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S – 5714

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, October 2023

Physics with Specialization in Nano Science / Physics with Specialization in Space Physics

PHNS 512/PHSP 512 : MATHEMATICAL PHYSICS

(2020 Admission Onwards)

Time : 3 Hours

Max. Marks : 75

PART – A

Answer any **five** questions and each question carries **3** marks.

1. What do you understand by taking curl of a vector, is it spatial or temporal, write you comment?

2. Form a two-by-two matrix A with a matrix elements $A = \lfloor a_{ij} = i - j \rfloor$.

3. Explain the translational invariance property of Green's function.

4. Calculate the value of given integral $\int_C \frac{1}{2\pi z} dz$ over a unit circle.

5. Find the Laplace transform of $F(t) = \sin h(kt)$.

6. Deduce the relation in Bessel Function $\frac{d}{dx}\{x^n J_n(x)\} = x^n J_{n-1}(x)$.

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7. Prove that $\Gamma_{ab,c} + \Gamma_{bc,a} = \frac{\partial g_{ac}}{\partial x_b}$.

8. Define a group and check the following matrices will form a group $\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}; \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}; I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

(5 × 3 = 15 Marks)

PART – B

Answer **all** the questions and each question carries **15** marks.

9. (a) State and prove Green's theorem. **6**

(b) Find fourier series expansion of $f(x) = x, -\pi < x < \pi$ and show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots = \frac{\pi}{4}$. **9**

OR

10. (a) Derive the Cauchy's Riemann condition for the analyticity of a function in a complex domain? **6**

(b) State and prove Cauchy's integral theorem. Deduce Cauchy's integral formula? **9**

11. (a) State and prove Laplace convolution theorem. **6**

(b) Rodrigue's Formula for Legendre function. **9**

OR

12. (a) Deduce the Eigen function expansion of Green function. **6**

(b) Analyse the forced harmonic oscillation problem by Laplace transform method, here the mass m on a spring, with damping and a driving force $F(t)$. The equation of motion is

$$mX''(t) + bX'(t) + kX(t) = F(t)$$

Initial conditions $X(0) = 0, X'(0) = 0$. **9**

13. (a) Deduce the transformation law for Christoffel Symbols? 9

(b) Differentiation of Covariant tensor. 6

OR

14. (a) If Γ^i and $\Gamma^j(R^*)$ are two irreducible, inequivalent, unitary representations of a group, then, prove $\sum_R \Gamma^i(R)_{\alpha\rho}\Gamma^j(R^*)_{\sigma\beta} = \frac{h}{l_i}\delta_{ij}\delta_{\rho\sigma}\delta_{\alpha\beta}$ where l_i is the dimensionality of representation and h is the order of group and Γ_i is summed over all the elements. 8

(b) Discuss the properties of Lie group and deduce an expression for finding the structure constants using the Lie algebra. 7

(3 × 15 = 45 Marks)

PART – C

Answer any **three** questions and each question carries **5** marks.

15. Deduce the relation $\nabla^2\left(\frac{1}{r}\right) = -4\pi\delta(r)$ for Cartesian coordinates.

16. Find the residues of a function $f(z) = 1/(z^2 - 1)$.

17. Why arithmetic mean but not $|x_i - \bar{x}|$, but sum of absolute deviation taken as the average value?

18. Deduce the recurrence relations in Hermite's Function $H_n(x) = 2nH'_{n-1}(x)$.

19. Show that $\Gamma^a_{ab} = \frac{\partial}{\partial x_b}\left(\log\sqrt{g}\right)$.

20. Define the Rotational group $S(O)_3$ and discuss its properties.

(3 × 5 = 15 Marks)