

The answer is (B).

DYNAMICS–35

The maximum capacity (occupant load) of an elevator is 1000 N. The elevator starts from rest, and its velocity varies with time as shown in the graph. What is most nearly the maximum additional tension in the elevator cable due to the occupants at full capacity? Neglect the mass of the elevator.



(A) 960 N (B) 1000 N (C) 1200 N (D) 1400 N

The maximum tension occurs during the period of maximum acceleration. This occurs for $0\ \text{s} < t < 2\ \text{s}$, with acceleration, a, equal to $v/t = 4\ \text{m/s}/2\ \text{s} = 2\ \text{m/s}^2$. The mass of the occupants is $m = 1000\ \text{N}/9.81\ \text{m/s}^2$. During this time,

$$\sum F = ma = T - W$$
$$T = W + ma = 1000\ \text{N} + \left(\frac{1000\ \text{N}}{9.81\ \frac{\text{m}}{\text{s}^2}}\right)\left(2\ \frac{\text{m}}{\text{s}^2}\right)$$
$$= 1204\ \text{N} \quad (1200\ \text{N})$$

The answer is (C).

DYNAMICS–36

What is most nearly the kinetic energy of a 3924 N motorcycle traveling at 40 km/h?

(A) 11 100 J (B) 12 300 J (C) 23 600 J (D) 24 600 J

$$m = \frac{W}{g} = \frac{3924\ \text{N}}{9.81\ \frac{\text{m}}{\text{s}^2}} = 400\ \text{kg}$$

$$\text{v} = \left(40\ \frac{\text{km}}{\text{h}}\right)\left(\frac{1000\ \frac{\text{m}}{\text{km}}}{3600\ \frac{\text{s}}{\text{h}}}\right) = 11.1\ \text{m/s}$$

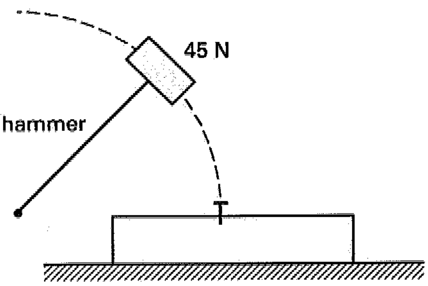
The kinetic energy is

$$E_k = \tfrac{1}{2}m\text{v}^2 = \left(\frac{1}{2}\right)(400\ \text{kg})\left(11.1\ \frac{\text{m}}{\text{s}}\right)^2$$
$$= 24\,640\ \text{J} \quad (24\,600\ \text{J})$$

The answer is (D).

DYNAMICS–37

A lead hammer weighs 45 N. In one swing of the hammer, a nail is driven 1.5 cm into a wood block. The velocity of the hammer's head at impact is 4.5 m/s. What is most nearly the average resistance of the wood block?



(A) 3090 N (B) 3100 N (C) 3920 N (D) 4090 N

Because energy is conserved, the kinetic energy of the hammer before impact is equal to the work done by the resistance force of the wood block. $m = 45\ \text{N}/9.81\ \text{m/s}^2 = 4.59\ \text{kg}$, $\text{v} = 4.5\ \text{m/s}$, and $x_{\text{nail}} = 0.015\ \text{m}$.

$$\tfrac{1}{2}m\text{v}^2 = Fx$$

$$F = \frac{\frac{1}{2}m\text{v}^2}{x} = \frac{\left(\frac{1}{2}\right)(4.59\ \text{kg})\left(4.5\ \frac{\text{m}}{\text{s}}\right)^2}{0.015\ \text{m}}$$

$$= 3098\ \text{N} \quad (3100\ \text{N})$$

The answer is (B).

DYNAMICS–38

An automobile uses 74.6 kW to maintain a uniform speed of 96 km/h. What is the thrust force provided by the engine?

(A) 0.87 kN (B) 2.8 kN (C) 3.2 kN (D) 5.6 kN

Power is defined as work done per unit time, which, for a linear system, is equivalent to force times velocity. Therefore,

$$P = F\text{v}$$

$$F = \frac{P}{\text{v}}$$

$$= \frac{(74.6\ \text{kW})\left(1000\ \frac{\text{W}}{\text{kW}}\right)}{\left(96\ \frac{\text{km}}{\text{h}}\right)\left(1000\ \frac{\text{m}}{\text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)}$$

$$= 2797.5\ \text{N} \quad (2.8\ \text{kN})$$

The answer is (B).

DYNAMICS–39

A 580 N man is standing on the top of a building 40 m above the ground. What is his potential energy relative to the ground?

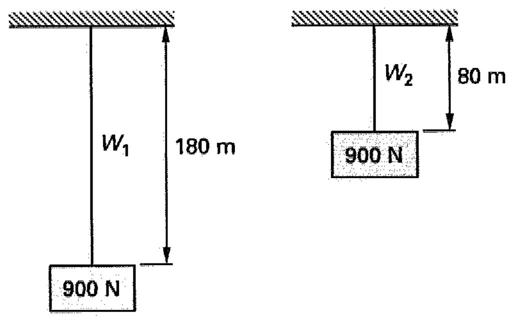
(A) 10 kJ (B) 12 kJ (C) 20 kJ (D) 23 kJ

$$E_p = Wy = (580\text{ N})(40\text{ m}) = 23\,200\text{ J}$$

The answer is (D).

DYNAMICS–40

A 900 N object is initially suspended on a 180 m long cable. The object is then raised 100 m. If the cable weighs 16 N/m, how much work is done?



(A) 100 000 J (B) 298 000 J (C) 320 000 J (D) 398 000 J

The weight of the extended cable for the two situations is

$$W_1 = (180\text{ m})\left(16\ \frac{\text{N}}{\text{m}}\right) = 2880\text{ N}$$

$$W_2 = (80\text{ m})\left(16\ \frac{\text{N}}{\text{m}}\right) = 1280\text{ N}$$

These weights may be considered to be concentrated at the midpoints of the extended cables. Choosing the datum to be at the top of the cable, and using the work-energy principle, the work done is equal to the difference in potential energies of the two situations.

$$E_p = (\text{weight})(\text{distance}) + W_1 L$$

$$E_{p1} = (900\ \text{N})(-180\ \text{m}) + (2880\ \text{N})\left(\frac{-180\ \text{m}}{2}\right) = -421\,200\ \text{J}$$

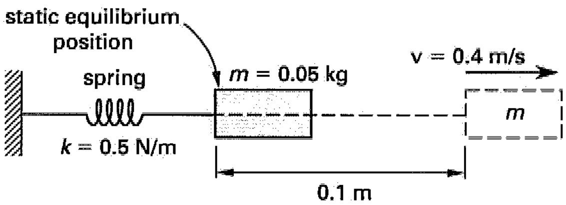
$$E_{p2} = (900\ \text{N})(-80\ \text{m}) + (1280\ \text{N})\left(\frac{-80\ \text{m}}{2}\right) = -123\,200\ \text{J}$$

$$\begin{aligned} W &= E_{p2} - E_{p1} = -123\,200\ \text{J} - (-421\,200\ \text{J}) \\ &= 298\,000\ \text{J} \end{aligned}$$

The answer is (B).

DYNAMICS–41

A 0.05 kg mass attached to a spring (spring constant, $k = 0.5$ N/m) is accelerated to a velocity of 0.4 m/s. What is the total energy for the body in the following diagram? Neglect the spring mass.



(A) 0.0025 J (B) 0.0040 J (C) 0.0065 J (D) 0.0092 J

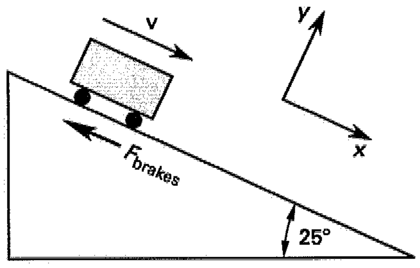
The total energy is the sum of the kinetic and potential energies.

$$\begin{aligned} E &= E_k + E_p \\ &= \tfrac{1}{2}m\text{v}^2 + \tfrac{1}{2}kx^2 \\ &= \left(\frac{1}{2}\right)(0.05\ \text{kg})\left(0.4\ \frac{\text{m}}{\text{s}}\right)^2 + \left(\frac{1}{2}\right)\left(0.5\ \frac{\text{N}}{\text{m}}\right)(0.1\ \text{m})^2 \\ &= 0.0065\ \text{J} \end{aligned}$$

The answer is (C).

DYNAMICS–42

A 1000 kg car is traveling down a 25° slope. At the instant that the speed is 13 m/s, the driver applies the brakes. What constant force parallel to the road must be generated by the brakes if the car is to stop in 90 m?



(A) 1290 N (B) 2900 N (C) 5080 N (D) 8630 N

The change in energy is equal to the work done by the brakes. The change in velocity squared is

$$v^2 - v_0^2 = 0 - \left(13\ \frac{\text{m}}{\text{s}}\right)^2 = -169\ \text{m}^2/\text{s}^2$$

The change in elevation of the car is

$$h - h_0 = 0 - (90\ \text{m})\sin 25° = -38\ \text{m}$$

$$\Delta E_k + \Delta E_p = Fx$$

$$\tfrac{1}{2}m(v^2 - v_0^2) + mg(h - h_0) = Fx$$

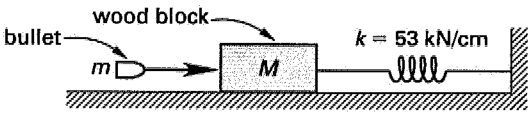
$$\left(\frac{1}{2}\right)(1000\ \text{kg})\left(-169\ \frac{\text{m}^2}{\text{s}^2}\right) + (1000\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(-38\ \text{m}) = F(90\ \text{m})$$

$$F = -5080\ \text{N}$$

The answer is (C).

DYNAMICS–43

A bullet of mass 100 g is fired at a wooden block resting on a horizontal surface. A spring with stiffness $k = 53$ kN/cm resists the motion of the block. If the maximum displacement of the block produced by the impact of the bullet is 3.4 cm, what is most nearly the velocity of the bullet at impact? Assume there are no losses at impact, and the spring has no mass.



(A) 250 km/h (B) 450 km/s (C) 630 km/h (D) 890 km/h

Due to the conservation of energy, the kinetic energy of the bullet before impact is equal to the potential energy of the spring-mass-bullet system at maximum compression.

$$E_{k,\text{bullet}} = E_{p,\text{system}}$$

$$\tfrac{1}{2}m_{\text{bullet}}\text{v}^2 = \tfrac{1}{2}kx^2$$

$$\text{v} = \sqrt{\frac{kx^2}{m_{\text{bullet}}}}$$

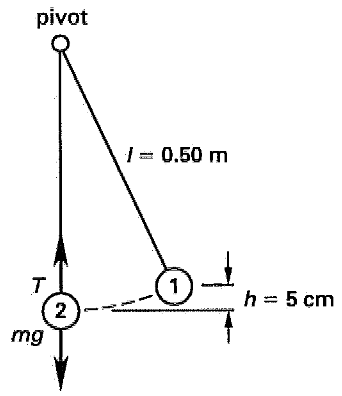
$$= \frac{\sqrt{\dfrac{\left(53\,000\ \dfrac{\text{N}}{\text{cm}}\right)(3.4\ \text{cm})^2\left(\dfrac{1\ \text{m}}{100\ \text{cm}}\right)}{0.1\ \text{kg}}}\left(60\ \dfrac{\text{s}}{\text{min}}\right)\left(60\ \dfrac{\text{min}}{\text{h}}\right)}{1000\ \dfrac{\text{m}}{\text{km}}}$$

$$= 891\ \text{km/h} \quad (890\ \text{km/h})$$

The answer is (D).

DYNAMICS–44

A simple pendulum consists of a 100 g mass attached to a weightless cord. If the mass is moved laterally such that $h = 5$ cm and then released, what is the maximum tension in the cord, T?



(A) 1.08 N (B) 1.12 N (C) 1.18 N (D) 1.25 N

The maximum tension will occur when the pendulum is at its lowest point, position 2 in the figure. The force balance in the vertical y direction gives

$$ma_y = T - mg$$
$$T = ma_y + mg$$
$$= \frac{m\mathrm{v}^2}{l} + mg \qquad \text{[I]}$$

From the conservation of energy,

$$E_{p1} = E_{k2}$$
$$mgh = \tfrac{1}{2}m\mathrm{v}^2$$
$$\mathrm{v} = \sqrt{2gh} \qquad \text{[II]}$$

Equations I and II give

$$T_{\max} = \frac{m\left(\sqrt{2gh}\right)^2}{l} + mg$$
$$= mg\left(\frac{2h + l}{l}\right)$$
$$= (100\ \mathrm{g})\left(9.81\ \frac{\mathrm{m}}{\mathrm{s}^2}\right)\left(\frac{(2)(5\ \mathrm{cm}) + 50\ \mathrm{cm}}{50\ \mathrm{cm}}\right)\left(\frac{1\ \mathrm{kg}}{1000\ \mathrm{g}}\right)$$
$$= 1.177\ \mathrm{kg{\cdot}m/s^2} \quad (1.18\ \mathrm{N})$$

The answer is (C).

DYNAMICS–45

A stationary passenger car of a train is set into motion by the impact of a moving locomotive. What is the impulse delivered to the car if it has a velocity of 11 m/s immediately after the collision? The weight of the car is 56.8 kN.

(A) 45.5 kN·s (B) 57.5 kN·s (C) 63.7 kN·s (D) 64.1 kN·s

From the impulse-momentum principle,

$$\text{Imp} = \Delta m\text{v}$$

$$m\text{v}_1 + \text{Imp} = m\text{v}_2$$

$$\text{Imp} = m(\text{v}_2 - \text{v}_1)$$

$$= \left(\frac{56.8 \text{ kN}}{9.81 \frac{\text{m}}{\text{s}^2}}\right)\left(11 \frac{\text{m}}{\text{s}} - 0 \frac{\text{m}}{\text{s}}\right)$$

$$= 63.7 \text{ kN·s}$$

The answer is (C).

DYNAMICS–46

Which of the following statements is FALSE?

(A) The time rate of change of the angular momentum about a fixed point is equal to the total moment of the external forces acting on the system about the point.

(B) The coefficient of restitution can be less than zero.

(C) The frictional force always acts to resist motion.

(D) Momentum is conserved during elastic collisions.

The coefficient of restitution is defined as the ratio of the impulses corresponding to the period of restitution and to the period of deformation of a body, respectively. Its value is always between 0 and 1.

The answer is (B).

DYNAMICS–47

Two identical balls hit head-on in a perfectly elastic collision. Given that the initial velocity of one ball is 0.85 m/s and the initial velocity of the other is −0.53 m/s, what is the relative velocity of each ball after the collision?

(A) 0.85 m/s and −0.53 m/s
(B) 1.2 m/s and −0.72 m/s
(C) 1.2 m/s and −5.1 m/s
(D) 1.8 m/s and −0.98 m/s

Let v_1 and v_2 be the velocities of balls 1 and 2, respectively, after the collision. The conservation of momentum equation is

$$\begin{aligned} m v_{01} + m v_{02} &= m v_1 + m v_2 \\ 0.85\ \frac{\text{m}}{\text{s}} + \left(-0.53\ \frac{\text{m}}{\text{s}}\right) &= v_1 + v_2 \\ v_1 + v_2 &= 0.32\ \text{m/s} \qquad \text{[I]} \end{aligned}$$

Since kinetic energy is conserved,

$$\begin{aligned} \tfrac{1}{2} m v_{01}^2 + \tfrac{1}{2} m v_{02}^2 &= \tfrac{1}{2} m v_1^2 + \tfrac{1}{2} m v_2^2 \\ \left(0.85\ \frac{\text{m}}{\text{s}}\right)^2 + \left(-0.53\ \frac{\text{m}}{\text{s}}\right)^2 &= v_1^2 + v_2^2 \\ v_1^2 + v_2^2 &= 1\ \text{m}^2/\text{s}^2 \qquad \text{[II]} \end{aligned}$$

Combining Eqs. I and II,

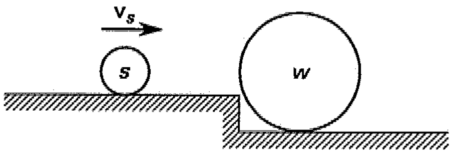
$$v_2^2 - 0.32 v_2 - 0.4488 = 0$$

$$\begin{aligned} v_2 &= \frac{0.32\ \frac{\text{m}}{\text{s}} \pm \sqrt{\left(-0.32\ \frac{\text{m}}{\text{s}}\right)^2 - (4)(1)\left(-0.4488\ \frac{\text{m}^2}{\text{s}^2}\right)}}{2} \\ &= 0.85\ \text{m/s or } -0.53\ \text{m/s} \quad \text{[negative value not used]} \\ v_1 &= 0.32 - v_2 = 0.32\ \frac{\text{m}}{\text{s}} - 0.85\ \frac{\text{m}}{\text{s}} \\ &= -0.53\ \text{m/s} \end{aligned}$$

The answer is (A).

DYNAMICS–48

A steel ball weighing 490 N strikes a stationary wooden ball weighing 490 N. If the steel ball has a velocity of 5.1 m/s at impact, what is its velocity immediately after impact? Assume the collision is central and perfectly elastic.



(A) −5 m/s (B) −2 m/s (C) 0 m/s (D) 5 m/s

Since the balls have the same weight, they have equal mass. Denoting the instances before and after the collision by the subscripts 1 and 2, respectively, $v_{s1} = 5.1$ m/s and $v_{w1} = 0$. Conservation of momentum gives

$$m_s v_{s1} + m_w v_{w1} = m_s v_{s2} + m_w v_{w2}$$
$$v_{s2} + v_{w2} = v_{s1} = 5.1 \text{ m/s} \qquad \text{[I]}$$

Conservation of energy gives

$$\tfrac{1}{2}m_s v_{s1}^2 + \tfrac{1}{2}m_w v_{w1}^2 = \tfrac{1}{2}m_s v_{s2}^2 + \tfrac{1}{2}m_w v_{w2}^2$$
$$v_{s2}^2 + v_{w2}^2 = v_{s1}^2 = \left(5.1\ \frac{\text{m}}{\text{s}}\right)^2$$
$$= 26.01\ \text{m}^2/\text{s}^2 \qquad \text{[II]}$$

Solving Eqs. I and II simultaneously,

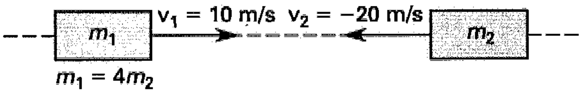
$$v_{s2}^2 + \left(26.01\ \frac{\text{m}^2}{\text{s}^2} - 10.2v_{s2} + v_{s2}^2\right) = 26.01\ \text{m}^2/\text{s}^2$$
$$2v_{s2}^2 - 10.2v_{s2} = 0$$
$$v_{s2}^2 - 5.1\ \frac{\text{m}}{\text{s}}v_{s2} = 0\ \frac{\text{m}}{\text{s}^2}$$
$$v_{s2} = 0\ \text{m/s},\ 5.1\ \text{m/s}$$

If $v_{s2} = 5.1$ m/s, then $v_{w2} = 0$ m/s, and no change has occurred during the collision. This is physically impossible, so $v_{s2} = 0$ m/s.

The answer is (C).

DYNAMICS–49

Two masses collide in a perfectly inelastic collision. Given the data in the illustration, find the velocity and direction of motion of the resulting combined mass.



(A) The mass is stationary.
(B) 4 m/s to the right
(C) 5 m/s to the left
(D) 10 m/s to the right

Let the positive direction of motion be to the right. Let m_3 be the resultant combined mass moving at velocity v_3 after the collision. Since momentum is conserved,

$$m_1 v_1 + m_2 v_2 = m_3 v_3$$

However, $m_3 = m_1 + m_2 = 4m_2 + m_2 = 5m_2$. Therefore,

$$4m_2\left(10\ \frac{\text{m}}{\text{s}}\right) + m_2\left(-20\ \frac{\text{m}}{\text{s}}\right) = 5m_2 v_3$$

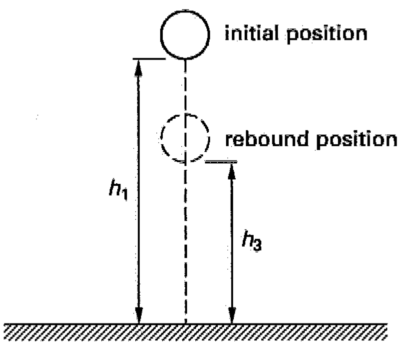
$$40m_2 - 20m_2 = 5m_2 v_3$$

$$v_3 = 4\ \text{m/s to the right}$$

The answer is (B).

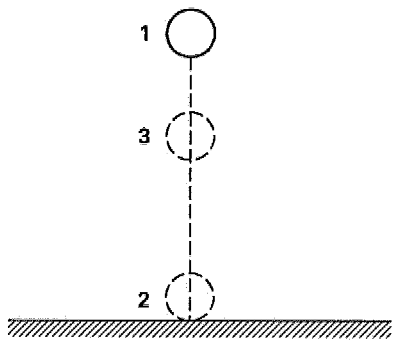
DYNAMICS–50

A ball is dropped onto a solid floor from an initial height, h_0. If the coefficient of restitution, e, is 0.90, how high will the ball rebound?



(A) $0.45h_1$ (B) $0.81h_1$ (C) $0.85h_1$ (D) $0.90h_1$

The subscripts 1, 2, and 3 denote the positions shown.



Conservation of energy gives, before impact,

$$E_{1,\text{total}} = E_{2,\text{total}}$$

Since the kinetic energy at position 1 and the potential energy at position 2 are zero,

$$mgh_1 = \tfrac{1}{2}m\text{v}_2^2$$
$$\text{v}_2 = \sqrt{2gh_1}$$

After impact, the kinetic energy at position 3 is zero.

$$\tfrac{1}{2}m\text{v}_2^2 = mgh_3$$
$$\text{v}_2 = \sqrt{2gh_3}$$

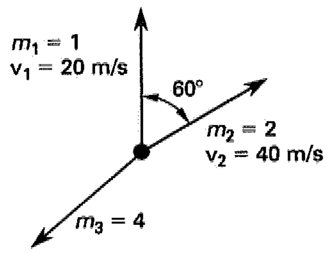
By definition, the coefficient of restitution is

$$e = \frac{\text{v}_{\text{ball}} - \text{v}_{\text{floor}}}{\text{v}_{1,\text{floor}} - \text{v}_{1,\text{ball}}} = -\frac{\text{v}}{\text{v}_1}$$
$$= -\frac{\sqrt{2gh_3}}{\sqrt{2gh_1}} = -\sqrt{\frac{h_3}{h_1}}$$
$$h_3 = e^2h_1 = (0.9)^2h_1$$
$$= 0.81h_1$$

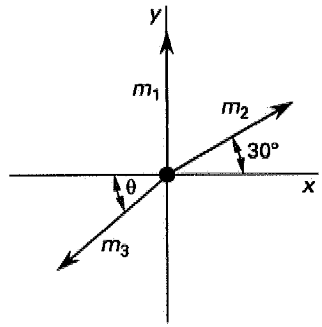
The answer is (B).

DYNAMICS–51

A mass suspended in space explodes into three pieces whose masses, initial velocities, and directions are given in the illustration. All motion is within a single plane. Find the velocity of m_3.



(A) 20 m/s (B) 23 m/s (C) 35 m/s (D) 40 m/s



Defining the x and y axes as shown, conservation of momentum for the x direction gives

$$m_2 v_2 \cos 30^\circ + m_3 v_3 \cos\theta = 0$$

$$2v_2 \cos 30^\circ + 4v_3 \cos\theta = 0$$

$$(2)\left(40\ \frac{\text{m}}{\text{s}}\right)\cos 30^\circ = -4v_3\cos\theta$$

$$20\ \frac{\text{m}}{\text{s}}\cos 30^\circ = -v_3\cos\theta$$

$$v_3 = -\frac{17.32\ \frac{\text{m}}{\text{s}}}{\cos\theta} \qquad \text{[I]}$$

For the y direction,

$$m_1 \mathrm{v}_1 + m_2 \mathrm{v}_2 \sin 30^\circ + m_3 \mathrm{v}_3 \sin\theta = 0$$

$$m_1 \left(20\ \frac{\mathrm{m}}{\mathrm{s}}\right) + 2m_1 \left(40\ \frac{\mathrm{m}}{\mathrm{s}}\right) \sin 30^\circ = -4m_1 \mathrm{v}_3 \sin\theta$$

$$-4\mathrm{v}_3 \sin\theta = 60\ \frac{\mathrm{m}}{\mathrm{s}}$$

$$\mathrm{v}_3 = -\frac{60\ \frac{\mathrm{m}}{\mathrm{s}}}{4\sin\theta} \qquad \text{[II]}$$

Equations I and II give

$$\tan\theta = \frac{60\ \frac{\mathrm{m}}{\mathrm{s}}}{\left(17.32\ \frac{\mathrm{m}}{\mathrm{s}}\right)(4)}$$

$$\theta = 40.9^\circ$$

$$\mathrm{v}_3 = \frac{-17.32\ \frac{\mathrm{m}}{\mathrm{s}}}{-\cos 40.9^\circ} = 22.9\ \mathrm{m/s}$$

The answer is (B).

DYNAMICS–52

Which of the following statements is FALSE?

(A) Kinematics is the study of the effects of motion, while kinetics is the study of the causes of motion.

(B) The radius of gyration for a mass of uniform thickness is identical to the radius of gyration for a planar area of the same shape.

(C) Angular momentum for rigid bodies may be regarded as the product of angular velocity and inertia.

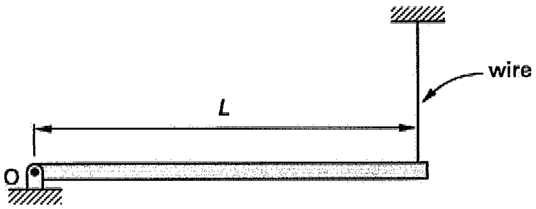
(D) The acceleration of any point within a homogenous body rotating with a constant angular velocity is proportional to the distance of that point to the center of mass.

A body rotating at a constant angular velocity has no angular acceleration.

The answer is (D).

DYNAMICS–53

A uniform beam of weight W is supported by a pin joint and a wire. What will be the angular acceleration, α, at the instant that the wire is cut?



(A) $\frac{g}{L}$ (B) $\frac{3g}{2L}$ (C) $\frac{2g}{L}$ (D) $\frac{Wg}{L}$

The only force on the beam is its weight acting at a distance of $L/2$ from point O. Taking the moment about O,

$$W\left(\frac{L}{2}\right) = I_O\alpha$$

For a slender beam rotating about its end,

$$I_O = \frac{1}{3}mL^2$$

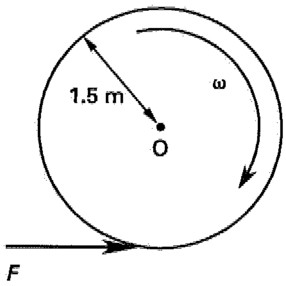
$$\alpha = \frac{WL}{2I_O} = \frac{WL}{2\left(\frac{1}{3}mL^2\right)}$$

$$= \frac{3g}{2L}$$

The answer is (B).

DYNAMICS–54

A thin circular disk of mass 25 kg and radius 1.5 m is spinning about its axis with an angular velocity of $\omega = 1800$ rpm. It takes 2.5 min to stop the motion by applying a constant force, F, to the edge of the disk. The force required is most nearly



(A) 7.2 N (B) 16 N (C) 24 N (D) 32 N

The relationship between the retarding moment, Fr, and the deceleration is

$$Fr = -I_O\alpha$$

Designating the positive rotational direction as counterclockwise, $\omega = -1800$ rpm. Therefore,

$$\begin{aligned} F &= -\frac{I_O\alpha}{r} = -\frac{\frac{1}{2}mr^2\alpha}{r} \\ &= -\tfrac{1}{2}mr\frac{\Delta\omega}{\Delta t} \\ &= \left(-\frac{1}{2}\right)(25\text{ kg})(1.5\text{ m})\left(\left(\frac{-1800\ \frac{\text{rev}}{\text{min}}}{2.5\text{ min}}\right)(2\pi)\left(\frac{1\text{ min}^2}{3600\text{ s}^2}\right)\right) \\ &= 23.6\text{ N} \quad (24\text{ N}) \end{aligned}$$

The answer is (C).

DYNAMICS–55

A mass, m, of 0.025 kg is hanging from a spring whose spring constant, k, is 0.44 N/m. If the mass is pulled down and released, what is the period of oscillation?

(A) 0.50 s (B) 1.2 s (C) 1.5 s (D) 2.1 s

By definition, the period T is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.025\ \text{kg}}{0.44\ \frac{\text{N}}{\text{m}}}}$$
$$= 1.5\ \text{s}$$

The answer is (C).

DYNAMICS–56

A body hangs from an ideal spring. What is the frequency of oscillation of the body if its mass, m, is 0.015 kg, and k is 0.5 N/m?

(A) 0.51 Hz (B) 0.66 Hz (C) 0.78 Hz (D) 0.92 Hz

By definition, the frequency, f, is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{0.5\ \frac{\text{N}}{\text{m}}}{0.015\ \text{kg}}}$$
$$= 0.92\ \text{Hz}$$

The answer is (D).

DYNAMICS–57

What is the natural frequency, ω, of an oscillating body whose period of oscillation is 1.8 s?

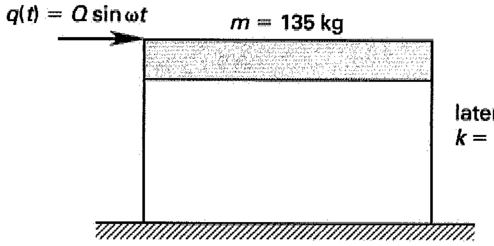
(A) 1.8 rad/s (B) 2.7 rad/s (C) 3.5 rad/s (D) 4.2 rad/s

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1.8\ \text{s}}$$
$$= 3.5\ \text{rad/s}$$

The answer is (C).

DYNAMICS–58

A one-story frame is subjected to a sinusoidal forcing function $q(t) = Q \sin \omega t$ at the transom. What is most nearly the frequency of $q(t)$, in hertz, if the frame is in resonance with the force?



(A) 2.6 Hz (B) 2.9 Hz (C) 3.6 Hz (D) 9.7 Hz

Resonance occurs when the forced frequency, ω, equals the natural frequency, ω_n.

$$m = 135 \text{ kg}$$

$$k = \left(5000 \ \frac{\text{N}}{\text{cm}}\right)\left(100 \ \frac{\text{cm}}{\text{m}}\right) = 500\,000 \text{ N/m}$$

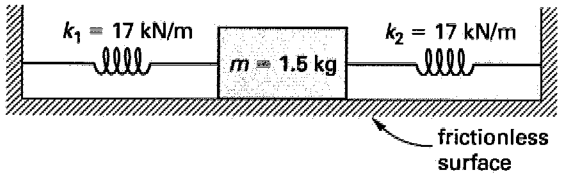
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{500\,000 \ \frac{\text{N}}{\text{m}}}{135 \text{ kg}}}$$

$$= 9.69 \text{ Hz} \quad (9.7 \text{ Hz})$$

The answer is (D).

DYNAMICS–59

In the mass-spring system shown, the mass, m, is displaced 0.09 m to the right of the equilibrium position and then released. Find the maximum velocity of m.



(A) 0.3 m/s (B) 5 m/s (C) 8 m/s (D) 14 m/s

The kinetic energy before the mass is released is zero. The maximum velocity will occur when the mass returns to the point of static equilibrium, where the deflection is zero and, hence, the potential energy equals zero. Therefore, since the total energy of the system is constant,

$$E_{p,1} = E_{k,2}$$

$$\tfrac{1}{2}k_1x_1^2 + \tfrac{1}{2}k_2x_2^2 = \tfrac{1}{2}m\mathrm{v}^2$$

The displacement of each spring is

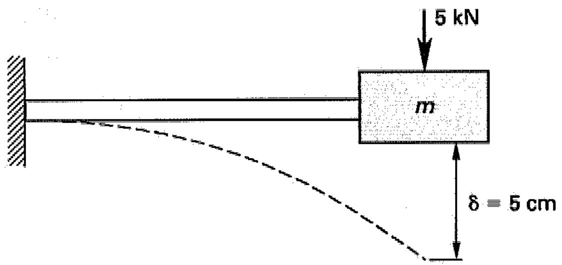
$$x = 0.09 \text{ m}$$

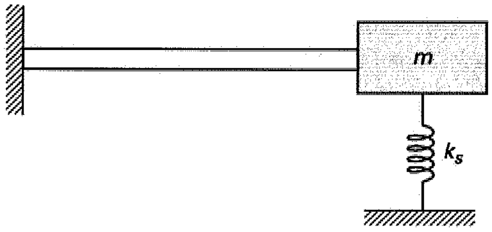
$$\begin{aligned} \mathrm{v} &= \sqrt{\frac{k_1x_1^2 + k_2x_2^2}{m}} \\ &= \sqrt{\frac{\left(\left(17\ \frac{\text{kN}}{\text{m}}\right)(0.09\text{ m})^2 + \left(17\ \frac{\text{kN}}{\text{m}}\right)(0.09\text{ m})^2\right)\left(1000\ \frac{\text{N}}{\text{kN}}\right)}{1.5\text{ kg}}} \\ &= 13.5 \text{ m/s} \quad (14 \text{ m/s}) \end{aligned}$$

The answer is (D).

DYNAMICS–60

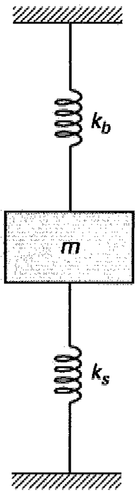
A cantilever beam with an end mass, $m = 7000$ kg, deflects 5 cm when a force of 5 kN is applied at the end. The beam is subsequently mounted on a spring of stiffness, $k_s = 1.5$ kN/cm. What is most nearly the natural frequency of the mass-beam-spring system?





(A) 1.5 rad/s (B) 3.1 rad/s (C) 6.0 rad/s (D) 6.3 rad/s

A cantilever with an end mass m can be modeled as follows.



$$k_b = \frac{5000 \text{ N}}{5 \text{ cm}} = 1000 \text{ N/cm}$$

For this model, both springs undergo the same deflection. Hence,

$$\begin{aligned} k &= k_b + k_s = 1000 \ \frac{\text{N}}{\text{cm}} + 1500 \ \frac{\text{N}}{\text{cm}} \\ &= 2500 \text{ N/cm} \end{aligned}$$

The natural frequency is, therefore,

$$\begin{aligned} \omega &= \sqrt{\frac{k}{m}} = \sqrt{\frac{\left(2500 \ \frac{\text{N}}{\text{cm}}\right)\left(100 \ \frac{\text{cm}}{\text{m}}\right)}{7000 \text{ kg}}} \\ &= 5.98 \text{ rad/s} \quad (6.0 \text{ rad/s}) \end{aligned}$$

The answer is (C).

12 DC ELECTRICITY

DC ELECTRICITY–1

Which statement about a charge placed on a dielectric material is true?

(A) The charge diffuses across the material's surface.
(B) The charge diffuses through the interior of the material.
(C) The charge is confined to the region in which the charge was placed.
(D) The charge increases the conductivity of the material.

In a dielectric, all charges are attached to specific atoms or molecules.

The answer is (C).

DC ELECTRICITY–2

The coulomb force, F, acts on two charges a distance, r, apart. What is F proportional to?

(A) r (B) r^2 (C) $\frac{1}{r^2}$ (D) $\frac{1}{r^3}$

The coulomb force is

$$F = \frac{q_1 q_2}{4\pi\epsilon r^2}$$

q_1 and q_2 are the charges, and ϵ is the permittivity of the surrounding medium. Hence, F is proportional to the inverse of r^2.

The answer is (C).

DC ELECTRICITY–3

The force between two electrons in a vacuum is 1×10^{-15} N. Approximately how far apart are the electrons?

(A) 1.4×10^{-12} m (B) 5.1×10^{-12} m (C) 4.8×10^{-7} m (D) 1.7×10^{-6} m

Coulomb's law is

$$F = \frac{q_1 q_2}{4\pi\epsilon_o r^2}$$

$\epsilon_o = 8.85 \times 10^{-12}\ \mathrm{C^2/N{\cdot}m^2}$. Also, for an electron, $q = 1.6 \times 10^{-19}$ C. Solving for r,

$$\begin{aligned} r &= q\sqrt{\frac{1}{4\pi\epsilon_o F}} \\ &= (1.6 \times 10^{-19}\ \mathrm{C})\sqrt{\frac{1}{4\pi\left(8.85 \times 10^{-12}\ \frac{\mathrm{C^2}}{\mathrm{N{\cdot}m^2}}\right)(1 \times 10^{-15}\ \mathrm{N})}} \\ &= 4.8 \times 10^{-7}\mathrm{m} \end{aligned}$$

The answer is (C).

DC ELECTRICITY–4

Two solid spheres have charges of 1 C and −8 C, respectively. The permittivity, ϵ_o, is $8.85 \times 10^{-12}\ \mathrm{C^2/N{\cdot}m^2}$, and the distance between the sphere centers, r, is 0.3 m. Determine the force on the spheres.

(A) -1×10^{13} N (B) -8×10^{11} N (C) 0 N (D) 8×10^{11} N

Because of their symmetry, charged spheres may be treated as point charges. Use Coulomb's law.

$$\begin{aligned} F = \frac{q_1 q_2}{4\pi\epsilon_o r^2} &= \frac{(1\ \mathrm{C})(-8\ \mathrm{C})}{4\pi\left(8.85 \times 10^{-12}\ \frac{\mathrm{C^2}}{\mathrm{N{\cdot}m^2}}\right)(0.3\ \mathrm{m})^2} \\ &= -8 \times 10^{11}\ \mathrm{N} \end{aligned}$$

The answer is (B).

DC ELECTRICITY–5

A parallel plate capacitor with plates of area A that are separated a distance d by air is initially charged with charge q_c. The energy stored in the capacitor initially is E. The plates are then separated by $2d$. What is the new energy stored in the capacitor?

(A) 0 (B) $0.5E$ (C) E (D) $2E$

The energy initially stored in the capacitor is

$$E = \frac{q_c^2}{2C}$$

C is the initial capacitance. After the increase in plate separation, the capacitance, C', is

$$C' = \frac{\varepsilon_0 A}{2d} = \tfrac{1}{2}C$$

Therefore the energy stored, E', after the plate distance is increased is

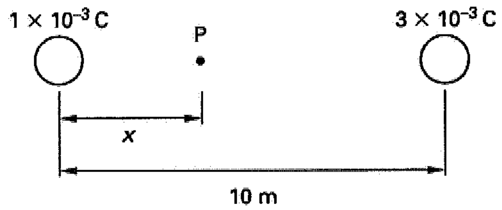
$$E' = \frac{q_c^2}{2C'} = \frac{q_c^2}{(2)\left(\frac{1}{2}C\right)} = 2E$$

The increased energy is added into the system when force is used to separate the plates against the electrostatic force between them.

The answer is (D).

DC ELECTRICITY–6

A 0.001 C charge is separated from a 0.003 C charge by 10 m. If P denotes the point of zero electric field between the charges, determine the distance, x, between the 0.001 C charge and point P.



(A) 2.2 m (B) 3.7 m (C) 6.3 m (D) 14 m

Electric field intensity E at point 2 due to a point charge, Q, at point 1 is

$$E = \frac{Q_1}{4\pi\epsilon r^2}$$

r is the distance between points 1 and 2.

At the point where E is zero, the electric field due to the 0.001 C charge equals the field due to the 0.003 C charge in magnitude.

$$\frac{0.001\text{ C}}{4\pi\epsilon_o x^2} = \frac{0.003\text{ C}}{4\pi\epsilon_o(10\text{ m} - x)^2}$$

$$(10\text{ m} - x)^2 = 3x^2$$

$$x^2 + 10x - 50 = 0$$

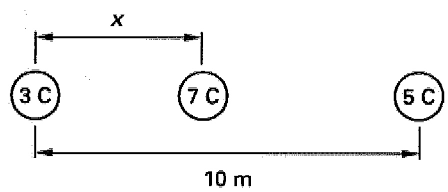
Solving for the positive x value,

$$\begin{aligned} x &= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-10\text{ m} + \sqrt{(10\text{ m})^2 - (4)(1\text{ m})(-50\text{ m})}}{2} \\ &= 3.66\text{ m} \quad (3.7\text{ m}) \end{aligned}$$

The answer is (B).

