

Reg. No. :

Name :

Third Semester B.Sc. Degree Examination, January 2023.

First Degree Programme under CBCSS

Statistics

Complementary Course for Physics

ST 1331.2 – PROBABILITY DISTRIBUTIONS AND STOCHASTIC PROCESS
(2019 Admission Onwards)

Time : 3 Hours

Max. Marks : 80

Use of calculator and statistical table is permitted

SECTION – A

Answer all questions. Each question carries 1 mark.

1. Binomial distribution is negatively skewed if
2. If X follows binomial distribution with parameters n and p , What is the distribution of $n - X$?
3. What is the value of $E(X^3)$ if X is a standard normal variate?
4. Name the probability distribution for which the first three moments are equal.
5. Which continuous distribution possessing the lack of memory property?
6. What is the variance of chi-square distribution with 6 degrees of freedom?
7. The mean of a Poisson distribution is 3.8, and then what is the mode?
8. What is the mean of student's t distribution?
9. If arrivals are according to Poisson process, then distribution of inter arrival times is _____
10. If Z follows $N(0, 1)$, what is $P(-2 < Z < 2)$?

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. Each question carries **2** marks

11. Obtain the moment generating function of binomial distribution.
12. A coin that is fair in nature is tossed n number of times and the probability of the occurrence of a head six times is the same as the probability that a head comes eight times. Find the value of n
13. The number of industrial injuries per working week in a particular factory is known to follow a Poisson distribution with mean 0.5, Find the probability that in a particular week, there are less than two accidents.
14. Distinguish between statistic and parameter
15. Define (a) sampling distribution and (b) standard error
16. Define t distribution
17. What is Bose Einstein statistics?
18. Describe a simple random walk process.
19. What are the basic features of Fermi Dirac statistics?
20. Define Markov chain
21. Distinguish between weakly and strictly stationary processes
22. Define exponential distribution. Find its mean.
23. A box of candies has many different colors in it. There is a 15% chance of getting a pink candy. What is the probability that exactly 4 candies in a box are pink out of 10?
24. Write down the binomial distribution if the mean and variance of the distribution are respectively 4 and 3.
25. Define gamma distribution with two parameters
26. Subway trains on a certain line run every half hour between midnight and six in the morning. What is the probability that a man entering the station at a random time during this period will have to wait at least 20 minutes?

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each question carries **4** marks

27. Find the mode of binomial distribution
28. If X and Y are independent Poisson variates, show that conditional distribution of X given $X+Y$ is binomial.
29. Define negative binomial distribution. Find its mean.

30. Show that geometric distribution satisfies the lack of memory property.
31. Assume the mean height of soldiers to be 68.22 inches with a variance of 10.8 inches. How many soldiers in a regiment of 1000 would you expect is being over six feet tall?
32. Write short notes on Maxwell-Boltzmann statistics
33. Define beta distribution. Find its mean.
34. Define transition probability. How transition probabilities together with initial distribution specify a Markov chain?
35. Explain Brownian motion process. What are its characteristics?
36. Write down the relation between chi-square, t and F distributions. Mention any two applications of chi-square distribution in Statistics.
37. State and establish the additive property of Poisson distribution
38. Let $\{X_n, n = 0, 1, 2, \dots\}$ be a Markov chain with states 1, 2, 3 having transition probability matrix

$$\begin{pmatrix} 1/2 & 1/4 & 1/4 \\ 1/3 & 0 & 2/3 \\ 1/2 & 1/2 & 0 \end{pmatrix} \text{ Given } P(X_1 = 1) = P(X_1 = 2) = \frac{1}{4}. \text{ Find } P(X_1 = 3, X_2 = 2, X_3 = 1)$$

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions. Each question carries **15** marks.

39. (a) Derive the recurrence relation for the moments of binomial distribution and hence find the first four moments of the distribution
- (b) A set of three similar coins are tossed 100 times with the following results:

No. of heads	0	1	2	3
Frequency	36	40	22	2

Fit a binomial distribution and calculate the expected frequencies. (9+6=15)

40. (a) Derive Poisson distribution as a limiting case of binomial distribution.
- (b) Obtain the MGF of Poisson distribution and hence or otherwise find the skewness and kurtosis of the distribution. (5+10=15)

41. (a) Write down any six properties of normal distribution.
 (b) Find the MGF of normal distribution. Show that If X and Y are independent normal variates, then for constants a and b , $aX + bY$ is also a normal variate. (6+9=15)
42. (a) Define standard Weibull distribution. Find its mean.
 (b) If X and Y are independent exponential random variables, show that $\text{Min}(X, Y)$ has exponential distribution
 (c) Find the MGF of gamma distribution having probability density function $f(x/\lambda) = \frac{e^{-\lambda} x^{\lambda-1}}{\Gamma(\lambda)}$; $\lambda > 0, 0 < x < \infty$. Hence or otherwise find the mean and variance of the distribution. (5+3+7=15)
43. (a) Define stochastic process. What are state space and parameter set? Explain classification of stochastic processes with respect to state space and time. Give examples.
 (b) Distinguish between irreducible and reducible Markov chains. What are transient and recurrent states in a Markov chain?
 (c) Classify the Markov chain having transition probability matrix
- $$\begin{pmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{pmatrix} \quad (8+4+3=15)$$
44. (a) Derive the moment generating function of chi-square distribution and hence find its mean.
 (b) Define F statistic. Write down its distribution. If F follows $F(m, n)$ distribution, what is the distribution of $1/F$?
 (c) Write down the sampling distribution of variance of a random sample drawn from normal distribution. (8+5+2=15)

(2 × 15 = 30 Marks)