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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course I for Physics

MM 1131.1 : MATHEMATICS I – CALCULUS AND SEQUENCE AND SERIES

(2021 Admission)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions.

1. Find $\lim_{x \to 2} \frac{5x^3 + 4}{x - 3}$.

2. Find the slope of the tangent line to the $y = \sqrt{x}$ at $x_0 = 4$.

3. Find x such that $\log_{10} x = \sqrt{2}$.

4. Evaluate $\int \cos 5x \, dx$.

5. Find $\frac{d}{dx}\left[\cosh\left(x^3\right)\right]$.

6. Find the level surface of $f(x, y, z) = x^3 + y^2 + z^2$.

P.T.O.

7. If $\nabla f \neq 0$ at point P, then among all possible directional derivatives of f at P, the derivative in the direction of ________ at P has the largest value. The value of this largest directional derivative is ________ at P.

8. Let $f(x,y) = x\sin(xy)$. Then find $f_x(x,y)$.

9. What does it mean to say that a sequence $\{a_n\}$ converges?

10. State Divergence Test.

(10 × 1 = 10 Marks)

PART – B

Answer any **eight** questions. These question carries **2** marks each.

11. Find the average rate of change of $y = x^2 + 1$ with respect to x over the interval [3, 5].

12. Find $\frac{d}{dx}\left[ln\left(x^2+1\right)\right]$.

13. Evaluate $\int \frac{3x^2}{x^3+5} dx$.

14. Find if $\frac{dy}{dx}$ if $y = \sin^{-1}\left(x^3\right)$.

15. Evaluate $\int \frac{\cos x}{\sin^2 x} dx$.

16. Evaluate $\int_0^2 x\left(x^2+1\right)^3 dx$.

17. Evaluate $\int xe^x\, dx$.

18. Find the area under the curve $f(x) = x^3$ over the interval [2, 3].

19. Let $f(x,y,z)=\sqrt{1-x^2-y^2-z^2}$. Find $f\left(0,\frac{1}{2},\frac{1}{2}\right)$ and the natural domain of *f*.

20. Consider the sphere $x^2+y^2+z^2=1$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the point $\left(\frac{2}{3},\frac{1}{3},\frac{2}{3}\right)$.

21. Find parametric equations of the line that is normal to the ellipsoid $x^2+4y^2+z^2=18$ at the point (1, 2, 1).

22. Find the directional derivative of $f(x,y,z)=x^2y-yz^3+z$ at the point (1, –2, 0) in the direction of the vector $a=2i+j-2k$.

23. Find all values of *x* for which the series $\sum_{k=0}^{\infty} x^k$ converges and find the sum of the series for those values of *x*.

24. Show that the integral test applies, and use the integral test to determine whether the series $\sum_{k=1}^{\infty}\frac{1}{k^2}$ converge or diverge.

25. Use the ratio test to determine whether the series $\sum_{k=1}^{\infty}\frac{k^k}{k!}$ converge or diverge.

26. Test the convergence of the series $\sum_{k=1}^{\infty}(-1)^k\frac{2^k}{k!}$.

(8 × 2 = 16 Marks)

PART – C

Answer any **six** questions. These question carries **4** marks each.

27. Find $\lim_{x\to+\infty}\frac{\sqrt{x^2+2}}{3x-6}$.

28. Find the derivative of $y = \dfrac{x^2\sqrt{7x-14}}{(1+x^2)^4}$.

29. Find $y'(x)$ for $y = \dfrac{x^3+2x^2-1}{x+5}$.

30. Evaluate $\int \left(\dfrac{1}{x^2} + \sec^2 \pi x\right) dx$.

31. Evaluate $\int_2^5 (2x-5)(x-3)^9\, dx$.

32. Evaluate $\int \sin^4 x \cos^5 x\, dx$.

33. Suppose that $w = \sqrt{x^2+y^2+z^2}$, $x = \cos\theta, y = \sin\theta, z = \tan\theta$. Use the chain rule to find $\dfrac{dw}{d\theta}$ where $\theta = \dfrac{\pi}{4}$.

34. Locate all relative and saddle points of $f(x,y) = 3x^2 - 2xy + y^2 - 8y$.

35. Let $L(x,y)$ denote the local linear approximation to $f(x,y) = \sqrt{x^2+y^2}$ at the point (3, 4). Compare the error in approximating $f(3.04, 3.98) = \sqrt{(3.04)^2 + (3.98)^2}$ by $L(3.04, 3.98)$ with the distance between the points (3, 4) and (3, 04, 3.98).

36. Find the sum of the series $\sum_{k=1}^{\infty}\left(\dfrac{3}{4^k} - \dfrac{2}{5^{k-1}}\right)$.

37. Find the nth Maclaurin polynomial for *sin* x.

38. Test for the convergence of $\sum_{k=1}^{\infty}(-1)^{k+1}\dfrac{k+3}{k(k+1)}$.

(6 × 4 = 24 Marks)

PART – D

Answer any **two** questions. These question carries **15** marks each.

39. (a) Prove that $\lim_{x \to 3} x^2 = 9$.

(b) Prove that $\lim_{x \to 0} \sqrt{x} = 9$.

40. (a) Use implicit differentiation to find $\frac{d^2y}{dx^2}$ if $4x^2 - 2y^2 = 9$.

(b) Find the slopes of the tangent lines to the curve $y^2 - x + 1 = 0$ at the points (2, –1) and (2, 1).

(c) Find $f''\left(\frac{\pi}{4}\right)$ if $f(x) = \sec x$.

41. Evaluate $\int \frac{x^2 + x - 2}{3x^3 - x^2 + 3x - 1} dx$.

42. (a) Let $f(x) = \begin{cases} -\frac{xy}{x^2 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$.

Show that $f_x(x,y)$ and $f_y(x,y)$ exist at all points (x, y).

(b) Show that the function $u(x,t) = \sin(x - ct)$ is a solution of one-dimensional wave equation.

43. The length width, and height of a rectangular box are measured with an error of at most 5%. Use a total differential to estimate the maximum percentage error that results if these quantities are used to calculate the diagonal of the box.

44. Find the interval of convergence and radius of convergence of the following series

(a) $\sum_{k=1}^{\infty} \frac{(x-5)^k}{k^2}$

(b) $\sum_{k=0}^{\infty} \frac{(x)^k}{k!}$.

(2 × 15 = 30 Marks)
