TRANSVERSE WAVE PROPAGATION ALONG A STRETCHED STRING

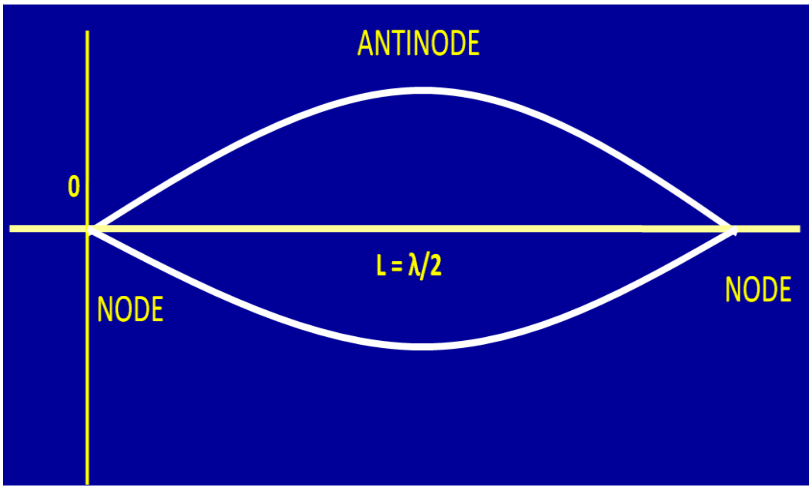
DEFINITION OF STRING

String is perfectly elastic, uniform and flexible cord having very large length in comparison to its diameter.

REASONS FOR TRANSVERSE VIBRATIONS

When a string stretched between two points is plucked in a direction perpendicular to its length, transverse vibrations are set up in the string.

This is due to the fact that tension in the string tends to bring to its mean position but due to inertia the string overshoots and goes to the other extreme. In this way, transverse vibrations are set up in the string.