DC ELECTRICITY–7

A 3 C charge and a 5 C charge are 10 m apart. A 7 C charge is placed on a line connecting the two charges, x meters away from the 3 C charge. If the 7 C charge is in equilibrium, find the value of x.



(A) 3.9 m (B) 4.4 m (C) 5.0 m (D) 5.7 m

At equilibrium, $F_{37} = F_{75}$. Using Coulomb's law,

$$\frac{(3\text{ C})(7\text{ C})}{4\pi\epsilon_o x^2} = \frac{(7\text{ C})(5\text{ C})}{4\pi\epsilon_o(10\text{ m} - x)^2}$$
$$(21\text{ C}^2)(10\text{ m} - x)^2 = 35x^2$$
$$x^2 + 30x - 150 = 0$$

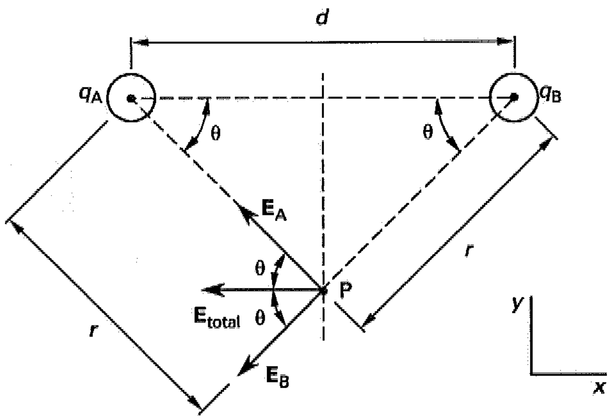
Solving for a positive value of x,

$$\begin{aligned} x &= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-30\text{ m} + \sqrt{(30\text{ m})^2 - (4)(1\text{ m})(-150\text{ m})}}{2} \\ &= 4.36\text{ m} \quad (4.4\text{ m}) \end{aligned}$$

The answer is (B).

DC ELECTRICITY-8

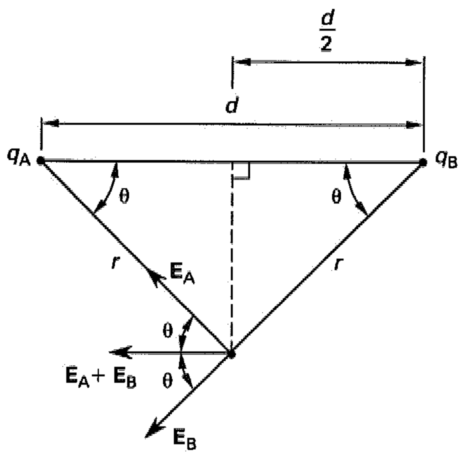
Two charges, A and B, of equal and opposite value are separated by a distance, d. r is the distance from a charge to any point, P, lying on the normal plane that bisects the length d. What is the electric field at point P if K is a constant equal to $1/4\pi\epsilon$?



(A) $\frac{Kqd}{r^3}$ (B) $\frac{Kq^2d}{r^3}$ (C) $\frac{Kq}{r^2}$ (D) $\frac{2Kq}{r^2}$

The total electric field will be in the x direction only, since the y components of the charges cancel each other out. By definition, with $\mathbf{a_r}$ denoting the unit radial vector,

$$\mathbf{E}_{\text{total}} = \left(\frac{Kq}{r^2}\right)\mathbf{a_r}$$



Therefore,

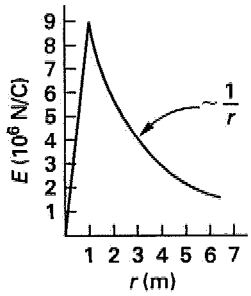
$$\begin{aligned} E_{\text{total}} = E_A + E_B &= \left(\frac{Kq_A}{r^2}\right)\cos\theta + \left(\frac{Kq_B}{r^2}\right)\cos\theta \\ &= \left(\frac{2Kq}{r^2}\right)\left(\frac{\frac{1}{2}d}{r}\right) \\ &= \frac{Kqd}{r^3} \end{aligned}$$

The answer is (A).

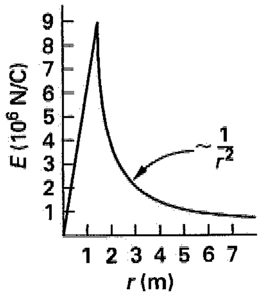
DC ELECTRICITY–9

A hollow metallic spherical shell has a charge of 0.001 C. The shell is 2 m in diameter. Which of the following correctly shows the variation of electric field with respect to the distance, r, from the center of the sphere?

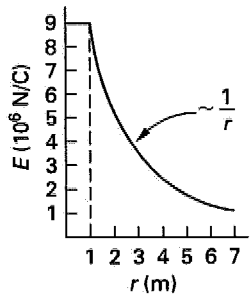
(A)



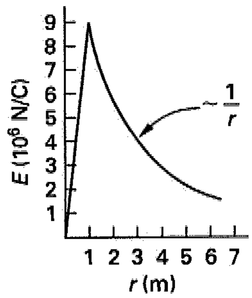
(B)



(C)



(D)



Outside the sphere, Coulomb's law can be used to find the electric field. Thus, the electric field varies as $1/r^2$ for $r > 1$ m. On the surface of the sphere, $r = 1$ m.

$$E = \frac{q}{4\pi\epsilon_o r^2} = \frac{0.001\ \text{C}}{4\pi\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(1\ \text{m})^2} = 9\times10^6\ \text{N/C}$$

Gauss' law states that the electric flux passing through a given closed surface is proportional to the charge enclosed by the surface. There is no charge within the sphere. Therefore, the electric field is zero for $r < 1$ m. Only (D) is correct.

The answer is (D).

DC ELECTRICITY–10

Approximately how far away must an isolated positive point charge of 1×10^{-8} C be in order for it to produce an electric potential of 100 V? The charge is in free space with $\epsilon_o = 8.85 \times 10^{-12}\ \mathrm{C^2/N{\cdot}m^2}$.

(A) 0.90 m (B) 1.2 m (C) 5.3 m (D) 8.6 m

At a distance, r, from a point charge, q,

$$V = -\int E\,dr\,\frac{q}{4\pi\epsilon r}$$

$$r = \frac{q}{4\pi\epsilon V}$$

$$= \frac{1 \times 10^{-8}\ \mathrm{C}}{4\pi\left(8.85 \times 10^{-12}\ \dfrac{\mathrm{C^2}}{\mathrm{N{\cdot}m^2}}\right)(100\ \mathrm{V})}$$

$$= 0.90\ \mathrm{m}$$

The answer is (A).

DC ELECTRICITY–11

A point charge, q, in a vacuum creates a potential, V, at a distance, r. A reference voltage of zero is arbitrarily selected when $r = a$. If $K = 1/4\pi\epsilon_o$, which of the following is the correct expression for V?

(A) $Kq\left(\frac{1}{r^2} - \frac{1}{a^2}\right)$ (B) $Kq\frac{1-a}{r^2}$ (C) $Kq\left(\frac{1}{r} - \frac{1}{a}\right)$ (D) $Kq\left(\frac{1}{r^3} - \frac{1}{a^3}\right)$

From Coulomb's law for a point charge,

$$E = \frac{Kq}{r^2}$$

The total voltage is measured between the reference voltage, a, and r.

$$V = -\int E\,dr = -\int_a^r \frac{Kq}{r^2}dr$$

$$= Kq\left(\frac{1}{r} - \frac{1}{a}\right)$$

The answer is (C).

DC ELECTRICITY–12

What accelerating voltage is required to accelerate an electron to a kinetic energy of 5×10^{-15} J? The charge of an electron is 1.6×10^{-19} C.

(A) 8 kV (B) 13 kV (C) 19 kV (D) 31 kV

For an electron, after the potential energy has been converted to kinetic energy, kinetic energy is

$$E_k = qV$$

$$V = \frac{E_k}{q} = \frac{5 \times 10^{-15}\ \text{J}}{1.6 \times 10^{-19}\ \text{C}}$$

$$= 31\,250\ \text{V} \quad (31\ \text{kV})$$

The answer is (D).

DC ELECTRICITY–13

A certain potential variation in the xy plane is given by the expression

$$\nabla V = \left(\frac{1}{\sqrt{x^2 + 4y^2}}\right)(\mathbf{i} + \mathbf{j})$$

Which of the following gives the magnitude and direction (angle made with the x-axis) of the electric field intensity at the point (2,1)?

(A) $-\sqrt{2}/4,\ \pi$ (B) $1/2,\ -\pi/4$ (C) $\sqrt{2}/2,\ \pi/4$ (D) $1/2,\ \pi/4$

$$V = \int E\, dr$$

$$\mathbf{E} = \nabla V = \frac{\partial V}{\partial x}\mathbf{i} + \frac{\partial V}{\partial y}\mathbf{j}$$

Since this is equivalent to the expression given,

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{\left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2}$$

$$= \sqrt{\left(\frac{1}{\sqrt{x^2 + 4y^2}}\right)^2 + \left(\frac{1}{\sqrt{x^2 + 4y^2}}\right)^2}$$

$$= \sqrt{\frac{2}{x^2 + 4y^2}}$$

Evaluating at the point (2,1),

$$E = \sqrt{\frac{2}{(2)^2 + (4)(1)^2}} = 1/2$$

The angle from horizontal that the **E** field is directed is

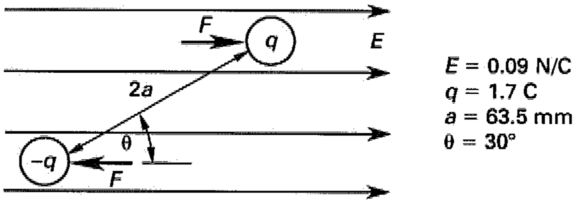
$$\tan\theta = \frac{E_x}{E_y} = \frac{\frac{1}{\sqrt{x^2+4y^2}}}{\frac{1}{\sqrt{x^2+4y^2}}} = 1$$

$$\theta = \pi/4$$

The answer is (D).

DC ELECTRICITY–14

An electric dipole is placed in a uniform electric field of intensity, E. Given the information in the figure, what is most nearly the torque acting on the dipole?



(A) 1.7×10^{-3} N·m (B) 3.3×10^{-3} N·m (C) 4.8×10^{-3} N·m (D) 9.5×10^{-2} N·m

The torque is

$$T = F(2a)\sin\theta$$

F is the force from the electric field.

$$F = Eq = 0.09\ \frac{\text{N}}{\text{C}} \times 1.7\ \text{C} = 0.15\ \text{N}$$

Solving for torque,

$$T = (0.15\ \text{N})(2)\left(\frac{63.5\ \text{mm}}{1000\ \frac{\text{mm}}{\text{m}}}\right)\sin 30^\circ$$
$$= 9.5 \times 10^{-2}\ \text{N}{\cdot}\text{m}$$

The answer is (D).

DC ELECTRICITY–15

Current i is applied to a long N-turn solenoid with cross-section area A and length d. The magnetic field intensity inside the solenoid is $H = Ni/d$ when d is very large. What is the inductance of this long solenoid in air?

(A) $\frac{\mu_0 NA}{d}$ (B) $\frac{\mu_0 N^2 A}{d}$ (C) $\frac{\mu_0 NA}{i}$ (D) $\frac{\mu_0 NA}{id}$

The magnetic flux passing through one turn of the solenoid is

$$\Phi_1 = \int \mathbf{B}\cdot d\mathbf{S} = \mu_0 HA$$

The total flux enclosed by the N turns is obtained by summing the contribution of all the turns.

$$\Phi = \sum_{\text{turns}} \Phi_1 = \mu_0 NHA$$

The inductance is

$$L = \frac{\Phi}{i} = \frac{\mu_0 NHA}{i} = \frac{\mu_0 N\left(\frac{Ni}{d}\right)A}{i} = \frac{\mu_0 N^2 A}{d}$$

The answer is (B).

DC ELECTRICITY–16

Which of the following is NOT a property of magnetic field lines?

(A) The field is stronger where the lines are closer together.
(B) The lines intersect surfaces of equal intensity at right angles.
(C) Magnetic field lines have no beginnings and no ends.
(D) The lines cross themselves only at right angles.

Magnetic field lines do not cross. Their direction at any given point is unique.

The answer is (D).

DC ELECTRICITY–17

The tesla is a unit of

(A) permittivity
(B) capacitance
(C) inductance
(D) magnetic flux density

The tesla is a unit of magnetic flux density.

The answer is (D).

DC ELECTRICITY–18

The south poles of two bar magnets are 7.5 cm apart in air. The magnets are of equal strength and repel each other with a force of 4.9×10^{-4} N. What is most nearly the strength of each magnet?

(A) 6.6×10^{-6} Wb (B) 0.86 Wb (C) 11 Wb (D) 53 Wb

The force between two magnets of strength M_1 and M_2 is

$$F = \frac{M_1 M_2}{4\pi\mu r^2}$$

$M_1 = M_2$. Therefore,

$$M = \sqrt{4\pi\mu r^2 F}$$
$$= \sqrt{4\pi\left(4\pi \times 10^{-7}\ \frac{\text{H}}{\text{m}}\right)(0.075\ \text{m})^2(4.9 \times 10^{-4}\ \text{N})}$$
$$= 6.60 \times 10^{-6}\ \text{Wb} \quad \text{[unit poles]}$$

The answer is (A).

DC ELECTRICITY–19

A conductor has length of 1 m, electrical resistivity of 0.1 Ω·m, and area of 0.01 m^2. A uniform direct current having a density of 100 A/m^2 flows through this conductor. What is the power loss in the conductor?

(A) 0 W (B) 1 W (C) 10 W (D) 100 W

The resistance of the conductor is

$$R = \frac{\rho L}{A} = \frac{(0.1\ \Omega\cdot\text{m})(1\ \text{m})}{0.01\ \text{m}^2} = 10\ \Omega$$

The current flows through the conductor is

$$I = JA = \left(100\ \frac{\text{A}}{\text{m}^2}\right)(0.01\ \text{m}^2) = 1\ \text{A}$$

Therefore, the power consumed in the conductor is

$$P = I^2 R = (1\ \text{A})^2(10\ \Omega) = 10\ \text{W}$$

The answer is (C).

DC ELECTRICITY–20

For a field given by $B = \mu H$ (Wb/m^2), what is the energy storage per unit volume?

(A) $U = \frac{B^2}{2\mu}$ (B) $U = \frac{H^2}{2}$ (C) $U = \frac{H^2}{2\mu}$ (D) $U = \frac{H^2}{2\mu^2}$

The energy stored in a magnetic field H per unit volume is $U = {}^1\!/_2 BH$. Since $B = \mu H$, $H = B/\mu$. Therefore,

$$U = \tfrac{1}{2}B\frac{B}{\mu} = \frac{B^2}{2\mu}$$

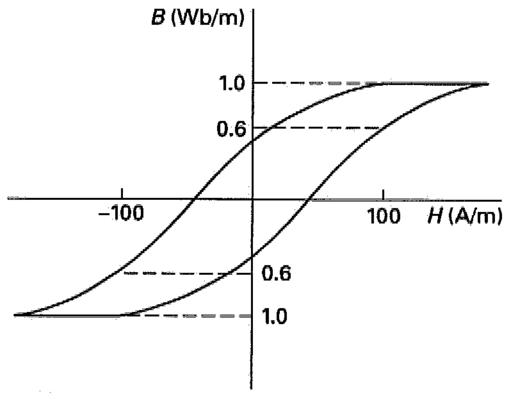
The answer is (A).

DC ELECTRICITY–21

The magnetic flux density, B, and the magnetic field intensity, H, have the following relationship.

$$B = \mu_o(H + M)$$

μ_o is the permeability of free space (in H/m), and M is the magnetic polarization of the material (in A/m). If B is increasing, which of the following may be true about the state of metal X at a value of $H = 100$ A/m? The B-H curve of the metal is as shown.



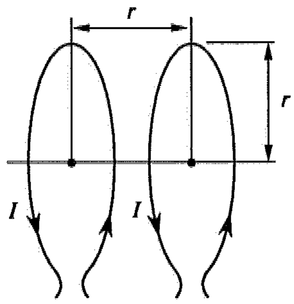
(A) $B = 0.6$ Wb/m; metal X is nonferrous
(B) $B = 0.6$ Wb/m; metal X is ferrous
(C) $B = 1.0$ Wb/m; metal X is nonferrous
(D) $B = 1.0$ Wb/m; metal X is ferrous

Nonferrous metals do not exhibit hysteresis; hence, metal X is ferrous. The hysteresis curve follows a counterclockwise path. Therefore, for B to be increasing at an H value of 100 A/m, $B = 0.6$ Wb/m.

The answer is (B).

DC ELECTRICITY–22

Two identical coils of radius r are placed at a distance r apart as shown. Such a configuration is called a Helmholtz coil. Which of the following describes the magnetic field created by passing a uniform current through the assembly?



(A) The magnetic field is negligible regardless of the magnitude of I.
(B) The magnetic field is zero midway between the two coils.
(C) The magnetic field is fairly uniform between the two coils.
(D) The magnetic field is zero at the centers of the coils.

The magnetic field between the two coils is the superposition of the field created by each coil. Since the currents in both coils are in the same direction, the direction of each individual magnetic field is also the same, using the right-hand rule. Therefore, the fields will not cancel each other.

Since the two coils are circular with their centers aligned, the field between them will be fairly uniform.

The answer is (C).

DC ELECTRICITY–23

Which statement is true?

(A) Magnetic flux lines have sources only.
(B) Magnetic flux lines have sinks only.
(C) Magnetic flux lines have both sources and sinks.
(D) Magnetic flux lines do not have sources or sinks.

Magnetic flux lines are closed loops with no sources or sinks. No known particle produces lines of magnetism.

The answer is (D).

DC ELECTRICITY–24

A charge of 0.75 C passes through a wire every 15 s. What is most nearly the current in the wire?

(A) 5.0 mA (B) 9.4 mA (C) 20 mA (D) 50 mA

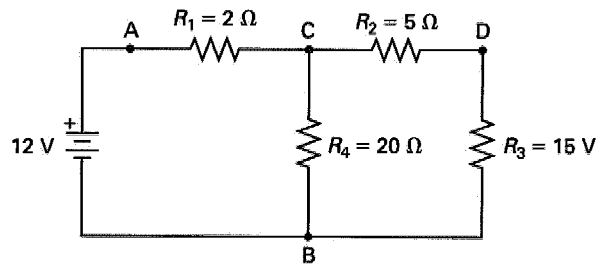
Current is the charge per unit time passing through the wire.

$$I = \frac{q}{t} = \frac{(0.75\ \text{C})\left(1000\ \frac{\text{mA}}{\text{A}}\right)}{15\ \text{s}}$$
$$= 50\ \text{mA}$$

The answer is (D).

DC ELECTRICITY–25

What is most nearly the total resistance between points A and B?



(A) 0 Ω (B) 12 Ω (C) 16 Ω (D) 22 Ω

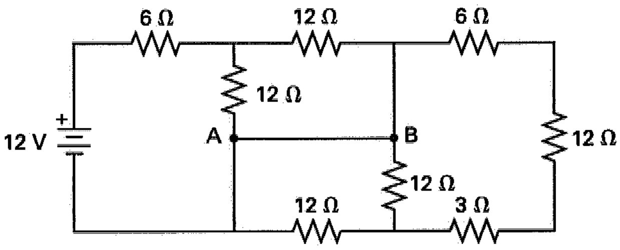
The total resistance is the sum of the resistance between points A and C, plus the equivalent resistance of the resistors in parallel between points C and B.

$$R_{\text{total}} = R_1 + R_4 \parallel (R_2 + R_3)$$
$$= R_1 + \frac{1}{\frac{1}{R_4} + \frac{1}{R_2 + R_3}}$$
$$= 2\ \Omega + \frac{1}{\frac{1}{20\ \Omega} + \frac{1}{5\ \Omega + 15\ \Omega}}$$
$$= 2\ \Omega + 10\ \Omega$$
$$= 12\ \Omega$$

The answer is (B).

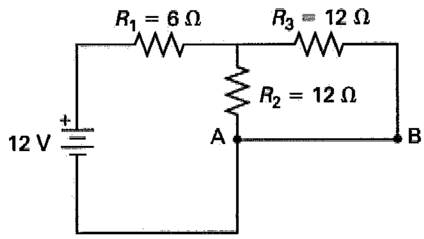
DC ELECTRICITY–26

What is the total resistance (as seen by the battery) of the following network?



(A) 6.0 Ω (B) 12 Ω (C) 15 Ω (D) 24 Ω

AB is a short circuit. Therefore, the rest of the circuit does not contribute to the resistance. The effective circuit is

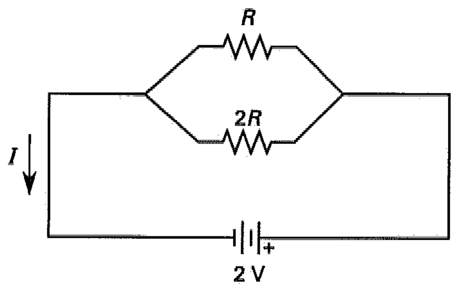


$$
\begin{aligned}
R_{\text{total}} &= R_1 + R_2 \parallel R_3 \\
&= R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} \\
&= 6\ \Omega + \frac{1}{\frac{1}{12\ \Omega} + \frac{1}{12\ \Omega}} \\
&= 6\ \Omega + 6\ \Omega \\
&= 12\ \Omega
\end{aligned}
$$

The answer is (B).

DC ELECTRICITY–27

In the circuit shown, $R = 10\ \Omega$, and the electromagnetic force, V, is 2 V. What is most nearly the current, I?



(A) 0.10 A (B) 0.30 A (C) 0.67 A (D) 3.3 A

$$R_{\text{total}} = R \parallel 2R = \frac{1}{\frac{1}{R} + \frac{1}{2R}}$$
$$= \frac{2R}{3}$$
$$= \frac{(2)(10\ \Omega)}{3}$$
$$= 6.67\ \Omega$$

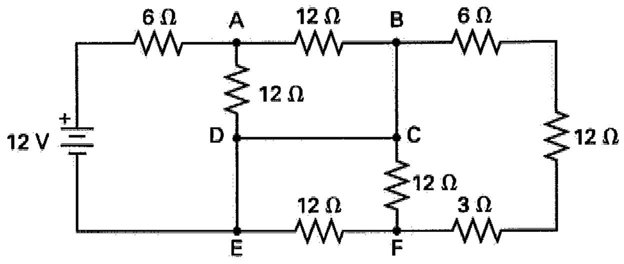
Use Ohm's law. Current is calculated as voltage divided by total resistance.

$$I = \frac{V}{R_{\text{total}}}$$
$$= \frac{2\ \text{V}}{6.67\ \Omega}$$
$$= 0.30\ \text{A}$$

The answer is (B).

DC ELECTRICITY–28

Find the current passing through the 3 Ω resistor.



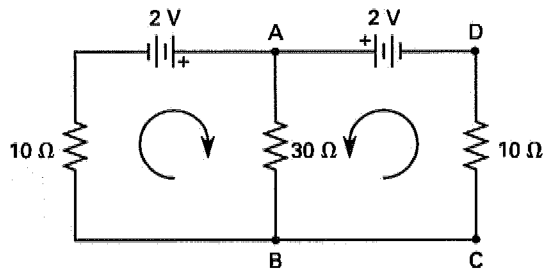
(A) 0 A (B) 0.3 A (C) 1 A (D) 4 A

Current from a battery will always follow a path of zero resistance in a circuit. Instead of flowing through the 3 Ω resistor and its neighboring resistors, the current will follow the path BCDE, a short circuit. There will be no current in the resistor.

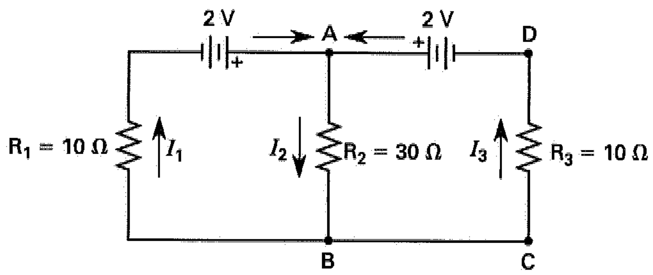
The answer is (A).

DC ELECTRICITY–29

What is most nearly the current passing through the 30 Ω resistor?



(A) 0.0 A (B) 29 mA (C) 50 mA (D) 57 mA



The circuit is symmetrical. Therefore, a current, I_1, flows through the resistors, R_1, and R_3. Another current, I_2, flows through resistor R_2. From Kirchhoff's current law at point A,

$$\sum I = 0$$
$$I_2 = 2I_1$$

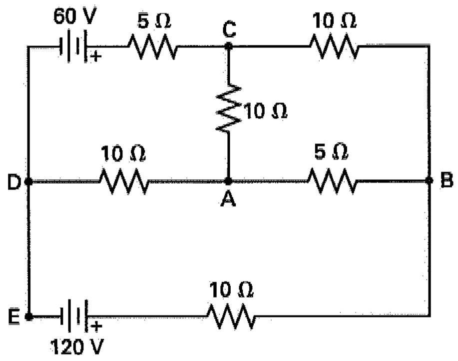
Using Kirchhoff's voltage law around the loop ABCDA.

$$V = R_2 I_2 + R_3 I_1$$
$$2V = (30\ \Omega)I_2 + (10\ \Omega)I_1$$
$$2V = (30\ \Omega)(2I_1) + (10\ \Omega)I_1$$
$$2V = (70\ \Omega)I_1$$
$$I_1 = 0.0286\ \text{A}$$
$$I_2 = 2I_1 = (2)(0.0286\ \text{A})\left(1000\ \frac{\text{mA}}{\text{A}}\right)$$
$$= 57\ \text{mA}$$

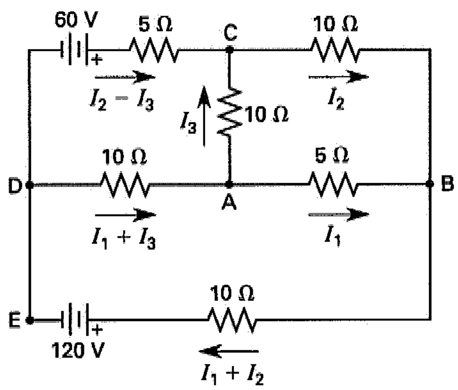
The answer is (D).

DC ELECTRICITY-30

What is most nearly the current through AB?



(A) 0.5 A (B) 1 A (C) 3 A (D) 4 A



By redrawing the circuit and designating the currents as shown in loop ACBA, the currents through the remaining loops can be expressed in terms of I_1, I_2, and I_3. Since voltage equals resistance multiplied by current, for loop CDAC,

$$60 \text{ V} = (5 \ \Omega)(I_2 - I_3) - (10 \ \Omega)(I_1 + I_3) - (10 \ \Omega)I_3$$

$$2I_1 - I_2 + 5I_3 = -12 \text{ A} \quad \text{[I]}$$

For loop BEDAB,

$$120 \text{ V} = (10 \ \Omega)(I_1 + I_2) + (10 \ \Omega)(I_1 + I_3) + (5 \ \Omega)I_1$$

$$0 = 25I_1 + 10I_2 + 10I_3 - 120$$

$$5I_1 + 2I_2 + 2I_3 = 24 \text{ A} \quad \text{[II]}$$

Around loop ACBA,

$$0 \text{ V} = -(5\ \Omega)I_1 + (10\ \Omega)I_2 + (10\ \Omega)I_3$$
$$I_1 - 2I_2 - 2I_3 = 0 \text{ A} \qquad \text{[III]}$$

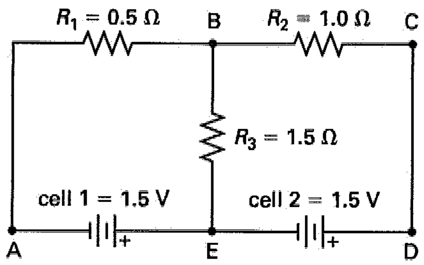
Observe that adding Eqs. II and III can directly solve I_1.

$$6I_1 = 24 \text{ A}$$
$$I_1 = 4 \text{ A}$$

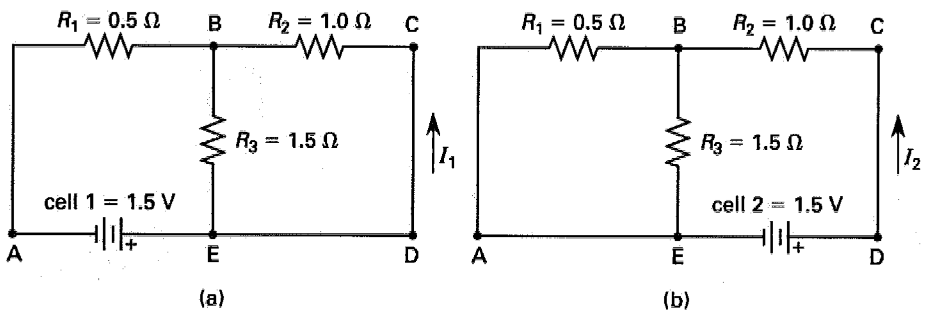
The answer is (D).

DC ELECTRICITY–31

In the circuit shown, what is the current through CD?



(A) 0.20 A (B) 0.60 A (C) 1.0 A (D) 1.9 A



The method of superposition is used to find the current, I. Let I_1 be the current from cell 1, and let I_2 be the current from cell 2. Then, $I = I_1 + I_2$. Short circuiting cell 2 to find I_1 as shown in illustration (a), the equivalent total resistance is

$$\begin{aligned} R_{\text{total},1} &= R_1 + R_2 \parallel R_3 \\ &= R_1 + \frac{R_2 R_3}{R_2 + R_3} \\ &= 0.5\ \Omega + \frac{(1.5\ \Omega)(1\ \Omega)}{2.5\ \Omega} \\ &= 1.1\ \Omega \\ I_1 &= \left(\frac{1.5\ \Omega}{2.5\ \Omega}\right)\left(\frac{1.5\ \text{V}}{1.1\ \Omega}\right) \\ &= 0.82\ \text{A} \end{aligned}$$

Short circuiting cell 1 to find I_2, as shown in illustration (b), the equivalent total resistance is

$$\begin{aligned} R_{\text{total},2} &= R_1 \parallel R_3 + R_2 \\ &= \frac{R_1 R_3}{R_1 + R_3} + R_2 \\ &= 1\ \Omega + \frac{(0.5\ \Omega)(1.5\ \Omega)}{2\ \Omega} \\ &= 1.375\ \Omega \\ I_2 &= \frac{1.5\ \text{V}}{1.375\ \Omega} \\ &= 1.1\ \text{A} \end{aligned}$$

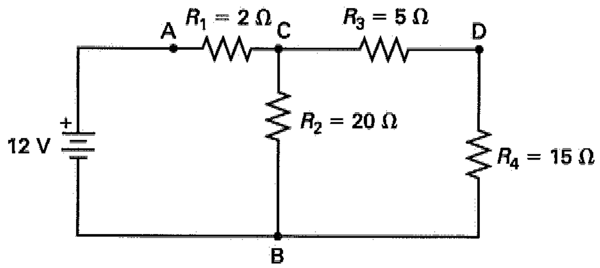
The total current is

$$\begin{aligned} I = I_1 + I_2 &= 0.82\ \text{A} + 1.1\ \text{A} \\ &= 1.92\ \text{A} \quad (1.9\ \text{A}) \end{aligned}$$

The answer is (D).

DC ELECTRICITY–32

For the network shown, find the voltage drop from C to D.



(A) 2.0 V (B) 2.5 V (C) 3.0 V (D) 8.0 V

The total resistance is

$$R_{\text{total}} = R_1 + R_2 \parallel (R_3 + R_4)$$
$$= R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3 + R_4}}$$
$$= 2\ \Omega + \frac{1}{\frac{1}{20\ \Omega} + \frac{1}{15\ \Omega + 5\ \Omega}}$$
$$= 12\ \Omega$$
$$I_{\text{total}} = \frac{V}{R_{\text{total}}} = \frac{12\ \text{V}}{12\ \Omega} = 1\ \text{A}$$

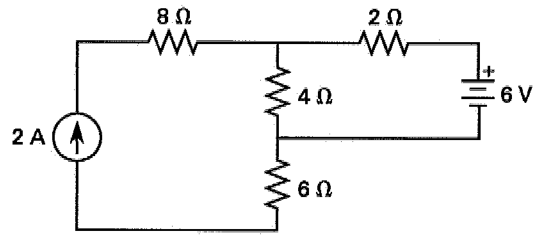
Use a current divider to find the current in section CDB.

$$I_{\text{CDB}} = I_{\text{total}}\left(\frac{R_2}{R_2 + R_3 + R_4}\right)$$
$$= (1\ \text{A})\left(\frac{20\ \Omega}{40\ \Omega}\right)$$
$$= 0.5\ \text{A}$$
$$V_{\text{CD}} = I_{\text{CDB}} R_3$$
$$= (0.5\ \text{A})(5\ \Omega)$$
$$= 2.5\ \text{V}$$

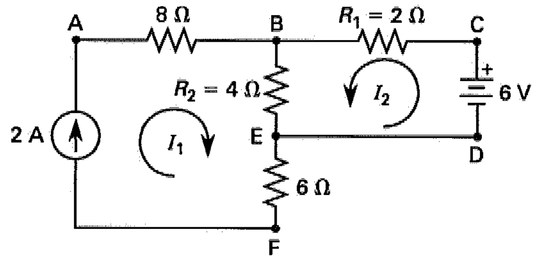
The answer is (B).

DC ELECTRICITY–33

Determine the voltage drop across the 4 Ω resistor in the network shown.



(A) 4.3 V (B) 6.7 V (C) 12 V (D) 24 V



The network is redrawn with the currents and circuit points labeled as shown. The current through BE is equal to the sum of currents from AB and CB.

$$I_{BE} = I_1 + I_2 = 2\text{ A} + I_2$$

Kirchhoff's voltage law around loop DCBE gives

$$V_{CD} = R_1 I_2 + R_2 I_{BC}$$

$$6V = (2\ \Omega)I_2 + (4\ \Omega)I_{BE} = (2\ \Omega)I_2 + (4\ \Omega)(2\text{ A} + I_2)$$

$$I_2 = -0.333\text{ A} \quad \text{[opposite to the direction that it was defined]}$$

$$I_{BE} = 2\text{ A} - 0.333\text{ A} = 1.67\text{ A}$$

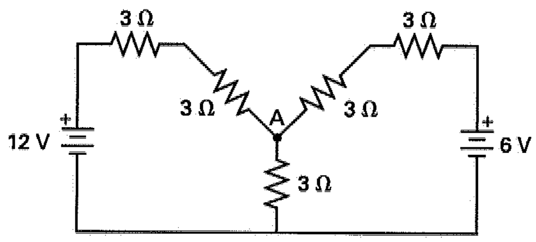
$$V_{BE} = (1.67\text{ A})(4\ \Omega)$$

$$= 6.68\text{ V} \quad (6.7\text{ V})$$

The answer is (B).

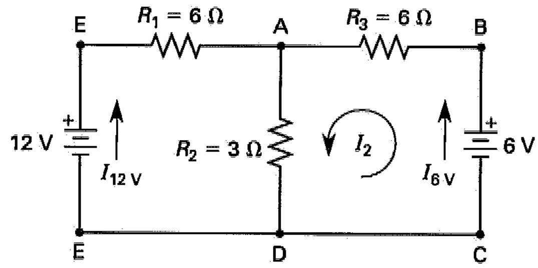
DC ELECTRICITY–34

The voltage at point A in the network shown is most nearly



(A) 1.0 V (B) 2.3 V (C) 3.0 V (D) 4.5 V

The circuit is redrawn.



Superposition is used to find I_2.

$$I_2 = I_{6\,\text{V}} - I_{12\,\text{V}}$$

$I_{6\,\text{V}}$ is the current through BA from the 6 V source, and $I_{12\,\text{V}}$ is the current through BA from the 12 V source. The equivalent resistances are calculated by short circuit for each voltage source.

$$\begin{aligned} R_{6\,\text{V}} &= R_3 + R_1 \parallel R_2 \\ &= R_3 + \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \\ &= 6\ \Omega + \frac{1}{\frac{1}{6\,\Omega} + \frac{1}{3\,\Omega}} \\ &= 8\ \Omega \end{aligned}$$

$$I_{6\,\text{V}} = \frac{V_{6\,\text{V}}}{R_{6\,\text{V}}} = \frac{6\ \text{V}}{8\ \Omega} = 0.75\ \text{A}$$

$$
\begin{aligned}
R_{12\,\mathrm{V}} &= R_1 + R_2 \parallel R_3 \\
&= R_1 + \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} \\
&= 6\ \Omega + \frac{1}{\frac{1}{6\,\Omega} + \frac{1}{3\,\Omega}} \\
&= 8\ \Omega
\end{aligned}
$$

$$
I_{12\,\mathrm{V}} = \left(\frac{V_{12\,\mathrm{V}}}{R_{12\,\mathrm{V}}}\right)\left(\frac{R_2}{R_2 + R_3}\right) = \left(\frac{3\ \Omega}{9\ \Omega}\right)\left(\frac{12\ \mathrm{V}}{8\ \Omega}\right) = 0.5\ \mathrm{A}
$$

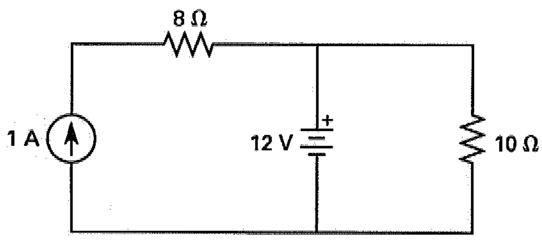
$$
\begin{aligned}
I_2 &= I_{6\,\mathrm{V}} - I_{12\,\mathrm{V}} \\
&= 0.75\ \mathrm{A} - 0.5\ \mathrm{A} \\
&= 0.25\ \mathrm{A}
\end{aligned}
$$

$$
\begin{aligned}
V_\mathrm{A} &= V_{6\,\mathrm{V}} - I_2 R_3 \\
&= 6\ \mathrm{V} - (0.25\ \mathrm{A})(6\ \Omega) \\
&= 4.5\ \mathrm{V}
\end{aligned}
$$

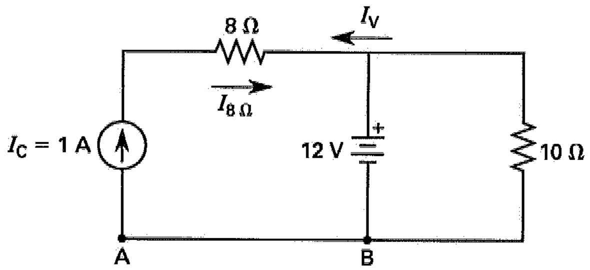
The answer is (D).

DC ELECTRICITY–35

What is the voltage drop across the 8 Ω resistor in the following circuit?



(A) 8 V (B) 12 V (C) 20 V (D) 22 V



Redrawing the circuit as shown, with I_C equal to the component of the current through the 8 Ω resistor due to the current source, and I_V equal to the component of the current through the resistor due to the voltage source,

$$I_{8\,\Omega} = I_C - I_V$$

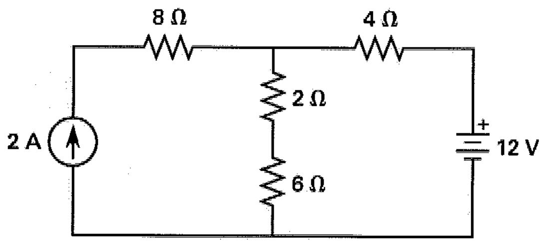
But, $I_C = 1$ A, and $I_V = 0$ A. Therefore,

$$\begin{aligned} I_{8\,\Omega} &= 1\text{ A} \\ V_{8\,\Omega} &= IR = (1\text{ A})(8\ \Omega) \\ &= 8\text{ V} \end{aligned}$$

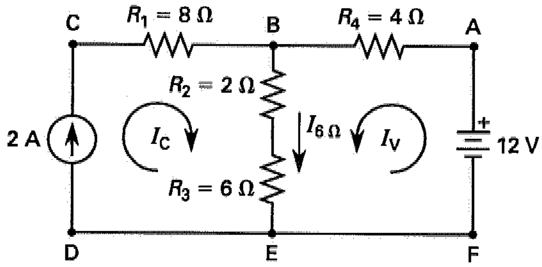
The answer is (A).

