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## Sharpness of resonance

At resonance the amplitude of ~~os~~ the oscillating system becomes maximum, it decreases from this maximum value with the change (decrease / increase) of the frequency of the applied force

- The term sharpness of resonance refers to the rate of falling in amplitude with the change of forcing frequency on the each side of the resonance frequency.

eg:- Sonometer.

resonance column is an example of flat resonance.

A simple harmonic motion of a body of mass 0.05 kg is represented by $x = 4 \sin(\pi t + \frac{\pi}{3})$ m

Calculate the following.

i) Amplitude

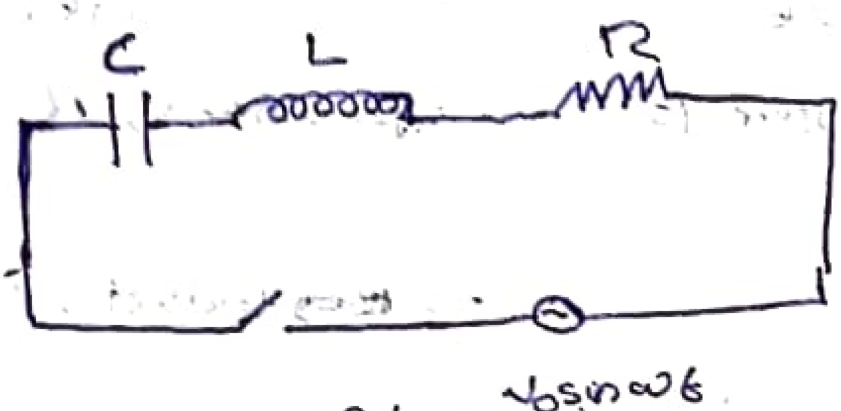
ii) Initial phase

$\div L$

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{CL} = \frac{V_0}{L}\sin\omega t.$$

$$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega^2 x = f_0 \sin\omega t.$$

→ LCR circuit is ... oscillator in which oscillations are sustained ... electromotive force.