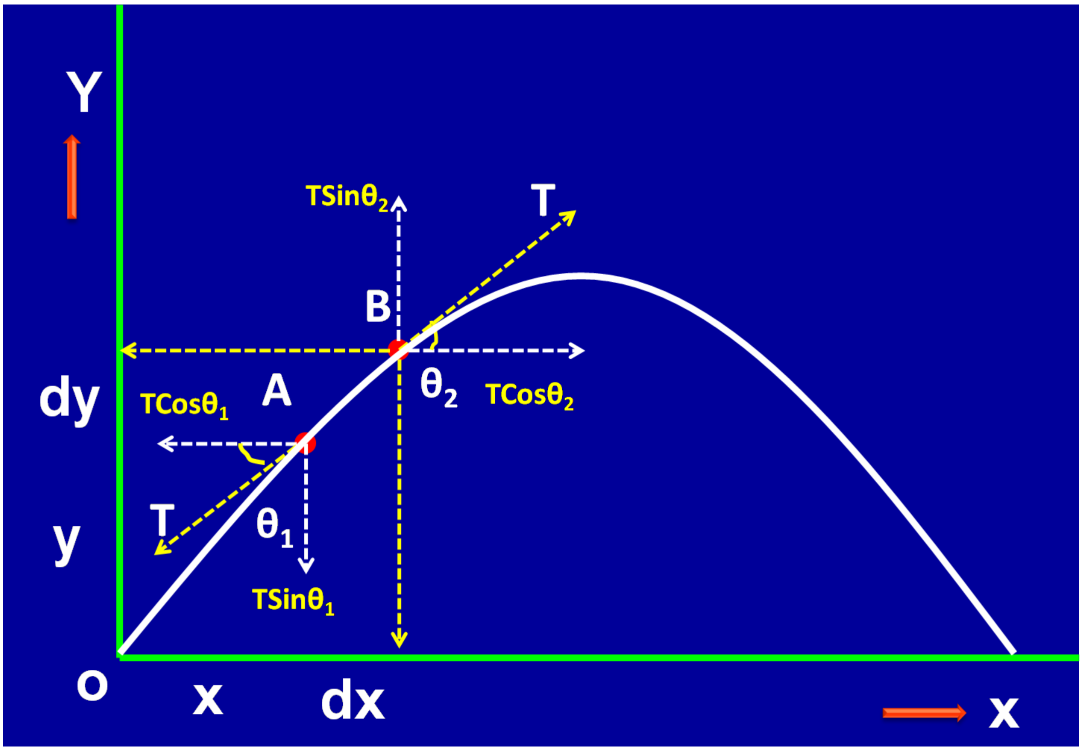


WAVE EQUATION FOR TRANSVERSE VIBRATIONS ALONG A STRING

The wave equation for the transverse vibrations of a string fixed between two rigid supports and stretched under a tension T along x-axis can be derived as follows

Consider an infinitesimal string element AB of length dx between the coordinates x and x+dx.

Let y be its displacement at time t. Let θ_1 and $\theta_2$be the angles which the tension makes with x-axis.



From the figure; at A the tension in the horizontal direction is $T\cos\theta_1$ and in the vertical direction is $T\sin\theta_1$. Similarly at B tension in the horizontal direction is $T\cos\theta_2$ and in the vertical direction is $T\sin\theta_2$. Hence $T\cos\theta_1$ and $T\cos\theta_2$ are nearly equal and balance each other.

The resultant vertical force $F_y = T\sin\theta_2 - T\sin\theta_1 = T(\sin\theta_2 - \sin\theta_1)$

Since the displacement AB is small and hence θ_1 and θ_2 are also small.

Hence $\sin\theta_1 \approx \tan\theta_1 \approx (\partial y/\partial x)_x$ and $\sin\theta_2 \approx \tan\theta_2 \approx (\partial y/\partial x)_{x+dx}$

$F_y = T\left((\partial y/\partial x)_{x+dx} - (\partial y/\partial x)_x \right)$

Using Taylor's series, we can expand

$(\partial y/\partial x)_{x+dx} = (\partial y/\partial x)_x + (\partial^2 y/\partial x^2)\, dx + (\partial^3 y/\partial x^3)(dx)^2/2! + \ldots$

Neglecting higher powers, we have

$(\partial y/\partial x)_{x+dx} = (\partial y/\partial x)_x + (\partial^2 y/\partial x^2)\, dx$

Substituting the value of $(\partial y/\partial x)_{x+dx}$ in the above equation, we get

$F_y = T\, [\, \{ (\partial y/\partial x)_x + (\partial^2 y/\partial x^2)\, dx \} - (\partial y/\partial x)_x \,]$

$F_y = T\, (\partial^2 y/\partial x^2)\, dx$ ------(1)

Let m be the mass per unit length of the wire. Then the mass of the element AB will be m dx. The force in the upward direction will be

$F_y =$ mass x acceleration $= m\, dx(\partial^2 y/\partial t^2)$ ----------(2)

Comparing eqs (1) and (2), we get

$m\, (\partial^2 y/\partial t^2)\, dx = T(\partial^2 y/\partial x^2) dx$

$(\partial^2 y/\partial t^2) = T/m\, (\partial^2 y/\partial x^2)$

The differential equation of a wave motion is given by

$(\partial^2 y/\partial t^2) = v^2\, (\partial^2 y/\partial x^2)$

Therefore the transverse wave velocity along the stretched string is

$$V = \sqrt{(T/m)}$$

The velocity of transverse vibrations of a stretched string depends on

i. directly on the square root of tension T
ii. Inversely on the square root of linear mass density m.

The wave equation for transverse wave motion along a stretched string is given by $(\partial^2 y/\partial t^2) = v^2\, (\partial^2 y/\partial x^2)$

This is the second order differential equation.

The solution of the above wave equation is

$$y(x,t) = a \sin 2\pi/\lambda(vt-x)$$