DC ELECTRICITY–36

Determine the voltage drop across the 6 Ω resistor.



(A) 6.0 V (B) 9.0 V (C) 10 V (D) 18 V



From superposition, with I_C designating the current through the resistor from the current source, and I_V designating the current through the resistor from the voltage source,

$$I_{6\ \Omega} = I_C + I_V$$

I_C and I_V are

$$\begin{aligned} I_C &= (2\ \text{A})\left(\frac{R_4}{R_2 + R_3 + R_4}\right) \\ &= (2\ \text{A})\left(\frac{4\ \Omega}{4\ \Omega + 2\ \Omega + 6\ \Omega}\right) \\ &= 0.667\ \text{A} \end{aligned}$$

$$I_V = \frac{12\ \text{V}}{R_2 + R_3 + R_4}$$

$$I_V = \frac{12\ \text{V}}{4\ \Omega + 2\ \Omega + 6\ \Omega} = 1\ \text{A}$$

$$I_{6\ \Omega} = 0.667\ \text{A} + 1\ \text{A} = 1.67\ \text{A}$$

$$V_{6\ \Omega} = I_{6\ \Omega}R = (1.67\ \text{A})(6\ \Omega) = 10\ \text{V}$$

The answer is (C).

DC ELECTRICITY–37

The rated voltage drop across a device is 50 V, and the current drawn is 30 A. What is most nearly the power rating of this device?

(A) 0.66 hp (B) 1.0 hp (C) 1.5 hp (D) 2.0 hp

Power is calculated as current multiplied by voltage.

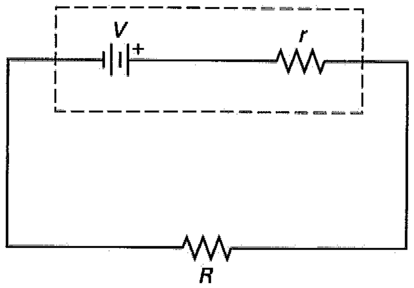
$$P = IV = \frac{(30 \text{ A})(50 \text{ V})}{746 \ \frac{\text{W}}{\text{hp}}}$$

$$= 2.01 \text{ hp} \quad (2.0 \text{ hp})$$

The answer is (D).

DC ELECTRICITY–38

In the circuit shown, $V = 6$ V, and the internal resistance of the source, r, is 1 Ω. For a specific value of R, the power output is the maximum possible. What is most nearly this maximum output power?



(A) 4.5 W (B) 6.0 W (C) 9.0 W (D) 18 W

Output power is given by the following expression.

$$P = I^2R$$

$$I = \frac{V}{R + r}$$

$$P = \frac{RV^2}{(R + r)^2}$$

Maximum power occurs when $dP/dR = 0$.

$$\frac{dP}{dR} = V^2(-2(R+r)^{-3}R + (R+r)^{-2}) = 0$$
$$(R+r)^{-2} = 2(R+r)^{-3}R$$
$$R+r = 2R$$
$$R = r = 1\ \Omega$$

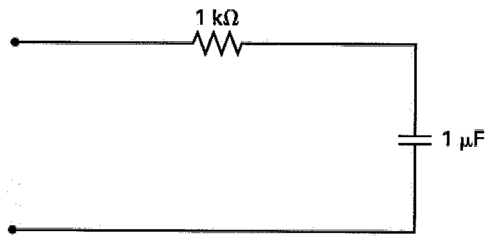
Therefore, the maximum power is

$$P_{max} = \frac{RV^2}{(R+r)^2} = \frac{(1\ \Omega)(6\ \text{V})^2}{(1\ \Omega + 1\ \Omega)^2} = \frac{36}{4}\ \text{W} = 9\ \text{W}$$

The answer is (C).

DC ELECTRICITY-39

What is the time constant of the network?



(A) 0.001 s (B) 10 s (C) 100 s (D) 1000 s

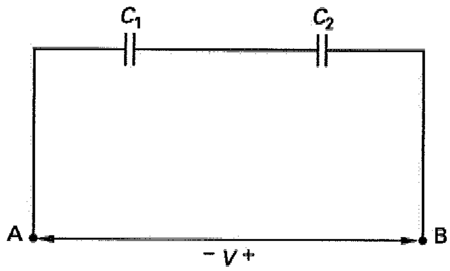
The time constant, τ, is

$$\tau = RC = (1000\ \Omega)(1 \times 10^{-6}\ \text{F}) = 0.001\ \text{s}$$

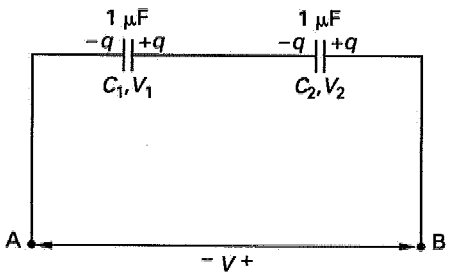
The answer is (A).

DC ELECTRICITY–40

For the two capacitors shown, $C_1 = 1\ \mu\text{F}$, and $C_2 = 3\ \mu\text{F}$. What is the equivalent capacitance between A and B?



(A) 0.75 μF (B) 1.0 μF (C) 2.0 μF (D) 4.0 μF



By definition, $q = CV$. For capacitors in series, the charge, q, is the same on each capacitor. Therefore,

$$V = V_1 + V_2$$

$$\frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{(1\ \mu\text{F})(3\ \mu\text{F})}{1\ \mu\text{F} + 3\ \mu\text{F}}$$

$$= 0.75\ \mu\text{F}$$

The answer is (A).

DC ELECTRICITY–41

The equivalent capacitance of capacitors C_1 and C_2 connected in series is 7.3 μF. If the capacitance of $C_1 = 9.6\ \mu$F, what is most nearly the capacitance of C_2?

(A) 2.3 μF (B) 31 μF (C) 35 μF (D) 49 μF

For capacitors in series, the equivalent capacitance, C, is

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

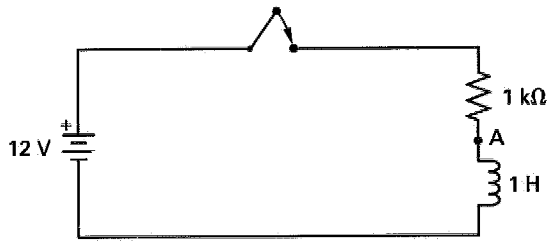
$$C_2 = \frac{C_1 C}{C_1 - C} = \frac{(9.6\ \mu\text{F})(7.3\ \mu\text{F})}{9.6\ \mu\text{F} - 7.3\ \mu\text{F}}$$

$$= 30.5\ \mu\text{F}$$

The answer is (B).

DC ELECTRICITY–42

Find the voltage at point A at the instant the switch is closed.



(A) 0.0 V (B) 1.0 V (C) 3.0 V (D) 12 V

The initial current at $t = 0^+$ is zero, so there is no voltage drop across the resistor. The full 12 V potential appears across the inductor.

$$V_A = 12\ \text{V}$$

The answer is (D).

DC ELECTRICITY–43

Which of the following statements regarding the motion of a conductor through a changing magnetic field is FALSE?

(A) The lines of magnetic flux pass from the north pole to the south pole of the magnet.

(B) When a conductor is "open circuited," no current flows despite its motion through the field.

(C) The conductor must move at constant velocity in order to generate a current.

(D) A current flowing counter to the direction of the conductor's motion will create a torque.

A varying amount of flux gives rise to a current. Flux through a conducting loop can be varied either by changing the magnetic field or by changing the speed of a conductor through the field. Thus, a conductor accelerating through a magnetic field will generate current.

The answer is (C).

DC ELECTRICITY–44

Modern locomotives are powered by DC motors driven by DC generators. These are, in turn, driven by diesel engines. Which of the following is the reason for using such a configuration instead of AC generator-motor sets?

(A) The DC configuration provides high torque and good incremental power at low speeds, and performs equally well at high speeds.

(B) The DC equipment is significantly less expensive.

(C) Historically, the DC engine configuration has been used. There is no reason to change this.

(D) By using the DC equipment, the power factor problems associated with AC equipment are avoided.

At low speeds, the DC system is best because it gives high torque and excellent control. DC motors or generators generally cost more than AC units because they have windings on the armature, which the AC units lack. The power factor problem has nothing to do with the decision not to use AC systems. At low speeds, the torque delivered by AC units is poor.

The answer is (A).

DC ELECTRICITY–45

In a DC motor, what is the definition of "field resistance"?

(A) It is the load resistance seen by a generator without considering inductance.
(B) It is the resistance of the excitation circuit.
(C) It is the resistance of the armature windings plus the load resistance.
(D) It is the static resistance of the motor.

The field circuit is the circuit that excites the pole pieces, thereby producing the flux cut by the armature windings. The field resistance is the resistance of this circuit.

The answer is (B).

DC ELECTRICITY–46

In a DC motor, which of the following does NOT cause sparking at a commutator?

(A) no load on the output leads
(B) frozen armature
(C) high brush contact resistance
(D) graphite brushes with good contact pressure

Sparking at the commutators results from having very low resistance at that point. It is normally associated with the use of copper or mainly copper brushes because the resistance of the brushes is nonlinear and drops as current increases.

The answer is (C).

DC ELECTRICITY–47

The armature in a DC generator has one or more pairs of conductors or coils in which current is produced. In general, which of the following is true about the amount of power produced?

(A) No gain in power is achieved beyond four pairs of coils.
(B) More coils give more power.
(C) Power is related only to the number of poles.
(D) Power is a function only of the output voltage and current.

Every coil that cuts the flux lines gives more power. It is desirable to place as many coils on the armature as possible.

The answer is (B).

DC ELECTRICITY–48

Which of the following limits the number of coils that may be placed on the armature of a DC motor or generator?

(A) the type of winding used
(B) the number of poles
(C) the size of the load on the motor
(D) coil to coil arcing due to the breakdown of insulation

Coils must be well insulated from each other when spaced close together, otherwise failure will occur due to arcing. There is also a physical limit on the number of coils that will fit on an armature. However, this is due to the volume of insulation material needed to prevent shorts.

The answer is (D).

DC ELECTRICITY–49

The overall torque of a DC motor is

$$T = K_T \phi I_a z$$

K_T is a constant for the particular machine, ϕ is the total magnetic flux per pole, I_a is the armature current, and z is the number of conductors on the surface of the armature. Which of the following is FALSE regarding the above equation?

(A) It applies to both motors and generators.
(B) It applies only to a machine having an even number of pairs of poles.
(C) It applies since torque is indirectly dependent upon the pole winding current.
(D) It applies whether or not there is a load on the system.

The equation is valid for generators or motors having any number of poles. Option (B) is false.

The answer is (B).

DC ELECTRICITY–50

A DC system can be protected from lightning by including a thyrite tube in the circuit that connects the high-voltage line with the ground. Which of the following is FALSE regarding a thyrite tube?

(A) It maintains a very high resistance at or below the system operating voltage.
(B) Its resistance becomes low at very high voltages.
(C) It has a very fast recovery time with regard to voltage change.
(D) Its fusable link melts at high voltages.

The resistance of thyrite tubes drops significantly near the operating voltage, allowing large currents to be discharged. They do not melt at high voltages, but instead recover immediately after passing large currents to the ground.

The answer is (D).

DC ELECTRICITY–51

The magnetic saturation curve limits the voltage at which a generator or motor can operate. Which of the following statements regarding saturation curves is FALSE?

(A) As field current increases, the hysteresis effect limits the increase in the flux produced.

(B) Poles that allow the production of more flux permit higher operating voltages.

(C) More flux at a constant field voltage can be produced by increasing the number of poles.

(D) Saturation does not depend upon the type of steel used in the poles.

The magnetic permeabilities of steels vary greatly.

The answer is (D).

DC ELECTRICITY–52

Series and shunt motors are connected like series and shunt generators, respectively. The terms refer to the manner in which the self-excitation of the poles is connected to the unit. Which of the following statements is FALSE?

(A) The torque curve of a shunt motor is linear.

(B) Field coils of the shunt motor or generator are in parallel with the armature windings.

(C) Field coils of the series motor or generator are in series with the armature windings.

(D) The torque curves of both shunt and series motors are not affected by the value of the armature current.

The torque for these motors is directly related to the armature current.

The answer is (D).

DC ELECTRICITY–53

In terms of efficiency, shunt and series motors or generators have similar characteristics. Which of the following statements is FALSE?

(A) Series motors have low torque at low speeds.

(B) Shunt and series motors have approximately 80% efficiency above one-third of the rated load.

(C) Efficiency decreases with lower speeds for both types of motors.

(D) The two types of motors have very similar efficiency curves.

Although the efficiency curves for the two types of motors both drop at lower operating speeds, they are still quite different.

The answer is (D).

DC ELECTRICITY–54

What is pole pitch?

(A) the mica used to insulate the poles from each other
(B) the space on the stator allocated to one pole
(C) the space on the stator allocated to two poles
(D) the angle at which the pole windings are wound

Pole pitch is defined as the periphery of the armature divided by the number of poles. Thus, it is the space on the stator allocated to one pole.

The answer is (B).

DC ELECTRICITY–55

Which of the following statements regarding a compound motor is FALSE?

(A) It has a shunt winding.
(B) It has a series winding.
(C) It has commutators, armature windings, and field windings.
(D) Its speed remains fairly constant when subjected to sudden loads.

The advantage of a compound motor is its ability to respond to sudden heavy loads. The motor's speed is reduced quickly when the load is applied, transferring the kinetic energy of the system to the work area.

The answer is (D).

DC ELECTRICITY–56

Which of the following does NOT contribute to core losses in DC motors?

(A) eddy currents in the armature
(B) hysteresis losses in the armature
(C) commutator losses
(D) eddy currents and hysteresis losses in the armature

The induced eddy currents together with the hysteresis losses constitute the core losses. Commutator losses do not contribute to the core losses.

The answer is (C).

DC ELECTRICITY–57

Which of the following are power losses in a DC motor?

(A) I^2R losses
(B) gear and frictional losses
(C) hysteresis losses
(D) all of the above

All of the choices are types of power losses in a DC motor. Note that core losses include hysteresis losses.

The answer is (D).

DC ELECTRICITY–58

Which of the following statements is FALSE regarding large DC motors?

(A) To avoid flashover, the voltage difference between adjacent commutators should not exceed 15 V DC.

(B) Two, four, six, or eight coils may be laid in a slot in the armature to make maximum use of the flux and to generate as much power as possible.

(C) The pitch of the winding of a given coil is the number of slots spanned by the coil.

(D) Wave winding does not pass under all poles at one time.

The lap winding loops several times under the same poles, while the wave winding lies under all poles at the same time.

The answer is (D).

DC ELECTRICITY–59

Which of the following statements is FALSE about the operation of parallel shunt generators?

(A) The drooping load or decreasing terminal voltage characteristic of shunt generators makes two or more units operating in parallel more stable.

(B) The use of several units makes maintenance and repair easier.

(C) The parallel configuration makes it possible to add units as needed, and to shut off unnecessary units at low load demands.

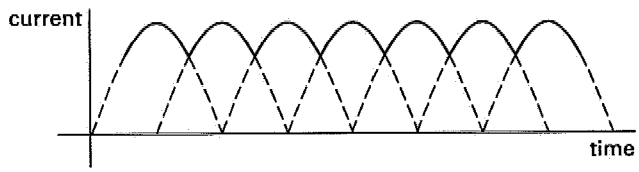
(D) One large unit would be more expensive than the use of several smaller ones, even if it ran at full load at all times.

Although it is rare to be able to load a unit fully at all times due to maintenance considerations, one large unit operating as such would be more economical.

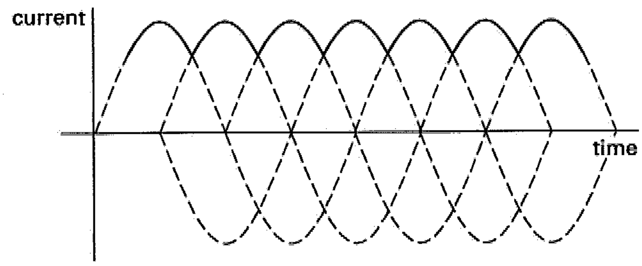
The answer is (D).

DC ELECTRICITY–60

How many commutators does a DC machine require to produce the waveform shown?



(A) two (B) three (C) four (D) five



Two commutators are needed for each coil. Each coil produces a single sine wave. If the alternate peaks in the figure are reversed, two full sine waves are described. Therefore, since there are two coils, four commutators are needed.

The answer is (C).

DC ELECTRICITY–61

A generator provides power to a load dissipating 160 kW. Current flows over 760 m of two-conductor copper feeder having a resistance of 0.026 Ω per 100 m. The generator voltage is constant at 800 V. What is most nearly the voltage delivered at the specified load?

(A) 612 V (B) 652 V (C) 702 V (D) 710 V

Since it is a two-conductor feeder, the total resistance of the feeder is

$$\begin{aligned} R &= 2L_{\text{total}}R \\ &= (2)(760\ \text{m})\left(\frac{0.026\ \Omega}{100\ \text{m}}\right) \\ &= 0.40\ \Omega \end{aligned}$$

By definition, $V = IR$.

$$V_{\text{load}} = 800\ \text{V} - I(0.40\ \Omega)$$

$$I = \frac{800\ \text{V} - V_{\text{load}}}{0.40\ \Omega}$$

Using this equation for I, the expression $P = VI$ gives

$$\begin{aligned} 160\,000\ \text{W} &= IV_{\text{load}} \\ &= \left(\frac{800\ \text{V} - V_{\text{load}}}{0.40\ \Omega}\right)V_{\text{load}} \\ 64\,000 &= 800V_{\text{load}} - V_{\text{load}}^2 \\ 0 &= V_{\text{load}}^2 - 800V_{\text{load}} + 64\,000 \end{aligned}$$

Solving for V_{load},

$$V_{\text{load}} = \frac{800\text{ V} \pm \sqrt{(800\text{ V})^2 - (4)(1)(64\,000)}}{2}$$
$$= 90.2\text{ V or } 710\text{ V}$$

Of the choices, the only reasonable voltage is 710 V.

The answer is (D).

DC ELECTRICITY-62

In a DC machine, if the field current I_f increases to $2I_f$, what will the new torque be as a function of the initial torque, T_m? Neglect saturation.

(A) T_m (B) $2T_m$ (C) $4T_m$ (D) ${T_m}^2$

The mechanical torque produced is

$$T_m = \frac{60}{2\pi} K_a \Phi I_a$$

The magnetic flux, Φ, generated by the field is

$$\Phi = K_f I_f$$

Therefore, the new torque is

$$T_{\text{new}} = \frac{60}{2\pi} K_a K_f 2 I_f I_a = 2T_m$$

The answer is (B).

13 AC ELECTRICITY

AC ELECTRICITY–1

An alternating current with a frequency of 60 Hz is passed through a moving coil galvanometer that measures DC current. What will the galvanometer reading be equal to?

(A) the peak value of the AC current
(B) the average value of the AC current
(C) the rms value
(D) a negligible amount

If the galvanometer is designed to measure DC current, it will not be able to respond quickly enough to measure an alternating current of 60 Hz. The reading will be negligible.

The answer is (D).

AC ELECTRICITY–2

Which of the following effects are generally less for an alternating current than for a direct current?

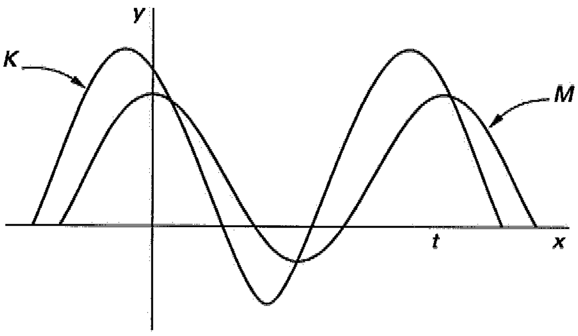
(A) heating effects (B) chemical effects (C) magnetic effects (D) impedance

Chemical effects are generally less for an AC current than for a DC current. Heating and magnetic effects are generally greater for an AC current than for a DC current. Impedance for an AC current is either larger than or the same as a DC current.

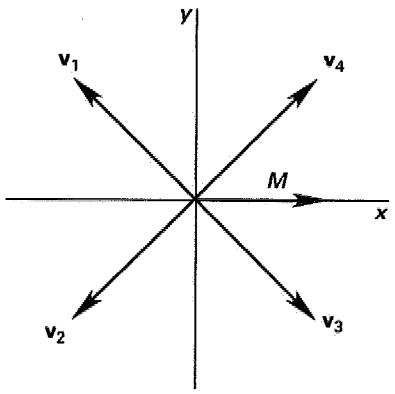
The answer is (B).

AC ELECTRICITY–3

The following two sine waves, K and M, are plotted as phasors. Determine which of the numbered vectors—$\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, or $\mathbf{v}_4$—corresponds to K.



$K = K_0 \cos(\omega t + \theta)$
$M = M_0 \cos(\omega t)$



(A) $K = \mathbf{v}_1$ (B) $K = \mathbf{v}_2$ (C) $K = \mathbf{v}_3$ (D) $K = \mathbf{v}_4$

The magnitude of the vector corresponds to the amplitude of the wave. Thus, the vector, K, is longer than M. All angles are measured from the positive x-axis, with a leading angle measured counterclockwise by convention. Therefore, since the peak of K leads that of M by less than 90°, the K vector lies in the first quadrant. The only choice satisfying these conditions is option (D).

The answer is (D).

AC ELECTRICITY–4

A wire carries an AC current of $3\cos 100\pi t$ A. What is the average current over 6 s?

(A) 0 A (B) $\pi/6$ A (C) 1.5 A (D) $6/\pi$ A

If T is the total period of time and $I(t)$ is the current as a function of time, the average current is

$$I_{\text{ave}} = \frac{1}{T}\int_0^T I(t)\,dt$$

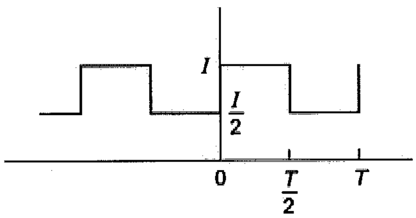
Therefore, for the particular AC current,

$$\begin{aligned} I_{\text{ave}} &= \frac{1}{6}\int_0^6 3\cos 100\pi t\,dt \\ &= \frac{1}{2}\int_0^6 \cos 100\pi t\,dt \\ &= \frac{1}{200\pi}\sin 100\pi t\Big|_0^6 \\ &= 0\text{ A} \end{aligned}$$

The answer is (A).

AC ELECTRICITY–5

What is the I_{rms} value for the waveform shown?



(A) $\frac{\sqrt{2}}{4}I$ (B) $\frac{\sqrt{3}}{4}I$ (C) $\frac{\sqrt{10}}{4}I$ (D) $\frac{\sqrt{3}}{2}I$

If T is the period and I is the current, the rms (effective) value is

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T I^2(t)dt}$$

For the square wave shown,

$$I(t) = I \qquad 0 \le t \le \frac{T}{2}$$
$$I(t) = \frac{I}{2} \qquad \frac{T}{2} \le t \le T$$

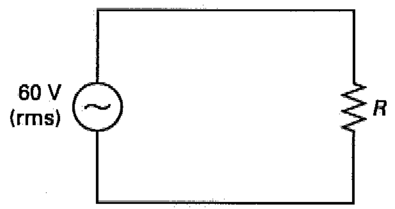
Therefore,

$$\begin{aligned} I_{\text{rms}} &= \sqrt{\frac{1}{T}\int_0^{T/2} I^2 dt + \frac{1}{T}\int_{T/2}^{T} \left(\frac{I}{2}\right)^2 dt} \\ &= \sqrt{\left(\frac{1}{T}\right)\left(\frac{I^2T}{2}\right) + \left(\frac{1}{T}\right)\left(\frac{I^2T}{8}\right)} \\ &= \sqrt{\frac{5}{8}I^2} \\ &= \frac{\sqrt{10}}{4}I \end{aligned}$$

The answer is (C).

AC ELECTRICITY–6

A sinusoidal AC voltage with an rms value of 60 V is applied to a purely resistive circuit as shown. What steady voltage most nearly generates the same power as the alternating voltage?



(A) 38 V (B) 42 V (C) 60 V (D) 85 V

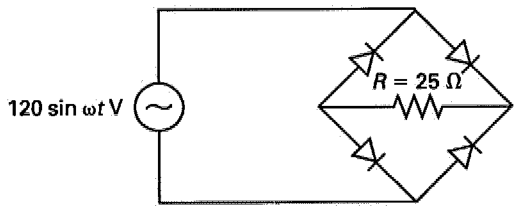
By definition of average power,

$$P_{\text{ave}} = \frac{E_{\text{rms}}^2}{R} = \frac{E^2}{R}$$
$$E = E_{\text{rms}}$$
$$= 60 \text{ V}$$

The answer is (C).

AC ELECTRICITY–7

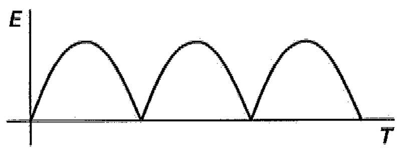
What is most nearly the average current through the resistor, R, in the rectifier shown? Assume ideal diodes.



(A) 0.0 A (B) 0.76 A (C) 3.1 A (D) 4.8 A

The type of rectifier shown is a "full wave" rectifier, with an average current of

$$I_{\text{ave}} = \frac{E_{\text{ave}}}{R}$$



E_{ave} for a full-wave rectifier is

$$E_{\text{ave}} = \frac{1}{T}\int_0^T (120 \text{ V}) \sin \omega t \, dt$$

In the preceding equation, $T = 2\pi/\omega$.

$$
\begin{aligned}
E_{\text{ave}} &= \frac{2}{T}\int_0^{T/2} (120\ \text{V}) \sin\omega t\, dt \\
&= \left(\frac{240\ \text{V}}{T}\right)\left(-\frac{\cos\omega t}{\omega}\right)\Bigg|_0^{T/2} \\
&= \left(\frac{240\ \text{V}}{T}\right)\left(\frac{2}{\omega}\right) \\
&= \frac{240\ \text{V}}{\pi}
\end{aligned}
$$

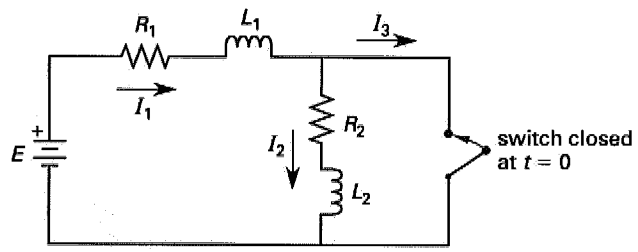
Therefore,

$$
\begin{aligned}
I_{\text{ave}} &= \left(\frac{240\ \text{V}}{\pi}\right)\left(\frac{1}{25\ \Omega}\right) \\
&= 3.06\ \text{A} \quad (3.1\ \text{A})
\end{aligned}
$$

The answer is (C).

AC ELECTRICITY–8

For the circuit shown, $I_1 = I_2$ before the switch is closed. If the switch is closed at time $t = 0$, what is the behavior of I_1 at $t = 0$?

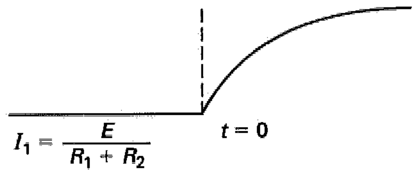


(A) I_1 is discontinuous and decreasing.
(B) I_1 is discontinuous and increasing.
(C) I_1 is continuous and decreasing.
(D) I_1 is continuous and increasing.

For $t < 0$, the current travels through L_1 and L_2. After the switch is closed, $I_1 = I_2 + I_3$, with I_2 slowly decaying through the short. As t goes to infinity, the current will travel around the outer loop with

$I_1 = I_3 = E/R_1$ and $I_2 = 0$. At $t < 0$, $I_1 = I_2 = E/(R_1 + R_2)$, but for $t \geq 0$,

$$\begin{aligned} I_1(t) &= I_1(0)e^{-R_1t/L_1} + \left(\frac{E}{R_1}\right)\left(1 - e^{-R_1t/L_1}\right) \\ &= \left(\frac{E}{R_1 + R_2}\right)e^{-R_1t/L_1} + \left(\frac{E}{R_1}\right)\left(1 - e^{-R_1t/L_1}\right) \\ &= \frac{E}{R_1} - \left(\frac{R_2E}{R_1(R_1 + R_2)}\right)e^{-R_1t/L_1} \end{aligned}$$

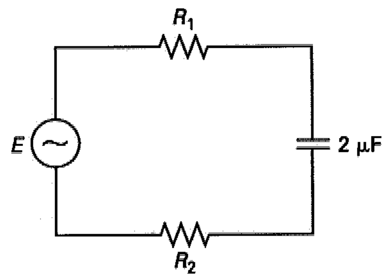


Therefore, I_1 is continuous and increasing at $t = 0$.

The answer is (D).

AC ELECTRICITY–9

A 2 μF capacitor in the circuit shown has a reactance of $X_C = 1500\ \Omega$. What is most nearly the frequency of the AC source?



(A) 3.0 Hz (B) 53 Hz (C) 60 Hz (D) 120 Hz

The reactance is

$$X_C = \frac{1}{\omega C}$$

Therefore,

$$\omega = \frac{1}{CX_C}$$

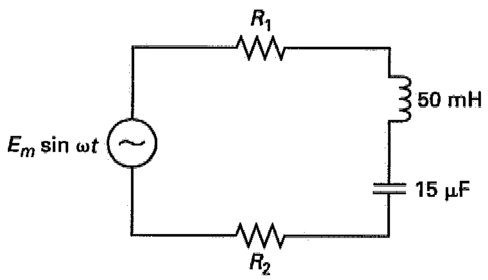
The frequency is

$$\begin{aligned} f &= \frac{\omega}{2\pi} = \frac{1}{2\pi C X_C} \\ &= \frac{1}{2\pi(2 \times 10^{-6}\ \text{F})(1500\ \Omega)} \\ &= 53\ \text{Hz} \end{aligned}$$

The answer is (B).

AC ELECTRICITY–10

If the capacitor and the inductor in the circuit shown have the same reactance, what is most nearly the frequency of the AC source?



(A) 27 Hz (B) 180 Hz (C) 210 Hz (D) 1200 Hz

If the inductor and capacitor have the same reactance, then

$$\begin{aligned} \frac{1}{\omega C} &= \omega L \\ \omega^2 &= \frac{1}{CL} \\ \omega &= \frac{1}{\sqrt{CL}} \end{aligned}$$

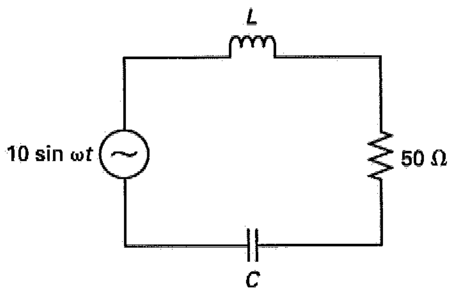
The frequency, f, is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{CL}}$$
$$= \frac{1}{2\pi\sqrt{(15 \times 10^{-6}\ \text{F})(50 \times 10^{-3}\ \text{H})}}$$
$$= 184\ \text{Hz} \quad (180\ \text{Hz})$$

The answer is (B).

AC ELECTRICITY–11

An alternating voltage of $E = 10 \sin \omega t$ V is applied to the RCL circuit shown. What is the effective current, I_{rms}, if the circuit is in resonance with the driving voltage?



(A) 0.141 A (B) 0.200 A (C) 7.07 A (D) 7.14 A

At resonance, the impedance of the circuit is equal to the impedance of the resistor. Therefore,

$$\omega L = \frac{1}{\omega C}$$

Additionally,

$$I = \frac{E}{\sqrt{R^2 + 0}} = \frac{E}{R}$$

$$I_{\text{rms}} = \frac{E_{\text{rms}}}{R} = \frac{\frac{E_{\text{max}}}{\sqrt{2}}}{R}$$

$$= \frac{\frac{10 \text{ V}}{\sqrt{2}}}{50 \ \Omega}$$

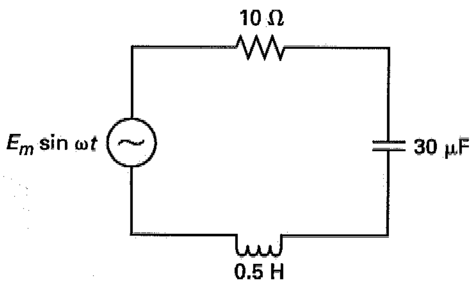
$$= 0.141 \text{ A}$$

The answer is (A).

AC ELECTRICITY-12

In the RCL circuit shown, $R = 10 \ \Omega$, $C = 30 \ \mu\text{F}$, and $L = 0.5$ H. At approximately what frequency will the rms current be one-third of the maximum possible rms current? The magnitude of the current is

$$I = E\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right)^{-1/2}$$



(A) 37 Hz (B) 41 Hz (C) 46 Hz (D) 160 Hz

The maximum rms current occurs at resonance. That is,

$$I_{\text{rms,max}} = \frac{E_{\text{rms}}}{R}$$

For the rms current to be one-third of the maximum,

$$\frac{1}{3}\left(\frac{E_{\text{rms}}}{R}\right) = \frac{E_{\text{rms}}}{\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right)^{1/2}}$$

$$\frac{1}{3R} = \frac{1}{\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right)^{1/2}}$$

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 9R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = 8R^2$$

$$\omega L - \frac{1}{\omega C} = 2\sqrt{2}R$$

$$\omega^2 - \frac{2\sqrt{2}R\omega}{L} - \frac{1}{LC} = 0$$

Solving for the positive ω value,

$$\begin{aligned}\omega &= \frac{\sqrt{2}R}{L} + \sqrt{\frac{2R^2}{L^2} + \frac{1}{LC}} \\ &= \frac{(\sqrt{2})(10\ \Omega)}{0.5\ \text{H}} + \sqrt{\frac{(2)(10\ \Omega)^2}{(0.5\ \text{H})^2} + \frac{1}{(0.5\ \text{H})(30 \times 10^{-6}\ \text{F})}} \\ &= 288\ \text{s}^{-1}\end{aligned}$$

$$\begin{aligned}f &= \frac{\omega}{2\pi} \\ &= \frac{\frac{288}{\text{s}}}{2\pi} \\ &= 45.8\ \text{Hz} \quad (46\ \text{Hz})\end{aligned}$$

The answer is (C).

AC ELECTRICITY–13

Which of the following statements regarding transformers is FALSE?

(A) The copper losses (I^2R) in the primary and secondary coils are equal.

(B) Transformer power losses are generally low, approximately 1–3%.

(C) Power losses in transformers are converted to heat, which is then dissipated.

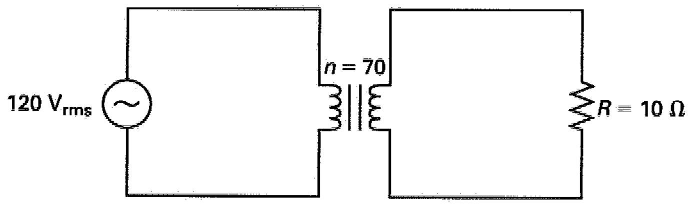
(D) One three-phase transformer weighs more than three equivalent single-phase transformers.

Power conversion using a three-phase transformer is more efficient than conversion using three separate single-phase units. Reduced weight and space requirements are obtained for the three-phase transformer.

The answer is (D).

AC ELECTRICITY–14

An ideal step-up transformer with a power factor of 1.0 is used in the circuit shown. The turns ratio is 70, and the primary rms voltage is 120 V. What is most nearly the average power dissipated due to the resistance, R?



(A) 17 W

(B) 29 W

(C) 8.4×10^4 W

(D) 7.1×10^6 W

For an ideal transformer, the turns ratio is

$$\frac{V_{\text{rms},2}}{V_{\text{rms},1}} = \frac{N_2}{N_1} = 70$$

$$\begin{aligned} V_{\text{rms},2} &= 70V_{\text{rms},1} \\ &= (70)(120 \text{ V}) \\ &= 8400 \text{ V} \end{aligned}$$

Since power is given by $P = I^2R = V^2/R$,

$$P_{\text{ave},2} = \frac{V^2_{\text{rms},2}}{R}$$
$$= \frac{(8400\ \text{V})^2}{10\ \Omega}$$
$$= 7.06 \times 10^6\ \text{W} \quad (7.1 \times 10^6\ \text{W})$$

There is no power loss in the primary circuit.

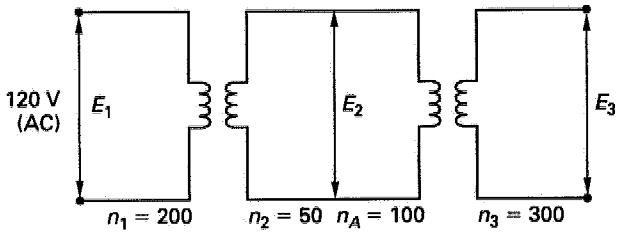
The answer is (D).

AC ELECTRICITY–15

In a transformer, the total voltage induced in each winding is proportional to the number of turns in that winding.

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

Disregarding all losses, determine E_3.



(A) 45 V (B) 65 V (C) 75 V (D) 90 V

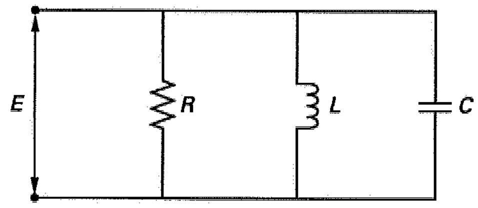
From the ratio given,

$$E_2 = \frac{n_2}{n_1}E_1$$
$$E_3 = \frac{n_3}{n_A}E_2$$
$$= \left(\frac{n_3}{n_A}\right)\left(\frac{n_2}{n_1}\right)E_1$$
$$= \left(\frac{300}{100}\right)\left(\frac{50}{200}\right)(120\ \text{V})$$
$$= 90\ \text{V}$$

The answer is (D).

AC ELECTRICITY–16

Determine the resonant frequency, ω, of the circuit shown.



(A) $\frac{1}{\sqrt{LC}}$ (B) $\frac{2}{\sqrt{LC}}$ (C) $\sqrt{\frac{LC}{3}}$ (D) $\sqrt{\frac{LC}{2}}$

Resonance occurs when $X_C = X_L$. Since $X_C = 1/j\omega C$ and $X_L = j\omega L$,

$$\frac{1}{j\omega C} = j\omega L$$

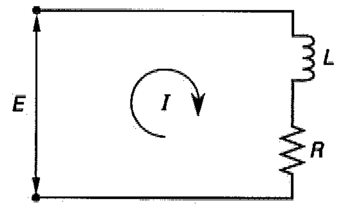
$$\omega^2 = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

The answer is (A).

AC ELECTRICITY–17

Determine most nearly the power angle, ϕ, in the AC circuit if $R = 25\ \Omega$, $L = 0.2$ H, $V = 200$ V, and $f = 30$ Hz.



(A) 36° (B) 46° (C) 52° (D) 57°

The power angle is the impedance angle. The impedance for the circuit is

$$\begin{aligned} Z &= R + jX_L \\ &= 25\ \Omega + j2\pi(30\ \text{Hz})(0.2\ \text{H}) \\ &= 25\ \Omega + j37.7\ \Omega \end{aligned}$$

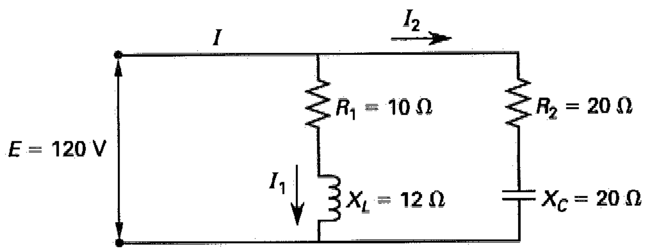
$$\tan\phi = \frac{X_L}{R}$$

$$\begin{aligned} \phi &= \tan^{-1}\left(\frac{37.7\ \Omega}{25\ \Omega}\right) \\ &= 56.5^\circ \quad (57^\circ) \end{aligned}$$

The answer is (D).