Applying kirchoff's law

$$I = \frac{dq}{dt}$$

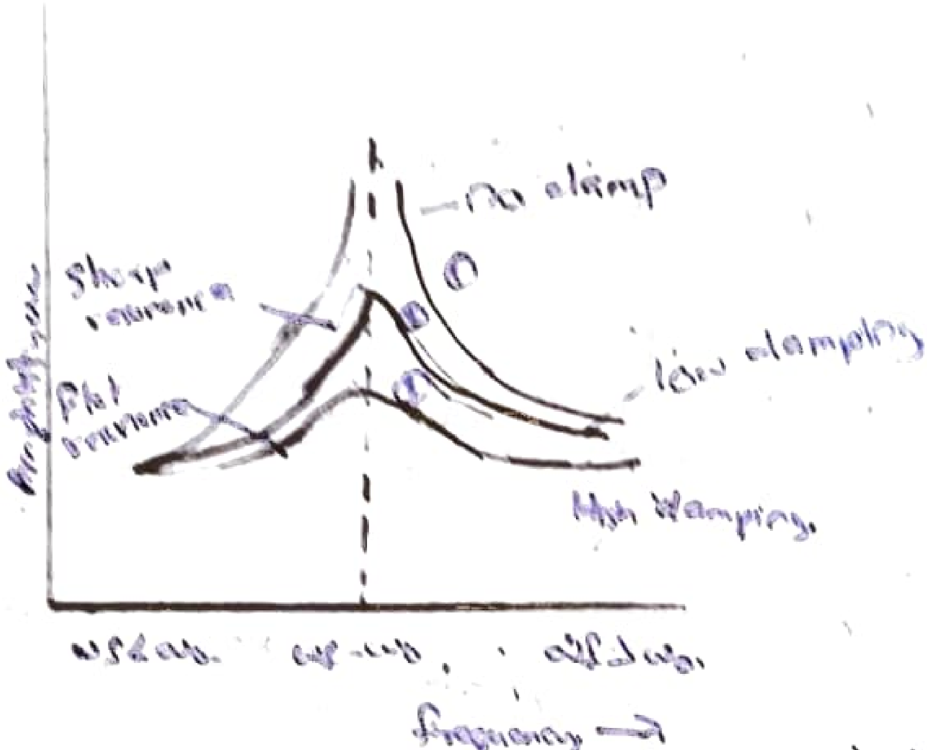
$$V_L + V_C + V_R = V_0 \sin\omega t$$

$$L\frac{dI}{dt} + \frac{q}{C} + IR = V_0 \sin\omega t$$

$$L\frac{d}{dt}\left(\frac{dq}{dt}\right) + \frac{R\,dq}{dt} + \frac{q}{C} = V_0 \sin\omega t$$

$$L\frac{d^2q}{dt^2} + \frac{R\,dq}{dt} + \frac{q}{C} = V_0 \sin\omega t.$$

- For low damping the fall of the curve about at resonance frequency is steeper than high damping.




Curve (1) shows the amplitude when there is no damping, i.e., $r=0$. In this case amplitude become infinite at $\omega_f = \omega_0$. This case is never attain, in practice due to frictional resistance a slight damping is always present.

Curve's (2) & (3) shows the effect of damping on the amplitude. It is observed that the peaks of the curves moves towards the left. It is also observed that the value of 'r', which different for different value of 'r' diminishing as the value of $\beta$ increases $\left(2r = \frac{\beta}{m}\right)$

The amphlitud is maximum

when $\sqrt{(\omega_0^2-\omega_f^2)+4r^2 2\omega_f^2}$ is minimum.

$$\frac{d}{d\omega_f}\left((\omega_0^2-\omega_f^2)^2+4r^2\omega_f^2\right)=0$$

$$2(\omega_0^2-\omega_f^2)\frac{2d\omega_0}{d\omega_f}$$

$$2(\omega_0^2-\omega_f^2)(-2\omega_f)+4r^2\omega_f=0.$$

$$\cancel{\omega_f=\sqrt{\omega_0}}$$

$$\cancel{\omega_f=\sqrt{\omega_0^2-2r^2}}$$

$F = F_R + F_D + F_A$

$F_R$ = Restoring force $= -kx$

$F_D$ = Damping force $= -\beta \frac{dx}{dt}$

$F_A$ = applied force. $= F_0 \sin \omega$

$ma = -kx - \beta \frac{dx}{dt} + F_0 \sin \omega_p t$

$F = ma$.

$a = \frac{d^2x}{dt^2}$

$$m\frac{d^2x}{dt^2} = -kx - \beta\frac{dx}{dt} + F_0 \sin \omega_p t.$$

$$m\frac{d^2x}{dt^2} + kx + \beta\frac{dx}{dt} = F_0 \sin \omega_p t$$

$\div m.$ $$\frac{d^2x}{dt^2} + \frac{kx}{m} + \frac{\beta}{m}\frac{dx}{dt} = \frac{F_0}{m}\sin \omega_p t.$$

$\frac{k}{m} = 2\lambda$

$\frac{\beta}{m} = \omega_0^2$.

$$\frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + \omega_0^2 x = \frac{F_0}{m}\sin \omega_p t$$

Forced harmonic oscillation.

defn:-

→ If an external force acts on a damped oscillator the oscillating system is called a forced harmonic oscillator.

→ The oscillations produced under the action of externa periodic force is called forced oscillations.

