

Reg. No. : .....

Name : .....

Third Semester B.Sc. Degree Examination, February 2024

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1331.1 MATHEMATICS III — LINEAR ALGEBRA, SPECIAL FUNCTIONS  
AND CALCULUS

(2021 Admission onwards)

Time : 3 Hours

Max. Marks : 80

SECTION – I

(All the first **ten** questions are compulsory. They carry **1** mark each).

1. Write the differential equation corresponding to  $y = a \cos nx$ .
2. State Cayley - Hamilton theorem.

3. Find the rank of the matrix  $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ .

4. Which one of the following matrices is in the reduced echelon form?

$$\begin{bmatrix} 1 & 2 & 3 & 7 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 5 & 1 \end{bmatrix}; \begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

5. Verify that  $y = e^{-2x}$  is a solution of  $y'' + y' - 2y = 0$ .
6. Find the particular integral of  $y'' + 9y = e^{2x}$ .
7. Define an inverse square field.
8. Determine whether the force field  $F = 4y i + 4x j$  is a conservative or not.
9. If  $\sigma$  any closed surface enclosing a volume  $V$  and  $r = xi + yj + zk$ , prove that 
$$\iiint_{\sigma} r \cdot n \, dS = 3V.$$
10. Find  $\beta(1,1)$ .

(10 × 1 = 10 Marks)

### SECTION – II

Answer **any eight** questions. Each question carries **2** marks.

11. Show that a square matrix  $A$  can be expressed as a sum of two matrices of which one is symmetric and the other is Skew symmetric.
12. Find  $A$  and  $B$  if  $A+B = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$  and  $A-B = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ .
13. If  $C$  is the straight line path from  $(0,0,0)$  to  $(1,1,1)$ , then evaluate 
$$\int_C dx + 2dy + 3dz.$$
14. Show that  $x = a \cos nt$  is a solution of the differential equation  $\frac{d^2 x}{dt^2} + n^2 x = 0$ .
15. State Greens theorem including all hypotheses.
16. Solve  $\frac{dy}{dx} + y \tan x = \cos x$ .
17. Solve  $(y'' + y' + 1)^2 = 0$ .
18. Find the work done in moving a particle in the force field  $F = 3x^2 i + (2xz - y)j - zk$  from  $t=0$  to  $t=1$  along the curve  $x = 2t^2$ ,  $y = t$ ,  $z = 4t^3$ .
19. If  $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$ , find  $A^2$ .

20. Find the sum and product of eigen values of the matrix  $\begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ .
21. If  $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ , find its characteristic equation of A.
22. Find the outward flux of the vector field  $F(x, y, z) = 2xi + 3yj + z^2k$  across the unit cube  $x=0, y=0, z=0, x=1, y=1, z=1$ .

(8 × 2 = 16 Marks)

### SECTION – III

Answer **any six** questions. Each question carries **4** marks.

23. Find the eigen values and eigen vectors of the matrix  $A = \begin{bmatrix} 4 & -5 \\ 1 & -2 \end{bmatrix}$ .
24. Using Cayley Hamilton theorem evaluate  $A^{-1}$  given  $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ .
25. Find the orthogonal trajectories of the family of curves  $x^2 - y^2 = c^2$ .
26. Find the general solution of the differential equation  $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ .
27. Solve  $(x+y)\frac{dy}{dx} = y - x$ .
28. Find the number of solutions of the following system of equations  
 $2x + 6y + 11 = 0$   
 $4x + 14y - z - 7 = 0$   
 $6y - 3z + 2 = 0$
29. Evaluate by stokes theorem  $\oint_C (e^x dx + 2ydy - dz)$ , where C is the curve  $x^2 + y^2 = 4, z = 2$ .
30. Using Gauss's divergence theorem evaluate  $\iiint_S F \cdot n ds$  for  $F = x^2i + y^2j + z^2k$  taken over the region V of the cube bounded by  $x=0, x=1, y=0, y=1, z=0, z=1$ .
31. Show that  $\beta(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$ .

(6 × 4 = 24 Marks)

## SECTION - IV

Answer **any two** questions. Each question carries **15** marks.

32. Diagonalize the symmetric matrix  $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix}$ .

33. (a) Find for what values of  $a$  and  $b$ , the equations

$$x + 2y + 3z = 6$$

$$3x + 3y + 5z = 9$$

$$2x + 5y + az = b$$

have

(i) no solution

(ii) a unique solution

(iii) more than one solution?

(b) Find the value of  $k$  for which the equations

$$2x + 3y + 4z = 0$$

$$x + 2y - 5z = 0$$

$$3x + 5y - kz = 0$$

have a non-trivial solution.

34. Verify Greens theorem for  $\int_C (xy + y^2) dx + x^2 dy$  where  $C$  is the closed curve of the region bounded by  $y = x$  and  $y = x^2$ .

35. (a) Solve  $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x$

(b) Solve  $\frac{dy}{dx} + \frac{x}{1-x^2}y = x\sqrt{y}$ .

(2 × 15 = 30 Marks)