AC ELECTRICITY–18

Approximate the impedance of the circuit. The line current is **I**, and the line voltage lies along the real axis (i.e., has a zero phase angle).



(A) 12 Ω (B) 13 Ω (C) 14 Ω (D) 15 Ω

$$\begin{aligned} \mathbf{I}_1 &= \frac{\mathbf{E}}{\mathbf{Z}_1} = \frac{\mathbf{E}}{R_1 + jX_L} = \left(\frac{\mathbf{E}}{R_1 + jX_L}\right)\left(\frac{R_1 - jX_L}{R_1 - jX_L}\right) \\ &= \mathbf{E}\left(\frac{R_1 - jX_L}{R_1^2 + X_L^2}\right) \\ &= (120\ \text{V})\left(\frac{10\ \Omega - j12\ \Omega}{(10\ \Omega)^2 + (12\ \Omega)^2}\right) \\ &= 4.92 - j5.9\ \text{A} \end{aligned}$$

$$\begin{aligned}\mathbf{I}_2 &= \frac{\mathbf{E}}{R_2 - jX_C} = \left(\frac{\mathbf{E}}{R_2 - jX_C}\right)\left(\frac{R_2 + jX_C}{R_2 + jX_C}\right) \\ &= \mathbf{E}\left(\frac{R_2 + jX_C}{R_2^2 + X_C^2}\right) \\ &= (120 \text{ V})\left(\frac{20 \ \Omega + j20 \ \Omega}{(20 \ \Omega)^2 + (20 \ \Omega)^2}\right) \\ &= 3 + j3.0 \text{ A}\end{aligned}$$

The total current is

$$\begin{aligned}\mathbf{I} &= \mathbf{I}_1 + \mathbf{I}_2 \\ &= (4.92 - j5.90 \text{ A}) + (3 + j3.0 \text{ A}) \\ &= 7.92 - j2.90 \text{ A}\end{aligned}$$

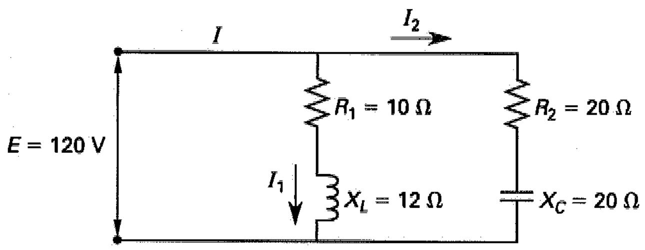
The impedance is

$$\begin{aligned}\mathbf{Z} &= \frac{\mathbf{E}}{\mathbf{I}} = \frac{120 + j0 \text{ V}}{7.92 - j2.9 \text{ A}} \\ &= 13.4 + j4.90 \ \Omega \\ Z &= |\mathbf{Z}| \\ &= \sqrt{x^2 + y^2} \\ &= \sqrt{(13.4 \ \Omega)^2 + (4.90 \ \Omega)^2} \\ &= 14.3 \ \Omega \quad (14 \ \Omega)\end{aligned}$$

The answer is (C).

AC ELECTRICITY–19

What is most nearly the power factor for the following circuit? The total impedance of the circuit is $\mathbf{Z} = 13.4 + j4.9 \ \Omega$.



(A) 74% (B) 79% (C) 84% (D) 94%

The power factor is

$$\cos\phi = \frac{P_{\text{real}}}{P_{\text{apparent}}} = \frac{R}{Z}$$

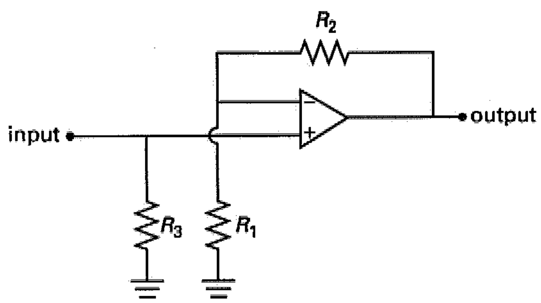
Since $R = 13.4\ \Omega$ and $Z = 14.3\ \Omega$,

$$\cos\phi = \frac{13.4\ \Omega}{14.3\ \Omega} \times 100\% = 93.7\% \quad (94\%)$$

The answer is (D).

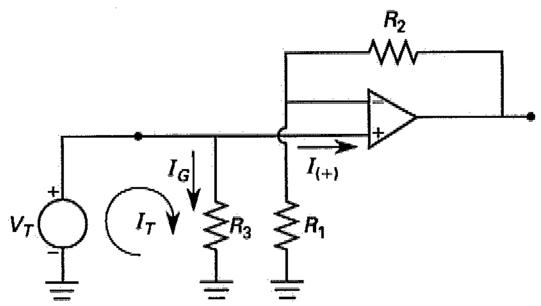
AC ELECTRICITY-20

What is the input impedance of the ideal op amp shown?



(A) R_1 (B) R_3 (C) $\frac{R_2}{R_1} + R_3$ (D) $\frac{R_1 R_3}{R_1 + R_3}$

To find the input impedace, a test voltage, V_T, is applied to the input. The resistance seen by the test voltage will be equal to the impedance: $R_{\text{in}} = V_T/I_T$. The circuit can be replaced with its equivalent.



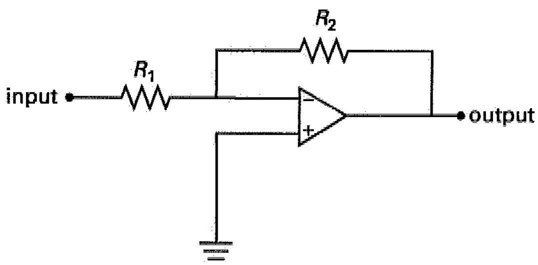
By Kirchoff's law, $I_T = I_G + I_{(+)}$. In an ideal op amp, there is no current drawn by the positive and negative terminals. Therefore, $I_{(+)} = 0$ and $I_T = I_G$. Around that loop,

$$I_T = \frac{V_T}{R_3}$$
$$\frac{V_T}{I_T} = R_3$$
$$R_{\text{in}} = \frac{V_T}{I_T}$$
$$= R_3$$

The answer is (B).

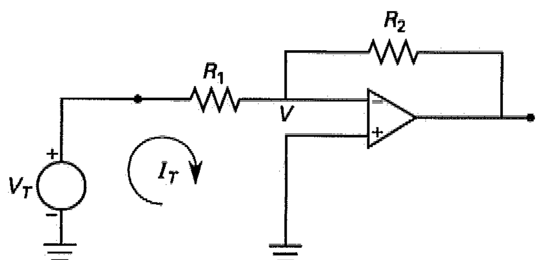
AC ELECTRICITY–21

What is the input impedance of the following ideal op amp?



(A) R_1 (B) R_2 (C) $\frac{R_2}{R_1}$ (D) $\frac{R_1}{R_1 + R_2}$

To find the input impedance or resistance, the circuit is examined using a test voltage, V_T, and a test current, I_T. The circuit diagram becomes



The input resistance will be

$$R_{\text{in}} = \frac{V_T}{I_T}$$

For an ideal op amp, the voltage at the (+) terminal equals the voltage of the (−) terminal. Therefore, $V = 0$ and

$$I_T = \frac{V_T - V}{R_1} = \frac{V_T}{R_1}$$

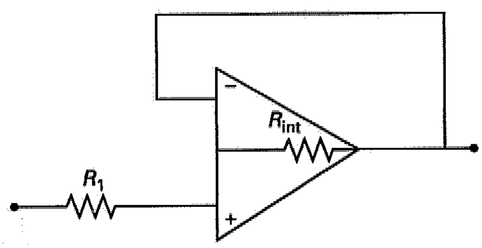
$$R_{\text{in}} = V_T \frac{R_1}{V_T}$$

$$= R_1$$

The answer is (A).

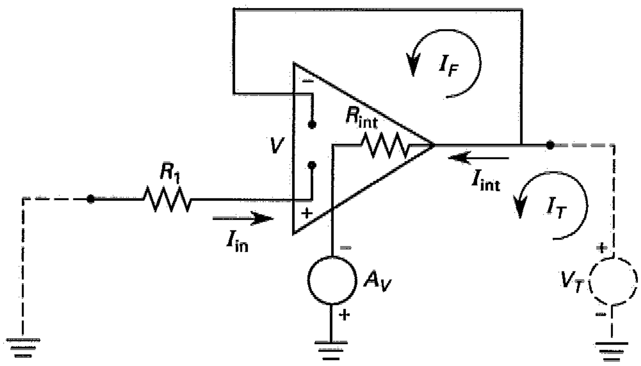
AC ELECTRICITY–22

What is most nearly the output impedance, R_{out}, of the circuit shown? Assume no current is drawn at the (+) and (−) inputs, and that the op amp has a small internal resistance, R_{int}, at the output.



(A) R_{int} (B) R_1 (C) $\frac{R_1 R_{\text{int}}}{R_1 + R_{\text{int}}}$ (D) 0

In terms of its operation, the op amp diagram is like the solid part in the following illustration.



A_V is very large. To find the output resistance, a test voltage, V_T, is attached to the output and the input is grounded. Then,

$$R_{out} = \frac{V_T}{I_T}$$

The test current, I_T, is equal to the internal current, I_{int}, plus the forced current, I_f. Since the inputs draw no current, $I_f = 0$. Therefore,

$$I_T = I_{int} = \frac{V_{int}}{R_{int}} = \frac{V_T - (-A_V V)}{R_{int}}$$

Since no current is drawn at the inputs, the (+) input is at 0 V, and the (−) input is at V_T, so that $V = V_T$. Thus,

$$I_{int} = \frac{V_T(1 + A)}{R_{int}}$$

$$\frac{V_T}{I_{int}} = \frac{R_{int}}{1 + A}$$

$$R_{out} = \frac{V_T}{I_{int}} = \frac{R_{int}}{1 + A}$$

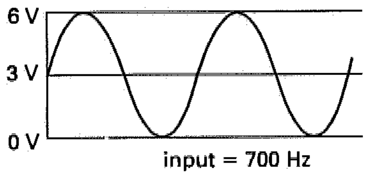
Since A_V is very large and R_{int} is very small,

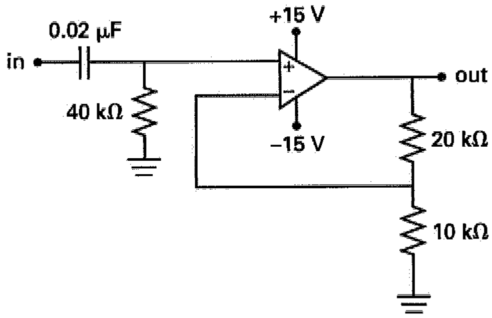
$$R_{out} \approx 0$$

The answer is (D).

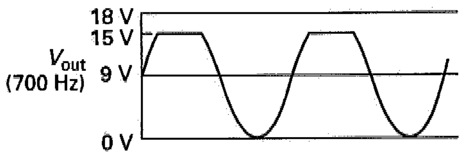
AC ELECTRICITY–23

The 700 Hz signal shown is injected into the circuit shown. What will be the output signal?

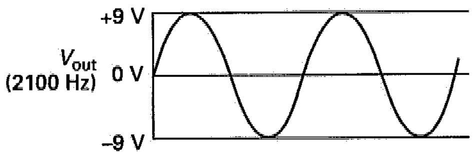




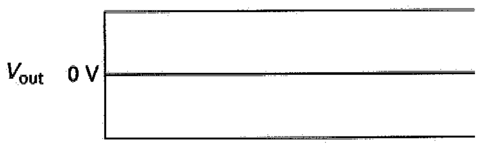
(A)



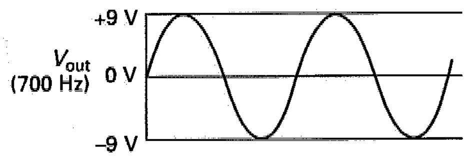
(B)



(C)



(D)



The op amp part of the circuit is a simple noninverting amplifier with a gain of

$$\frac{V_{out}}{V_{in}} = \frac{10\text{ V} + 20\text{ V}}{10\text{ V}} = 3\text{ V}$$

The input into the amplifier is a high-pass filter with a cutoff frequency of

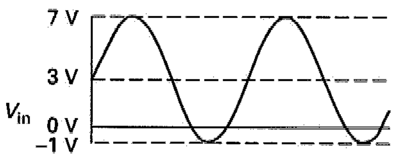
$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(40 \times 10^3\ \Omega)(0.02 \times 10^{-6}\ \text{F})}$$
$$= 200\ \text{Hz}$$

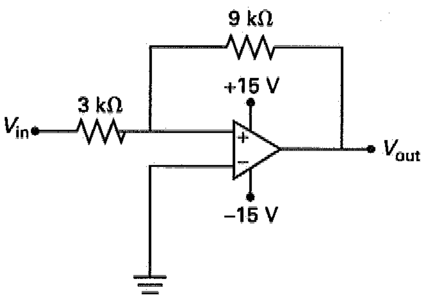
Thus, the AC component of the signal will pass through and be amplified three times, while the DC component will be cut out, resulting in a 9 V amplitude sinusoid centered about 0 V. This is known as an active high-pass filter. Thus, the correct output is shown in option (D).

The answer is (D).

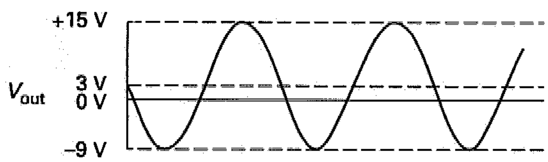
AC ELECTRICITY–24

The signal shown is the input to the ideal op amp. Which of the choices is the output signal?

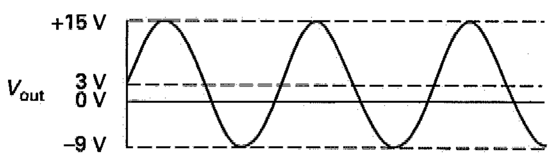




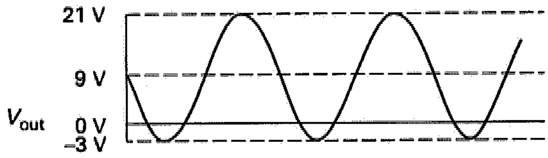
(A)



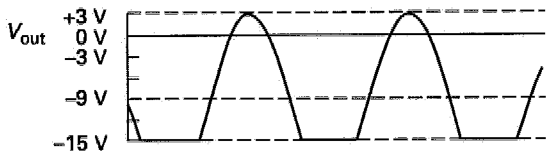
(B)



(C)



(D)



Since the amplifier is an inverting amplifier,

$$V_{\text{out}} = -\left(\frac{9\ \text{k}\Omega}{3\ \text{k}\Omega}\right) V_{\text{in}} = -3V_{\text{in}}$$

Both the DC and the AC components will be amplified. The DC component is $(-3)(3\ \text{V}) = -9\ \text{V}$, so the new waveform is centered at $V = -9\ \text{V}$. The AC component is $(3)(8\ \text{V}) = 24\ \text{V}$ peak-to-peak. Since the amplifier has only a 15 V source, though, the voltage will be clipped at $\pm 15\ \text{V}$. Since it never goes to $+15\ \text{V}$, the upper half of the output signal will be intact, and the lower half will be clipped at $-15\ \text{V}$.

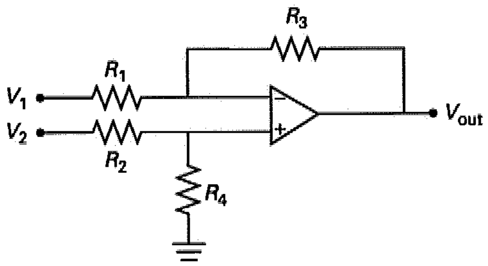
The answer is (D).

AC ELECTRICITY–25

Two AC signals, V_1 and V_2, are to be combined such that

$$V_{\text{out}} = \tfrac{3}{2}V_2 - \tfrac{5}{2}V_1$$

The subtracting amplifier circuit shown is used. What must be the values of R_1, R_2, R_3, and R_4?



(A) $R_1 = 2\ \text{k}\Omega,\ R_2 = 2\ \text{k}\Omega,\ R_3 = 5\ \text{k}\Omega,\ R_4 = 3\ \text{k}\Omega$
(B) $R_1 = 2\ \text{k}\Omega,\ R_2 = 4\ \text{k}\Omega,\ R_3 = 5\ \text{k}\Omega,\ R_4 = 3\ \text{k}\Omega$
(C) $R_1 = 4\ \text{k}\Omega,\ R_2 = 8\ \text{k}\Omega,\ R_3 = 10\ \text{k}\Omega,\ R_4 = 2\ \text{k}\Omega$
(D) $R_1 = 5\ \text{k}\Omega,\ R_2 = 3\ \text{k}\Omega,\ R_3 = 4\ \text{k}\Omega,\ R_4 = 2\ \text{k}\Omega$

The output for this op amp configuration is

$$\begin{aligned} V_{\text{out}} &= V_{(-)} - \left(\frac{V_1 - V_{(-)}}{R_1}\right) R_3 \\ &= \left(1 + \frac{R_3}{R_1}\right) V_{(-)} - \frac{R_3}{R_1} V_1 \end{aligned}$$

Additionally,

$$V_{(-)} = V_{(+)} = V_2 \left(\frac{R_4}{R_2 + R_4}\right)$$

Therefore,

$$V_{\text{out}} = \left(\frac{R_1 + R_3}{R_1}\right)\left(\frac{R_4}{R_2 + R_4}\right) V_2 - \frac{R_3}{R_1} V_1$$

The ratio $R_3{:}R_1$ must be 5:2. Therefore, the initial values $R_1 = 2\ \text{k}\Omega$ and $R_3 = 5\ \text{k}\Omega$ are chosen. Thus, the coefficient of V_2 is

$$\begin{aligned} \left(\frac{R_1 + R_3}{R_1}\right)\left(\frac{R_4}{R_2 + R_4}\right) &= \left(\frac{2\ \text{k}\Omega + 5\ \text{k}\Omega}{2\ \text{k}\Omega}\right)\left(\frac{R_4}{R_2 + R_4}\right) = 3/2 \\ \left(\frac{7}{2}\right)\left(\frac{R_4}{R_2 + R_4}\right) &= 3/2 \\ \frac{R_4}{R_2 + R_4} &= 3/7 \\ 7R_4 &= 3R_2 + 3R_4 \\ 4R_4 &= 3R_2 \\ R_4 &= \tfrac{3}{4} R_2 \end{aligned}$$

Try the values of R_2 and R_4 in the four answer choices to see if they satisfy the relation.

$R_2 = 4\ \text{k}\Omega$ and $R_4 = 3\ \text{k}\Omega$ are chosen. Checking the results,

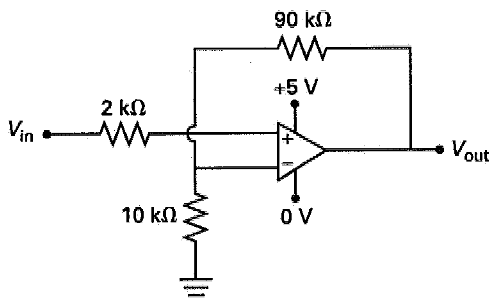
$$\begin{aligned} V_{\text{out}} &= \left(\frac{2\ \text{k}\Omega + 5\ \text{k}\Omega}{2\ \text{k}\Omega}\right)\left(\frac{3\ \text{k}\Omega}{4\ \text{k}\Omega + 3\ \text{k}\Omega}\right)V_2 - \tfrac{5}{2}V_1 \\ &= \tfrac{3}{2}V_2 - \tfrac{5}{2}V_1 \end{aligned}$$

The answer is (B).

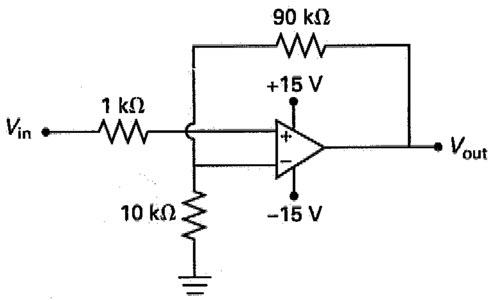
AC ELECTRICITY–26

A sinusoidal signal with maximum voltage, $V_0 = 30$ mV, is to be amplified without inversion to at least 0.3 V. Which of the following operational amplifier configurations will best achieve this? Assume ideal op amps.

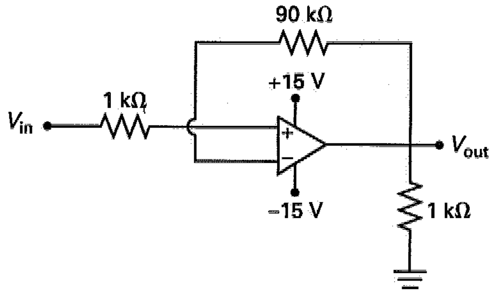
(A)



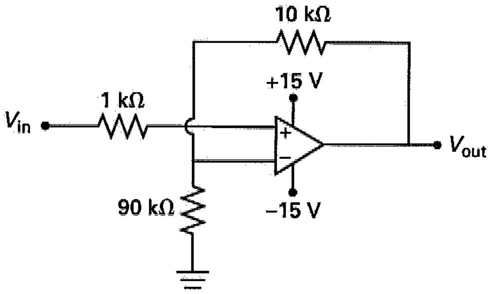
(B)



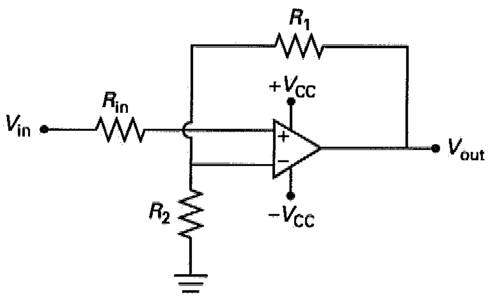
(C)



(D)



A noninverting topology is required. The resistances and voltages are labeled in the following illustration.



For this topology,

$$V_{out} = \left(\frac{R_1 + R_2}{R_2}\right) V_{in}$$

The voltage has to be amplified by a factor of 10 in order to get 30 mV up to 0.3 V.

$$\frac{R_1 + R_2}{R_2} = 10$$

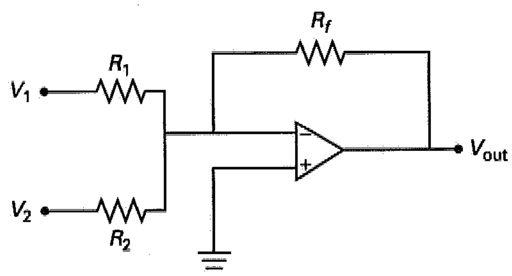
$$R_1 = 9R_2$$

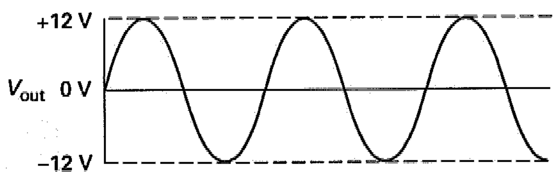
R_{in} is not important, since very little current is drawn through it. Of the answer choices, options (A), (B), and (D) are the correct topologies, and options (A) and (B) have the correct R_1 to R_2 ratio. The next criterion is that the supply voltage, $\pm V_{CC}$, must be greater in magnitude than the output voltage, or clipping will occur. Since option (A) has a negative input supply of 0 V, it will clip the output. Only option (B) will satisfy all requirements.

The answer is (B).

AC ELECTRICITY–27

In the ideal op-amp configuration shown, $V_{out} = 12$ V sinusoidal as shown in the waveform, $R_f = 60$ kΩ, $R_1 = 30$ kΩ, and $R_2 = 10$ kΩ. Nothing is known about the inputs except that $V_1 = 5V_2$, and that they are 180° out of phase with the output. From this information, what are the maximum voltages of the inputs?





(A) $V_1 = 0.75$ V, $V_2 = 3.75$ V
(B) $V_1 = 5.00$ V, $V_2 = 1.00$ V
(C) $V_1 = 12.5$ V, $V_2 = 2.50$ V
(D) $V_1 = 3.75$ V, $V_2 = 0.75$ V

This is an adding amplifier with an output of

$$V_{\text{out}} = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2\right)$$

It is known that $V_1 = 5V_2$. Substituting this into the equation for V_{out} and evaluating with the given R values and V_{out},

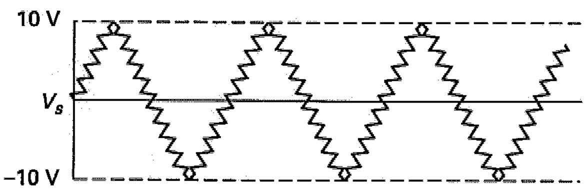
$$\begin{aligned} 12 &= -\left(\left(\frac{60\ \text{k}\Omega}{30\ \text{k}\Omega}\right)5V_2 + \left(\frac{60\ \text{k}\Omega}{10\ \text{k}\Omega}\right)V_2\right) \\ V_2 &= -0.75\ \text{V} \\ V_1 &= (5)(-0.75\ \text{V}) \\ &= -3.75\ \text{V} \end{aligned}$$

Since the output is a sinusoid, the inputs must also be sinusoids. It is known that they are 180° out of phase, which is confirmed by the negative sign of the voltage. Thus, the maximum voltages are $V_1 = 3.75$ V and $V_2 = 0.75$ V.

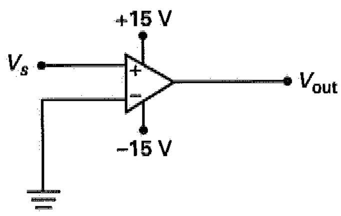
The answer is (D).

AC ELECTRICITY–28

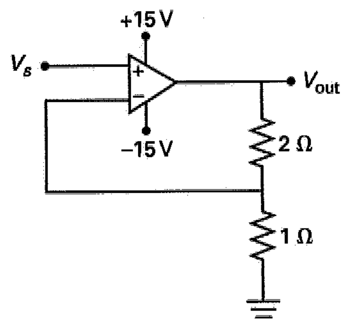
A zero-crossing detector is needed for the noisy circuit shown. The phase of the detector output is not important. (That is, the detector can show a time lag.) Which of the following op amp configurations would be best?



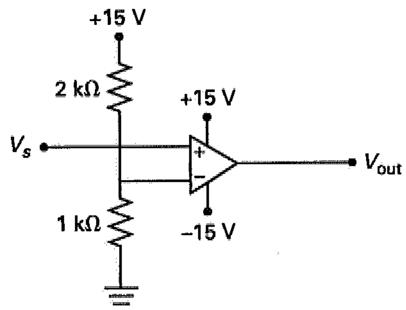
(A)



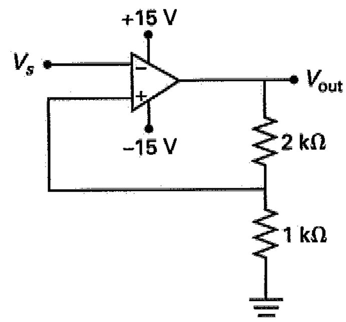
(B)



(C)

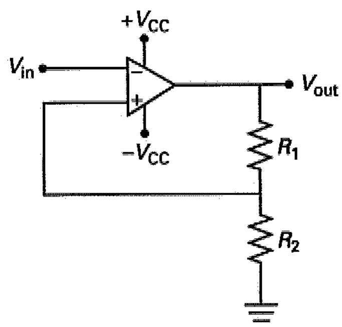


(D)



Because of the noise, a simple comparator such as in option (A) will not work. There will be false zero crossings where the noise crosses zero at the signal crossing.

The configuration shown in option (C) will not work for the same reason as the comparator. The device shown in option (B) is a noninverting amplifier, which will amplify the entire signal. A device with hysteresis, such as a Schmitt trigger, is needed. Such a device will not change until a threshold is reached, and will not change again until the negative threshold is reached. The configuration is



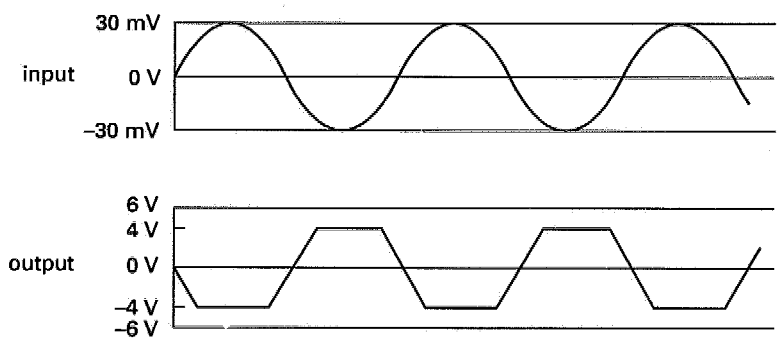
$$V_{\text{threshold}} = \left(\frac{\pm R_2}{R_1 + R_2}\right) V_{CC}$$

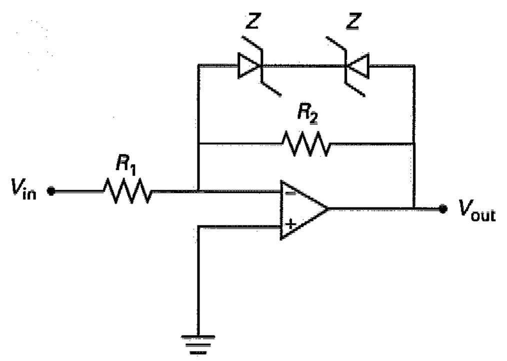
The only Schmitt trigger circuit is given in option (D). With the R_1 and R_2 resistances shown, it will trigger at $V = (1\ \text{k}\Omega/2\ \text{k}\Omega + 1\ \text{k}\Omega)(15\ \text{V}) = 5\ \text{V}$, which will work. There will be some delay, and the signal will be inverted.

The answer is (D).

AC ELECTRICITY–29

A 30 mV sinusoidal signal must be inverted, amplified to 6 V, and chopped at 4 V. If the following circuit is used, what are the values of R_1, R_2, and the avalanche voltage of the zener diodes, Z? There is a forward voltage drop of −0.7 V for the diodes.





(A) $R_1 = 1\ \text{k}\Omega,\ R_2 = 20\ \text{k}\Omega,\ Z = 4.0\ \text{V}$

(B) $R_1 = 1\ \text{k}\Omega,\ R_2 = 200\ \text{k}\Omega,\ Z = 4.0\ \text{V}$

(C) $R_1 = 2\ \text{k}\Omega,\ R_2 = 400\ \text{k}\Omega,\ Z = 3.3\ \text{V}$

(D) $R_1 = 2\ \text{k}\Omega,\ R_2 = 800\ \text{k}\Omega,\ Z = 3.3\ \text{V}$

The amplification is similar to that found in the normal inverting amplifier.

$$V_{\text{out}} = -\frac{R_2}{R_1}V_{\text{in}}$$

$$\frac{R_2}{R_1} = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{6\ \text{V}}{30 \times 10^{-3}\ \text{V}}$$

$$= 200$$

This leaves options (B) and (C) to be the possible choices.

When $\pm V_{\text{out}} < Z$, one of the diodes will be reverse biased, but not avalanched. There is essentially an open circuit across the diodes, and the amplifier is a simple inverting amplifier. When $\pm V_{\text{out}} \geq Z$, one of the diodes is forward biased with a voltage drop of 0.7 V, while the other diode is avalanched at the zener voltage. This keeps V_{out} constant. Thus, $V_{\text{out}} = 4$ V is equal to the zener voltage plus the 0.7 V voltage drop of the other diode.

$$V_{\text{out}} = Z + 0.7\ \text{V}$$

$$Z = 4\ \text{V} - 0.7\ \text{V} = 3.3\ \text{V}$$

So, $R_2 = 200R_1$ and $Z = 3.3$ V.

The answer is (C).

AC ELECTRICITY–30

An AC alternator operated as a motor is called a synchronous motor. Which of the following statements regarding synchronous motors is FALSE?

(A) The average speed, regardless of load, does not decrease, since the motor must operate at a constant speed.

(B) When a load is increased, the increased torque is a result of the shift in the relative positions of the fields on the rotor and stator.

(C) The relationship between speed, frequency, and number of poles is the same for the rotating field of the induction motor and for the alternator.

(D) The poles of a synchronous motor must be salient.

Salient poles have laminated pole pieces. Although salient poles are generally used, either salient or nonsalient poles can be used in a synchronous motor.

The answer is (D).

AC ELECTRICITY–31

Which of the following statements about induction motors is FALSE?

(A) They are used to increase the line power factor.
(B) They have no slip rings, no brushes, and no excited field current.
(C) They have no commutators and no windings on the armature.
(D) Squirrel-cage induction motors operate at essentially constant speeds.

Induction motors degrade the power factor. All the other answer choices are true.

The answer is (A).

AC ELECTRICITY–32

A single-phase induction motor is not self-starting. Instead, auxiliary methods must be used, such as varying inductance, resistance, and capacitance. Which of the following is FALSE regarding this situation?

(A) A capacitor motor uses capacitance to split the phase, resulting in two phases almost 90° apart.

(B) Capacitor motors have lower starting torque than comparably sized single-phase induction motors.

(C) To obtain a higher reactance, a capacitor can be used when starting and then be switched out of the circuit by mechanical means.

(D) If the capacitor remains in the circuit, the power factor will have a value close to unity.

Due to the favorable phase relationship, the torque is higher for a capacitor motor than for other types of single-phase motors. For example, a capacitive phase split motor gives better torque than a resistively split motor. Therefore, option (B) is false.

The answer is (B).

AC ELECTRICITY–33

A squirrel-cage motor has such low resistance that it draws excessive currents when starting. Which of the following actions will NOT reduce this problem?

(A) connecting the windings as in a three-phase, wye, transformer, taking 58% of the normal line voltage; then, at sufficient motor speed, switching to a delta connection

(B) using an in-line rheostat

(C) using an autotransformer to reduce line voltage

(D) using a class A motor

A class A motor draws a heavy starting current, usually 200–300% of the normal load. The other alternatives reduce the effective voltage across the windings, thus reducing the problem of excessive currents.

The answer is (D).

AC ELECTRICITY–34

Which of the following statements about AC generators is FALSE?

(A) The poles of an AC generator are located on the rotor.

(B) The three main types of AC generator are direct-connect engine driven, water driven, and turbine driven.

(C) An AC generator uses commutators.

(D) Large turbine driven generators usually have two pole rotors to accommodate the high speed of the turbine.

Commutators are not used in AC machines. It is the relative motion between the rotor and the stationary armature located on the stator that generates the power.

The answer is (C).

AC ELECTRICITY–35

Which of the following is FALSE regarding rotating machinery?

(A) The avoidance of harmonics in the production of a sine wave can be achieved by using a coil having multiple loops passing through adjacent slots rather than using only one pair of slots.

(B) Uniformity in the production of flux on a pole can be obtained by using distributed field windings over a portion of the rotor surface.

(C) AC generator ratings are usually given in units of kVA (kilovolt amps).

(D) At zero power factor, the generator delivers real power to a load.

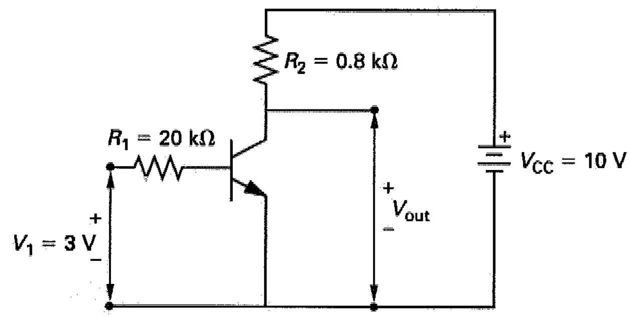
A power factor of 1 delivers only real power and no reactive power to a resistive load. A zero power factor is associated with a nonresistive load or with no-load conditions.

The answer is (D).

AC ELECTRICITY–36

In the following transistor circuit, β is 100, and the DC base-to-emitter voltage is 0.6 V. What is the output voltage, V_{out}?

(A) 0.3 V (B) 2 V (C) 3 V (D) 10 V



$$I_B = \frac{V_1 - V_{be}}{R_1} = \frac{3\text{ V} - 0.6\text{ V}}{20 \times 10^3\ \Omega} = 0.000\,12\text{ A} \quad (0.12\text{ mA})$$

$$I_C = (1+\beta)I_B = (1+100)(0.12\text{ mA}) = 12.1\text{ mA}$$

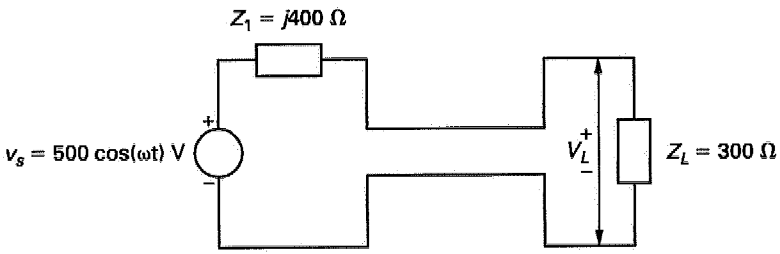
$$V_{out} = V_{CC} - I_c R_2 = 10\text{ V} - (12.1\text{ mA})\left(\frac{1\text{ A}}{1000\text{ mA}}\right)(0.8\text{ k}\Omega)\left(\frac{1000\ \Omega}{1\text{ k}\Omega}\right)$$

$$= 0.3\text{ V}$$

The answer is (A).

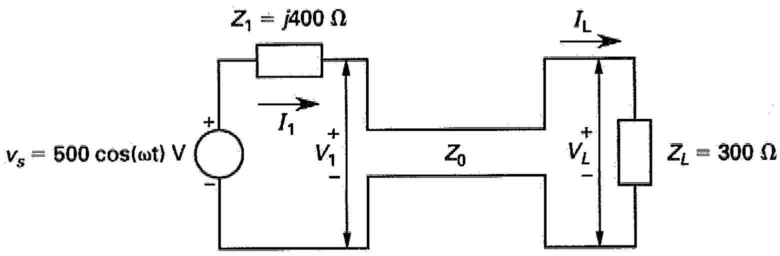
AC ELECTRICITY–37

The circuit shown represents a matched lossless transmission line. What is the maximum load voltage, V_L?



(A) 200 V (B) 300 V (C) 400 V (D) 500 V

For a matched lossless transmission line, the characteristic impedance, Z_0, equals the load impedance, Z_L, with $V_1 = V_L$ and $I_1 = I_L$ as shown in the following illustration .



$$I_1 = \frac{v_s}{Z_1 + Z_0}$$

$$V_L = I_1 Z_L = \left(\frac{v_s}{Z_1 + Z_0}\right) Z_L$$

$$V_{L,\text{max}} = v_{s,\text{max}} \left|\frac{Z_L}{Z_1 + Z_0}\right| = (500 \text{ V}) \left(\left|\frac{300 \ \Omega}{300 \ \Omega + j400 \ \Omega}\right|\right) = 300 \text{ V}$$

The answer is (B).

AC ELECTRICITY–38

A 10-pole synchronous motor operates on a 60 cycle voltage. What is the speed of the motor?

(A) 520 rpm (B) 620 rpm (C) 660 rpm (D) 720 rpm

The synchronous speed for AC motors is

$$\begin{aligned} n_s &= \frac{120f}{p} = \frac{(120)(60\ \text{Hz})}{10} \\ &= 720\ \text{rpm} \end{aligned}$$

The answer is (D).

AC ELECTRICITY–39

The core of a 400 Hz aircraft transformer has a net cross-sectional area of 13 cm^2. The maximum flux density is 0.9 T, and there are 70 turns in the secondary coil. What is most nearly the rms voltage induced in the secondary coil?

(A) 130 V (B) 150 V (C) 170 V (D) 1500 V

The induced voltage is

$$V = -N\frac{d\phi}{dt} = -N\frac{d}{dt}\int_s B\,ds$$

In the preceding equation, N is the number of turns, and B is the flux density in Teslas. $B = 0.9\sin\omega t$, where $\omega = 2\pi f = 2\pi(400\ \text{rad/s})$.

Therefore,

$$V = -N\frac{d}{dt}BA$$

In the preceding formula, A is the cross-sectional area of the core.

Therefore,

$$\begin{aligned} V_{\text{rms}} &= \frac{N\omega BA}{\sqrt{2}} = \frac{(70)(2\pi)(400\ \text{Hz})(0.9\ \text{T})(13\times 10^{-4}\ \text{m}^2)}{\sqrt{2}} \\ &= 146\ \text{V} \quad (150\ \text{V}) \end{aligned}$$

The answer is (B).