→ when a body or particle executing oscillations, an oscillatory external force, three forces are acting on

$$\eta = \frac{f_0}{\omega_0^2}$$

(g) resonance frequency

$$A_{max} = \frac{f_0}{2\eta\omega_0}$$

$$\frac{A_{max}}{A} = \frac{\frac{f_0}{2\eta\omega_0}}{\frac{f_0}{\omega_0^2}}$$

$$= \frac{\omega_0}{2\eta} = \omega_0\tau = \frac{\pi\gamma}{\eta}$$

5. The frequency of a tuning fork is 400Hz and its ... is $6\times10^4$. Find the relaxation time. Also calculate the time ... which its energy becomes $\frac{1}{10}$ of its initial-un...

ans.

$$Q = 2\pi\nu\times\tau \rightarrow \tau = \frac{Q}{2\pi\nu} = \frac{6\times10^4}{2\times\pi\times400} = 23.873$$

$$E = E_0 e^{-t/\tau} \rightarrow \frac{E_0}{10} = E_0 e^{-t/\tau} \rightarrow \frac{1}{10} = \frac{1}{e^{t/\tau}} \rightarrow$$

$$e^{t/23.873} = 10$$

$$\frac{t}{23.87}\log(e) = \log 10 = 1$$

$$t = \frac{23.87}{\log(e)} = 54.97$$

Quality factor = $\dfrac{2\pi \text{ energy stored}}{\text{Energy dissipated in time period.}}$

Quality factor is defined as the $2\pi$ times the ratio energy stored in the system to the average energy per period.

→ The avg per power dissipation $p$ is given by $-ve$ of the rate of change of energy

power dessipation per sec $= \dfrac{-dE}{dt} = E = E_0 e^{-2\gamma t}$

$$= \frac{-d E_0 e^{-2\gamma t}}{dt}$$

$$= E_0 e^{-2\gamma t}(-2\gamma)$$

$$= E_0 e^{-\gamma}(2\gamma)$$

$$Q.\text{factor} = \frac{2\pi E_0 e^{-t/\tau}}{E_0 e^{-t/\tau}(2\gamma)T}$$

$$Q.\text{factor} = \frac{2\pi}{2\gamma t}$$

$$= \frac{\omega}{2\gamma} = \omega\tau$$

$$Q.\text{factor} = \omega.\tau$$

$$Q.\text{factor} = \frac{A_{max}}{A_r}$$

Amplitude of oscilation decreases with time

- Frequancy of oscilation reduced from natural frequen

as to $\omega = \sqrt{\omega_0^2 - \gamma^2}$.

3- Energy of the damped ocilator decreases exponen

with time.

4. A part of stored energy is dissipated in e

cycle, then the average energy over a cycle

given by

$$E = E_0 e^{-2\gamma t}$$

$$E = E_0 e^{-t/\tau}$$

where $\tau = 1/2\gamma$ this is called relaxation time

$p-2$

$$2B = \frac{a - a\gamma}{\sqrt{\gamma^2 - \omega_0^2}}$$

$$B = \frac{a}{2}\left[1 - \frac{\gamma}{\sqrt{\gamma^2 - \omega_0^2}}\right]$$

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$$

$$= \frac{a}{2}\left(1 + \frac{\gamma}{\sqrt{\gamma^2 - \omega_0^2}}\right)e^{(-\gamma + \sqrt{\gamma^2 - \omega_0^2})t} + \frac{a}{2}\left(1 - \frac{\gamma}{\sqrt{\gamma^2 - \omega_0^2}}\right)e$$

$$= \frac{a}{2}\left(1 + \frac{\gamma}{\sqrt{\gamma^2 - \omega_0^2}}\right)e^{(-\gamma + \sqrt{\gamma^2 - \omega_0^2})t} + \frac{a}{2}\left(1 - \frac{\gamma}{\sqrt{\gamma^2 - \omega_0^2}}\right)e^{(-\gamma \ldots}$$

Critically damped

$\alpha_1 = -\gamma + \sqrt{\gamma^2 - \omega_0^2}$

$\alpha_2 = -\gamma - \sqrt{\gamma^2 - \omega_0^2}$

$\sqrt{\gamma^2 - \omega_0^2} = h$

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t}$$

$$= Ae^{(-\gamma + h)t} + Be^{(\gamma - ht)}$$

$$= Ae^{\gamma t}e^{ht} + Be^{\gamma t}e^{-ht}$$

$$= Ae^{-\gamma t}\left(1 + \frac{ht}{1!} + \frac{(ht)^2}{2!}\right) + Be^{\gamma t}\left(1 - \frac{ht}{1!} + \frac{(ht)^2}{2!}\right)$$

