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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, August 2021

First Degree Programme under CBCSS

Mathematics

Complementary Course I for physics

MM 1131.1 : MATHEMATICS I – CALCULUS WITH APPLICATIONS IN PHYSICS I

(2020 Admission Regular)

Time : 3 Hours Max. Marks : 80

PART – I

Answer **all** questions. **Each** question carries **1** mark.

1. Find the derivative with respect to x of $f(x) = 7x^4 - 5x^3 + 4$.

2. Find the derivative of $\sin(x^2 + x)$ with respect to x.

3. Differentiate $(\cos x)\ \ln x$.

4. Find $\int \ln x\, dx$.

5. Write the equation of an ellipse in plane polar coordinates.

6. What is the total length of a curve $y = f(x)$ between the points $x = a$ and $x = b$?

P.T.O.

7. Define an arithmetic series.

8. Sum the series $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$

9. Find a vector orthogonal to $3i-2j+k$.

10. Write the equation of a line passing through two fixed points A and C with position vectors $\bar{a}$ and $\bar{c}$.

PART – II

Answer any **eight** questions. **Each** question carries **2** marks.

11. Find the second derivative of $\cos x \sin 2x$.

12. Find $\frac{dy}{dx}$ if $x^3+y^3-9xy=0$.

13. State mean value theorem.

14. Using the quotient rule, find the derivative of $y=\frac{t^2-1}{t^2+1}$.

15. Evaluate $I=\int_0^2 (2-x)^{-1/4}\,dx$.

16. Find the mean value of $f(x)=x^2-1$ between the limits $x=0$ and $x=\sqrt{3}$.

17. Evaluate $\int_0^\infty xe^{-x}dx$.

18. Find the area of the cardioid $\rho = a(1 - \sin\phi)$.

19. Sum the series : $2 + \frac{5}{2} + \frac{8}{2^2} + \frac{11}{2^3} + \ldots$

20. Evaluate the sum $\sum_{n=1}^{N} \frac{1}{n(n+1)}$.

21. Write the Maclaurin series for e^x.

22. State d'Alembert's ratio test for the convergence of a power series.

23. Find a unit vector in the direction of $3i - 4j$.

24. Find the angle between the vectors $\bar{a} = i - 2j - 2k$ and $\bar{b} = 6\bar{i} + 3\bar{j} + 2k$.

25. Show that if $\bar{a} = \bar{b} + \lambda\bar{c}$ for some scalar λ, then $\bar{a} \times \bar{c} = \bar{b} \times \bar{c}$.

26. Find the volume of the parallelepiped determined by $\bar{a} = \bar{i} + 2\bar{j} - \bar{k}$, $\bar{b} = -2\bar{i} + 3\bar{k}$ and $\bar{c} = 7\bar{j} - 4\bar{k}$.

PART – III

Answer any **six** questions. **Each** question carries **4** marks.

27. Determine inequalities satisfied by $\ln x$ for suitable ranges of the real variable x.

28. Show that the curve $x^3 + y^3 - 12x - 8y - 16 = 0$ touches the x- axis.

29. Verify Rolle's theorem for $f(x) = x(x+3)e^{-x/2}$ in $[-3, 0]$.

30. Show that the total length of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ is $6a$.

31. Find the surface area of a cone formed by rotating about the x-axis the line $y = 2x$ between $x = 0$ and $x = h$.

32. Find the volume of the solid generated by revolving the region between the curve $y = \sqrt{x}$, $0 \le x \le 4$, and the x-axis.

33. Determine whether the series $\sum_{n=1}^{\infty} \frac{2}{n^2}$ converge.

34. Identify the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n}}{(2n-1)!}$ and then by integration deduce the value S of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{(2n-1)!}$.

35. Determine the range of values of x for which the power series $P(x) = 1 + 2x + 4x^2 + 8x^2 + \ldots$ converges.

36. Four points A, B, C, D are positioned such that the line AD is perpendicular to BC and BD is perpendicular to AC. Show that CD is perpendicular to AB.

37. Find a vector parallel to the line of intersection of the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.

38. Find the angle between the planes $x + y = 1$ and $2x + y - 2z = 2$.

PART – IV

Answer any **two** questions. **Each** question carries **15** marks.

39. Show that the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the curve $x^3 + y^3 = 3axy$ is $-3a/8\sqrt{2}$.

40. Show that the value of the integral $I = \int_0^1 \frac{1}{\left(1 + x^2 + x^3\right)^{1/2}} dx$ lies between 0.810 and 0.882.

41. Find the first three non-zero terms in the Maclaurin series for the following function

    (a) $\exp(\sin x)$

    (b) $\ln\left[(2+x)^3\right]$

    (c) $\tan^{-1} x$

42. Using known series, find the first three terms of the Taylor series for the given functions using power series operations

    (a) $\frac{1}{3}(2x + x\cos x)$

    (b) $e^x \cos x$

43. Find the radius ρ of the circle that is the intersection of the plane $\overline{n}.\overline{r} = \overline{p}$ and the sphere of radius a centred on the point with position vector $\overline{c}$.

44. (a) Find the distance from the point P with coordinates $(1, 2, 3)$ to the plane that contains the points $A(0, 1, 0)$, $B(2, 3, 1)$ and $C = (5, 7, 2)$.

(b) A line is given by $\overline{r} = \overline{a} + \lambda \overline{b}$, where $\overline{a} = i + 2j + 3k$ and $\overline{b} = 4i + 5j + 6k$. Find the coordinates of the point P at which the line intersects the plane $x + 2y + 3z = 6$.