AC ELECTRICITY–40

A 150 kVA, 1000 V single-phase alternator has an open circuit emf of 750 V. When the alternator is short circuited, the armature current is 460 A. What is most nearly the synchronous impedance?

(A) 1.6 Ω (B) 2.2 Ω (C) 2.6 Ω (D) 3.2 Ω

Synchronous impedance, Z, is

$$Z = \frac{V_{oc}}{I_a}$$

V_{oc} is the open-circuit voltage, and I_a is the armature current when the alternator is short circuited. Therefore,

$$Z = \frac{750 \text{ V}}{460 \text{ A}} = 1.63 \ \Omega \quad (1.6 \ \Omega)$$

The answer is (A).

AC ELECTRICITY–41

In a balanced three-phase system with a power factor of unity, the line voltage, E_l, and the line current, I_l, deliver normal AC power. What is the expression for the power, P?

(A) $P = E_l I_l$

(B) $P = \frac{1}{2} E_l I_l$

(C) $P = \frac{1}{\sqrt{2}} E_l I_l$

(D) $P = \sqrt{3} E_l I_l$

The power developed by a three-phase generator is three times the coil voltage, E_c, multiplied by the coil current, I_c.

$$P = 3E_c I_c$$

The line voltage has the following relationship with the coil voltage.

$$E_l = \sqrt{3} E_c$$

Therefore, since $I_c = I_l$ for a power factor of 1,

$$P = \frac{3}{\sqrt{3}} E_l I_l$$
$$= \sqrt{3} E_l I_l$$

The answer is (D).

AC ELECTRICITY–42

A three-phase alternator has three armature coils, each rated at 1200 V and 120 A. What is most nearly the kVA rating of this unit?

(A) 430 kVA (B) 440 kVA (C) 520 kVA (D) 540 kVA

The kVA rating is equal to the power output.

$$\text{kVA} = 3E_c I_c$$
$$= (3)(1200 \text{ V})(120 \text{ A})$$
$$= 432\,000 \text{ VA} \quad (430 \text{ kVA})$$

The answer is (A).

AC ELECTRICITY–43

What is the relationship between the line current, I_l, and the coil current, I_c, in a balanced delta system?

(A) $I_l = \frac{I_c}{\sqrt{3}}$ (B) $I_l = \frac{I_c}{\sqrt{2}}$ (C) $I_l = I_c$ (D) $I_l = \sqrt{3} I_c$

The three-phase line-phase relations for a balanced three-phase delta system are

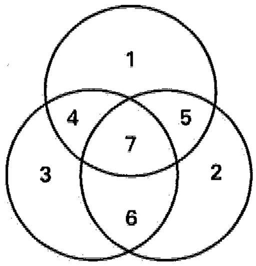
$$I_l = \sqrt{3} I_p = \sqrt{3} I_c$$

The answer is (D).

14 PHYSICS

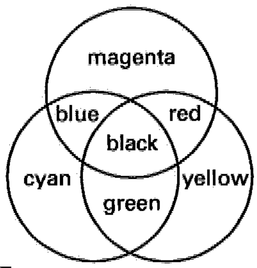
PHYSICS-1

The figure shown is used to indicate combinations of color primaries for subtractive mixing of colors. Which one of the following is true?



(A) 1 = green, 6 = magenta, 4 = yellow
(B) 1 = yellow, 4 = blue, 7 = white
(C) 1 = cyan, 5 = green, 7 = white
(D) 1 = magenta, 5 = red, 7 = black

This figure could be rotated so that 1 = magenta, yellow, or cyan. However, the only choice that has all three colors in the proper places is option (D).



The answer is (D).

PHYSICS–2

Which of the following statements is FALSE?

(A) wavelength of visible light > wavelength of microwaves
(B) frequency of radio waves < frequency of infrared waves
(C) wavelength of x-rays > wavelength of gamma rays
(D) frequency of ultraviolet > frequency of infrared

The electromagnetic spectrum is

radio waves	micro-waves	infrared	visible	ultraviolet	x-rays	gamma rays

increasing frequency ⟶

⟵ increasing wavelength

The wavelength of microwaves is greater than the wavelength of visible light. Therefore, option (A) is false.

The answer is (A).

PHYSICS–3

A light source emits a total luminous flux of 1000 lm distributed uniformly over a quarter of a sphere. What is most nearly the luminous intensity 2.5 m from the source?

(A) 42 lm/m^2 (B) 51 lm/m^2 (C) 58 lm/m^2 (D) 62 lm/m^2

Luminous intensity, I, is

$$I = \frac{L_o}{A}$$

In the preceding equation, L_o is the luminous flux and A is the surface area of sphere around the light source. The area of a quarter of a sphere is

$$A = \frac{4\pi r^2}{4} = \pi(2.5\ \text{m})^2$$
$$= 19.63\ \text{m}^2$$

$$I = \frac{1000\ \text{lm}}{19.63\ \text{m}^2} = 50.94\ \text{lm/m}^2 \quad (51\ \text{lm/m}^2)$$

The answer is (B).

PHYSICS–4

A 100 W lightbulb emits a total luminous flux of 1500 lm, distributed uniformly over a hemisphere. What is most nearly the illuminance at a distance of 2 m?

(A) 11 lm/m^2 (B) 21 lm/m^2 (C) 34 lm/m^2 (D) 60 lm/m^2

The illumination, E, is

$$E = \frac{\Phi}{A}$$

In the preceding equation, Φ is the luminous flux and A is the area.

$$\begin{aligned} A &= \frac{4\pi r^2}{2} \\ &= 2\pi(2\text{ m})^2 \\ &= 25.13\text{ m}^2 \end{aligned}$$

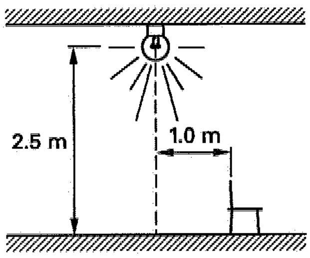
Therefore,

$$\begin{aligned} E &= \frac{1500\text{ lm}}{25.13\text{ m}^2} \\ &= 59.7\text{ lm/m}^2 \quad (60\text{ lm/m}^2) \end{aligned}$$

The answer is (D).

PHYSICS–5

A lightbulb is used to light a stage 2.5 m below. A chair sits on the stage 1.0 m from a spot directly below the bulb. If the bulb has a luminous intensity of 150 lm, what is most nearly the illumination on the floor around the chair?

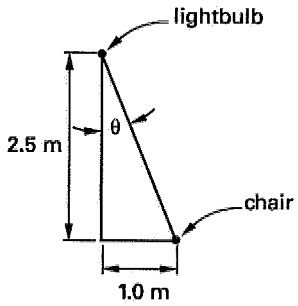


(A) 7.7 lm/m^2 (B) 19 lm/m^2 (C) 21 lm/m^2 (D) 51 lm/m^2

The illumination, E, is given by the following formula.

$$E = \frac{I\cos\theta}{r^2}$$

In the preceding equation, I is the luminous intensity of the source, θ is the angle from the normal to the surface the light strikes, and r is the distance from the light source.



$$\cos\theta = \frac{2.5 \text{ m}}{2.7 \text{ m}}$$

$$r = \sqrt{(2.5 \text{ m})^2 + (1 \text{ m})^2} = \sqrt{7.25 \text{ m}^2}$$

$$E = \frac{(150 \text{ lm})\left(\frac{2.5 \text{ m}}{2.7 \text{ m}}\right)}{7.25 \text{ m}^2}$$

$$= 19.2 \text{ lm/m}^2 \quad (19 \text{ lm/m}^2)$$

The answer is (B).

PHYSICS–6

Light of wavelength λ and intensity I_0 passes through a 0.05 m thick slab of glass whose absorption coefficient for that wavelength is 15 m^{-1}. What is most nearly the intensity, I, of the light after passing through the slab?

(A) $0.3I_0$ (B) $0.5I_0$ (C) $0.6I_0$ (D) $0.8I_0$

$$I = I_0 e^{-\alpha x}$$

The absorption coefficient, α, is 15 m^{-1}. Therefore,

$$I = I_0 e^{-(15/\text{m})(0.05 \text{ m})}$$

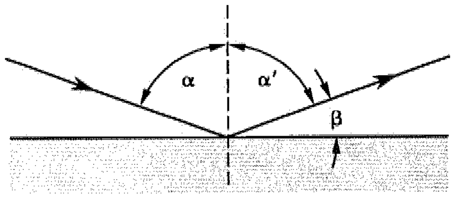
$$= 0.47I_0 \quad (0.5I_0)$$

The answer is (B).

PHYSICS–7

A light ray passing through air ($n = 1$) strikes a glass surface ($n_{\text{glass}} = 1.52$) at an angle of $\alpha = 60°$ from the normal to the surface. What is the angle, β, between the reflected light and the surface?

(A) 7.5° (B) 15° (C) 30° (D) 45°



The reflection law states that the angle of incidence is equal to the angle of reflection ($\alpha = \alpha'$). Therefore, $\alpha' = 60°$ and $\beta = 30°$.

The answer is (C).

PHYSICS–8

Which material type usually has a higher index of refraction?

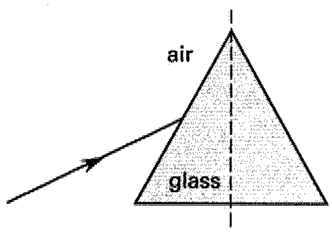
(A) lighter materials (B) heavier materials (C) denser materials (D) less-dense materials

In general, denser materials have higher indices of refraction.

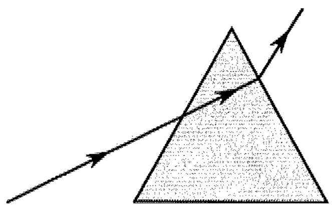
The answer is (C).

PHYSICS–9

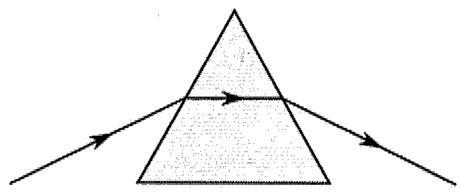
What is the path of the refracted ray in the following illustration?



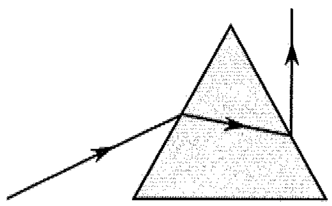
(A)



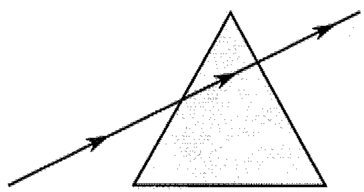
(B)



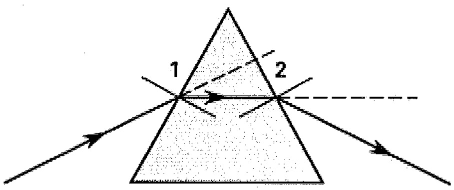
(C)



(D)



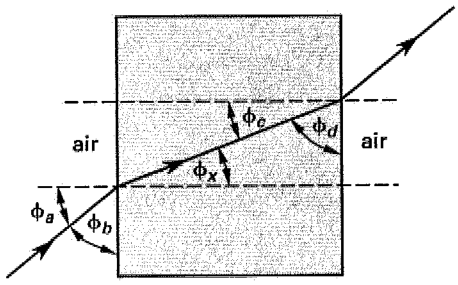
The light ray is refracted at interface 1. It is not normal to the surface of the glass, so its path changes direction. This eliminates options (A) and (D). Since $n_{\text{glass}} > n_{\text{air}}$, the ray is bent toward the normal to the surface of the glass at interface 1, and away from the normal to the surface at interface 2. Thus, option (B) is the correct path of the ray.



The answer is (B).

PHYSICS–10

How can the index of refraction of material x be defined?



material *x*

(A) $n_x = \frac{\sin \phi_a}{\sin \phi_b}$ (B) $n_x = \frac{\sin \phi_a}{\sin \phi_c}$ (C) $n_x = \frac{\sin \phi_c}{\sin \phi_d}$ (D) $n_x = \frac{\sin \phi_b}{\sin \phi_c}$

Indices of refraction are defined such that $n_a \sin \phi_a = n_x \sin \phi_x$, where n_a and n_x are the indices of refraction of materials a and x, and ϕ_a and ϕ_b are the angles between the light ray and the normal to the interface between the two materials.

The reference index for air, n_a, is 1. Therefore,

$$n_x = \frac{\sin \phi_a}{\sin \phi_x}$$

From the illustration,

$$\phi_x = \phi_c$$

$$n_x = \frac{\sin \phi_a}{\sin \phi_c}$$

The answer is (B).

PHYSICS–11

What is the index of refraction of a material if the speed of light through the material is 2.37×10^8 m/s?

(A) 1.10 (B) 1.19 (C) 1.27 (D) 1.34

The index of refraction of a material is

$$n = \frac{c}{\mathrm{v}}$$

$$n = \frac{3 \times 10^8 \ \frac{\mathrm{m}}{\mathrm{s}}}{2.37 \times 10^8 \ \frac{\mathrm{m}}{\mathrm{s}}} = 1.27$$

The answer is (C).

PHYSICS–12

What is most nearly the speed of light through glass that has an index of refraction of 1.33?

(A) 1.1×10^8 m/s (B) 2.3×10^8 m/s (C) 2.5×10^8 m/s (D) 2.8×10^8 m/s

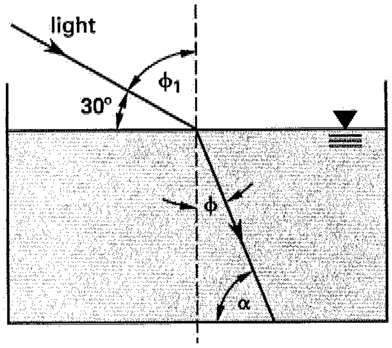
$$n = \frac{c}{\mathrm{v}}$$

$$\mathrm{v} = \frac{c}{n} = \frac{3.0 \times 10^8 \ \frac{\mathrm{m}}{\mathrm{s}}}{1.33} = 2.26 \times 10^8 \ \mathrm{m/s} \quad (2.3 \times 10^8 \ \mathrm{m/s})$$

The answer is (B).

PHYSICS–13

Light hits the surface of a trough of water at an angle of 30° from horizontal. The index of refraction of water is 1.333. What is most nearly the angle, α, in the illustration?



(A) 30° (B) 34° (C) 41° (D) 50°

First, use Snell's law to find ϕ. Then, use ϕ to find α. Since $n_a = 1$ for air,

$$n_a \sin\phi_1 = n_w \sin\phi$$

$$\sin\phi_1 = n\sin\phi$$

$$\phi_1 = 90^\circ - 30^\circ = 60^\circ$$

$$\sin\phi = \frac{\sin 60^\circ}{1.333} = 0.650$$

$$\phi = 40.5^\circ$$

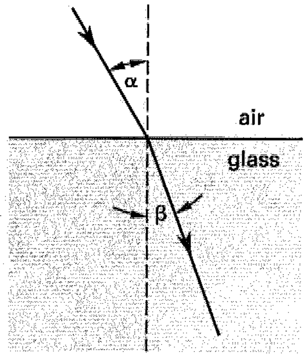
$$\alpha = 90^\circ - \phi = 90^\circ - 40.5^\circ = 49.5^\circ \quad (50^\circ)$$

The answer is (D).

PHYSICS–14

A light ray in air ($n_{\text{air}} = 1$) is incident on a glass surface ($n_{\text{glass}} = 1.52$) at an angle of 30° from the normal. What is most nearly the angle between the refracted light ray and the normal?

(A) 16° (B) 19° (C) 30° (D) 45°



$$n_a \sin\alpha = n_g \sin\beta$$

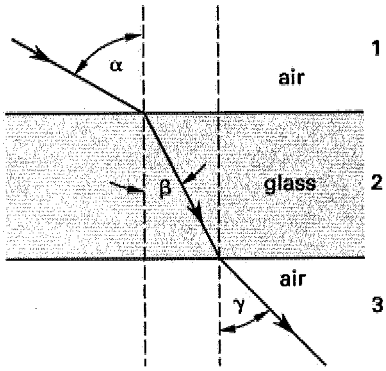
$$\sin\beta = \frac{n_a \sin\alpha}{n_g} = \frac{1 \sin 30^\circ}{1.52} = 0.3289$$

$$\beta = 19.2^\circ \quad (19^\circ)$$

The answer is (B).

PHYSICS–15

Given that $\alpha = 60°$, $n_{\text{air}} = 1$, and $n_{\text{glass}} = 1.52$, find the angle, γ, in the illustration.



(A) 15° (B) 30° (C) 45° (D) 60°

Use Snell's law at each interface.

$$n_{\text{air}} \sin\alpha = n_{\text{glass}} \sin\beta$$

$$n_{\text{glass}} \sin\beta = n_{\text{air}} \sin\gamma$$

Therefore,

$$n_{\text{air}} \sin\alpha = n_{\text{air}} \sin\gamma$$

$$\sin\gamma = \sin\alpha$$

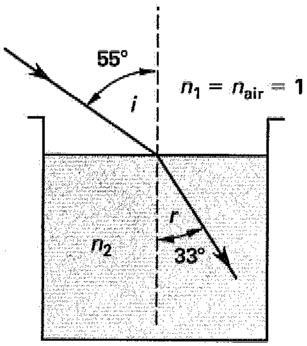
$$= \sin 60°$$

$$\gamma = 60°$$

The answer is (D).

PHYSICS–16

A student has a beaker of unknown liquid. When a beam of light of wavelength $\lambda = 0.59$ Å shines into the liquid at an angle of 55° from the normal to the surface, the refracted beam continues at an angle of 33° from the normal to the surface. What is the liquid?



(A) acetic acid, $n = 1.30$
(B) water, $n = 1.33$
(C) nitric acid, $n = 1.40$
(D) benzene, $n = 1.50$

$$n_1 \sin i = n_2 \sin r$$

In the preceding equation, i is the angle of incidence, and r is the angle of refraction.

$$\begin{aligned} n_2 &= n_1 \frac{\sin i}{\sin r} \\ &= (1)\left(\frac{\sin 55°}{\sin 33°}\right) \\ &= 1.50 \end{aligned}$$

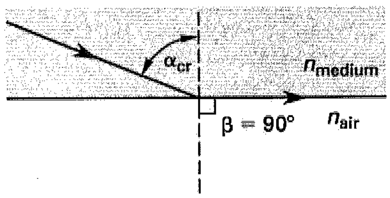
The only liquid listed with an index of refraction of 1.50 is benzene.

The answer is (D).

PHYSICS–17

A light ray in a medium ($n_{medium} = 1.7$) is totally reflected when it strikes the interface between the medium and air. In order for this phenomenon to occur, what should most nearly be the critical (i.e., minimum) angle between the light ray and the normal to the surface?

(A) 0° (B) 15° (C) 18° (D) 36°



Since the index of refraction is greater for the medium than for air, total internal reflection may occur. For total internal reflection to occur, the angle of refraction, β, must be at least 90°.

$$n_{medium} \sin \alpha_{cr} = n_{air} \sin \beta$$

$$\sin \alpha_{cr} = \frac{n_{air} \sin 90°}{n_{medium}}$$

$$= \frac{(1)(1)}{1.7}$$

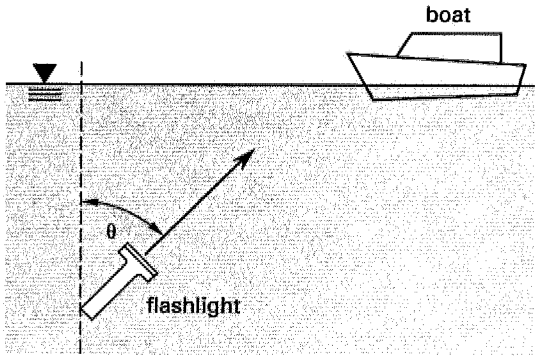
$$= 0.5882$$

$$\alpha_{cr} = 36°$$

The answer is (D).

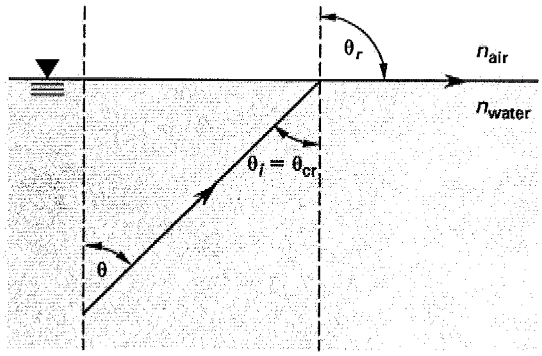
PHYSICS–18

An underwater diver signals his partner in a boat using a very bright flashlight. Assuming the water is crystal clear and the surface is perfectly calm, at what angle from vertical, θ, can the diver hold his flashlight and still have its beam visible above the surface? (The indices of refraction are $n_{\text{water}} = 1.33$ and $n_{\text{air}} = 1.00$.)



(A) 32.5° (B) 41.2° (C) 45.0° (D) 48.8°

Since $n_{\text{air}} < n_{\text{water}}$, total internal reflection can occur. This happens when the incident angle, θ, is large enough so that the refracted angle is at least 90°. The incident angle for which the refracted angle is exactly 90° is called the critical angle, θ_{cr}. The critical angle can be found from Snell's law.



$$\theta = \theta_i$$

$$\sin\theta_{cr} = \frac{n_{air}}{n_{water}}\sin 90^\circ$$

$$= \frac{n_{air}}{n_{water}}$$

$$= \frac{1.00}{1.33}$$

$$\theta_{cr} = 48.8^\circ$$

For the light to be seen above the surface, θ must be less than θ_{cr}.

The answer is (D).

PHYSICS–19

The radius of curvature of a convex spherical mirror is 48 cm. What are the focal length and focal type?

(A) −12 cm, virtual focus
(B) −24 cm, virtual focus
(C) 12 cm, real focus
(D) 24 cm, real focus

For a convex mirror, the radius of curvature, R, is negative. The equation for the focal length of a spherical mirror is

$$\frac{1}{f} = \frac{2}{R}$$

$$f = \frac{R}{2}$$

$$= \frac{-48 \text{ cm}}{2}$$

$$= -24 \text{ cm}$$

The negative sign indicates that the mirror has a virtual focus. The focal length is −24 cm.

The answer is (B).

PHYSICS–20

An object 6 cm high is placed 12 cm away from a concave mirror whose focal length is 36 cm. What is the height of the image?

(A) 2 cm (B) 4 cm (C) 5 cm (D) 9 cm

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

In the preceding equation, q is the image distance, p is the object distance, and f is the focal length.

$$\begin{aligned} q &= \frac{pf}{p-f} \\ &= \frac{(12\text{ cm})(36\text{ cm})}{12\text{ cm} - 36\text{ cm}} \\ &= -18\text{ cm} \end{aligned}$$

$$\begin{aligned} \frac{I}{O} &= \left|\frac{q}{p}\right| \\ &= \frac{18\text{ cm}}{12\text{ cm}} \\ &= 1.5 \end{aligned}$$

In the preceding equation, I is the image size and O is the object size.

$$\begin{aligned} I &= (1.5)O \\ &= (1.5)(6\text{ cm}) \\ &= 9\text{ cm} \end{aligned}$$

The answer is (D).

PHYSICS–21

An object is placed 10 cm away from a concave mirror with a focal length of 30 cm. What image is formed?

(A) a real image 10 cm in front of the mirror
(B) a virtual image 15 cm behind the mirror
(C) a real image 20 cm in front of the mirror
(D) a virtual image 25 cm behind the mirror

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

In the preceding equation, p is the object distance, q is the image distance, and f is the focal length.

$$\begin{aligned} q &= \frac{pf}{p-f} \\ &= \frac{(10\ \text{cm})(30\ \text{cm})}{10\ \text{cm} - 30\ \text{cm}} \\ &= -15\ \text{cm} \end{aligned}$$

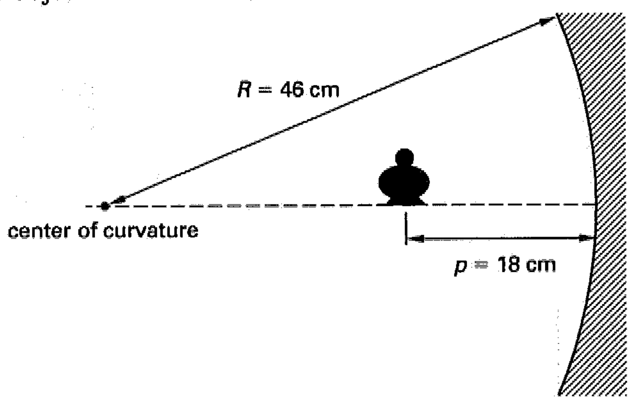
The negative sign indicates that this is a virtual image, 15 cm behind the mirror.

The answer is (B).

PHYSICS–22

A chess piece is placed 18 cm in front of a concave mirror that has a radius of curvature of 46 cm. Which of the following statements about the image is/are true?

I. The image is larger than the object.
II. The image is real.
III. The image is upright.
IV. The image is beyond the center of curvature.
V. The object is at the focus.



(A) V only (B) I and III (C) II and IV (D) I, III, and IV

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

In the preceding equation, p is the object distance, q is the image distance, and f is the focal length.

$$\begin{aligned} f &= \frac{R}{2} \\ &= \frac{46 \text{ cm}}{2} \\ &= 23 \text{ cm} \end{aligned}$$

$$\begin{aligned} q &= \frac{pf}{p - f} \\ &= \frac{(18 \text{ cm})(23 \text{ cm})}{18 \text{ cm} - 23 \text{ cm}} \\ &= -82.8 \text{ cm} \end{aligned}$$

Thus, the image is virtual and behind the mirror. Statements II, IV, and V are incorrect.

Determine the magnification, m.

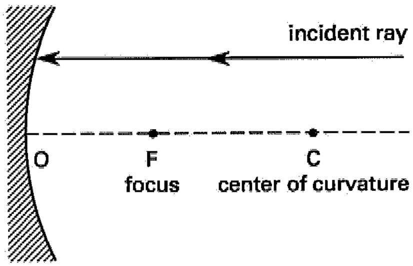
$$\begin{aligned} m &= -\frac{q}{p} \\ &= -\left(\frac{-82.8 \text{ cm}}{18 \text{ cm}}\right) \\ &= +4.6 \end{aligned}$$

The positive sign indicates that the image is upright. Therefore, III is true. Since $m > 1$, the image is larger than the object. Thus, I is also true.

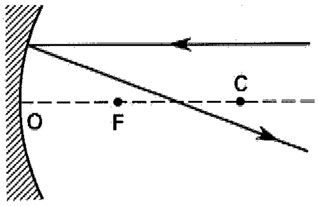
The answer is (B).

PHYSICS–23

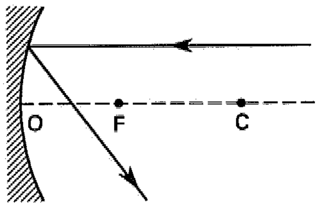
Consider the concave spherical mirror in the illustration. What is the path of the reflected ray?



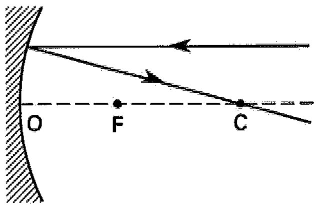
(A)



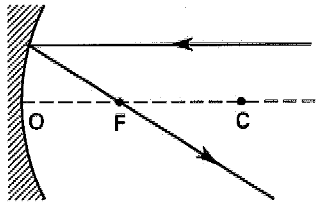
(B)



(C)



(D)

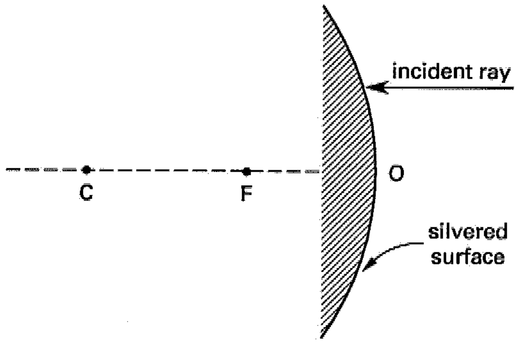


If an incident ray is parallel to the principal axis of a concave mirror, the reflected ray will pass through the focus point, F. Therefore, option (D) is correct.

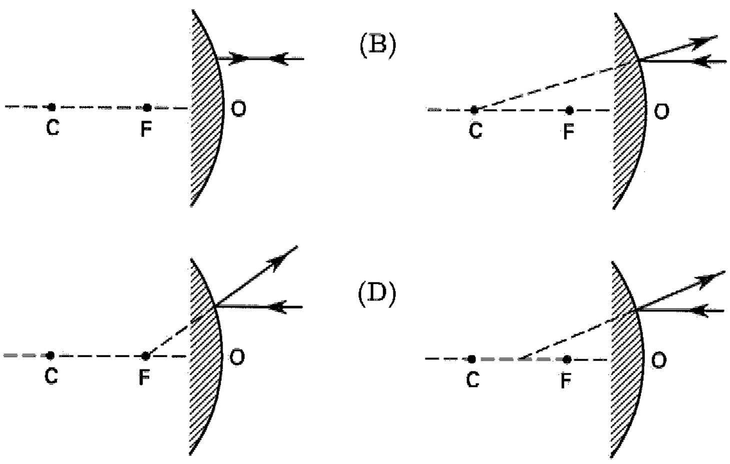
The answer is (D).

PHYSICS–24

Consider the convex spherical mirror shown. What is the path of the reflected ray?



(A) (B) (C) (D)

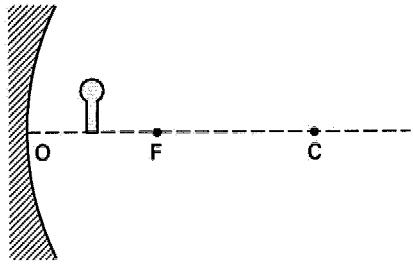


If the incident ray is parallel to the principal axis of a convex mirror, the reflected ray follows a path such that its extension passes through the focus. Therefore, option (C) is correct.

The answer is (C).

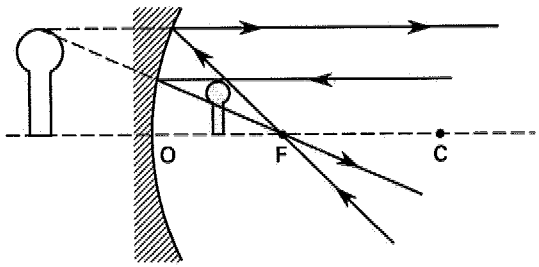
PHYSICS–25

Which of the following describes the image of an object that is placed between the focus and the concave spherical mirror?



(A) real, larger, inverted
(B) virtual, larger, not inverted
(C) virtual, smaller, not inverted
(D) real, smaller, inverted

Construct a ray diagram to find the image.

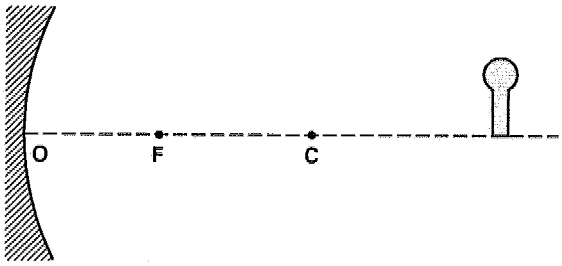


The image is virtual, behind the mirror, larger, and not inverted. Therefore, option (B) is the correct answer.

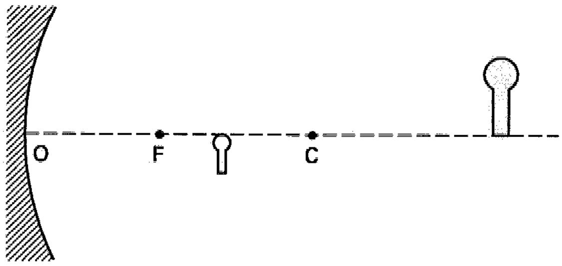
The answer is (B).

PHYSICS–26

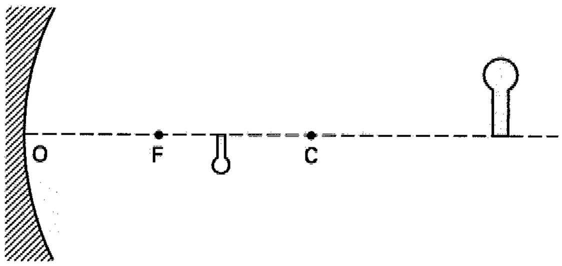
Which of the following options correctly depicts the image of the object as shown in the concave spherical mirror?



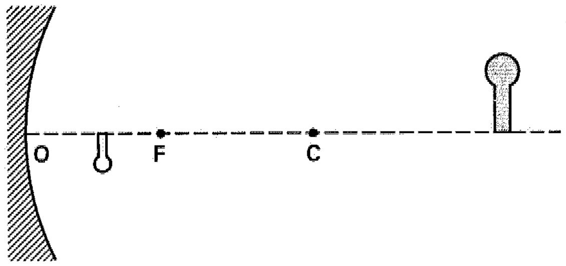
(A)



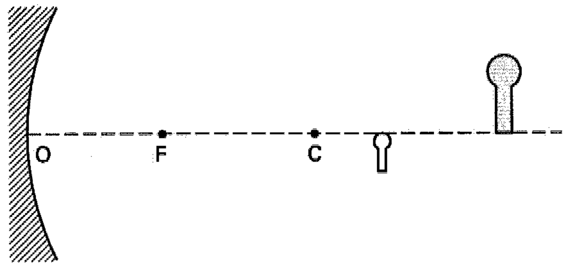
(B)



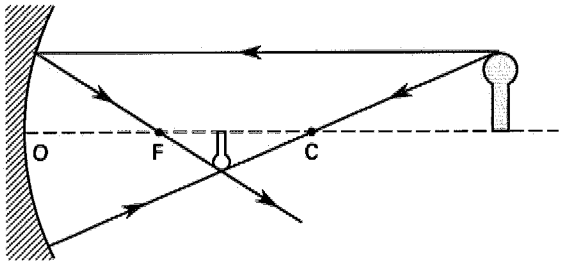
(C)



(D)



Construct a ray diagram to find the image.



Thus, option (B) gives the correct location and type of the image.

The answer is (B).

PHYSICS–27

A thin lens is made from glass with $n = 1.5$. It has a convex face with a 25 cm radius of curvature and a concave face with a 35 cm radius of curvature. What are the focal length and type of the lens?

(A) diverging lens, virtual focus, focal length of 100 cm
(B) converging lens, real focus, focal length of 125 cm
(C) diverging lens, virtual focus, focal length of 150 cm
(D) converging lens, real focus, focal length of 175 cm

The radius of curvature for the convex face, R_1, is positive, but for the concave face, R_2, it is negative.

$$\begin{aligned}\frac{1}{f} &= (n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\\ &= (1.5-1)\left(\frac{1}{25\text{ cm}}-\frac{1}{35\text{ cm}}\right)\\ &= \frac{1}{175\text{ cm}}\\ f &= 175\text{ cm}\end{aligned}$$

The focal length is positive, indicating a real focus and, therefore, a converging lens with a focal length of 175 cm.

The answer is (D).

PHYSICS–28

An object 0.31 m tall is placed 0.61 m from a converging lens whose focal length is 0.46 m. Most nearly where is the image formed?

(A) 0.61 m from the lens
(B) 0.92 m from the lens
(C) 1.2 m from the lens
(D) 1.9 m from the lens

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

In the preceding equation, f is the focal length, p is the object distance, and q is the image distance.

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p}$$

$$q = \frac{pf}{p-f}$$

$$= \frac{(0.61\ \text{m})(0.46\ \text{m})}{0.61\ \text{m} - 0.46\ \text{m}}$$

$$= 1.87\ \text{m} \quad (1.9\ \text{m})$$

The answer is (D).

PHYSICS–29

What is the image position of an object placed 15 cm away from a thin, spherical converging lens with a focal length of 10 cm?

(A) 15 cm beyond the lens
(B) 20 cm beyond the lens
(C) 25 cm beyond the lens
(D) 30 cm beyond the lens

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

In the preceding equation, p is the object distance, q is the image distance, and f is the focal length.

$$\frac{1}{q} = \frac{p-f}{pf}$$

$$q = \frac{pf}{p-f}$$

$$= \frac{(15\ \text{cm})(10\ \text{cm})}{15\ \text{cm} - 10\ \text{cm}}$$

$$= 30\ \text{cm}$$

The positive value of q means that there is a real image on the opposite side of the lens from the object (beyond the lens).

The answer is (D).

PHYSICS–30

For of an object placed 25 cm from a diverging lens with a focal length of 15 cm, most nearly where is the image? What type of image is it?

(A) 9.4 cm behind the lens; virtual image
(B) 13 cm behind the lens; real image
(C) 15 cm behind the lens; virtual image
(D) 18 cm behind the lens; real image

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

In the preceding equation, p is the object distance, q is the image distance, and f is the focal length. Determine the focal length.

$$f = -15 \text{ cm} \quad [f \text{ is negative for a diverging lens}]$$

$$\frac{1}{q} = \frac{p-f}{pf}$$

$$q = \frac{pf}{p-f}$$

$$= \frac{(25 \text{ cm})(-15 \text{ cm})}{25 \text{ cm} - (-15 \text{ cm})}$$

$$= -9.375 \text{ cm} \quad (-9.4 \text{ cm})$$

This is a virtual image, located about 9.4 cm behind the lens.

The answer is (A).

PHYSICS–31

A magnifying glass has a plastic lens with an index of refraction of $n = 5.4$ and radii of curvature of 0.9 m and 1.3 m for the two faces. What is the magnification of the lens when it is held 0.1 m from an object being viewed?

(A) 1.5 (B) 4.5 (C) 6.0 (D) 6.5

A magnifying glass uses a biconvex lens. If f is the focal length of the lens, R_1 and R_2 are the radii of curvature, and n is the index of refraction, then

$$\begin{aligned}\frac{1}{f} &= (n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\\ &= (5.4-1)\left(\frac{1}{0.9\text{ m}}+\frac{1}{1.3\text{ m}}\right)\\ &= 8.3\text{ m}^{-1}\\ f &= \frac{1}{8.3\,\frac{1}{\text{m}}} = 0.12\text{ m}\end{aligned}$$

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

In the preceding equation, p is the object distance, and q is the image distance.

$$\begin{aligned}\frac{1}{q} &= \frac{1}{f}-\frac{1}{p}\\ q &= \frac{pf}{p-f}\\ &= \frac{(0.1\text{ m})(0.12\text{ m})}{0.1\text{ m}-0.12\text{ m}}\\ &= -0.6\text{ m}\end{aligned}$$

The magnification, m, is

$$\begin{aligned}m &= -\frac{q}{p}\\ &= -\frac{-0.6\text{ m}}{0.1\text{ m}}\\ &= 6\end{aligned}$$

The answer is (C).

PHYSICS–32

An astronomer observing the night sky has a telescope with an 0.2 m objective lens. If two stars 700 ly away are barely resolved by the telescope, approximately how far apart are they? Assume the light from both stars has a wavelength of $\lambda = 5500$ Å.

(A) 2.2×10^9 m (B) 1.1×10^{10} m (C) 2.2×10^{10} m (D) 2.2×10^{13} m

The minimum resolvable distance, d_0, between two objects is

$$d_0 = 1.22\frac{\lambda L}{D}$$

In the preceding equation, λ is the wavelength of light, L is the distance of the objects from the lens, and D is the diameter of the lens.

In the above problem,

$$\lambda = (5500\ \text{Å})\left(1.0 \times 10^{-10}\ \frac{\text{m}}{\text{Å}}\right) = 5.5 \times 10^{-7}\ \text{m}$$

$$L = 700\ \text{ly}$$

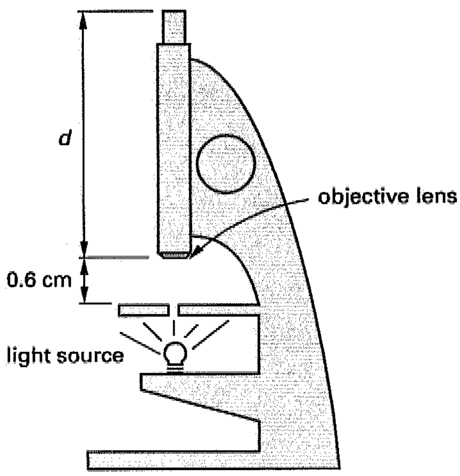
$$D = 0.2\ \text{m}$$

$$d_0 = \frac{(1.22)(5.5 \times 10^{-7}\ \text{m})(700\ \text{ly})\left(9.44 \times 10^{15}\ \frac{\text{m}}{\text{ly}}\right)}{0.2\ \text{m}} = 2.22 \times 10^{13}\ \text{m} \quad (2.2 \times 10^{13}\ \text{m})$$

The answer is (D).