At $t=0$

$v=0$

$$v = \frac{dx}{dt}$$

$$v = \frac{d}{dt}A e^{\alpha_1 t} + \frac{d}{dt} B e^{\alpha_2 t}.$$

$$v = A\alpha_1 e^{\alpha_1 t} + B\alpha_2 e^{\alpha_2 t}.$$ $t=0$

$e^{\alpha t_0} =$

$$0 = A\alpha_1 + B\alpha_2.$$

$$= A(-\gamma + \sqrt{\gamma^2 - \omega_0^2}) + B(-\gamma - \sqrt{\gamma^2 - \omega_0^2})$$

$$= -\gamma(A+B) + \sqrt{\gamma^2 - \omega_0^2}\ A-B.$$

$$A - B = \frac{a\gamma}{\sqrt{\gamma^2 - \omega_0^2}} \quad \text{—(2)}$$

1+2

$$A+B = a$$

$$A-B = \frac{a\gamma}{\sqrt{\gamma^2-\omega_0^2}}$$

$$2A = a + \frac{a\gamma}{\sqrt{\gamma^2-\omega_0^2}}$$

$$A = \frac{a}{2}\left(1 + \frac{\gamma}{\sqrt{\gamma^2-\omega_0^2}}\right)$$

Damped ~~oscilla~~ oscilation.

$\vec{F}_R = -kx$

$F_d = -\beta \frac{dx}{dt}$

$F = F_R + F_d$

$m\ddot{x} = -kx + -\beta \frac{dx}{dt}$

$\beta$ = damping Constant

$$m\frac{d^2x}{dt^2} = -kx - \beta \frac{dx}{dt}$$

$\frac{\beta}{m}$ = damping co efficient

$= 2\gamma$

$$m\frac{d^2x}{dt^2} + kx + \beta \frac{dx}{dt} = 0$$

$\frac{k}{m} = \omega_0^2$

$$\div m \quad \frac{d^2x}{dt^2} + \frac{\beta}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

$$\frac{d^2x}{dt^2} + 2\gamma \frac{dx}{dt} + \omega_0^2 x = 0$$

over damped

we assume body start from extreme position

$v = 0$

$x = a$

$t = 0$

$x = Ae^0 + Be^0$

$a = A + B$ — ①

$$K = \frac{1}{2} m v^2$$

$$= \frac{1}{2} m \omega^2 (a^2 - x^2)$$

$$KE = \frac{1}{2} k (a^2 - x^2)$$

$$P.E = -F dx$$

$$\cancel{F\ kdx} \quad -(-kx) dx = kx\,dx$$

$$W = \int_0^x kx\,dx$$

$$= k\,x\,dx$$

$$= k\frac{x^2}{2}$$

$$P.E = \frac{1}{2} k x^2$$

$$T.E = P.E + K.E$$

$$= \frac{1}{2} k x^2 + \frac{1}{2} k (a^2 - x^2)$$

$$= \cancel{\frac{1}{2} k x^2} + \frac{1}{2} k a^2 - \cancel{\frac{1}{2} k x^2}$$

$$T.E = \frac{1}{2} k a^2$$

$$x = a\sin(\omega t + \phi)$$

$$\text{velocity} = \frac{dx}{dt}$$

$$= \frac{d}{dt}\left(a\sin(\omega t+\phi)\right)$$

$$= a\frac{d}{dt}\left(\sin(\omega t+\phi)\right)$$

$$= a\omega\cos(\omega t+\phi)$$

$$\sin(\omega t+\phi) = \frac{x}{a}$$

$$= \pm a\omega\sqrt{1-\sin^2(\omega t+\phi)}$$

$$= \pm a\omega\sqrt{1-\frac{x^2}{a^2}}$$

$$= \pm \cancel{a}\omega\sqrt{\frac{a^2-x^2}{\cancel{a^2}}}$$

$$\text{velocity} = \pm\omega\sqrt{a^2-x^2}$$