(Pages : 6) **P – 3842**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, January 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1331.1 : MATHEMATICS III – LINEAR ALGEBRA, SPECIAL FUNCTIONS AND CALCULUS

(2021 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Find the rank of the matrix $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

2. If $AB = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} -1 & 3 \\ 0 & 2 \end{bmatrix}$, find A.

3. Find the sum of eigen values of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$.

4. Find the inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}$.

P.T.O.

5. Verify that $y = e^{-3x}$ is a solution of $y''+y'-6y = 0$.

6. Find the integrating factor $y'-y = e^{2x}$.

7. Solve $y''-5y'+6y = 0$.

8. What is the outward flux of the vector field $F = xi + yj + zk$, across any unit cube?

9. Prove that the force field $F = i\,e^{y} + j\,x\,e^{y}$ is conservative in the entire xy- plane.

10. State the recurrence relation for Gamma function.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. Each question carries **2** marks.

11. Form the differential equation from the equation $y = A\cos x + B\sin x$.

12. Show that if A is a square matrix.

(a) $A + A'$ is symmetric

(b) $A - A'$ is skew symmetric

13. If A and B are matrices such that $A + B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$ and $A - B = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$, find A and B.

14. IF C is the straight line path from (1,2,3) to (4,5,6) then evaluate $\int_C dx + 2dy + 3dz$.

15. State Stokes theorem.

16. Evaluate $\int_0^1 \frac{1}{\sqrt{-\log x}}\,dx$.

17. Solve the initial value problem $y' = 2x$ given $y(0) = 1$.

18. Solve $(x + y + 1)^2 \frac{dy}{dx} = 1$.

19. Solve $\frac{dy}{dx} - y \tan x = e^x \sec x$.

20. Solve the differential equation $(e^y + 1)\cos x dx + e^y \sin x dy = 0$.

21. Solve $y' = \frac{-y}{x}$, given that $y(1) = 1$.

22. Find the divergence of the inverse square field

$$F(x, y, z) = \frac{c}{(x^2 + y^2 + z^2)^{3/2}}(xi + yj + zk).$$

23. Find the outward flux of the vector field $F(x, y, z) = 2x\,i + 3y\,j + 4z\,k$ across the unit cube $x = 0,\ y = 0,\ z = 0,\ x = 1,\ y = 1,\ z = 1$.

24. Find the sum and product of eigen values of the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

25. If $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$, show that $A^2 - 4A - 5I = 0$.

26. Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. Each question carries **4** marks.

27. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.

28. Find the general and singular solutions of $y = px + \frac{a}{p}$.

29. Find the Orthogonal trajectories of the family of co-axial circles $x^2 + y^2 + 2\lambda x + c = 0$ where λ is the parameter.

30. Solve $1 + yx\frac{dx}{dy} + x^2 = 0$.

31. Solve $(y''+2y'+3)^2 = 0$.

32. Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ 4 & 1 & 0 \\ 8 & 1 & 1 \end{bmatrix}$. Hence find A^{-1}.

33. Find the work done by the force $F = x\,i + 2y\,j$, when it moves a particle on the curve $2y = x^2$ from (0,0) to (1,1)

34. Use divergence theorem to evaluate $\iint_S F.n\,ds$ where $F = (x^2 - yz)i + (y^2 - xz)j + (z^2 - yz)k$ taken over the region bounded by $x = 0,\ x = a,\ y = 0,\ y = b,\ z = 0,\ z = c$.

35. Use Green's theorem to evaluate $\int_C x^2y\,dx + x\,dy$ where C is the triangle with vertices (0,0), (1,0) and (1,2)

36. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, find A^2 and hence find A^n.

37. Show that $\beta(m,n) = \beta(n,m)$.

38. Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. **Each** question carries **15** marks.

39. Diagonalize the symmetric matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.

40. (a) Solve $x\frac{dy}{dx} + y = x^4y^4$.

(b) Solve $\frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3}$.

41. (a) Evaluate $\iint_S F.n\,ds$ where $F = 4xi - 2y^2j + z^2k$ taken over the cylindrical region bounded by $x^2 + y^2 = 4, z = 0, z = 3$.

(b) Verify Green's theorem for $f(x,y) = y^2 - 7y$, $g(x,y) = 2xy + 2x$ and C is the circle $x^2 + y^2 = 1$.

42. (a) Find for what values of a and b, the equations

$x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + az = b$ have

(i) no solution

(ii) a unique solution

(iii) more than one solution?

(b) Find the value of k for which the equations

$$3x + y - kz = 0$$
$$4x - 2y - 3z = 0$$
$$2kx + 4y + kz = 0$$ may possess non – trivial solution

43. Verify Stokes' Theorem for the vector field $F(x, y, z) = 2zi + 3xj + 5yk$, taking σ to be the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \geq 0$ with upward orientation, and C to be the positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of σ in the xy-plane.

44. (a) Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}$.

(b) Show that the equation

$(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y + 2)dy = 0$ is exact and hence solve it.

(2 × 15 = 30 Marks)