PHYSICS–33

A microscope has an eyepiece with a focal length, f_e, of 2.5 cm and a magnification of 5. If the objective lens is 0.6 cm from the object being viewed and has a magnification of 10, what is the distance, d, between the two lenses?



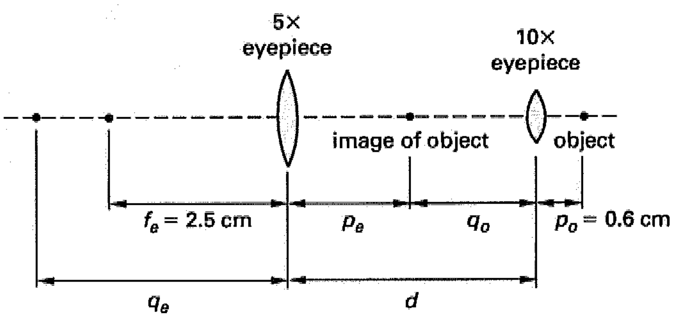
(A) 2.0 cm (B) 2.5 cm (C) 8.0 cm (D) 8.5 cm

For a lens with p as the object distance, q as the image distance, and f as the focal length, the lens equation is

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

The magnification, m, is

$$m = -\frac{q}{p}$$



The sign of m indicates whether or not the image is inverted. From the illustration, it can be seen that $d = |p_e| + |q_o|$. The subscripts denote the eyepiece, e, and the objective lens, o. First, find the distance between the image of the object and the objective lens.

$$m = -\frac{q_o}{p_o}$$

$$q_o = -mp_o$$

$$= -(10)(0.6\ \text{cm})$$

$$= -6.0\ \text{cm}$$

$$|q_o| = 6.0\ \text{cm}$$

Next, find p_e for the eyepiece.

$$\frac{1}{p_e} + \frac{1}{q_e} = \frac{1}{f_e}$$

$$m_e = -\frac{q_e}{p_e}$$

$$|q_e| = m_e p_e$$

$$\frac{1}{p_e} - \frac{1}{m_e p_e} = \frac{1}{f_e}$$

$$p_e = f_e - \frac{f_e}{m_e}$$

$$= 2.5\ \text{cm} - \frac{2.5\ \text{cm}}{5}$$

$$= 2.0\ \text{cm}$$

$$d = |q_o| + |p_e|$$

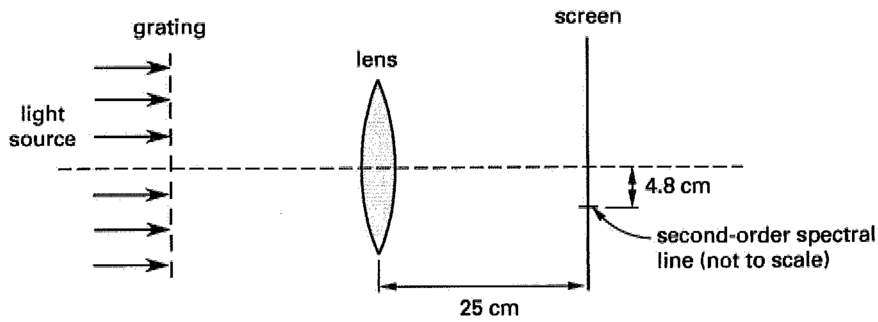
$$= 6.0\ \text{cm} + 2.0\ \text{cm}$$

$$= 8.0\ \text{cm}$$

The answer is (C).

PHYSICS–34

A diffraction grating set up as shown is used to find the wavelength of a light source. If the grating has 4000 lines/cm, the lens used to focus the light from the grating is 25 cm from the screen where the light is focused, and the second-order spectral line is 4.8 cm from the center position, what is the wavelength of the light?

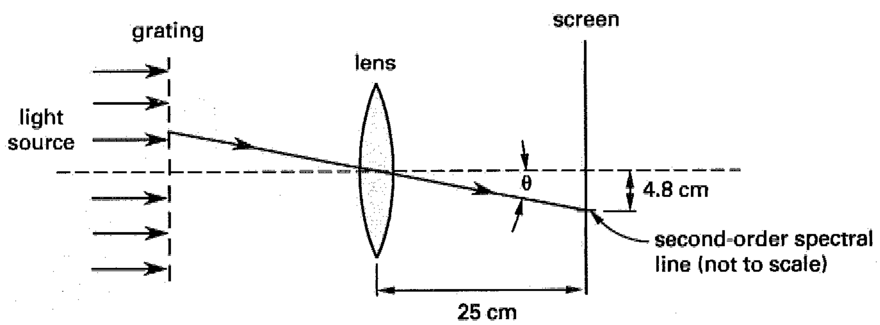


(A) 2000 Å (B) 2400 Å (C) 3200 Å (D) 7300 Å

The relationship between the wavelength, the position of the spectral line, and the diffraction grating is determined by the sine of the angle of two lines drawn from the center of the lens straight to the screen and the spectral line.

$$\sin\theta = \frac{n\lambda}{d}$$

In the preceding equation, n is the order of spectral line and d is the spacing of the grating.



The following information can be determined from the diagram.

$$\sin\theta = \frac{4.8 \text{ cm}}{\sqrt{(25 \text{ cm})^2 + (4.8 \text{ cm})^2}}$$
$$= 0.189$$
$$d = \frac{1 \text{ line}}{4000 \, \frac{\text{lines}}{\text{cm}}}$$
$$= 2.5 \times 10^{-4} \text{ cm}$$
$$n = 2 \quad \text{[for a second-order spectral line]}$$
$$\lambda = \frac{d\sin\theta}{n}$$
$$= \frac{(2.5 \times 10^{-4} \text{ cm})(0.189)}{2}$$
$$= 2.36 \times 10^{-5} \text{ cm} \quad (2400 \text{ Å})$$

The answer is (B).

PHYSICS–35

What is the photon energy associated with green-blue light of wavelength 500 nm? ($\hbar = 6.626 \times 10^{-34}$ J·s, and $c = 3.0 \times 10^8$ m/s.)

(A) 1×10^{-19} J (B) 2×10^{-19} J (C) 3×10^{-19} J (D) 4×10^{-19} J

$$E = \hbar f$$
$$= \frac{\hbar c}{\lambda}$$
$$\lambda = (500 \text{ nm})\left(1.0 \times 10^{-9} \, \frac{\text{m}}{\text{nm}}\right)$$
$$= 5.00 \times 10^{-7} \text{ m}$$
$$E = \frac{(6.626 \times 10^{-34} \text{ J·s})\left(3 \times 10^8 \, \frac{\text{m}}{\text{s}}\right)}{5 \times 10^{-7} \text{ m}}$$
$$= 3.98 \times 10^{-19} \text{ J} \quad (4 \times 10^{-19} \text{ J})$$

The answer is (D).

PHYSICS–36

What is most nearly the energy of a photon of wavelength 0.1 nm?

(A) 0.090×10^{-14} J (B) 0.11×10^{-14} J (C) 1.6×10^{-15} J (D) 2.0×10^{-15} J

$$\begin{aligned} E &= \hbar f \\ &= \frac{\hbar c}{\lambda} \\ &= \frac{(6.626 \times 10^{-34}\ \text{J·s})\left(3.0 \times 10^{8}\ \frac{\text{m}}{\text{s}}\right)}{1 \times 10^{-10}\ \text{m}} \\ &= 1.99 \times 10^{-15}\ \text{J} \quad (2.0 \times 10^{-15}\ \text{J}) \end{aligned}$$

The answer is (D).

PHYSICS–37

Light at wavelength 6493 Å is visible red light. It is, however, very close to the limits of the human eye. What is most nearly the energy of this light?

(A) 1.9×10^{-19} J (B) 2.5×10^{-19} J (C) 2.9×10^{-19} J (D) 3.1×10^{-19} J

$$\begin{aligned} E &= \hbar f \\ &= \frac{\hbar c}{\lambda} \end{aligned}$$

Planck's constant, $\hbar$, is

$$\begin{aligned} \hbar &= 6.626 \times 10^{-34}\ \text{J·s} \\ \lambda &= 6.493 \times 10^{-10}\ \text{m} \\ E &= \frac{(6.626 \times 10^{-34}\ \text{J·s})\left(3.0 \times 10^{8}\ \frac{\text{m}}{\text{s}}\right)}{6493 \times 10^{-10}\ \text{m}} \\ &= 3.06 \times 10^{-19}\ \text{J} \quad (3.1 \times 10^{-19}\ \text{J}) \end{aligned}$$

The answer is (D).

PHYSICS–38

To be effective, an omnidirectional alarm must have a minimum loudness of 70 dB. If it is to be effective 60 m away, what is most nearly the minimum power required?

(A) 0.1 W (B) 0.2 W (C) 0.3 W (D) 0.5 W

The intensity level, I, is related to distance and power according to

$$I = \frac{W}{4\pi r^2}$$

In the preceding equation, W is power and r is the distance from the source.

In decibels,

$$I = 10 \log \frac{I}{I_0}$$

$$I_0 = 1 \times 10^{-12} \text{ W/m}^2$$

$$I = I_0 10^{I/10}$$

$$W = I_0 10^{I/10}(4\pi r^2)$$

$$= \left(1 \times 10^{-12} \frac{\text{W}}{\text{m}^2}\right)(10^{70/10})(4\pi)(60 \text{ m})^2$$

$$= 0.45 \text{ W} \quad (0.5 \text{ W})$$

The answer is (D).

PHYSICS–39

A stationary observer hears a siren approaching. The siren has a sound frequency of 700 Hz and is approaching the observer at 80 km/h. What is the frequency heard by the observer? Assume the velocity of sound in air is 332 m/s.

(A) 600 Hz (B) 650 Hz (C) 700 Hz (D) 750 Hz

The frequency heard by the observer, f_o, is

$$f_o = f_s \left(\frac{\text{v} + \text{v}_o}{\text{v} - \text{v}_s}\right)$$

In the preceding equation, f_s is the frequency of the source, v is the velocity of wave transmission in the medium, v_o is the component of observer velocity directed toward the source, and v_s is the component of source velocity directed toward the observer.

$$v_s = \left(80\ \frac{\text{km}}{\text{h}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)\left(1000\ \frac{\text{m}}{\text{km}}\right)$$
$$= 22.2\ \text{m/s}$$
$$f_o = (700\ \text{Hz})\left(\frac{332\ \frac{\text{m}}{\text{s}} + 0\ \frac{\text{m}}{\text{s}}}{332\ \frac{\text{m}}{\text{s}} - 22.2\ \frac{\text{m}}{\text{s}}}\right)$$
$$= 750\ \text{Hz}$$

The answer is (D).

PHYSICS-40

A policeman waiting at a speed trap hears a car going by honking its horn. Being an amateur musician with perfect pitch, the policeman recognizes that the pitch of the horn is G$^\sharp$ (f = 415.30 Hz), a half-step lower than its normal pitch of A (f = 440.00 Hz). The velocity of sound in the air is 332 m/s. At what speed was the car traveling?

(A) 35 km/h (B) 67 km/h (C) 71 km/h (D) 83 km/h

The change in pitch is due to the Doppler effect. The frequency heard by the observer, f_o, is

$$f_o = f_s\left(\frac{v + v_o}{v - v_s}\right)$$

In the preceding equation, f_s is the frequency of source, v is the velocity of sound, v_o is the velocity of the observer, and v_s is the velocity of the source.

$$v = 332\ \text{m/s}$$
$$v_o = 0\ \text{m/s}$$

Solve for v_s.

$$\begin{aligned} v_s &= v - \frac{f_s}{f_o}(v + v_o) \\ &= \left(332\ \frac{\text{m}}{\text{s}} - \left(\frac{440.00\ \text{Hz}}{415.30\ \text{Hz}}\right)\left(332\ \frac{\text{m}}{\text{s}} + 0\ \frac{\text{m}}{\text{s}}\right)\right)\left(3600\ \frac{\text{s}}{\text{h}}\right)\left(\frac{1\ \text{km}}{1000\ \text{m}}\right) \\ &= -71\ \text{km/h} \quad [71\ \text{km/h away from the policeman}] \end{aligned}$$

The answer is (C).

PHYSICS–41

Solar collectors generate 1 kW/m^2 of thermal energy using the sun's energy. A collector with an area of 1 m^2 heats water. The water flow rate is 30 L/min. What is most nearly the temperature rise in the water? The specific heat of water is 4200 J/kg·°C.

(A) 0.1°C (B) 0.5°C (C) 5°C (D) 30°C

$$\begin{aligned} P &= \left(1\ \frac{\text{kW}}{\text{m}^2}\right)(1\ \text{m}^2)\left(1000\ \frac{\text{W}}{\text{kW}}\right) \\ &= 1000\ \text{W} \\ P &= \rho Q c_p \Delta T \end{aligned}$$

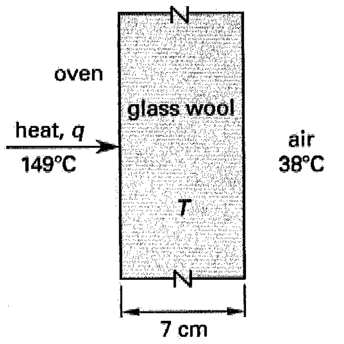
$$\begin{aligned} 1000\ \text{W} &= \left(1000\ \frac{\text{kg}}{\text{m}^3}\right)\left(30\ \frac{\text{L}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{1\ \text{m}^3}{1000\ \text{L}}\right)\left(4200\ \frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}\right)\Delta T \\ &= \left(2100\ \frac{\text{W}}{{}^\circ\text{C}}\right)\Delta T \end{aligned}$$

$$\begin{aligned} \Delta T &= \frac{1000\ \text{W}}{2100\ \frac{\text{W}}{{}^\circ\text{C}}} \\ &= 0.48^\circ\text{C} \quad (0.5^\circ\text{C}) \end{aligned}$$

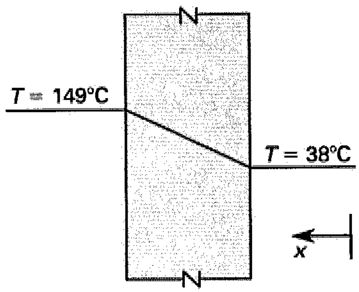
The answer is (B).

PHYSICS–42

A sketch of an oven wall is shown. What is most nearly the temperature in the center of the glass wool?



(A) 46°C (B) 54°C (C) 94°C (D) 98°C



For steady-state flow, the temperature profile is linear.

$$\frac{dT}{dx} = A$$

$$T = Ax + B$$

$$A = \frac{149°\text{C} - 38°\text{C}}{7 \text{ cm}} = 15.86°\text{C/cm}$$

$$B = 38°\text{C}$$

$$T = \left(15.86 \ \frac{°\text{C}}{\text{cm}}\right) x + 38°\text{C}$$

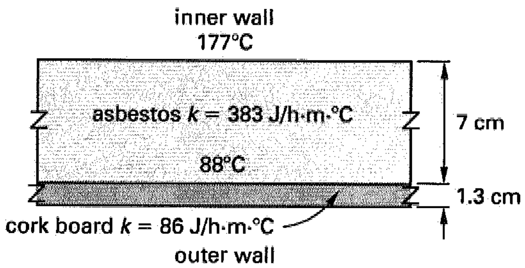
At the center,

$$x = 3.5 \text{ cm}$$
$$T = \left(15.86 \ \frac{°\text{C}}{\text{cm}}\right)(3.5 \text{ cm}) + 38°\text{C}$$
$$= 93.5°\text{C} \quad (94°\text{C})$$

The answer is (C).

PHYSICS–43

As a measure to reduce the outer wall temperature of an oven, an extra layer of cork insulation is added as shown. What is most nearly the temperature of the outer wall if the temperature between the asbestos and cork is 88°C?



(A) 14°C (B) 22°C (C) 31°C (D) 37°C

$$q_{\text{oven to asbestos}} = q_{\text{asbestos to cork}}$$
$$= -kA\frac{\Delta T}{\Delta x}$$

$$-k_{\text{asbestos}}A\left(\frac{88°\text{C} - 177°\text{C}}{\dfrac{7 \text{ cm}}{100 \ \frac{\text{cm}}{\text{m}}}}\right) = -k_{\text{cork}}A\left(\frac{T_{\text{out}} - 88°\text{C}}{\dfrac{1.3 \text{ cm}}{100 \ \frac{\text{cm}}{\text{m}}}}\right)$$

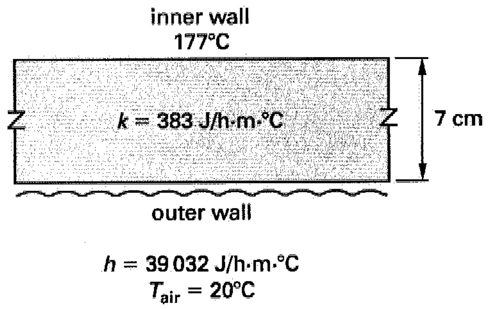
$$-\left(383 \ \frac{\text{J}}{\text{h·m·°C}}\right)\left(\frac{(-89°\text{C})(100 \text{ cm})}{7 \text{ cm}}\right) = -\left(86 \ \frac{\text{J}}{\text{h·m·°C}}\right) \times \left(\frac{(T_{\text{out}} - 88°\text{C})(100 \text{ cm})}{1.3 \text{ cm}}\right)$$

$$T_{\text{out}} = 14.4°\text{C} \quad (14°\text{C})$$

The answer is (A).

PHYSICS–44

An oven has an inner wall temperature of 177°C. The insulation is 7 cm thick and has a thermal conductivity of 383 J/h·m·°C. If the film coefficient of the outer wall is 39 032 J/h·m^2·°C, and the air is at 20°C, what is most nearly the temperature of the outer wall surface?



(A) 22°C (B) 32°C (C) 39°C (D) 42°C

$$q_{\text{oven to insulation}} = q_{\text{outer wall to air}}$$

$$-k\frac{(T_{\text{outer}} - T_{\text{inner}})}{t_{\text{insul}}}A = hA(T_{\text{outer}} - T_{\text{air}})$$

Rearrange to solve for T_{outer}.

$$T_{\text{outer}} = \frac{\left(\frac{k}{t_{\text{insul}}h}\right)T_{\text{inner}} + T_{\text{air}}}{1 + \frac{k}{t_{\text{insul}}h}}$$

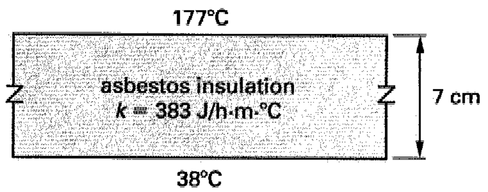
$$= \frac{\left(\frac{\left(383\ \frac{\text{J}}{\text{h·m·°C}}\right)}{(7\ \text{cm})\left(\frac{1\ \text{m}}{100\ \text{cm}}\right) \times \left(39\,032\ \frac{\text{J}}{\text{h·m}^2\text{·°C}}\right)}\right)(177°\text{C}) + 20°\text{C}}{1 + \frac{383\ \frac{\text{J}}{\text{h·m·°C}}}{(7\ \text{cm})\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)\left(39\,032\ \frac{\text{J}}{\text{h·m}^2\text{·°C}}\right)}}$$

$$= 39.3°\text{C} \quad (39°\text{C})$$

The answer is (C).

PHYSICS–45

A 0.03 m^3 (interior capacity) oven is modeled as a six-sided cubical box. The inside and outside wall temperatures at steady state are shown. The insulating material is asbestos with a thermal conductivity of 383 J/h·m·°C. What is most nearly the power dissipated by the oven? Neglect heat transfer through the corners and edges.



(A) 2.5×10^5 J/h (B) 3.7×10^5 J/h (C) 3.9×10^5 J/h (D) 4.4×10^5 J/h

The heat transfer, Q, is

$$Q = -kA\frac{dT}{dx}$$

At steady state, the temperature profile is linear.

$$dx = 0.07 \text{ m}$$

Therefore,

$$\frac{dT}{dx} = \frac{177°\text{C} - 38°\text{C}}{0.07 \text{ m}} = 1986 \text{ °C/m}$$

The oven is modeled as a cubical box with equal sides.

$$s = \sqrt[3]{V} = \sqrt[3]{0.03 \text{ m}^3} = 0.31 \text{ m}$$

$$A = ns^2 = (6)(0.31 \text{ m})(0.31 \text{ m}) = 0.58 \text{ m}^2$$

$$Q = -\left(383 \frac{\text{J}}{\text{h·m·°C}}\right)(0.58 \text{ m}^2)\left(1986 \frac{\text{°C}}{\text{m}}\right)$$
$$= -4.41 \times 10^5 \text{ J/h} \quad (4.4 \times 10^5 \text{ J/h})$$

The negative value indicates that heat is lost to the surroundings.

The answer is (D).

PHYSICS–46

A copper sphere 0.3 m in diameter radiates power through a vacuum to an environment at 273K. The emissivity of copper is 0.15, and the sphere is at 60°C. The power radiated is most nearly

(A) 28 W (B) 30 W (C) 37 W (D) 53 W

The emitted energy rate is

$$E = \epsilon\sigma T^4 A = \epsilon\sigma T^4 4\pi r^2$$
$$= (0.15)\left(5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2\text{·K}^4}\right)(60\text{°C} + 273\text{°})^4(4\pi)\left(\frac{0.30 \text{ m}}{2}\right)^2$$
$$= 29.6 \text{ W} \quad (30 \text{ W})$$

The answer is (B).

PHYSICS–47

An oxidized copper sphere 0.3 m in diameter with an emissivity of 0.78 is filled with water. If the water temperature is 88°C, and the environment is at 273K, what is most nearly the instantaneous rate of cooling of the water? At 88°C, the density of water is 966.6 kg/m^3, and the specific heat is 4190 J/kg·°C.

(A) 10°C/h (B) 11°C/h (C) 12°C/h (D) 13°C/h

The radiated energy is

$$\begin{aligned} E &= \epsilon\sigma T^4 A \\ &= \epsilon\sigma T^4 4\pi r^2 \end{aligned}$$

The heat lost from the water is

$$\begin{aligned} Q &= mc_p\frac{dT}{dt} = V\rho c_p\frac{dT}{dt} \\ &= \frac{4\pi r^3}{3}\rho c_p\frac{dT}{dt} \\ E &= Q \end{aligned}$$

$$4\pi\epsilon r^2\sigma T^4 = \frac{4\pi\rho r^3 c_p\frac{dT}{dt}}{3}$$

$$\begin{aligned} \frac{dT}{dt} &= \frac{3\epsilon\sigma T^4}{rc_p\rho} \\ &= \left(\frac{(3)(0.78)\left(5.67\times 10^{-8}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)(88^\circ\text{C} + 273^\circ)^4}{\left(\frac{0.3\ \text{m}}{2}\right)\left(4190\ \frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}\right)\left(966.6\ \frac{\text{kg}}{\text{m}^3}\right)}\right) \\ &\quad \times\left(3600\ \frac{\text{s}}{\text{h}}\right) \\ &= 13.4^\circ\text{C/h} \quad (13^\circ\text{C/h}) \end{aligned}$$

The answer is (D).

15 SYSTEMS MODELING

SYSTEMS–1

Which of the following matrices has an inverse?

$$\mathbf{A}_1 = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 2 & 2 & 1 \end{pmatrix} \qquad \mathbf{A}_2 = \begin{pmatrix} 1 & 3 & 1 \\ 2 & 2 & 3 \\ 0 & 1 & 1 \end{pmatrix} \qquad \mathbf{A}_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ -1 & 2 & 3 \end{pmatrix}$$

(A) $\mathbf{A}_2$ only (B) $\mathbf{A}_1$ and $\mathbf{A}_2$ (C) $\mathbf{A}_1$ and $\mathbf{A}_3$ (D) $\mathbf{A}_2$ and $\mathbf{A}_3$

If, for matrix $\mathbf{A}$, the determinant is nonzero, the inverse matrix, $\mathbf{A}^{-1}$, exists.

$$\mathbf{D}_1 = \begin{vmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 9 + 2 + 2 - 6 - 6 - 1 = 0$$

$\mathbf{A}_1^{-1}$ does not exist.

$$\mathbf{D}_2 = \begin{vmatrix} 1 & 3 & 1 \\ 2 & 2 & 3 \\ 0 & 1 & 1 \end{vmatrix} = 2 + 2 - 6 - 3 = -5$$

$\mathbf{A}_2^{-1}$ exists.

$$\mathbf{D}_3 = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ -1 & 2 & 3 \end{vmatrix} = 12 - 2 - 12 - 2 = -4$$

$\mathbf{A}_3^{-1}$ exists.

Only $\mathbf{A}_2$ and $\mathbf{A}_3$ have inverses.

The answer is (D).

SYSTEMS–2

An investor is considering a stock portfolio that costs \$55. If he invests in the portfolio, there is a 0.5 probability that he will receive a total revenue of \$20. If that event does not occur, he will receive a total revenue of \$100. What will be the investor's expected profit if he decides to invest?

(A) \$5 (B) \$15 (C) \$55 (D) \$60

The expected profit is found by multiplying the expected revenues by their respective probabilities, adding them, and subtracting the initial cost.

$$\text{profit} = (0.5)(\$100) + (0.5)(\$20) - \$55 = \$5$$

> The answer is (A).

SYSTEMS–3

For a function of a single variable, $f(x)$, to be convex, which of the following must be true?

(A) For each pair of values of x_1 and x_2, with $0 \leq \lambda \leq 1$,

$$f\big(\lambda x_2 + (1-\lambda)x_1\big) \leq \lambda f(x_2) + (1-\lambda)f(x_1)$$

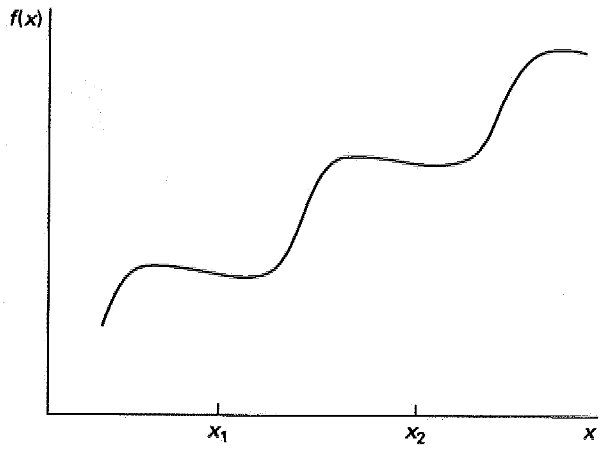
(B) For each pair of values of x_1 and x_2, with $0 \leq \lambda \leq 1$,

$$f\big(\lambda x_2 + (1-\lambda)x_1\big) \geq \lambda f(x_2) + (1-\lambda)f(x_1)$$

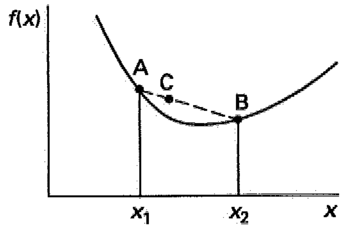
(C) For each pair of values of x_1 and x_2, with $0 \leq \lambda \leq 1$,

$$f\big(\lambda x_2 + (1-\lambda)x_1\big) = \lambda f(x_2) + (1-\lambda)f(x_1)$$

(D) Graphically, $f(x)$ is



A convex function always has a minimum value.



point A: $(x_1, f(x_1))$
point B: $(x_2, f(x_2))$
point C: $(x_1(1 - \lambda) + x_2, f(x_1(1 - \lambda) + x_2)$

The relationship in option (A) is the definition of a convex function, which implies that the function has a minimum value. For each pair of points A and B on the curve, the line segment joining these two points lies entirely above or on the graph of $f(x)$.

The answer is (A).

SYSTEMS–4

For a function of two variables, $f(x_1, x_2)$, and for all possible values of x_1 and x_2, which of the following conditions must exist in order for the function to be convex?

I. $\left(\frac{\partial^2 f(x_1,x_2)}{\partial x_1^2}\right)\left(\frac{\partial^2 f(x_1,x_2)}{\partial x_2^2}\right) - \left(\frac{\partial^2 f(x_1,x_2)}{\partial x_1 x_2}\right)^2 \geq 0$

II. $\frac{\partial^2 f(x_1,x_2)}{\partial x_1^2} \geq 0$

III. $\frac{\partial^2 f(x_1,x_2)}{\partial x_2^2} \geq 0$

(A) I only (B) I and II (C) I and III (D) I, II, and III

Second partial derivatives can be used to check functions of more than one variable to see if they are convex or concave. For two-variable functions, the partial derivatives in I, II, and III make up the determinant of the 2×2 Hessian matrix, which should be greater than or equal to zero. Thus, all three conditions must exist for the function to be convex.

The answer is (D).

SYSTEMS–5

If a function of n variables, $f(x_1, \ldots, x_n)$, is convex, which of the following is true about its $n \times n$ Hessian matrix?

(A) It is semidefinite.
(B) It is negative semidefinite.
(C) It is positive semidefinite.
(D) It is indefinite.

A positive semidefinite Hessian matrix implies two conditions.

1. For all values of x, the function $f(x_1, \ldots, x_n) \geq 0$.
2. There is at least one set of nonzero values of $x_1, \ldots, x_n$ such that $f(x_1, \ldots, x_n) = 0$.

These conditions are met when the determinant of the Hessian matrix is greater than or equal to zero, which occurs if and only if the function f is convex. Thus (C) is the correct answer.

The answer is (C).

SYSTEMS–6

Which of the following statements about linear programming is FALSE?

(A) In mathematical notation, linear programming problems are often written in the following form.

optimize:

$$Z = \sum_j C_j x_j$$

subject to the constraints:

$$\sum_i \sum_j a_{ij} x_j \leq b_i$$

($x_j \geq 0$, and a_{ij}, b_i, and C_j are constants.)

(B) Linear programming uses a mathematical model composed of a linear objective function and a set of linear constraints in the form of inequalities.

(C) The decision variables have physical significance only if they have integer values. The solution procedure yields integer values only.

(D) The simplex method is a technique used to solve linear programming problems.

By definition, $x_j \geq 0$ implies noninteger as well as integer values for the decision variable. Although it is sometimes the case that only integer values of the decision variables have physical significance, the solution procedure does not necessarily yield integer values.

The answer is (C).

SYSTEMS–7

Consider a nontrivial linear programming problem in one variable, x, with only lower- and upper-bound constraints on x. At optimum, where will x be in relation to these constraints?

(A) at its upper bound
(B) at its lower bound
(C) between its upper and lower bounds
(D) at its upper or lower bound

The constraints prevent the variable of a linear program from increasing or decreasing forever during maximization or minimization. The maximum or minimum will occur at either the upper or lower bound.

The answer is (D).

SYSTEMS–8

If all variables in a linear programming problem are restricted to be integers, which, if any, basic assumption of linear programming is violated?

(A) certainty (B) additivity (C) divisibility (D) proportionality

Divisibility implies that fractional levels of the decision variables must be permissible. By restricting all variables in the problem to be integers, divisibility is lost.

The answer is (C).

SYSTEMS–9

If a project that has diminishing returns with scale is modeled using a linear program, which basic assumption of linear programming will be violated?

(A) certainty (B) additivity (C) divisibility (D) proportionality

Proportionality assumes that a variable multiplied by a constant is equal to the contribution, regardless of scale.

The answer is (D).

SYSTEMS–10

Consider the following linear programming model.

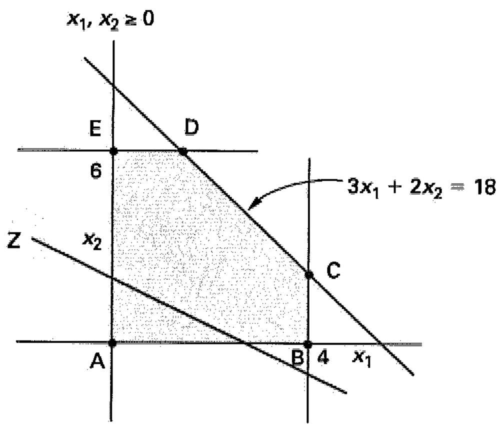
maximize:

$$Z = 3x_1 + 5x_2$$

subject to the constraints:

$$3x_1 + 2x_2 \leq 18$$
$$x_1 \leq 4$$
$$x_2 \leq 6$$

The graphical solution is



At what point does the optimum solution occur?

(A) A (B) B (C) C (D) D

If the objective function Z is plotted and moved along within the feasible region, the last point of contact will be point D. Therefore, Z will be maximized while satisfying all constraints at point D.

The answer is (D).

SYSTEMS–11

For which of the following linear programming problems can an optimum solution be found?

I. maximize:

$$Z = 20x + 10y$$

subject to the constraints:

$$\begin{aligned} x + y &\leq 4 \\ 3x + y &\leq 6 \\ x, y &\geq 0 \end{aligned}$$

II. maximize:

$$Z = 20x + 10y$$

subject to the constraints:

$$\begin{aligned} x + y &\geq 4 \\ 3x + 2y &\leq 6 \\ x, y &\geq 0 \end{aligned}$$

III. maximize:

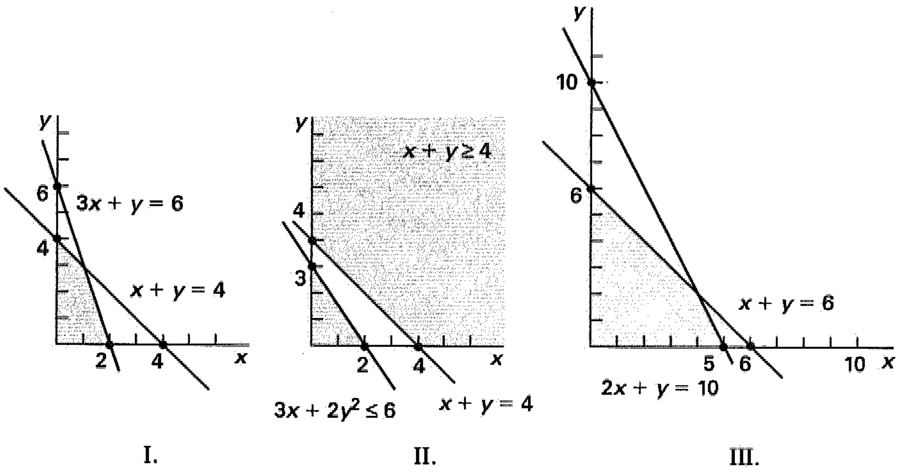
$$Z = 20x + 10y$$

subject to the constraints:

$$\begin{aligned} 2x + y &\leq 10 \\ x + y &\leq 6 \\ x, y &\geq 0 \end{aligned}$$

(A) I only (B) I and II (C) I and III (D) II and III

For an optimum solution to exist, there must be a feasibility region. The graphs of the feasibility regions I, II, and III are as shown.



For II, there is no region where all four conditions are met. I and III have feasibility regions and, therefore, have optimum solutions.

The answer is (C).

SYSTEMS–12

Which of the following linear programming problems have multiple optimum points that yield the same optimum solution?

I. maximize:

$$Z = 20x + 10y$$

subject to the constraints:

$$x + y \leq 4$$
$$3x + y \leq 6$$
$$x, y \geq 0$$

II. maximize:

$$Z = 20x + 10y$$

subject to the constraints:

$$x + y \geq 4$$
$$3x + 2y \leq 6$$
$$x, y \geq 0$$

III. maximize:

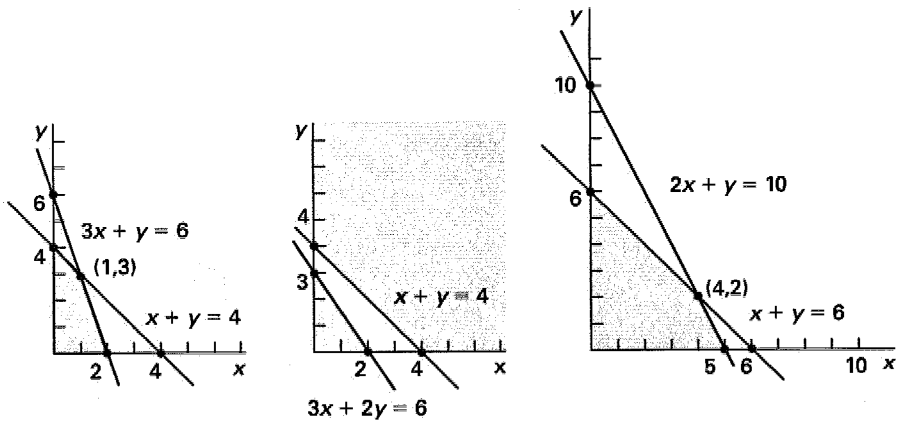
$$Z = 20x + 10y$$

subject to the constraints:

$$2x + y \leq 10$$
$$x + y \leq 6$$
$$x, y \geq 0$$

(A) I only (B) II only (C) III only (D) I and III

The graphs of I, II, and III are as shown.



I. II. III.

For I, the optimum solution is at the point (1,3) where $Z = 20 + 30 = 50$; this solution has a unique optimum point. II has no feasibility region and, thus, has no optimum solution. For III, the points (4,2) and (5,0) both yield the optimum solution $Z = 100$; III is the only choice with multiple optimum points. In fact, any point on the line segment adjoining (5,0) and (4,2) will yield the optimum solution $Z = 100$.

The answer is (C).

SYSTEMS–13

What is the maximum value of Z for the following integer linear programming problem? (x and y are integers.)

maximize:

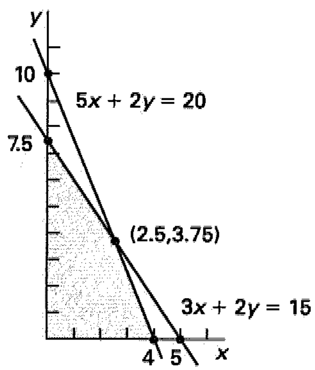
$$Z = 6x + 5y$$

subject to the constraints:

$$\begin{aligned} 5x + 2y &\leq 20 \\ 3x + 2y &\leq 15 \\ x, y &\geq 0 \\ x, y &\quad \text{integers} \end{aligned}$$

(A) 28 (B) 33 (C) 35 (D) 36

The maximum Z value is found from the extreme points in the illustration.



extreme point	Z
(0,0)	0
(4,0)	24
(0,7.5)	37.5
(2.5,3.75)	33.75

The largest value of Z over all real numbers is given at (0,7.5). Since x and y are integers, the largest Z value will be given at either $x = 0$ and $y = 7$, or $x = 1$ and $y = 6$. The first combination gives $Z = 35$, but the second gives $Z = 36$.

The answer is (D).

SYSTEMS–14

The simplex method is extremely efficient in solving which of the following classic problems?

(A) the transportation problem
(B) the assignment problem
(C) the transhipment problem
(D) the allocation problem

Theoretically, all problem categories can be solved using the simplex method. However, the assignment, transportation, and transhipment problems are numerically inefficient when solved by the general simplex method. For allocation problems, the variables are continuous and, therefore, manageable enough in size to be solved using the simplex method.

The answer is (D).

SYSTEMS–15

Upon which of the following properties of linear programming is the simplex method based?

I. The collection of feasible solutions constitutes a convex set.
II. If a feasible solution exists, a basic solution exists where the feasible solutions correspond to the extreme points of the set of feasible solutions.
III. Only a finite number of basic feasible solutions exist.
IV. If the objective function possesses a finite maximum, at least one optimum solution is a basic feasible solution.

(A) I only
(B) IV only
(C) I and II
(D) I, II, III, and IV

The simplex method is based upon all of the given linear programming properties.

The answer is (D).

SYSTEMS–16

As the simplex algorithm progresses from one solution to the next in a linear programming maximization problem, what will happen to the value of the objective function?

(A) It will increase and then decrease.
(B) It will decrease and then increase.
(C) It will increase or stay the same.
(D) It will decrease or stay the same.

