(Pages : 4) **S – 6281**

**Reg. No. : .....**

**Name : .....**

**First Semester M.Sc. Degree Examination, April 2024**

**Physics/Physics with Specialization in Nano Science/Physics with Specialization in Space Physics**

**PH 212/PHNS 512/PHSP 512 : MATHEMATICAL PHYSICS**

**(2020 Admission Onwards)**

Time : 3 Hours Max. Marks : 75

PART – A

Answer **any five** questions. Each question carries **3** marks.

1. State the properties of the members of a vector space with respect to vector addition and scalar multiplication.
2. Explain the concept of analytic continuation.
3. Obtain the relation between the Fourier transform of a function and its sine and cosine transforms.
4. Given a set of sample measurements $x_i\ i = 1,2,\cdots, n$ of a random variable $X$, write down and explain the terms in the expression for the standard deviation of the distribution of $X$.
5. Distinguish between the three classes of PDEs.
6. State and prove Cayley - Hamilton theorem.
7. Briefly describe kinematics in Riemannian space. Write down the equation of motion for a free particle.
8. Define a Lie group. Give two examples.

**(5 × 3 = 15 Marks)**

P.T.O.

## PART – B

Answer **three** questions. Each question carries **15** marks.

9. (a) Derive the expression for the curl of a vector in general curvilinear co-ordinates.

   (b) Prove the residue theorem.

OR

10. (a) Develop the integral transform of a function $f(x)$ starting from its Fourier series expansion.

   (b) Determine the mean and standard deviation of the outcome of $n$ independent trials of an experiment which has only two outcomes with probability $p$ and $q = (1-p)$ using moment generating function.

11. (a) Develop the expression for the Laplace transform of the $n^{th}$ derivative of a function $f(t)$ in terms of $L[f(t)]$. Illustrate its use for determining $L[f(t)]$ by applying it to the equation $\frac{d^2}{dt^2}\sin kt = -k^2 \sin kt$

   (b) Obtain the series expansion for $J_n(x)$ starting from its generating function. Prove that $J_{-n}(x) = (-1)^n J_n(x)$.

OR

12. (a) Obtain the expression for the covariant derivative of a covariant vector. Hence obtain the expression for the covariant derivative of a contravariant vector.

   (b) Illustrate the use of group theory in particle physics using two instances of its use.

13. (a) Expand $f(x) = x$ in the interval $(0, 2L)$ in a Fourier series.

(b) Prove the addition theorem that if random variables $X$ and $Y$ are normally distributed with the same mean and variance then $Z = X + Y$ has a normal distribution with twice the variance of $X$ and $Y$.

OR

14. (a) Using partial fraction expansion show that

$$L^{-1}\left\{\frac{l}{(s^2+a^2)(s^2+b^2)}\right\} = \frac{1}{a^2-b^2}\left\{\frac{\sin at}{a} - \frac{\sin bt}{b}\right\} \text{ and}$$

$$L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\} = \frac{1}{a^2-b^2}\{a\sin at - b\sin bt\}$$

(b) Given the following Rodrigues formula develop the first four Legendre polynomials.

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n(x^2-1)^n}{dx^n}.$$

**(3 × 15 = 45 Marks)**

PART – C

Answer any **three** questions. Each question carries **five** marks.

15. Show that complex numbers $(a + ib)$ are isomorphic with $2\times2$ matrices $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

16. Examine the singularities of the function $f(z) = (Z^2-1)^{1/2}$ and identify the cut lines possible in the complex plane.

17. Solve the wave equation $\frac{\partial\psi}{\partial t} = a^2\frac{\partial^2\psi}{\partial x^2}$ using Fourier series expansion. Hence obtain the dispersion relation.