A characteristic of the simplex algorithm is that the value of the objective function improves (does not worsen) between iterations. For a maximization problem, only option (C) can be true.

The answer is (C).

SYSTEMS–17

In the following simplex tableau, the first row of numbers represents the objective function of a maximization problem, and subsequent rows represent constraints. What are the basic variables?

x_1	x_2	x_3	x_4	x_5	right side
−10	−6	0	0	0	0
1	0	1	0	0	3
0	1	0	1	0	15
2	3	0	0	1	17

(A) x_1 and x_2
(B) x_1, x_4, and x_5
(C) x_2, x_4, and x_5
(D) x_3, x_4, and x_5

The system of equations represented by the tableau implies that $Z = 10x_1 + 6x_2$, and

$$\mathbf{A}x = \begin{pmatrix} 3 \\ 15 \\ 17 \end{pmatrix}$$

Given the negative coefficients of the objective function, Z, the values of the variables must be $x_1 = 0$, $x_2 = 0$, $x_3 = 3$, $x_4 = 15$, and $x_5 = 17$. x_3, x_4, and x_5 are "used" by the columns of the basis and are, therefore, the basic variables.

The answer is (D).

SYSTEMS–18

After several iterations, the following simplex tableau is developed.

	Z	x_1	x_2	x_3	x_4	x_5	x_6	x_7	right side
Z	1	−4	6	0	0	−3	1	0	14
x_4	0	7	2	0	1	−2	−6	0	3
x_7	0	6	2.5	0	0	4	0.8	1	12
x_3	0	3	3.5	1	0	2.5	5	0	2

Which of the following describes the solution found?

(A) It is feasible but not optimum.
(B) It is optimum but not feasible.
(C) It is optimum and feasible.
(D) It is neither optimum nor feasible.

The solution is feasible because there are no negative numbers on the right side. It is not, however, an optimum solution since there are negative numbers in the objective function.

The answer is (A).

SYSTEMS–19

In the following simplex tableau, the first row of numbers represents the objective function of a maximization problem. What are the current values of x_1 and x_2?

x_1	x_2	x_3	x_4	x_5	right side
−7.5	0	−4.5	0	0	0
1	1	0	0	0	3
0	0	1	1	0	15
2	0	3	0	1	17

(A) $x_1 = 0, x_2 = 0$ (B) $x_1 = 0, x_2 = 3$
(C) $x_1 = 2, x_2 = 15$ (D) $x_1 = 3, x_2 = 3$

Since the coefficients of x_1 and x_3 in the objective function are negative, x_1 and x_3 must be zero. The tableau represents a system of equations with the following solution: $x_1 = 0$, $x_2 = 3$, $x_3 = 0$, $x_4 = 15$, and $x_5 = 17$.

The answer is (B).

SYSTEMS–20

In the following simplex tableau, identify the current entering and exiting basic variables.

	Z	x_1	x_2	x_3	x_4	x_5	x_6	right side
Z	1	4	−6	−1	0	0	0	0
x_4	0	−3	2	−3	1	0	0	10
x_5	0	4	−8	3	0	1	0	24
x_6	0	7	3	6	0	0	1	30

(A) x_2 entering, x_4 exiting
(B) x_2 entering, x_6 exiting
(C) x_3 entering, x_5 exiting
(D) x_3 entering, x_4 exiting

The first row of the tableau gives

$$Z = -4x_1 + 6x_2 + x_3$$

x_2 will cause the greatest increase in Z when it goes from 0 to 1. The entering variable is, therefore, x_2. With $x_1 = x_3 = 0$, the x_4 row gives

$$2x_2 + x_4 = 10$$
$$x_2 = 5$$
$$x_4 = 0$$

The x_6 row gives

$$3x_2 + x_6 = 30$$
$$x_2 = 10$$
$$x_6 = 0$$

The exiting variable is the one that goes to zero first as x_2 is increased from zero. The exiting variable is x_4.

The answer is (A).

SYSTEMS–21

In the following simplex tableau, the first row of numbers represents the objective function of a maximization problem. What will be the next variable to enter the basis?

x_1	x_2	x_3	x_4	x_5	right side
−5	−3	0	0	0	0
1	0	1	0	0	3
0	1	0	1	0	15
2	3	0	0	1	17

(A) x_1 (B) x_2 (C) x_3 (D) none of the above

If the first row represents the objective function of a maximization problem, then the tableau is already optimum, since for a maximization problem a tableau is optimum when all $C_i \leq 0$. If this had been the tableau for a minimization problem, x_1 would have been the next variable to enter the basis.

The answer is (D).

SYSTEMS–22

Find the "pivot" value in the following simplex tableau.

	x_1	x_2	u	v	
u	3	2	1	0	7
v	7	5	0	1	12
	−50	−40	0	0	

(A) 2 (B) 3 (C) 5 (D) 7

The first step in the method of pivot searching is to select the pivotal column by determining the column with the most negative entry in the objective row. In this problem it is the x_1 column.

Next, find the pivotal row by dividing each row's rightmost value by that row's pivotal column value. The row that has the lower quotient is the pivotal row. The quotient for row 1 is $^7/_3$, and for row 2 it is $^{12}/_7$. Thus, the second row is the pivotal row.

Finally, the pivot is the value that is at the intersection of the pivotal row and column. For this problem, it is 7.

The answer is (D).

SYSTEMS–23

Find the optimum value of the slack variable x_3.

	x_1	x_2	x_3	x_4	x_5	
x_3	1	−1	1	0	0	2
x_4	2	1	0	1	0	4
x_5	−3	2	0	0	1	6
	−5	−3	0	0	0	0

(A) 7/38 (B) 36/7 (C) 38/7 (D) 39/7

The simplex tableau becomes

	x_1	x_2	x_3	x_4	x_5	
x_3	1	−1	1	0	0	2
x_4	2	1	0	1	0	4
x_5	−3	2	0	0	1	6
	−5	−3	0	0	0	0

	x_1	x_2	x_3	x_4	x_5	
x_1	1	−1	1	0	0	2
x_4	0	3	−2	1	0	0
x_5	0	−1	3	0	1	12
	0	−8	5	0	0	10

	x_1	x_2	x_3	x_4	x_5	
x_1	1	0	$\frac{1}{3}$	$\frac{1}{3}$	0	2
x_2	0	1	$-\frac{2}{3}$	$\frac{1}{3}$	0	0
x_5	0	0	$\frac{7}{3}$	$\frac{1}{3}$	1	12
	0	0	$-\frac{1}{3}$	$\frac{8}{3}$	0	10

	x_1	x_2	x_3	x_4	x_5	
x_1	1	0	0	$\frac{2}{7}$	$-\frac{1}{7}$	$\frac{2}{7}$
x_2	0	1	0	$\frac{3}{7}$	$\frac{2}{7}$	$\frac{24}{7}$
x_3	0	0	1	$\frac{1}{7}$	$\frac{3}{7}$	$\frac{36}{7}$
	0	0	0	$\frac{59}{21}$	$\frac{1}{7}$	$\frac{82}{7}$

The slack variables are, therefore, $x_3 = {}^{36}/_{7}$, $x_4 = 0$, and $x_5 = 0$. The optimum value is ${}^{36}/_{7}$.

The answer is (B).

SYSTEMS–24

The heights of several thousand fifth grade boys in Santa Clara County were measured. It was found that the mean of the height was 1.20 m and the variance was $25 \times 10^{-4}\ \text{m}^2$. Approximately what percentage of these boys has a height greater than 1.23 m?

(A) 27% (B) 31% (C) 69% (D) 73%

To convert the normal distribution to unit normal distribution, the new variable, z, is constructed from the height h, mean μ, and variance σ^2.

$$z = \frac{h - \mu}{\sigma}$$

For a height greater than 1.23 m,

$$z = \frac{1.23\ \text{m} - 1.20\ \text{m}}{\sqrt{25 \times 10^{-4}}\ \text{m}} = 0.6$$

From a unit normal distribution table, the cumulative distribution function at $z = 0.6$ is 0.726. Therefore, the percentage of boys having height greater than 1.23 m is

$$\text{percentage taller than 1.23 m} = 100\% - 72.6\% = 27.4\%$$

The answer is (A).

SYSTEMS–25

Using the simplex method, in what form would one write the given objective function in order to maximize it?

$$Z = |x_1| - 3x_2$$

(A) $Z = x_1^+ + x_1^- - 3x_2$; subject to $x_1^+, x_1^- \geq 0$
(B) $Z - x_1^+ + x_1^- + 3x_2 = 0$; subject to $x_1^+, x_1^- \geq 0$
(C) $Z + x_1^+ - x_1^- - 3x_2 = 0$; subject to $x_1^+, x_1^- \geq 0$
(D) $Z - |x_1| + 3x_2 = 0$; subject to $x_1 \geq 0$

The absolute value term is written as $x_1^+ - x_1^-$, with x_1^+ and x_1^- greater than or equal to zero. The proper form for the simplex method is to have all terms on the same side of the equal sign equal to zero.

The answer is (B).

SYSTEMS–26

One constraint for a linear program is as follows: $3x_1 - 2x_2 + 4x_3 \geq 6$. What is the proper form of this constraint for use in the simplex method?

(A) $3x_1 - 2x_2 + 4x_3 - 6 = 0$
(B) $3x_1 - 2x_2 + 4x_3 + 6 \leq 0$
(C) $3x_1 - 2x_2 + 4x_3 + x_4 \geq -6$
(D) $3x_1 - 2x_2 + 4x_3 + x_4 = 6$

The proper form for a constraint uses a slack variable to account for the inequality. For this problem, the slack variable x_4 is added.

The answer is (D).

SYSTEMS–27

How would the following problem be written if the simplex solution method is to be used?

maximize:

$$Z = 12x_1 - 4x_2$$

subject to the constraints:

$$x_1 + 3x_2 = 42$$
$$x_1, x_2 \geq 0$$

(A) $Z = 12x_1 - 4x_2$; $x_1 + 3x_2 - 42 = 0$
(B) $Z - 12x_1 + 4x_2 = 0$; $x_1 + 3x_2 + x_3 = 42$
(C) $Z - 12x_1 + 4x_2 + Mx_3 = 0$; $x_1 + 3x_2 + x_3 = 42$; M is some large number.
(D) $Z - 12x_1 + 4x_2 = -Mx_3$; $x_1 + 3x_2 - 42 = \bar{x}_3$

The slack variable x_3 is added to the objective function, multiplied by a constant, M. The restriction becomes $x_1 + 3x_2 + x_3 = 42$. Although option (D) has all the correct terms, they are not in the proper position.

The answer is (C).

SYSTEMS–28

Consider the following linear programming problem.

maximize:

$$Z = 6x_1 + 5x_2$$

subject to the constraints:

$$x_1 + x_2 \leq 4$$
$$5x_1 + 3x_2 \leq 15$$
$$x_1, x_2 \geq 0$$

If $x_1 + x_2 + x_3 = 4 + \Delta b$, find the range of Δb over which the basis remains unchanged.

(A) $-1 \leq \Delta b \leq 1$ (B) $0 \leq \Delta b \leq 1$ (C) $-2 \leq \Delta b \leq 1$ (D) $-2 \leq \Delta b \leq 2$

Using the slack variables x_3 and x_4, the constraints become

$$x_1 + x_2 + x_3 = 4$$
$$5x_1 + 3x_2 + x_4 = 15$$

The simplex tableaus are shown.

	x_1	x_2	x_3	x_4	
x_3	1	1	1	0	4
x_4	5	3	0	1	15
	−6	−5	0	0	0

	x_1	x_2	x_3	x_4	
x_3	0	$\frac{2}{5}$	1	$-\frac{1}{5}$	1
x_1	1	$\frac{3}{5}$	0	$\frac{1}{5}$	3
	0	$-\frac{7}{5}$	0	$\frac{6}{5}$	18

	x_1	x_2	x_3	x_4	
x_2	0	1	$\frac{5}{2}$	$-\frac{1}{2}$	$\frac{5}{2}$
x_1	1	0	$-\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$
	0	0	$\frac{7}{2}$	$\frac{1}{2}$	$\frac{43}{2}$

For the basis to remain unchanged, Z must also remain unchanged. Thus, for Z to remain at $^{43}/_2$,

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{5}{2} \\ \frac{3}{2} \end{pmatrix}$$

$$x' = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \Delta b \begin{pmatrix} \frac{5}{2} \\ -\frac{3}{2} \end{pmatrix} \geq 0$$

This gives

$$\frac{5}{2} + \Delta b \left(\frac{5}{2}\right) \geq 0$$

$$\Delta b \geq -1$$

$$\frac{3}{2} - \Delta b \left(\frac{3}{2}\right) \geq 0$$

$$\Delta b \leq 1$$

Therefore, $-1 \leq \Delta b \leq 1$.

The answer is (A).

SYSTEMS–29

Which of the following statements is INCORRECT for the primal linear programming problem in the form given?

maximize:

$$Z_x = \sum_j C_j x_j$$

subject to the constraints:

$$\sum_j \sum_i a_{ij} x_j \leq b_i$$

$$x_j \geq 0$$

(A) The dual problem is

minimize:

$$Z_y = \sum_i b_i y_i$$

subject to the constraints:

$$\sum_j \sum_i a_{ij} y_i \geq C_j$$

$$y_i \geq 0$$

(B) The dual problem is the same as in option (A), but with the inequality signs reversed.

(C) y_i is unrestrictive in sign if the inequality signs in the primal problem are replaced by equality signs.

(D) x_j is unrestrictive in sign if the inequality signs in the dual problem are replaced by equality signs.

By definition, the dual of a primal linear programming problem is exactly the reverse of the primal, including the reversal of the inequality signs.

The answer is (B).

SYSTEMS–30

For the following problem, what are the constraints of the dual problem?

maximize:

$$Z = 6x_1 + 3x_2 + 4x_3$$

subject to the constraints:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &\leq 12 \\ x_1 + 4x_2 + 3x_3 &= 15 \\ x_1, x_3 &\geq 0 \end{aligned}$$

The dual problem statement is

minimize:

$$Z' = 12w_1 + 15w_2$$

(A)
$$\begin{aligned} w_1 + w_2 &= 6 \\ 2w_1 + 4w_2 &\geq 3 \\ 3w_1 + 3w_2 &\geq 4 \\ w_1, w_2 &\geq 0 \end{aligned}$$

(B)
$$\begin{aligned} w_1 + w_2 &\geq 6 \\ 2w_1 + 4w_2 &\geq 3 \\ 3w_1 + 3w_2 &= 4 \\ w_2 &\geq 0 \end{aligned}$$

(C)
$$\begin{aligned} w_1 + w_2 &\geq 6 \\ 2w_1 + 4w_2 &= 3 \\ 3w_1 + 3w_2 &\geq 4 \\ w_1 &\geq 0 \end{aligned}$$

(D)
$$\begin{aligned} w_1 + w_2 &\geq 6 \\ 2w_1 + 4w_2 &\geq 3 \\ 3w_1 + 3w_2 &= 4 \\ w_1, w_2 &\geq 0 \end{aligned}$$

Each of the constraints, C_i, in the primal problem corresponds to a respective variable, w_i, in the dual problem. The coefficients of the objective function in the primal problem are the constants on the right-hand side of the constraints in the dual problem. The coefficients of the ith constraint in the primal problem are the coefficients of the ith variable in the dual problem. If the jth variable in the primal problem is restricted, the jth constraint in the dual problem is an inequality. If the primal constraint is an equality, then the corresponding variable will be unrestricted in the dual problem.

Thus, w_2 is unrestricted in the dual problem, eliminating options (A), (B), and (D). In the dual problem, if a variable w_i is unrestricted, the ith constraint is an equality. If the variable is restricted, the ith constraint is an inequality. Therefore, $2w_1 + 4w_2 = 3$.

> The answer is (C).

SYSTEMS–31

The mathematical model for the classic transportation problem is as follows.

Find $x_{ij}(i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n)$ in order to maximize

$$\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij}$$

subject to the constraints:

$$\sum_{j=1}^{n} x_{ij} = a_i \quad [i = 1, \ldots, m]$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad [j = 1, \ldots, n]$$

$$x_{ij} \geq 0$$

Which of the following is FALSE?

(A) m can be regarded as the number of factories supplying n warehouses with a certain product.

(B) a_i is the number of units produced at factory i, while b_j is the number of units required for delivery to warehouse j.

(C) C_{ij} is the shipping cost from factory i to warehouse j. x_{ij} is the decision variable, the amount shipped from factory i to warehouse j.

(D) x_{ij} has physical significance only for noninteger values.

A transportation problem has physical significance only when decision variables are integers.

The answer is (D).

SYSTEMS–32

Which of the following describes the optimum solution to a transportation problem?

(A) It can be determined using the simplex algorithm.

(B) It cannot be found if there are no upper-bound constraints on supplies from several sources.

(C) It can only be found if there are no upper bounds on supplies from several sources.

(D) It is trivial if the demand is unstable.

Transportation problems are special linear programming problems that allow for the presence or lack of upper bounds on variables and assume constant demand. Only option (A) is true.

The answer is (A).

SYSTEMS–33

How must the following cost and requirements table for a transportation problem be altered so that linear programming methods can be used to find an optimum solution?

	destination				
source	A	B	C	D	supply
1	C_{1A}	C_{1B}	$\cdots$		4
2	C_{2A}	C_{2B}			6
3	$\vdots$				2
	6	2	7	3	
	demand				

(A) A dummy source must be added to supply six units.

(B) A dummy destination must be added to increase the demand by six units.

(C) The sum of the costs in each row must be made equal by inclusion of a dummy cost.

(D) The sum of the costs in each column must be made equal by inclusion of a dummy cost.

For the model to have a feasible solution, the supply and demand must be equal. A dummy source must be created which will supply an additional six units.

The answer is (A).

SYSTEMS–34

How many basic variables are there for the following transportation problem cost and requirements table?

	destination				
source	A	B	C	D	supply
1	C_{1A}	C_{1B}	$\cdots$		s_1
2	C_{2A}	C_{2B}			s_2
3	$\vdots$				s_3
	d_1	d_2	d_3	d_4	
	demand				

(A) three (B) four (C) six (D) seven

The number of basic variables is equal to the number of sources plus the number of destinations minus one. Thus, there are $3 + 4 - 1 = 6$ basic variables.

The answer is (C).

SYSTEMS–35

The *northwest corner rule* is to be used to find an initial solution to the following transportation simplex problem. What is the value of the fourth basic variable?

	destination					
source	A	B	C	D	E	supply
1						20
2						30
3						40
4						30
	40	20	10	30	20	
	demand					

(A) 10 (B) 20 (C) 30 (D) 40

The procedure under the *northwest corner rule* for obtaining an initial basic feasible solution is as follows.

1. Start with the cell in the upper left-hand corner.
2. Allocate the maximum feasible amount.
3. If there is supply remaining, move one cell to the right. If there is no remaining supply, move one cell down. Stop when it is impossible to do either of these. Repeat the process beginning at step 2 for the new cell. Each new cell represents a new basic variable.

Carrying out this procedure for the given problem results in the following table.

	destination					
source	*A*	*B*	*C*	*D*	*E*	supply
1	20					20
2	20	10				30
3		10	10	20		40
4				10	20	30
	40	20	10	30	20	
	demand					

The fourth basic variable is equal to 10.

The answer is (A).

SYSTEMS–36

Four technicians—Tom, Scott, Ed, and Jeri—are each assigned a project on which to work. The costs for each technician to complete each project are estimated as follows.

	project			
	1	2	3	4
Tom	10	14	15	12
Scott	9	13	17	10
Ed	8	12	14	11
Jeri	12	15	12	12

What is the optimum project assignment scheme such that all projects are completed at the minimum cost? (The order of technicians listed in the answer choices corresponds to project 1, project 2, project 3, and project 4.)

(A) Tom, Scott, Ed, Jeri
(B) Scott, Tom, Jeri, Ed
(C) Ed, Jeri, Scott, Tom
(D) Ed, Tom, Jeri, Scott

The cost matrix can be reduced by subtracting any constant from a row, as long as the row entries remain greater than or equal to zero. Subtract eight from each row. The matrix is then

	job			
	1	2	3	4
Tom	2	6	7	4
Scott	1	5	9	2
Ed	0	4	6	3
Jeri	4	7	4	4

Thus, for minimum cost, Ed should do job 1, and Scott should do job 4. Jeri should do job 3, and Tom should do job 2. The correct order is Ed, Tom, Jeri, and Scott.

There is another assignment scheme that will result in the same minimum cost: Tom, Ed, Jeri, Scott. However, this is not one of the options.

The answer is (D).

SYSTEMS–37

If an integer programming problem is solved as a linear programming problem and the resulting values of the decision variable are rounded off, which of the following will result?

(A) an optimum integer solution
(B) a noninteger solution that is optimum
(C) an integer solution that may be optimum, or close to it
(D) an integer solution that is not optimum

After rounding off, the solution will be an integer. However, it will no longer be an exact solution. Thus, it may provide an optimum solution or only one that is close to optimum.

The answer is (C).

SYSTEMS–38

Which of the following is NOT a good application of network analysis?

(A) electrical engineering
(B) information theory
(C) the study of transportation systems
(D) inventory theory

Inventory problems are generally not solved using network analysis. Network analysis involves maximizing the flow through a network connecting a source and a destination. Inventory theory involves the optimization of the problem of stocking goods; it is not concerned with the flow through a network.

The answer is (D).

SYSTEMS–39

For which of the following is the Program Evaluation and Review Technique (PERT) NOT used?

(A) construction projects
(B) computer programming assignments
(C) preparation of bids and proposals
(D) queueing problems

PERT is used to predict the completion time for large projects. All of the choices given except option (D) are projects that have a finite completion time.

The answer is (D).

SYSTEMS–40

Identify the FALSE statement.

(A) The primary objective of PERT is to determine the probability of meeting specified deadlines.

(B) PERT identifies the activities that are most likely to "bottleneck" and the activities that are most likely to stay on schedule.

(C) PERT evaluates the sensitivity to changes in the program.

(D) To apply PERT, one should develop a network representation of the project plan.

Linear programming automatically performs a sensitivity analysis of the variables as a by-product of the solution process. PERT, however, cannot provide a similar sensitivity analysis to evaluate the effect of changes in the program parameters.

The answer is (C).

SYSTEMS–41

What are the basic features of dynamic programming problems?

I. The problem can be divided into stages with a policy decision required at each stage.

II. Each stage has a number of states associated with it.

III. The effect of the policy decisions at each stage is to transform the current state into a state associated with the next stage.

IV. The problem formulation is dependent on the probability distribution associated with it.

(A) I only (B) IV only (C) I and II (D) I, II, and III

Statement IV is irrelevant. There may not be a probability distribution when deterministic problems are solved using dynamic programming. The formulation of the problem depends only on the first three statements.

The answer is (D).

SYSTEMS–42

Which of the following statements about dynamic programming is FALSE?

(A) Dynamic programming is a mathematical technique that is often useful for making a sequence of interrelated decisions.

(B) Dynamic programming provides a systematic procedure for determining the combination of decisions that maximize overall effectiveness.

(C) Dynamic programming can be represented in standard mathematical formulation.

(D) Dynamic programming is a conceptual approach to problem solving.

Dynamic programming is a conceptual approach to problem solving, not a mathematical one. Its formulation depends on the specifics of the problem.

The answer is (C).

SYSTEMS–43

Queueing theory provides a large number of alternative mathematical models for describing which of the following?

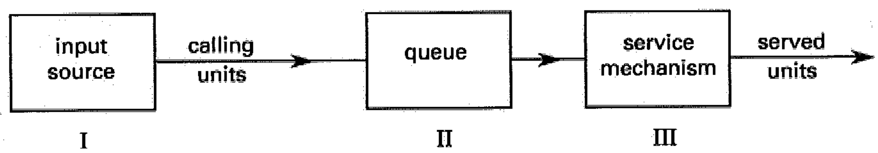
(A) network problems
(B) probabilistic arrivals
(C) probabilistic service facilities
(D) waiting line problems

Queueing theory involves the mathematical study of waiting lines or "queues."

The answer is (D).

SYSTEMS–44

The various elements of the queueing process are depicted as follows.



Which elements of the figure make up the queueing system?

(A) I only (B) II only (C) III only (D) II and III

The queue (II) and the service mechanism (III) make up the actual queueing system.

The answer is (D).

SYSTEMS–45

In a queueing process of customers in a store, what statistical pattern will most likely describe the arrival of customers over time?

(A) the normal law of probability
(B) the Poisson distribution
(C) the uniform law of probability
(D) the exponential distribution

The common assumption is that calling units in a queueing process arrive according to a Poisson distribution.

The answer is (B).

SYSTEMS–46

In a queueing process of customers in a store, what type of distribution most likely governs the time between consecutive arrivals of customers?

(A) a normal probability distribution
(B) an exponential distribution
(C) a uniform probability distribution
(D) a Poisson distribution

The "interarrival time" is commonly assumed to be exponentially distributed.

The answer is (B).

SYSTEMS–47

The jobs to be performed by a particular machine arrive according to a Poisson input process with a mean rate of 1 per hour. If the machine breaks down and requires 2 h to be repaired, what is the probability that the number of new jobs that arrive during the 2 h period is zero?

(A) e^{-2} (B) e^{-1} (C) 1 (D) e

For a Poisson distribution, the probability, p, of x jobs arriving in a time, t, is given by

$$p\{x\} = \lambda t^x \left(\frac{e^{-\lambda t}}{x!} \right)$$

λ is the mean arrival rate. Therefore,

$$\begin{aligned} p\{x\} &= \left(1 \; \frac{\text{job}}{\text{h}}\right) (2 \text{ h})^0 \left(\frac{e^{-(1/\text{h})(2 \text{ h})}}{0!} \right) \\ &= e^{-2} \end{aligned}$$

The answer is (A).

SYSTEMS–48

In a queueing system that has an arrival rate of 5 customers/h, the expected waiting time for any customer in the system, including service time, is 40 min. What is the expected number of customers in the system under steady-state conditions?

(A) 5/40 customers (B) 2/15 customers (C) 10/3 customers (D) 8 customers

Little's formula states

$$L = \lambda W$$

L is the expected number of customers in the system, λ is the mean arrival rate of customers per hour, and W is the expected waiting time for each customer. Thus,

$$\begin{aligned} L &= \left(5 \; \frac{\text{customers}}{\text{h}}\right) \left(\frac{2}{3} \text{ h}\right) \\ &= 10/3 \text{ customers} \end{aligned}$$

The answer is (C).

SYSTEMS–49

Consider a queueing system with three servers, such that the mean service rate for each busy server is 2 customers/h. If the mean arrival rate of customers is 5/h, what is the expected fraction of total time that all servers will be busy? Assume steady-state conditions.

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{2}{3}$ (D) $\frac{5}{6}$

The expected fraction of time that all servers will be busy is

$$\begin{aligned}\text{fraction of } t &= (\text{arrival rate})\left(\frac{1}{(\text{no. of servers})(\text{service rate})}\right)\\ &= \left(5\ \frac{\text{customers}}{\text{h}}\right)\left(\frac{1}{(3)\left(2\ \frac{\text{customers}}{\text{h}}\right)}\right)\\ &= 5/6\end{aligned}$$

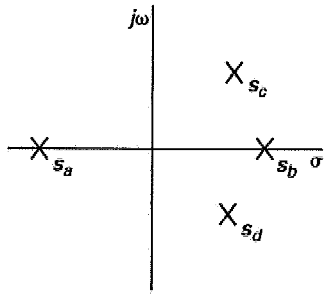
The answer is (D).

SYSTEMS–50

Consider the following equation.

$$F(s) = \frac{A_1}{s-a_1} + \frac{A_2}{s-a_2} + \frac{A_3}{s+a_3}$$

If s_1, s_2, and s_3 are the poles corresponding to the three terms in $F(s)$, respectively, which point on the graph may represent s_3 if a_3 is real and $a_3 > 0$?



(A) s_a (B) s_b (C) s_c (D) s_d

$s_3 = -a_3$. Since $a_3 > 0$ and real, s_3 is located at the negative real axis. The only point that may represent s_3 is s_a.

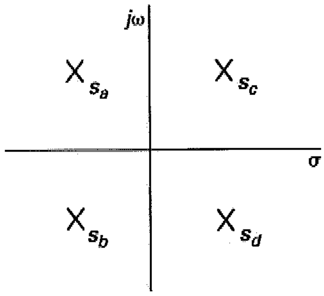
The answer is (A).

SYSTEMS–51

The pole diagram is shown for the following equation.

$$F(s) = \frac{P(s)}{Q(s)} = \frac{P(s)}{(s^2 + 2\zeta\omega_n s + \omega_n^2)(s - s_3)}$$

If s_1, s_2, and s_3 are the poles corresponding to the solution of $(s^2 + 2\zeta\omega_n s + \omega_n^2)$ and $(s - s_3)$, respectively, what points on the diagram correspond to s_1 and s_2? ($\zeta > 0$, $\omega_n > 0$, and $\zeta < 1$. s_3 is not plotted.)



(A) s_a and s_b (B) s_a and s_c (C) s_a and s_d (D) s_b and s_c

$$s_1,\ s_2 = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2}$$
$$= -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

The roots of $s^2 + 2\zeta\omega_n s + \omega_n^2$ must be either s_a and s_b or s_c and s_d, since the $j\omega$ components are of the same magnitude but different in sign. The algebraic expression gives negative roots. Therefore, the solution is on the left side of the plot.

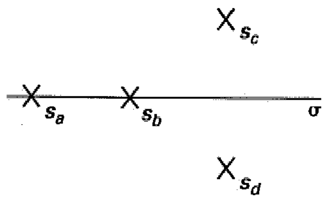
The answer is (A).

SYSTEMS–52

The following function is plotted on a pole-zero diagram.

$$F(s) = \frac{K(s - z_1)}{s(s - p_1)(s - p_2)}$$

The z_1 value and the $j\omega$ axis are not shown. The magnitude of p_1 is larger than the magnitude of p_2, and both are positive numbers. Determine which of the following statements is true.



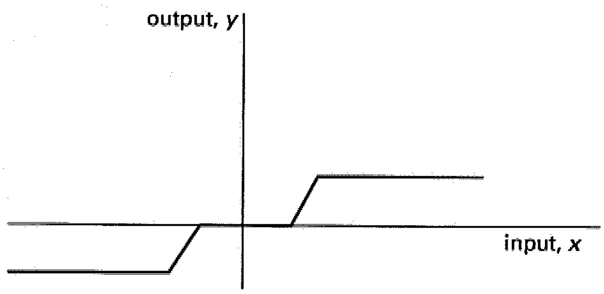
(A) $p_1 = s_a$, $p_2 = s_b$
(B) $p_1 = s_b$, $p_2 = s_a$
(C) $p_1 = s_c$, $p_2 = s_d$
(D) p_1 and p_2 are real and described by s_c and s_d.

Since p_1 and p_2 are positive numbers, they are real numbers. Therefore, p_1 and p_2 fall on the σ-axis. p_1 is of greater magnitude than p_2 and is, therefore, to the right of p_2.

The answer is (B).

SYSTEMS–53

Which of the following best describes the function shown?



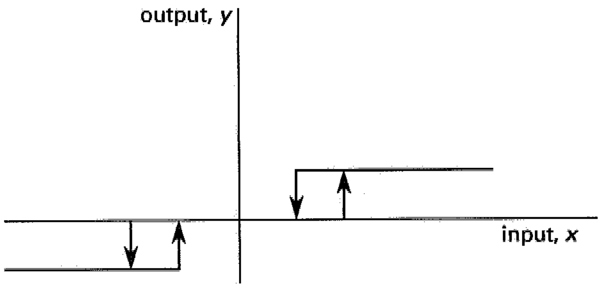
(A) It has a dead zone.
(B) It is a saturated zone system.
(C) There is both a dead zone and a saturated system zone.
(D) It is an impulse zone system.

The dead zone occurs in the region where there is no amplitude near the origin. The positive and negative saturation occurs after a short linear increase or decrease, respectively. A ramp would only have a single, continuously increasing function. An impulse is a narrow major increase, such as a single square-wave pulse.

The answer is (C).

SYSTEMS–54

Which of the following is true about the function shown?



(A) It has a dead zone with linear output outside the dead zone.
(B) It has a dead zone with saturation.
(C) There is no dead zone.
(D) It has a dead zone with hysteresis.

This is a classic hysteresis input/output curve with a dead zone. There is no information given about the stability of the system. There is no ramp present.

The answer is (D).

SYSTEMS–55

The frequency response of a system is given by

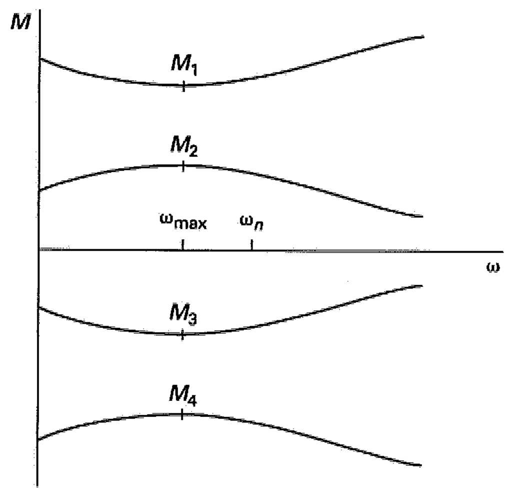
$$\frac{M}{\alpha} = \frac{X_2(j\omega)}{X_1(j\omega)}$$

By differentiation, the peak value of M, M_{max}, and the frequency at which it occurs, ω_{max}, are expressed in terms of the damping ratio, ζ, and natural frequency, ω_n.

$$M_{\text{max}} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$\omega_{\text{max}} = \omega_n\sqrt{1-2\zeta^2}$$

Determine which curve in the figure is a correct representation for M to be the largest response.



(A) $M = M_1$ (B) $M = M_2$ (C) $M = M_3$ (D) $M = M_4$

As ω goes to ω_{max}, M should increase to its peak value. Therefore, the shape of M_2 and M_4 are both correct. However, since M_{max} is always positive, only M_2 is correct.

The answer is (B).

SYSTEMS–56

The frequency response of a system is given by

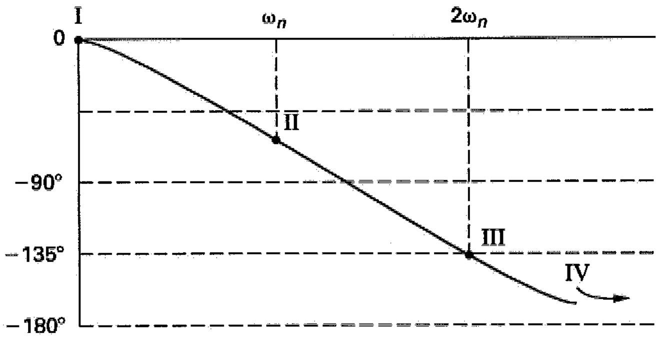
$$\frac{M}{\alpha} = \frac{X_2(j\omega)}{X_1(j\omega)}$$

By differentiation, the peak value of M, $M_{\max}$, and the frequency at which it occurs, $\omega_{\max}$, are expressed in terms of the damping ratio, ζ, and natural frequency, ω_n.

$$M_{\max} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$\omega_{\max} = \omega_n\sqrt{1-2\zeta^2}$$

Considering the polar plot shown, for what range of ω does the peak response in amplitude occur?



(A) ω is at point II.
(B) ω is at point IV.
(C) ω is between points I and II.
(D) ω is between points II and III.

A plot of $M_{\max}$ shows that the peak falls between 0 and ω_n. The polar plot shows two conjugate poles for a simple second-order system. The damping ratio, ζ, and the natural frequency are used to determine $\omega_{\max}$. The peak is at $\omega_{\max} = \omega_n\sqrt{1-2\zeta^2}$, except when $\zeta = 0$ (completely undamped), $\omega_{\max} < \omega_n$.

The answer is (C).

SYSTEMS–57

Which of the following $Q(s)$ equations can be stable?

I. $4s^4 + 8s^2 + 3s + 2 = 0$

II. $4s^4 + 2s^3 + 8s^2 + 3s + 2 = 0$

III. $4s^4 + 2s^3 + 8js^2 + 5s + 2 = 0$

IV. $4s^4 + 2s^3 + 8s^2 - 3s + 2 = 0$

(A) I only (B) II only (C) I and IV (D) II and III

The Routh test indicates that the necessary conditions for a polynomial to have all its roots in the left-hand plane (i.e., the system is stable) are (a) all of the terms must have the same sign; and (b) all of the powers between the highest and the lowest value must have nonzero coefficients, unless all even-power or all odd-power terms are missing. Condition (a) also implies that the coefficient cannot be imaginary. Equation II is the only equation that meets the criteria.

The answer is (B).

SYSTEMS–58

The Routhian array for the following equation is given.

$$Q(s) = s^4 + 6s^3 + 13s^2 + (20 + K)s + K = 0$$

s^4	1	13	K
s^3	6	$20 + K$	0
s^2	$\frac{58 - K}{6}$	K	0
s^1	$\frac{(58 - K)(20 + K) - 36K}{58 - K}$	0	0
s^0	K	0	0

For what range of K will the system be stable?

(A) $0 < K < 33$ (B) $33 < K < 58$ (C) $58 < K < 116$ (D) $0 < K < 116$

For the system to be stable, there can be no sign changes in the first column of the Routhian array. Since the first two entries of that column are positive, the last three entries must also be positive. The entry in the s^2 row gives $K < 58$, while the entry in the s^0 row gives $K > 0$. The numerator in the s^1 row is equal to $-K^2 + 2K + 116$, with roots of -35.1 and 33.1, or $-35.1 < K < 33.1$. For K to satisfy all these restrictions, $0 < K < 33$.

The answer is (A).

SYSTEMS–59

The characteristic equation for a system is

$$Q(s) = s^4 + 5s^3 + 10s^2 + Ks - 1 = 0$$
$$K > 0$$

The Routhian array is

s^4	1	10	-1
s^3	5	K	0
s^2	$\frac{50-K}{5}$	-1	0
s^1	$\frac{K(50-K)+25}{(5)(50-K)}$	0	0
s^0	5	0	0

Which of the following statements is true?

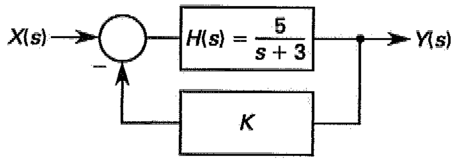
(A) The system is unstable at all points.
(B) The system is unstable for $K < 50$.
(C) The system is stable for $0 < K < 50$.
(D) The system is stable for some point above $K = 50$.

For the system to be stable, there can be no sign changes in the first column of the Routhian array. The $(50-K)$ term in the s^2 row means that the system should be stable for $0 < K < 50$. The term in the s^1 row requires that $-0.49 < K < 50.5$. Thus, the range of values for K for the system to be stable is $0 < K < 50$.

The answer is (C).

SYSTEMS–60

A control system is constructed from linear time-invariant elements as shown. What is the requirement of the constant K so that the closed-loop system is stable (i.e., so that bounded input yields bounded output)?



(A) $K \leq -5/3$ (B) $K \leq -3/5$ (C) $K \geq -3/5$ (D) $K \geq 0$

System transfer function $G(s)$ for the closed-loop system is

$$\begin{aligned} G(s) &= \frac{H(s)}{1 + KH(s)} \\ &= \frac{\frac{5}{s+3}}{\frac{1+5K}{s+3}} \\ &= \frac{\frac{5}{s+3}}{1 + K\left(\frac{5}{s+3}\right)} \\ &= \frac{5}{s+3+5K} \end{aligned}$$

In order for the system to be stable, the pole of the closed-loop system transfer function needs to be at the left hand side of the plane. Therefore,

$$\begin{aligned} s_1 &= -3 - 5K \leq 0 \\ K &\geq -3/5 \end{aligned}$$

The answer is (C).

SYSTEMS–61

Given the following transfer functions, which of these statements is true? K is a constant.

$$G_1(s) = \frac{K}{s\left(s+\frac{1}{B_1}\right)}$$

$$G_2(s) = \frac{K\left(s+\frac{1}{B_2}\right)}{s\left(s+\frac{1}{B_1}\right)}$$

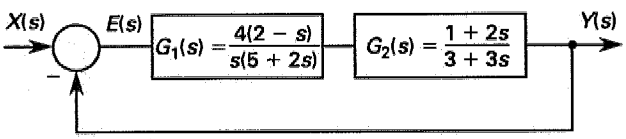
(A) $G_2(s)$ is as stable as $G_1(s)$.
(B) $G_2(s)$ is less stable than $G_1(s)$.
(C) $G_2(s)$ has slower transients than $G_1(s)$.
(D) none of the above

A system's stability is determined by its poles. $G_1(s)$ and $G_2(s)$ have the same poles, 0 and $-1/B_1$. The addition of another zero at $1/B_2$ in system $G_2(s)$ does not change the stability. The addition of the zero also does not change the decay rate of the function. Therefore, the correct answer is option (A).

The answer is (A).

SYSTEMS–62

For the following control system, what is the steady-state error $e_{ss}(t)$ for a ramp input function?



(A) 0 (B) 1/4 (C) 15/8 (D) ∞

Rearrange the open-loop transfer function to Canonic form.

$$G(s) = G_1(s)G_2(s) = \left(\frac{4(2-s)}{s(5+2s)}\right)\left(\frac{1+2s}{3+3s}\right)$$

$$= \frac{\left(\frac{8}{15}\right)\left(\frac{1-s}{2}\right)\left(\frac{1+s}{0.5}\right)}{s\left(\frac{1+s}{2.5}\right)(1+s)}$$

Therefore, $K_b = 8/15$ and $T = 1$.

This is a type 1 system. Using the steady-state error analysis table, the steady-state error $e_{ss}(t)$ for a ramp input function is

$$e_{ss}(t) = \frac{1}{K_B} = 15/8$$

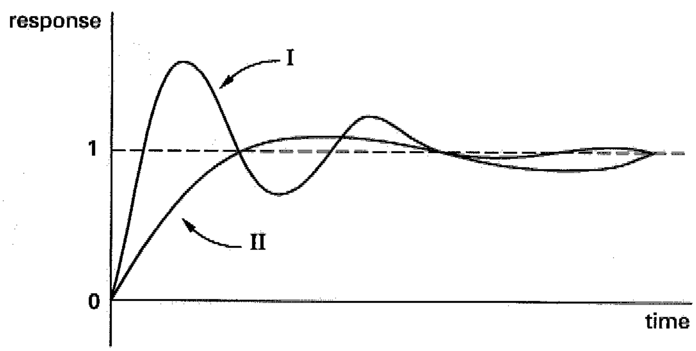
The answer is (C).

SYSTEMS–63

Both of the curves shown represent a system response of the form

$$G(s) = \frac{K}{s(1+s)(1+0.5s)}$$

K_1 is the gain for curve I, and K_2 is the gain for curve II. Which of these statements is true?



(A) K_1 is greater than K_2.
(B) K_1 is less than K_2.
(C) The size of K has no effect on the response.
(D) K is the same for both functions.

Generally, the gain, K, has a direct effect on the type of response obtained. Larger values of gain give larger overshoots or longer settling times. Thus, the gain of curve I is larger than the gain of curve II.

The answer is (A).

SYSTEMS–64

A control system has a control response ratio of

$$\frac{C(s)}{R(s)} = \frac{1}{s^2 + 0.3s + 1}$$

Given that the damping ratio is $\zeta < 0.707$, how many peaks occur in the transient prior to reaching steady-state conditions?

(A) zero
(B) one
(C) two
(D) The function gives a minimum.

The control ratio has two complex poles, which are dominant, but it has no zeros. The poles are $(-0.15 + j0.88)$ and $(-0.15 - j0.88)$. For a damping ratio of $\zeta < 0.707$, a peak occurs. The control response is in the form

$$\frac{C(s)}{R(s)} = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Then,

$$\begin{aligned} \omega_n^2 &= 1 \\ 2\zeta &= 0.3 \\ \zeta &= 0.15 \end{aligned}$$

The system is underdamped. Therefore, there is only one peak value given by the formula.

$$\begin{aligned} M_{\max} &= \frac{1}{2\zeta\sqrt{1-\zeta^2}} = \frac{1}{(2)(0.15)\sqrt{1-(0.15)^2}} \\ &= 3.371 \end{aligned}$$

The answer is (B).

SYSTEMS–65

Which of the following is the correct transform for the following function?

$$F(s) = \frac{1}{s(s+a)(s+b)}$$

(A) $\left(\frac{1}{ab}\right)\left(1+\frac{be^{-at}}{a-b}-\frac{ae^{-bt}}{a-b}\right)$

(B) $\left(\frac{1}{ab}\right)\left(1+\frac{be^{-at}}{b-a}+\frac{ae^{-bt}}{b-a}\right)$

(C) $\left(\frac{1}{ab}\right)\left(1+\frac{be^{-at}}{b-a}-\frac{ae^{-bt}}{b-a}\right)$

(D) $\left(\frac{1}{ab}\right)\left(1-\frac{be^{-at}}{b-a}-\frac{ae^{-bt}}{b-a}\right)$

$$\begin{aligned} F(s) &= \frac{A}{s} + \frac{B}{s+a} + \frac{C}{s+b} \\ A &= \left(\frac{1}{s(s+a)(s+b)}\right) s \,\Big|_{s=0} = \frac{1}{ab} \\ B &= \left(\frac{1}{s(s+a)(s+b)}\right) (s+a) \,\Big|_{s=-a} = \frac{1}{a(a-b)} \\ C &= \left(\frac{1}{s(s+a)((s+b)}\right) (s+b) \,\Big|_{s=-b} = -\frac{1}{b(a-b)} \end{aligned}$$

$$s_0f(t) = L^{-1}(F(s)) = A + Be^{-at} + ce^{-bt} \quad [\text{for } t \geq 0]$$
$$= \frac{1}{ab} + \left(\frac{1}{a(a-b)}\right)e^{-at} - \left(\frac{1}{b(a-b)}\right)e^{-bt}$$
$$= \left(\frac{1}{ab}\right)\left(1 + \left(\frac{b}{a-b}\right)e^{-at} - \left(\frac{a}{a-b}\right)e^{-bt}\right)$$

The answer is (A).

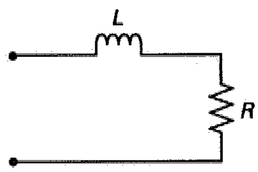
SYSTEMS–66

A circuit with inductance L and resistance R has the transfer function

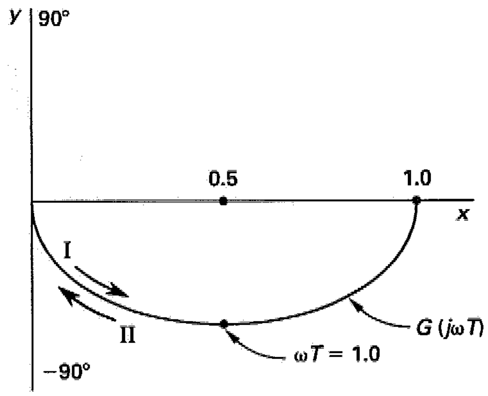
$$G(j\omega) = \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$

Alternatively,

$$G(j\omega T) = \frac{1}{1 + j\omega T}$$



Which of these statements is true about the polar plot of the function?



(A) I shows ω going to zero.

(B) II shows ω going to zero.

(C) If the inductor is replaced by a capacitor, the plot will be in the upper half of the phase plane.

(D) Both options (A) and (C) are true.

As ω approaches ∞, $|G(j\omega)|$ approaches 0. As ω approaches 0, $|G(j\omega)|$ approaches 1. I shows ω going to zero. If the inductor is a capacitor, the plot would be "reflected" on the horizontal axis into the top half of the plane.

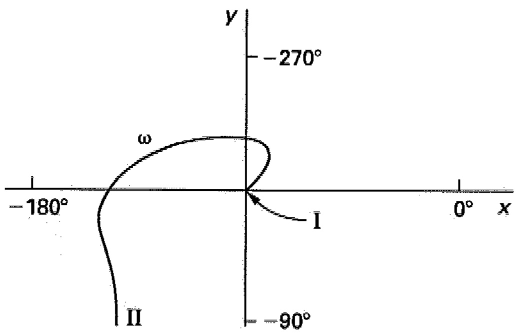
The answer is (D).

SYSTEMS–67

A typical transfer function is as follows.

$$G(j\omega) = \frac{C(j\omega)}{E(j\omega)} = \frac{K}{j\omega(1 + j\omega T_1)(1 + j\omega T_2)(1 + j\omega T_3)}$$

The plot is as shown.



Which of the following is true?

(A) I is where $G(j\omega) = 0$.
(B) I is where $G(j\omega)$ approaches ∞.
(C) II is where $G(j\omega)$ approaches ∞.
(D) II is where $G(j\omega)$ approaches $-\infty$.

In the equation, as ω approaches 0, $G(j\omega)$ approaches ∞, and as ω approaches ∞, $G(j\omega)$ approaches 0. Therefore, point I is where $G(j\omega)$ approaches ∞.

The answer is (B).

16 COMPUTER SCIENCE

COMPUTER SCIENCE–1

Where is the error in the following FORTRAN program?

```
670   READ L,M,N
680   DATA 33 40 60
690   PRINT L+M/N
700   PRINT L*N+M
800   END
```

(A) line 670 (B) line 680 (C) line 690 (D) line 700

Line 680 should have commas separating the data points.

The answer is (B).

COMPUTER SCIENCE–2

What is the output of the following FORTRAN program?

```
100   DATA 'June',15,1955
200   READ N$
300   DATA 1948
400   READ C,X,X1
500   PRINT X1,N$,X,C
```

(A) 15,1955,June,1955
(B) June 15 1955 1948
(C) 1948 June 1955 15
(D) 1948 June 15 1955

June is assigned to N$, 15 is assigned to C, 1955 is assigned to X, and 1948 is assigned to X_1. The output will be: 1948 June 1955 15.

The answer is (C).

COMPUTER SCIENCE–3

In the given program, what is the value assigned to N?

```
10  N=0
15  GO TO 50
20  FOR X=−1 TO 7
25  N=6
30  FOR Y=3 TO 6
35  N=X*Y+16
40  NEXT Y
45  NEXT X
50  PRINT N
55  END
```

(A) −3678 (B) −356 (C) 0 (D) 98

The statement of line 15 bypasses the loops, and N remains the value assigned to it in line 10: N=0.

The answer is (C).

COMPUTER SCIENCE–4

Determine the output of the given program.

```
10  FOR X=−1 TO 8 STEP 3
20  S=0
30  FOR Y=0 TO 4 STEP 2
40  S=S+X*Y+1
50  NEXT Y
60  PRINT S
70  NEXT X
80  END
```

(A) −5 15 18 24 (B) −3 15 33 51 (C) 0 17 23 84 (D) 72 12 21 28

S gets assigned a value three times during the first loop of X, and is then printed.

$$\left.\begin{array}{r} X = -1 \\ S = 0 \\ Y = 0 \end{array}\right\} \quad S = 0 + (-1)(0) + 1 = 1$$

$$\left.\begin{array}{r} X = -1 \\ S = 1 \\ Y = 2 \end{array}\right\} \quad S = 1 + (-1)(2) + 1 = 0$$

$$\left.\begin{array}{r} X = -1 \\ S = 0 \\ Y = 4 \end{array}\right\} \quad S = 0 + (-1)(4) + 1 = -3$$

This may be continued for the subsequent loop values of X. The corresponding S values printed are 15, 33, and 51.

> The answer is (B).

COMPUTER SCIENCE–5

What are the respective values of A_1, A_2, and A_3, given the following assignments?

A_1=INT(5)
A_2=INT(10.4)
A_3=INT(−10.4)

(A) A_1=5, A_2=10, A_3=−10
(B) A_1=5, A_2=11, A_3=−10
(C) A_1=5, A_2=10, A_3=−11
(D) A_1=5, A_2=11, A_3=−11

The INT function assigns the largest integer value not greater than the original value. 5, 10, and −11 are the respective A values.

> The answer is (C).

COMPUTER SCIENCE–6

What would be the value printed as a result of the following instructions?

```
10  DEF FNA(X)=X**2+1/X
20  PRINT FNA(2)
30  END
```

(A) 3.2 (B) 4.5 (C) 6.5 (D) 7.2

$$\text{FNA}(2) = (2)^2 + \frac{1}{2} = 4.5$$

The answer is (B).

COMPUTER SCIENCE–7

What would be the output of the following FORTRAN program?

```
10  DEF FNA(X,Y)=X*2+X*3−X*Y
20  READ B(1),C,D
30  DATA 3,2,4
40  PRINT FNA(B(1),B(1))
50  PRINT FNA(C/D,C*D)
60  END
```

(A) 6,−1.5 (B) 8,3 (C) 9,−11 (D) 11,−9

From the READ and DATA statements, B(1) is 3, C is 2, and D is 4. Thus, in line 40, B(1) replaces X and Y in the function statement of line 10. The output of line 40 is

$$(3)(2) + (3)(3) - (3)(3) = 6$$

Line 50 assigns X = 2/4 = 1/2 and Y = (2)(4) = 8, and then prints the function output

$$\left(\frac{1}{2}\right)(2) + \left(\frac{1}{2}\right)(3) - \left(\frac{1}{2}\right)(8) = -1.5$$

The answer is (A).

COMPUTER SCIENCE–8

If table Y represents an array, and B is 2, what is the value of Y(B+2)?

I	table Y
1	90
2	−20
3	410
4	70
5	17

(A) −22 (B) −18 (C) 17 (D) 70

When B is 2, Y(B+2)=Y(4). The value for Y(4) is 70.

The answer is (D).

COMPUTER SCIENCE–9

Why is the following expression NOT acceptable as a FORTRAN integer constant?

237, 100

(A) The first character must be a letter.
(B) There are more than six characters.
(C) There is no decimal point.
(D) It contains a comma.

Commas are not allowed in integer constants.

The answer is (D).

COMPUTER SCIENCE–10

Which of the following are acceptable as FORTRAN integer variables?

I. NINA II. A+10 III. INTEGER IV. 2813 V. A160

(A) I only (B) I and V (C) I and III (D) I, III, and V

Variable names can be formed from up to six characters, the first character being a letter. Integer variables must begin with the letters I, J, K, L, M, or N; all other variable names represent real variables. The characters must be alphanumeric, hence the + character in II is not allowed. I is the only acceptable choice.

The answer is (A).

COMPUTER SCIENCE–11

Which of the following is NOT acceptable as a FORTRAN integer variable name?

(A) IRISH (B) KOST (C) MAPLE (D) INSERTS

INSERTS exceeds the limit of six characters.

The answer is (D).

COMPUTER SCIENCE–12

Which of the following are NOT acceptable as FORTRAN integer constants?

I. 5000 II. −14 III. A15 IV. 16.52 V. +999

(A) I and II (B) II and III (C) III and IV (D) II, III, IV, and V

An integer constant cannot begin with a letter character and cannot have a decimal point. Thus, III and IV are unacceptable.

The answer is (C).

COMPUTER SCIENCE–13

Which of the following are NOT acceptable as FORTRAN real constant names?

I. APPLE II. LAMB III. 4ABC IV. WATER V. TREE

(A) II and III (B) IV and V (C) I, II, and III (D) II, III, and V

LAMB is incorrect since the initial character L denotes an integer variable. 4ABC is incorrect since the first character should be a letter.

The answer is (A).

COMPUTER SCIENCE–14

What is a correct declaration form for the logical variable AZ?

(A) AZ=.TRUE. (B) AZ=1 (C) AZ=1. (D) AZ='TRUE'

For a logical variable, TRUE is enclosed in periods only.

The answer is (A).

COMPUTER SCIENCE–15

Which of the following is NOT acceptable as a FORTRAN statement?

(A) A=B (B) A=B+C (C) AB=C*D (D) A*B=C*D

The left side of an assignment statement must be a variable and not an operation. Option (D) is unacceptable.

The answer is (D).

COMPUTER SCIENCE–16

What value is assigned to A in the following expression?

A=3.4+(18*(4**2+17.2*8/6))−2/4

(A) 92.5 (B) 178.1 (C) 502.1 (D) 703.7

The expressions within parentheses are evaluated first. This is followed by exponentiation, multiplication and division, and addition and subtraction. Within each level of operation, the expressions are performed left to right.

$$A = 3.4 + \big((18)(16 + 22.93)\big) - 0.5 = 703.7$$

The answer is (D).

COMPUTER SCIENCE–17

What value would the computer assign to the FORTRAN variable D8=1.0E8−1.5E6?

(A) -0.5×10^2 (B) 1.5×10^2 (C) 9.85×10^7 (D) 10.5×10^7

The letter E denotes scientific notation. Therefore,

$$D8 = (1 \times 10^8) - (1.5 \times 10^6) = 9.85 \times 10^7$$

The answer is (C).

COMPUTER SCIENCE–18

Which of the following is a valid FORTRAN statement for the algebraic expression below?

$$\frac{3x^2 - 5x}{y^2 + 1}$$

(A) 3.0*X**2.0−5.0*X/Y**2.0+1.0
(B) (3.0X**2.0−5.0X)/Y**2.0+1.0
(C) (3.0*X**2.0−5.0*X)/(Y**2.0+1.0)
(D) (3.0*X**2.0−5.0X)/(Y**2.0+1.0)

The numerator and denominator must be enclosed in parentheses. Each operation must be explicitly and unambiguously stated. 3.0X is not a substitute for 3.0*X.

The answer is (C).

COMPUTER SCIENCE–19

Which of the following is a FORTRAN expression corresponding to the given formula?

$$\frac{\ln\left(a+\sqrt{b^3}\right)}{\sin\left(c^2+1\right)}$$

(A) ALOG10(A+SQRT(B**3.0))/SIN(C**2.0+1.0)
(B) LOG(A+SQRTB**3.0)/SIN(C**2.0+1.0)
(C) ALOG(A+SQRT(B**3.0)/SIN(C**2.0+1.0))
(D) ALOG(A+SQRT(B**3.0))/SIN(C**2.0+1.0)

The FORTRAN expression for the natural logarithm is ALOG, followed by the term enclosed in parentheses. The natural logarithm term in option (C) includes the denominator due to a misplaced parenthesis.

The answer is (D).

COMPUTER SCIENCE–20

How would the algebraic expression below be written in FORTRAN?

$$\left(1+\frac{x}{y}\right)^{x-1}$$

(A) (1.0+X/Y)*(X−1.0)
(B) (1.0+X)/Y**(X−1.0)
(C) (1.0+X/Y)**(X−1.0)
(D) (1.0+X/Y)**X−1.0

The exponent must be enclosed in parentheses in this case. Option (A) has a multiplication sign instead of an exponent sign, and the first term in option (B) is incorrect. Only option (C) is correct.

The answer is (C).

COMPUTER SCIENCE–21

How would the following expression be written in FORTRAN?

$$2 - \frac{x}{\sqrt{x^2 - \frac{1}{y}}}$$

(A) (2.0–X)/SQRT(X**2.0–1.0/Y)

(B) 2.0–(X/SQRTX**2.0–1.0/Y)

(C) 2.0–X/SQRTX**2.0–1.0/Y

(D) 2.0–X/SQRT(X**2.0–1.0/Y)

The parentheses in options (A) and (B) are wrong, and option (C) is missing parentheses around the SQRT term.

The answer is (D).

COMPUTER SCIENCE–22

Which of the following is the mathematical equivalent of the given FORTRAN expression?

SIN(2.0PI)**ABS(Y–1.0)

(A) $(\sin 2\pi)^{y-1}$ (B) $(\sin 2\pi)^{|y-1|}$ (C) $\sin 2\pi ** |y-1|$ (D) $\sin(2\pi)^{|y-1|}$

The exponent in option (A) should have absolute value signs, the (**) character in option (C) is meaningless in math, and $(\sin 2\pi)$ should be enclosed in parentheses in option (D).

The answer is (B).

COMPUTER SCIENCE–23

What is the output of the following FORTRAN program? (␣ denotes a blank space.)

```
      F=50.0*4.6
      WRITE(6,100)F
 100  FORMAT(' ',T6,E9.3)
```

(A) 0.230E␣03 (B) ␣0.230E␣03 (C) ␣␣␣␣␣␣2.30E␣02 (D) ␣␣␣␣␣0.230E␣03

The T format in line 100 specifies the column location where the output starts. Thus, there are five blank spaces before the output. E9.3 means there are to be nine character positions reserved for the output, with three digits printed to the right of the decimal point in the scientific notation.

The answer is (D).

COMPUTER SCIENCE–24

Which of these FORTRAN format specifications reads the input −567610 as −5676.10?

(A) F6.2 (B) F7.2 (C) F7.3 (D) F8.2

There are a total of eight character positions used, including the negative sign and the decimal point. Since there are two digits to the right of the decimal, the correct format specification is F8.2.

The answer is (D).

COMPUTER SCIENCE–25

In a spreadsheet, the number in cell A2 is set to 2. Then cell B2 is set to A2*4/$A$2, where $ indicates the absolute cell address. This formula is copied into cells C2 and D2. The number shown in cell D2 is

(A) 2 (B) 4 (C) 8 (D) 16

The formula and value of each cell is as follows.

cell	formula	value of cell
A2	2	2
B2	A2*4/$A$2	4
C2	B2*4/$A$2	8
D2	C2*4/$A$2	16

The answer is (D).

COMPUTER SCIENCE–26

Which of the following is the corresponding FORTRAN FORMAT statement for this output? (␣ denotes a blank space.)

␣␣␣␣␣SUM=␣2230.

(A) 10 FORMAT(5X,'SUM=',F5.1)

(B) 10 FORMAT(5X,'SUM=',1X,F5.0)

(C) 10 FORMAT(5X,'SUM=',F6.1)

(D) 10 FORMAT(5X,'SUM=',1X,F5)

For the output, there are five blanks followed by text, then one blank, and finally a five-character number with no digits to the right of the decimal point. Option (B) is the correct choice.

The answer is (B).

COMPUTER SCIENCE–27

What should be the READ and FORMAT statements for this data output?

input data = 315.66

(A)
```
      READ(5,20)I
20    FORMAT(I6)
```

(B)
```
      READ(5,20)S
20    FORMAT(F6.2)
```

(C)
```
      READ(5,20)S
      FORMAT(F6.2)
```

(D)
```
      READ(5,30)S
20    FORMAT(F6.2)
```

Since the input is a real number, neither an integer variable I nor an integer field I6 can be used, thus option (A) is wrong. Option (C) is missing its FORMAT statement number, and the FORMAT statement number in option (D) does not match the READ statement number.

> The answer is (B).

COMPUTER SCIENCE–28

Which of the following is the corresponding output of this WRITE statement? (␣ denotes a blank space.)

```
      KX=130
      SUM=125.3869
      WRITE(6,3)KX,SUM
3     FORMAT(I6,2X,F8.2)
```

(A) 130␣␣␣␣␣␣␣␣125.39
(B) ␣␣130␣125.39␣␣␣␣
(C) ␣␣␣130␣␣␣␣␣125.39
(D) ␣␣␣130125.39␣␣␣␣

The integer 130 will be written first, with six character positions allotted to it. There will be three blank spaces preceding the numerals, since numbers are always flush with the right side of the specified column block. After 130, two blanks are printed due to the 2X in the FORMAT statement. Finally, the real number SUM is written, with two digits to the right of the decimal and eight character positions overall. The number will be flush right in its block.

The answer is (C).

COMPUTER SCIENCE–29

Which of the following are correct WRITE and FORMAT statements for the output A=␣0.023E␣05? (␣ denotes a blank space.)

(A)
```
     WRITE(6,2)A
   2 FORMAT(T2,'A=',1X,I9)
```

(B)
```
     WRITE(6,2)A
     FORMAT(T2,'A=',T4,E9.3)
```

(C)
```
     WRITE(6,2)A
   2 FORMAT(T2,'A=',1X,F9.3)
```

(D)
```
     WRITE(6,2)A
   2 FORMAT(T2,'A=',1X,E9.3)
```

For an output in scientific notation, the FORMAT statement cannot use an I- or F-mask as in options (A) and (C). There is no FORMAT statement number in option (B).

The answer is (D).

COMPUTER SCIENCE–30

Which of the following is the output of this WRITE statement?

```
      WRITE(6,10)
   10 FORMAT(1X,'THE NUMBER=',2X,'33.15')
```

(␣ denotes a blank space.)

(A) ␣THE␣NUMBER␣=␣␣33.15
(B) ␣THE␣NUMBER=␣␣33.15
(C) ␣THE␣NUMBER=␣␣␣33.15
(D) THE␣NUMBER=␣␣␣␣33.15

There is a blank space before "THE NUMBER=," then two more blanks, and then 33.15. There is no space between the letter R and the equal sign.

The answer is (B).

COMPUTER SCIENCE–31

Numerical characters arranged as 22.2␣55184.5␣750 are available to be read. (␣ denotes a blank space.) What numbers will the computer read using the following FORMAT statement?

```
      READ(5,8) X1, X2, X3, X4
    8 FORMAT(4F4.2)
```

(A) 22.2, 5.51, 8.45, 7.50
(B) 2.22, 5.51, 84.5, 7.50
(C) 22.2, 55.10, 84.5, 7.50
(D) 22.2, 5.51, 84.5, 7.50

The FORMAT statement specifies that the input is to be read four character spaces at a time from left to right, four times successively. The number read in will consist of four characters, with a maximum of two decimal places. If a decimal point was one of the initial four characters, it will be kept. Thus, the input is read as: 22.2, ␣551, 84.5, and ␣750, which become evaluated as 22.2, 5.51, 84.5, and 7.50.

The answer is (D).

COMPUTER SCIENCE–32

Which of the following is the correct WRITE statement to print the even numbers from 2 to 40, assuming conventional output device reference numbers?

(A)
```
      WRITE(6,8) (I=1,40,2)
    8 FORMAT(20I2)
```

(B)
```
      WRITE(5,8) (I=2,40,2)
    8 FORMAT(20I2)
```

(C)
```
      WRITE(6,8) (I=2,40,2)
    8 FORMAT(20I2)
```

(D)
```
      WRITE(6,8) (I=2,40,2)
    6 FORMAT(20I2)
```

The first number in the WRITE line should be 6, the conventional reference number for a printer; the WRITE line in option (B) refers to an input device instead of a printer. The second number is the FORMAT line number which is to be used; option (D) does not have the correct number in this case. Both options (A) and (C) have errors in their loop specifications: the first two numbers specify the beginning and end of the loop, and the third number specifies the increment within the loop. The correct loop specification should be (I=2,40,2) as in option (C).

The answer is (C).

COMPUTER SCIENCE–33

A data line is as follows: 21.459214.5307421557.82134524. Which of the following is an incorrect storage of this data using the following READ statement?

```
      READ(5,98)A,B,I,J,C,K
   98 FORMAT(2F7.4,2I2,F6.2,I3)
```

(A) A=21.4592 (B) B=14.5307 (C) J=15 (D) K=134

The FORMAT statement specifies that the data is to be read from left to right in two blocks of seven character spaces, two blocks of two character spaces, one block of six, and one last block of three spaces. Since each block corresponds to a variable, K=345 and not 134.

The answer is (D).

COMPUTER SCIENCE–34

Which of the following is a correct IF statement?

(A) IF.(I.GE.5) GO TO 7
(B) IF (I,GE,5) GO TO 7
(C) IF (I.GE.5)
(D) IF (I.GE.5) GO TO 7

The logical IF statement has the form

IF [**le**] [**statement**]

[**le**] is a logical expression and [**statement**] is any executable statement except DO or IF. The operator within the logical expression should be of the form .[op]., and there should be no periods or commas between the components of the statement. There are no commas as in option (B).

The answer is (D).

COMPUTER SCIENCE–35

What value is assigned to Q in this program?

```
    R=18.0
    S=6.0
    T=2.0
    Q=R/S**T-T
    IF Q 10,20,30
10  Q=10
15  GO TO 40
20  Q=100
25  GO TO 40
30  Q=1000
40  END
```

(A) −1.5 (B) 10 (C) 100 (D) 1000

Before the numbered lines, Q gets the value

$$Q = \frac{R}{S^T} - T$$
$$= \frac{18}{(6)^2} - 2$$
$$= 0.5 - 2$$
$$= -1.5$$

Since Q is less than zero, the IF statement passes control of the program to line 10, where Q is reassigned the value of 10. The program then skips to the END line.

The answer is (B).

COMPUTER SCIENCE–36

Which of the following DO statements counts J from 1 to 11 in increments of 2?

(A) DO 5 J=1,2,11 (B) DO 5, J=1,11,2
(C) DO 5 J=1.11.2 (D) DO 5 J=1,11,2

The general form of the DO statement is DO s $i = j, k, l$. s is a statement number, i is the integer loop variable, j is the initial value assigned to i, k is an inclusive upper bound on i, which must exceed j, and l is the increment for i. Thus, the correct form of the statement is DO 5 J $= 1, 11, 2$.

The answer is (D).

COMPUTER SCIENCE–37

What is wrong with this program?

```
      DO 20 I=1,5
      A(I)=5.8
      I=5
   20 CONTINUE
```

(A) A cannot be a real number.
(B) I cannot be an integer.
(C) There is an excess comma in the DO statement.
(D) The value of I cannot be changed in the DO loop.

The integer loop variable I is made a constant, creating an error in the DO loop.

The answer is (D).

COMPUTER SCIENCE–38

Which of these expressions for a one-dimensional array, A, is acceptable? Assume B, H, I, J, K, and L are greater than zero.

(A) A(I,K) (B) A(H+J) (C) A(2*L−1) (D) A(−3)

Option (C) is the only acceptable one-dimensional array. Since the array variable or element must be an integer variable or a positive integer, options (B) and (D) are wrong. Option (A) is an acceptable form for a two-dimensional array.

The answer is (C).

COMPUTER SCIENCE–39

Which is the correct DIMENSION statement for a 9×9 matrix, A, and a vector, B, with nine elements?

(A) DIMENSION A(9.9),B(9)
(B) DIM A(9,9),B(9)
(C) DIMENSION A(9,9),B(9)
(D) DIMENSION, A(9,9),B(9)

The correct choice is option (C). The DIMENSION statement does not use periods, thus option (A) is wrong. Option (B) does not have DIMENSION written completely, and option (D) has an extra comma.

The answer is (C).

COMPUTER SCIENCE–40

A 2×2 matrix, Z, is loaded from a DATA statement in the following order: 15.0, 10.0, 7.0, 20.0. List the matrix elements of Z in order of descending data magnitude.

(A) Z(1,1), Z(1,2), Z(2,1), Z(2,2)
(B) Z(2,2), Z(1,1), Z(1,2), Z(2,1)
(C) Z(1,1), Z(2,1), Z(1,2), Z(2,2)
(D) Z(2,2), Z(1,1), Z(2,1), Z(1,2)

The matrix elements are assigned in the order Z(1,1), Z(2,1), Z(1,2), and Z(2,2). These elements get the respective values of 15.0, 10.0, 7.0, and 20.0. Thus, in order of descending magnitude, the matrix elements are Z(2,2), Z(1,1), Z(2,1), Z(1,2).

The answer is (D).

COMPUTER SCIENCE–41

Find the values of X and Y after this program segment has been performed.

```
X=2
Y=4

Z=X
X=Y
Y=Z
```

(A) X=2, Y=4 (B) X=4, Y=4 (C) X=4, Y=2 (D) X=2, Y=2

The logic sequence is

Z=X=2
X=Y=4
Y=Z=2

The final X and Y values are 4 and 2, respectively.

The answer is (C).

COMPUTER SCIENCE–42

What is the value of A after this program segment is performed?

```
    A=0
    DO 77 I=1,4
77  A=A+I
```

(A) 4 (B) 5 (C) 8 (D) 10

A table of the value of A after each loop is

A	I	A=A+I
0	1	1
1	2	3
3	3	6
6	4	10

The final value of A is 10.

The answer is (D).

COMPUTER SCIENCE–43

Matt is downloading an 800 kB file from the internet using his 28.8k modem. How long will it take him to finish downloading?

(A) 0.46 min (B) 3.7 min (C) 3.8 min (D) 4.1 min

The number of bits to be transmitted is

$$\text{no. of bits transmitted} = (800 \text{ kB})\left(1024 \, \frac{\text{bytes}}{\text{kB}}\right)\left(8 \, \frac{\text{bits}}{\text{byte}}\right)$$
$$= 6.55 \times 10^6 \text{ bits}$$

The modem transmits at 28 800 bits/s. The time required for downloading is

$$t = \frac{\text{total bits to be transmitted}}{\text{transmission speed}} = \frac{6.55 \times 10^6 \text{ bits}}{\left(28\,800 \, \frac{\text{bits}}{\text{s}}\right)\left(60 \, \frac{\text{s}}{\text{min}}\right)}$$
$$= 3.8 \text{ min}$$

The answer is (C).

COMPUTER SCIENCE–44

Find the value of J after executing the following program.

```
    I=1
    J=0
15  J=J**2+I**2
    IF (I.EQ.3) GO TO 16
    I=I+1
    GO TO 15
16  CONTINUE
```

(A) 9 (B) 15 (C) 34 (D) 40

The value of J after each loop is

I	$J=J^2+I^2$
1	1
2	5
3	34

The answer is (C).

COMPUTER SCIENCE–45

Which of the following sets of statements gives a value for ISUM different than the others?

(A)
```
      ISUM=0
      DO 10 I=1,5
      ISUM=ISUM+I*2-1
   10 CONTINUE
```

(B)
```
      ISUM=0
      ISIG=1
      DO 20 I=1,9
      IF (ISIG.EQ.-1) GO TO 10
      ISUM=ISUM+I
   10 ISIG=-ISIG
   20 CONTINUE
```

(C)
```
      ISUM=O
      DO 10 I=2,10,2
      ISUM=ISUM+I
   10 CONTINUE
      ISUM=ISUM-10
```

(D)
```
      ISUM=0
      DO 10 I=1,9,2
      ISUM=ISUM+I
   10 CONTINUE
```

Option (C) gives ISUM=20. All other options give ISUM=25.

The answer is (C).

COMPUTER SCIENCE–46

What is the purpose of the following program?

```
      DIMENSION KK(100)
 10   KK(I)=I
      DO 60 J=2,7
      DO 30 K=J,50
      N=J*K
 30   KK(N)=0
 60   CONTINUE
      DO 80 I=1,100
      IF (KK(I).EQ.0) GO TO 80
      WRITE(5,101) KK(I)
101   FORMAT(3X,I5)
 80   CONTINUE
```

(A) to find a sum of the even numbers between 1 and 100
(B) to find a sum of the odd numbers between 1 and 100
(C) to find the prime numbers between 1 and 100
(D) to sort the numbers of KK(100) in descending order

This program finds the prime numbers between 1 and 100. It sets all nonprime values of the array KK equal to zero, and then writes out what remains.

The answer is (C).

COMPUTER SCIENCE–47

Given the following statements, what is the value of R in the MAIN program?

```
EXTERNAL             EXP      main program
A=0.0
R=FUNCT(A,EXP)+20.0

FUNCTION FUNCT(X,FX)          function definition
FUNCT=FX(X)
RETURN
END
```

(A) 0.0 (B) 1.0 (C) 20.0 (D) 21.0

The main program reads a value of 0 into the function FUNCT. This function is the natural exponential function e^x. Therefore, the value $e^0 = 1$ is returned to the main program, where it is summed with the value 20.0. The final value of R is 21.0.

The answer is (D).

COMPUTER SCIENCE–48

Of the following, which is the only acceptable FORTRAN statement?

(A) DIMENSION Z(TEN)
(B) WRITE(6,101) (MATRIX(J,J=1,8))
(C) READ(5,99) (TIME(X),X=1,10)
(D) READ(5,99) (TOTAL(N),N=1,20)

The only correct statement is option (D). The others are not allowed in FORTRAN.

The answer is (D).

COMPUTER SCIENCE–49

If A and B are false, and C is true, which of the following logical expressions will be true?

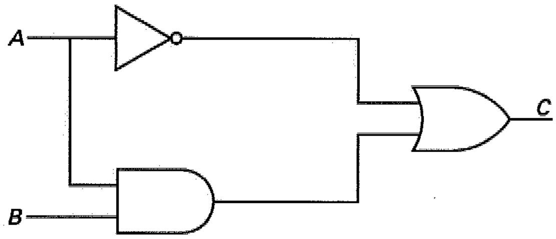
(A) .NOT.C.AND.A (B) B.AND.C.OR.A (C) .NOT.(A.AND.B) (D) .NOT.C

Only option (C) is true. Since (A.AND.B) is false, NOT(A.AND.B) is true.

The answer is (C).

COMPUTER SCIENCE–50

A set of standard logic gates receives binary logic (values of 1 or 0) signals A and B. What is the logic representation of output signal C?



(A) $C = B$ (B) $C = 1$ (C) $C = \overline{A} + AB$ (D) $C = \overline{A}(A + B)$

The output of the NOR gate is $\overline{A}$, the output of the AND gate is AB. These two outputs go to the OR gate and the output is $\overline{A} + AB$.

The answer is (C).

COMPUTER SCIENCE–51

What are the values assigned to A and B in this subroutine?

COMMON ALPHA,BETA **main program**
ALPHA=4.0
BETA=3.0

COMMON B,A **subroutine**

(A) A=3.0, B=3.0 (B) A=4.0, B=3.0 (C) A=3.0, B=4.0 (D) A=4.0, B=4.0

It is the order of the common variables that determines their value. In the subroutine, B corresponds to ALPHA=4.0 in the main program while A corresponds to BETA=3.0.

The answer is (C).

COMPUTER SCIENCE–52

Which of the following are true statements regarding user-defined functions?

I. The function is more versatile than a subroutine as it is not limited to mathematical calculations.
II. The function is defined as a variable in the main program.
III. The function may be used repeatedly and anywhere in the program

(A) I only (B) I and II (C) I and III (D) II and III

I is false, since the subroutine is more versatile. II and III are true.

The answer is (D).

COMPUTER SCIENCE–53

Which of the following would have to be added to the main program as a result of the variable XINCH being changed to INCH? Before the change, the function is

```
FUNCTION FOOT(XINCH)
FOOT=XINCH/12.0
RETURN
END
```

(A) a COMMON statement
(B) a CALL statement
(C) a DIMENSION specification
(D) a REAL variable declaration

Without a REAL variable declaration, the variable INCH would be read by the computer as an integer variable, and FOOT would have an error.

The answer is (D).

COMPUTER SCIENCE–54

What will be the output of the following program? (␣ denotes a space in the output.)

```
      REAL K
      DATA K,X/1.3,14.5
      READ(5,100) K,X
100   FORMAT(F5.1,3X,F5.1)
      F=K*X
      WRITE(6,200)K,X,F
200   FORMAT(1X,'K=',E9.2,3X,'X=',E10.3,3X,'F=',E9.2)
      STOP
999   END
```

(A) ␣K=␣0.1E␣01␣␣␣X=␣0.1E␣02␣␣␣F=␣0.2E02

(B) K=10␣␣␣X=100␣␣␣F=20

(C) ␣K=␣0.13E␣01␣␣␣X=␣0.145E␣02␣␣␣F=␣0.19E␣02

(D) ␣K=0.13E01␣␣␣X=0.145E02␣␣␣F=0.19E02

The key line in the program is line 200. Line 100 is a FORMAT line for the data read into the program, but line 200 is the FORMAT line for the output.

The answer is (C).

COMPUTER SCIENCE–55

Of the following statements, which is FALSE?

(A) The FORMAT statement may or may not be executed by the program.

(B) More than one STOP statement is permitted in a program.

(C) The arithmetic IF can only be used to evaluate an arithmetic expression.

(D) No FORMAT statement is required with unformatted READ or WRITE statements.

The FORMAT statement must be executed by the program. Therefore, option (A) is false.

The answer is (A).

COMPUTER SCIENCE–56

What is the output of the program given? (␣ denotes a space, and ⊏ denotes a blank line.)

```
      DO 30 I=1,3
      WRITE(6,10) X(I)
10    FORMAT(1X,E8.2)
      WRITE(6,20)
20    FORMAT(2X,/)
30    CONTINUE
```

(A) ␣0.41E␣04␣␣␣0.68E␣04␣␣␣0.58E␣04

(B) ␣0.41E␣04
␣0.68E␣04
␣0.58E␣04

(C) ␣0.41E␣04
⊏
␣0.68E␣04
⊏
␣0.58E␣04

(D) ␣0.41E␣04
⊏
⊏
␣0.68E␣04
⊏
⊏
␣0.58E␣04

The "/" symbol designates a skipped line. The 2X that precedes the line skip symbol in line 20 results in two skipped lines between output lines. Option (D) is the correct choice.

The answer is (D).